Semiconductor Devices and Circuits
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Lecture - 27
PN Junctions: Small Signal Impedance

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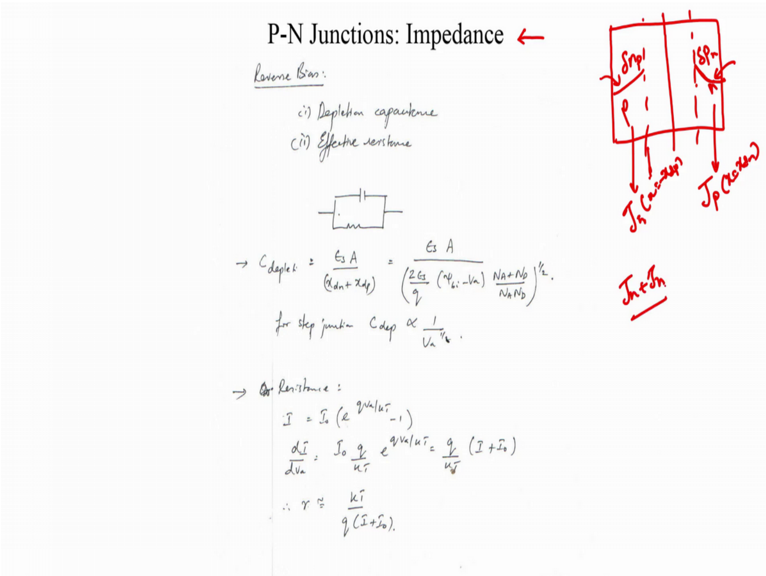

Now, let us move on to another topic which is to try and identify the effective Impedance of a PN junction. The case, just summarize the current voltage characteristics, we found that in a PN junction, you had a depletion region; we derived the electrostatics for it. And we found that when you apply a forward bias, we had excess holes being injected in the n type and excess electrons being injected into the p type and these excess holes would recombine and therefore, you had a concentration gradient and similar likewise with the and injected electrons.

And this concentration gradient gave rise to a diffusion current for holes and a diffusion current for electrons. And we computed the diffusion current at this boundary at equal to minus x d p and at x equal to x d n and therefore, we identify the complete current voltage characteristics for a PN junction diode so, that is what we did.

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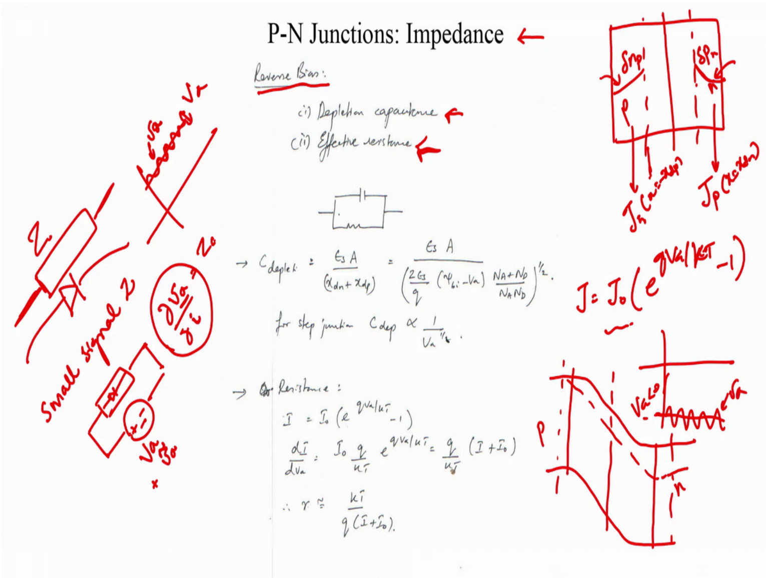

So, now we know the current voltage characteristics and we found that J was you know some constant into e to the power Q V a by k T minus 1 and just to recollect, what this constant was it had all these terms in it ok.

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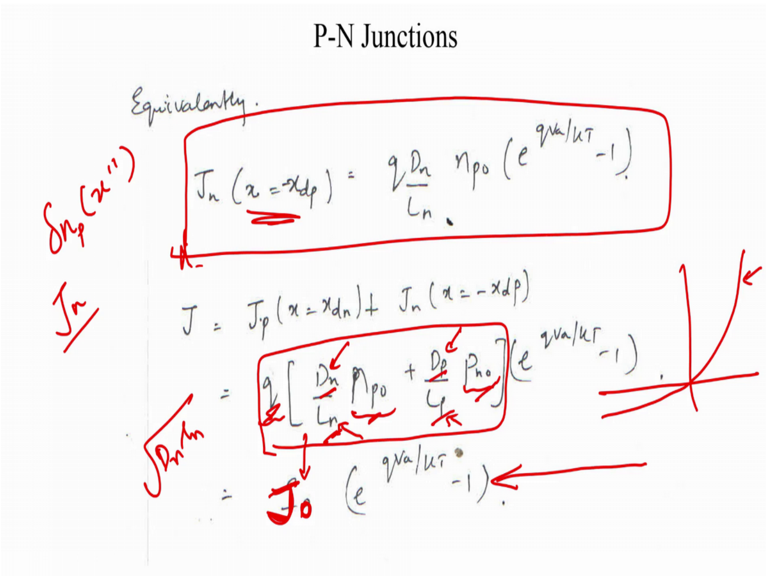

It had q D n L n n p o D p L p and P n o ok, L p is the diffusion length, D p is the diffusion coefficients. So, this is the current voltage characteristics, now however, there are some finer points to all these discussions ok. And we will sort of discuss these finer points by looking at the calculation of the impedance of a PN junction ok. So, if we have a PN junction in a circuit, what is the effective representation for this device? So, this very strongly depends on whether you are operating in reverse bias or in forward bias ok.

So, in reverse bias so, if you recollect your metal semiconductor junctions, we had a depletion capacitance and this PN junction is also going to offer an effective resistance ok. So, what we are interested in is something called as the small signal impedance and we have discussed this before and what it implies is that if you have this device in a black box.

So, let us say here you have your PN junction diode in a black box and you have these two terminals and you apply a you apply maybe so, you say apply a DC bias voltage firstly and on top of that bias voltage you apply a small signal AC ok. So, you have a DC voltage and on top of that you have a small signal acc AC voltage and around the small signal behavior you can linearize this device.

And therefore, the small signal impedance that, we are looking for is basically your response of the device to this small signal, which is how does the current through the device vary in correspondence with the applied small signal voltage. So, this is the small signal impedance, which were interested in. So, if you think about the PN junction reverse bias so, if this is the band bending diagram, you have a junction here and you have your depletion region boundaries here, so, that is a p side that is the n side so, let us say this is a equilibrium condition.

So, in reverse bias the depletion region increases a little right and the band bending becomes a little steeper. So, if I were to fluctuate, if I were to keep my device in reverse bias so, which means that I apply large negative voltage. So, V a is less than 0 and on top of that I apply a small signal AC voltage, what is the effective small signal impedance of this PN junction? So, that is the question asked on the slide ok. And the answer is there are two components here, you have a heavily doped n side and you have heavily doped p side and at the interface you have a depleted region ok.

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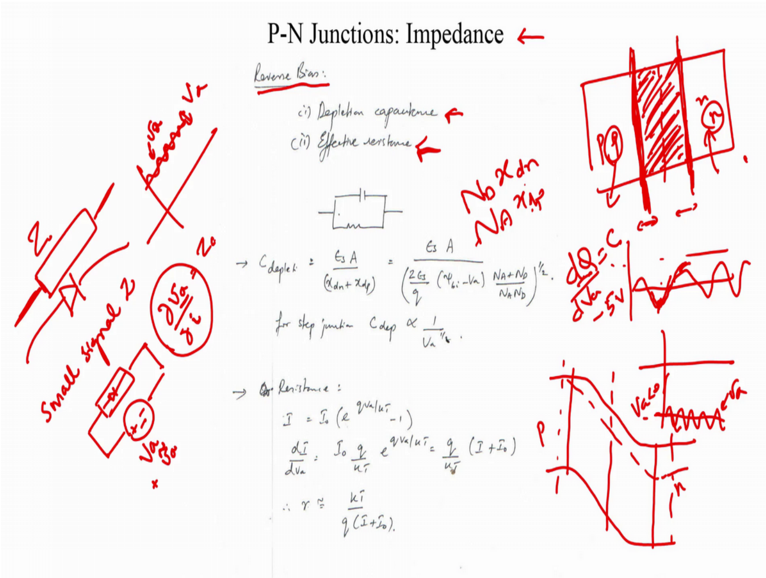

So, you have your p you have your p, you have your n and this depleted region has got no free carriers. But, this these two regions are almost metallic right, you have a large number of electrons, you have a large number of holes and this region is almost metallic, but here in the middle you have a depleted region. So, when I say almost metallic it is not to be taken in the sense of it being treated as a metal, what I mean is the resistance is very low. So, you have a device that looks like a capacitor ok and we have already looked at this in the metal semiconductor junction.

Wherein you have these two regions and then you have this depleted region and because of the small signal fluctuation the depletion region widths fluctuate ok. So, if the voltage goes more negative then so, let us say we are this is our reverse bias. So, let us say we have applied say minus 5 volts and let us say we have a small signal AC here.

So, if the voltage is in this region it is low it is more reverse bias than 5 volts and therefore, the depletion region increases and uncovers more fixed ions. And then as it goes to 5 volts it reaches it is you know mean depletion width and as it goes to above slightly above 5 volts, then it covers some of the ions and it reduces in size and it performs little fluctuation. And it is this fluctuation that results in an effective fluctuation in the charge and since there is an effective fluctuation in the charge, we can calculate d Q by d V a and say that this is your effective small signal capacitance.

So, that is what we did in the case of a metal semiconductor junction. So, what is d Q by d V a Q is nothing, but N D into x d n on the n side and NA into x d p on the p side. Since, x d n and x d p are changing with respect to the bias ok, you have your dx d n by d V a plus dx dp by d V a giving you this capacitive component. Now, that is one method to calculate the capacitance, the other method to calculate the capacitance is to simply think of it as a parallel plate capacitor in a very crude manner ok.

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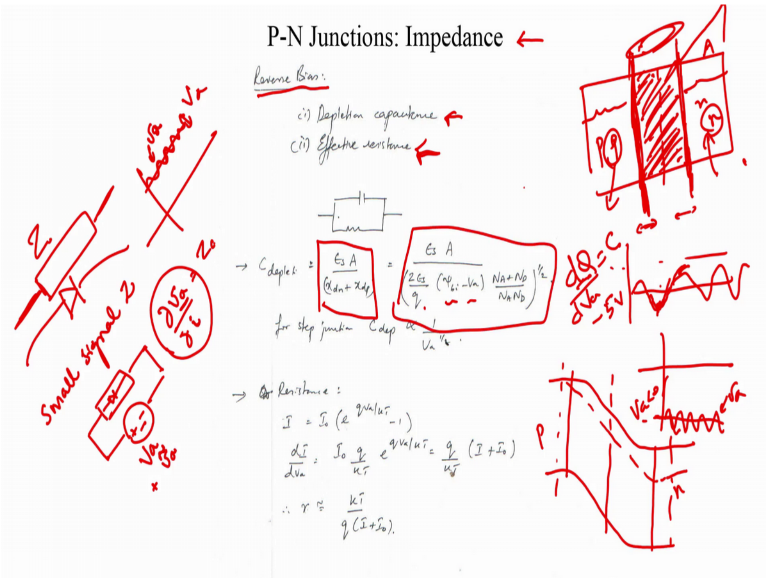

Because, you know these two regions have got low resistance and this region here is depleted. And therefore, if the junction has got an effective cross section area of A my capacitance is simply epsilon A by x d which is x d n plus x d p. And since x d n and x d p are dependent on the built in potential and the applied voltage in this manner, this is my total small signal capacitance ok.

And we saw the similar argument even in the case of a capacitance of a schottky junction a metal semiconductor schottky junction. So, that is the small signal capacitance of a diode in reverse bias ok, in forward bias there is something more interesting that happens we will come to that. What about the resistance of a diode in reverse bias we already know the current voltage characteristics.

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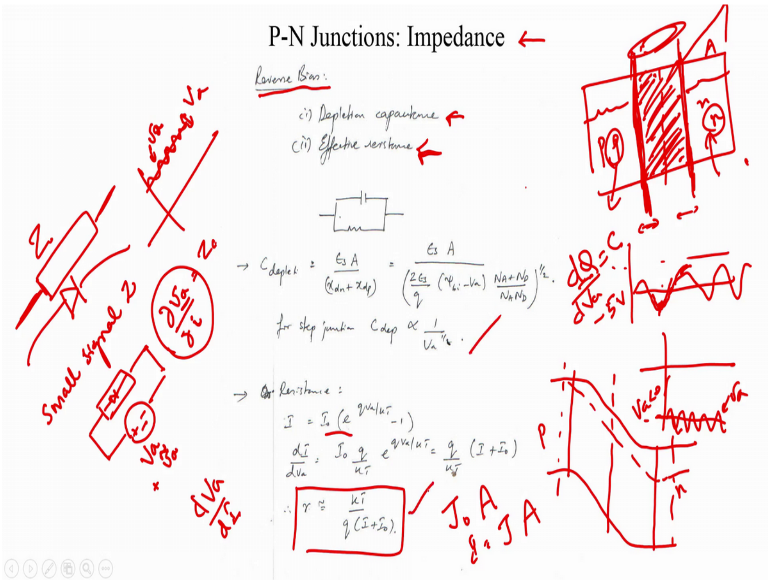

And therefore my small signal resistance is d V a by d I and that is easily obtained by taking the derivative of d I by d V a and then taking the inverse of that. And you will find that the small signal resistance is k T by q into I plus I naught, where what is I naught I naught is nothing, but your J naught times the cross section area and I is nothing, but the current density into the cross section area ok. And we have already seen what J naught is all the parameters it depends upon. So, in the in reverse bias this is the small signal capacitance and that is the small signal resistance ok.

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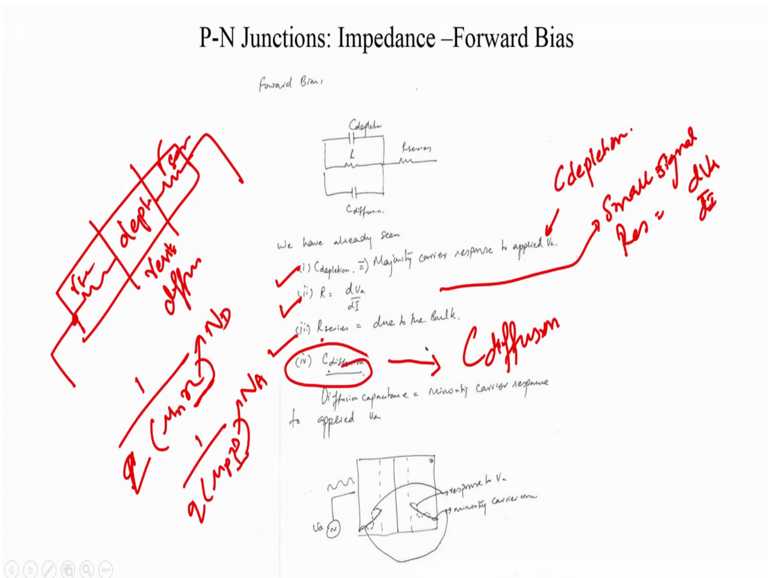

But in forward bias ok, let us sorry that is a repeat of the slide in forward bias, you do have the same capacitor component ok. That capacitance which we discussed is something called as a depletion capacitance, you do have a depletion capacitance, you once again do have a small signal resistance.

But you have an additional capacitance term which is called as the diffusion capacitance, and I will tell you what this is. And another parameter that now starts playing an important role, because the currents are now larger is simply the series resistance due to the P and N junctions. This was there even in the case of a reverse bias, but for practical purposes it need not be considered, because the currents were not very large, but if the currents grow very large then this series resistance can cause a huge significant, it can cause a significant voltage drop across the PN junction.

And that series resistance is simply because, the doping of your n n p type. And due to any metal contacts that you may have or any external circuit now this adds in a series resistance ok. But apart from that you have your depletion capacitance, you have your small signal resistance should be the diode behavior and you also have something called as a diffusion capacitance.

Now, we have already looked at depletion capacitance, we will not worry about that we have already calculated what d V a by d I is which is your diode resistance, we will not worry about that. The series resistance is quite simple to calculate, because the resistivity is simply q 1 by q into mu n on the inside and in the case of p side is 1 by q mu p on the p side mu n into n where n is the electron concentration and mu p into p v p is the hole concentration ok.

So, that is your series resistance, if you only consider the diode alone, but what we have never discussed so, far is this component called as diffusive capacitance diffusion capacitance. So, what is the diffusion capacitance very intuitively ok?

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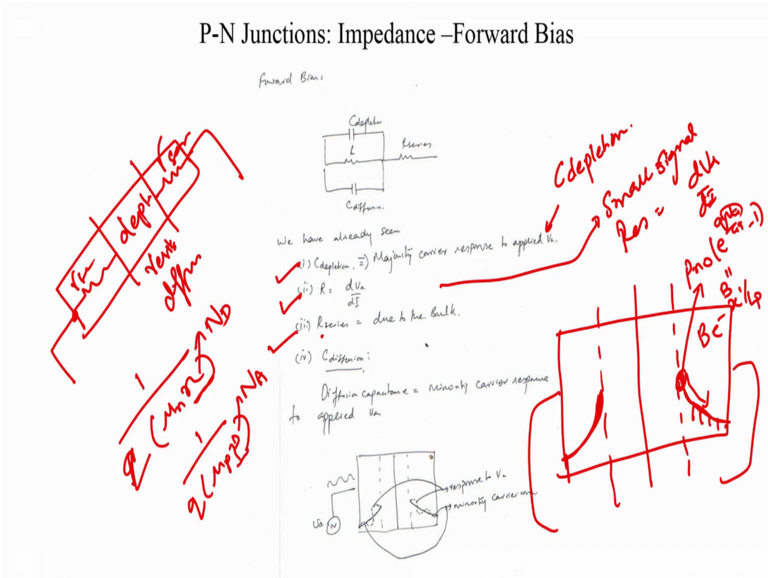

Now, in forward bias what we have done is we have injected the minority carriers all right. So, we now have a minority carrier injection, because the large number of large amount of carriers diffusing across the junction. So, if we have to apply if we were to change and we already saw, you already noticed that this term was P n naught into e to the power q V a by k T minus 1.

And this function that was also my component which is B and this function was given by B e to the power minus x dash by L p ok, just to recollect. So, obviously, this minority carrier concentration depends upon V a and the concentration at every point in space after that depends upon V a ok. So, if you take a very broad definition of what capacitance means, it is if I were to put this diode in a black box ok so, if I were to put this diode in a black box.

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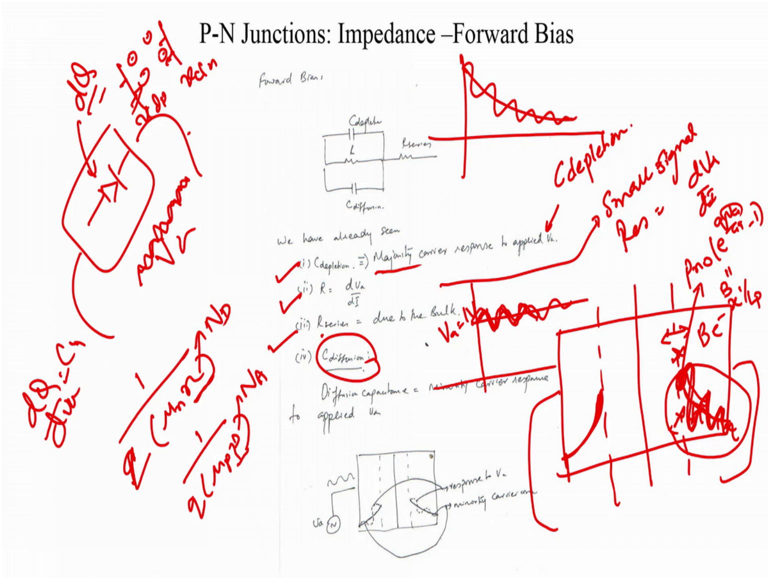

And I vary the voltage across and the amount of charge so, as the voltage varies if the voltage small signal fluctuates, the total charge content inside this black box fluctuates ok. And it is this ratio d Q by d V a which is my capacitance that is the that is my small signal capacitance. In reverse bias there was no minority carriers being injected, I mean it was it is not considered at all ok, because the current is purely drift paced and we really never worried about the injected minority carriers.

And therefore, the only capacitance we considered in reverse bias was the depletion capacitance; we never bothered about this new component here the diffusion capacitance. And therefore, this d Q was only due to the movement of x d n x d n and x d p. So, any voltage I apply the way that, the black box would respond is by uncovering or covering the fixed ions. It would uncover or cover the fixed ions and that was the only response the device gave and therefore, we could model it as a parallel plate capacitor and get away with everything.

But now, we have a rather tricky component here ok, but in forward bias not only do we have the depletion capacitance due to the depletion width change, which is already accounted for, but this minority carrier concentration will also change. So, if I were to apply my small signal if the small signal so, now, we are in forward bias let us say we have got a forward bias, let us say its plus one volt now drop of that I have a small signal fluctuation.

So, as I as the small signal goes above 1 volt, this minority carrier concentration will increase and as it goes below one volt it will decrease ok. So, therefore, any fluctuation here will be seen like a wave like response in the minority carrier concentration, the injected minority carriers. So, as this applied voltage fluctuates, not only would the diffusion a depletion width fluctuate, but also the injected minority carrier concentration would also fluctuate. And this fluctuation in charge was never accounted for in the calculation of the depletion capacitance and that fluctuation in charge is accounted for in what we call as a diffusion capacitance ok.

So, now that is a very qualitative understanding of what diffusion capacitances, but how do we quantify this ok, we want to write a nice mathematical expression for the diffusion capacitance. And what we will do is we will quickly look at how to go about calculating the diffusion capacitance. So, clearly it arises from the concentration variation the transience of the injected minority carriers right so, that is what is causing the diffusion capacitance.

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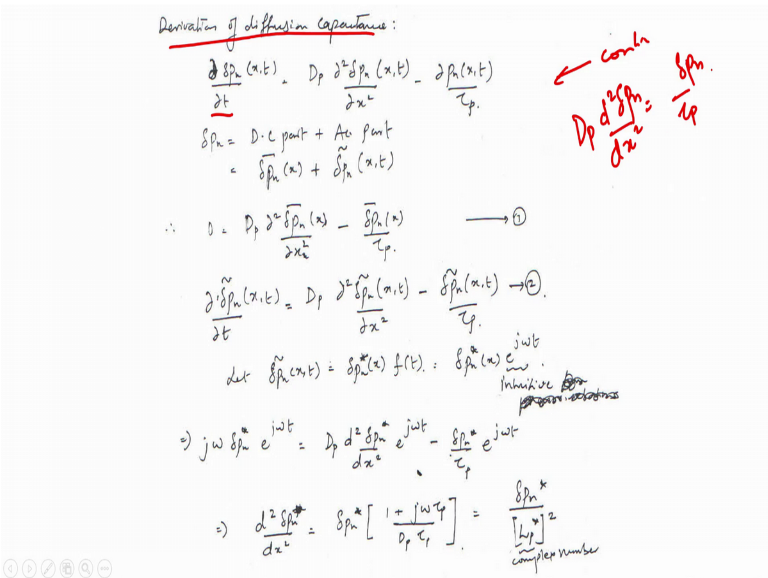

So, therefore, we are starting point must be in order to derive the diffusion capacitance starting point must be the continuity equation, but not in steady state, we must take into account that transient ok. So, we are no longer interested if it was steady state, we could simply say that you know this component was equal to your d delta P n by d delta P n by

I am sorry delta P n by tau p and just get away with it. But now we want to look at the transient response ok.

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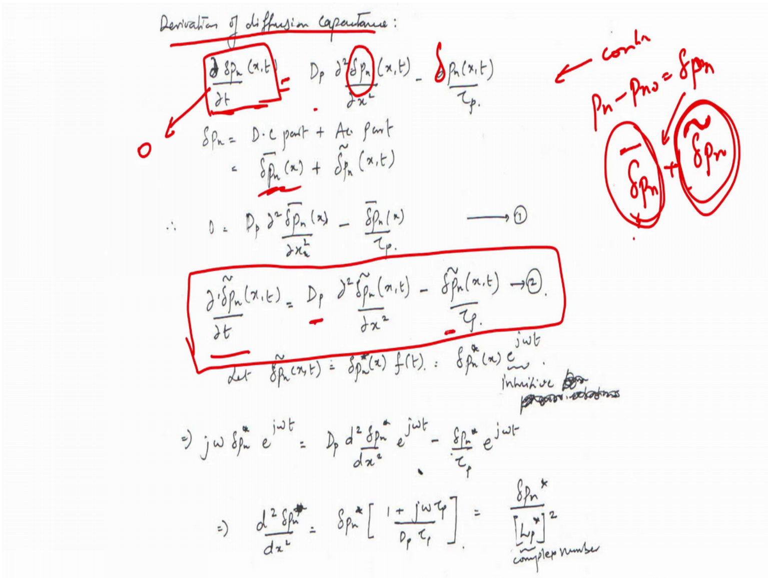

So, transient response of the injected carriers and therefore, we have a time dependent derivative term on the left side, it is not steady state and therefore, if you recollect you continuity equation you will find that this becomes your continuity equation ok. Now, this is the total injected this is the difference between the total injected charge and the equilibrium charge. So, that is my delta PN, now because of the DC bias is called a DC component and because of the applied AC signal it is got an AC component also. So, this is the component that we are interested in is fluctuating.

The DC component is already been accounted for calculating the DC current ok so, we looked at this already. So, in general this delta P and the injected carriers has got a DC part and it is got an AC part and I will define this AC part with this little tilde that at the top ok. And the DC part with just an over bar or just a straight line there ok, so, this v v line on top of delta P and describes the AC part.

So, now, I can split this continuity equation for the DC and AC part. So, if the DC part this time dependence is all 0 and we are talking about steady state. And therefore, this is my continuity equation that we have already looked at and solved, that is what we used for the current there was no fluctuating voltage component there.

But, now since we have a fluctuating voltage the AC part alone that is if you get rid of the DC component of it. And we take only the AC component we find that, this is the continuity equation of interest, there is a transient, there is a diffusive term and there is this recombination term ok.

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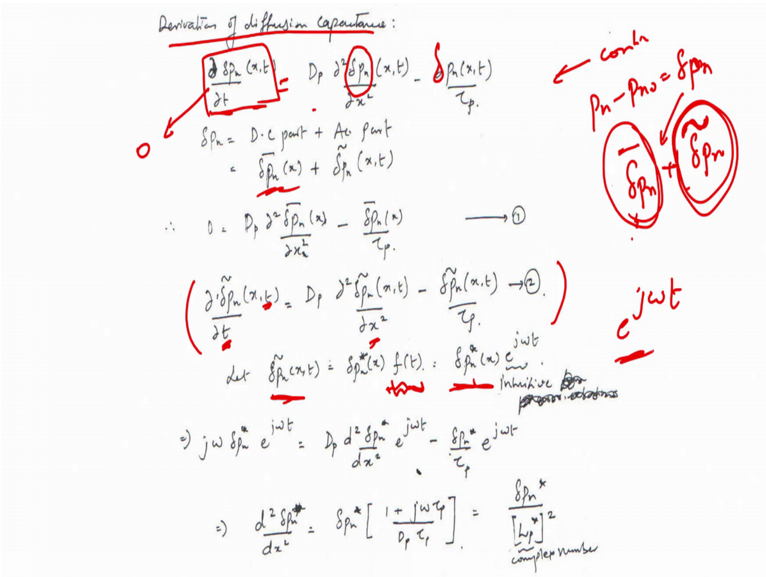

So, now how do we solve this continuity equation, it is got a time and it is got a time and space dependence ok. So, usually what we do that is the best approach for such things is to use separation of these two variables ok. So, we will say that if these two are truly independent and it so, happens so, that is a it is a good enough assumption and we can get away with it ok. We can truly separate the time and spatial component and get away with it and therefore, we will do that now I will tell you why very intuitively ok.

So, this is the AC component of the injected charge. And we will say that that is equal to delta P n star ok, which is the magnitude which is got only the spatial part of it into e to the power j omega t, which is got the time dependence ok. So, very intuitively you can think of it is a wave right it is a it is, your fluctuating it is like somebody holding a string, which is tied to one end and it is tied to one end implies it is like, if you take the long base approximation at x equal to infinite at x dash equal to infinite you have that end tied to delta P n equal to 0 that is not moving ok.

And then on the other side they are fluctuating the voltage, in other hand somebody is pulling string up and down or you can think of it as water ok, either way. And you have this little you know variation of delta P n in space and in time and it is got this wave like behavior. And therefore, very intuitively we will just say that this time dependence is simply e to the power j omega t ok, we getting a wave without explaining a lot of things, but we are using strong intuition here and we are actually fine with these approximations.

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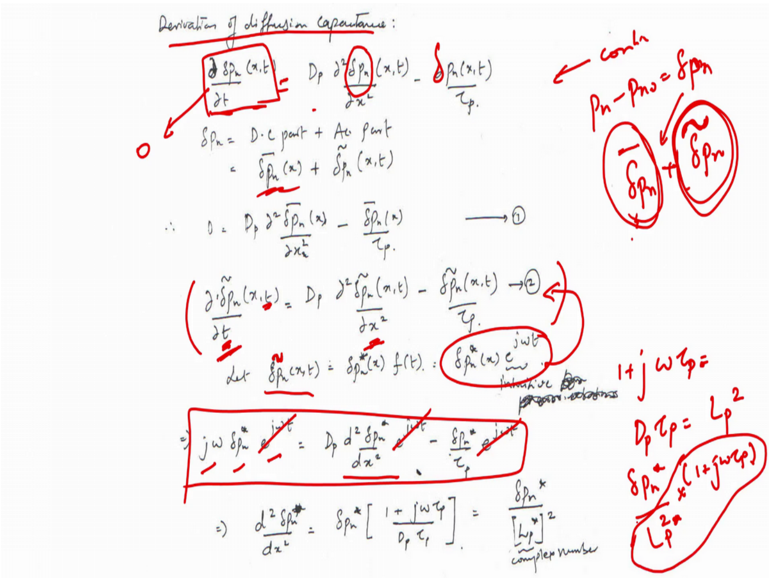

So, now we will just substitute this expression, for delta P n the AC component of delta P n is given by this and we will substitute this approximation inside our continuity equation. And when you make the substitution, you see that this time derivative becomes j omega delta P n e to the power j omega t, this term here will give you this component it is that is delta P n star there ok.

And this component is simply a direct substitution here and you find that all the e to the power j omega t terms conveniently cancel away and you will end up with this expression ok, which is neatly written out over here ok. So, that is what the continuity equation boils down to if you were to take only the AC component and use a separation of variables.

Now, if you look at this term ok, inside the bracket inside the square bracket, it is a complex number it is 1 plus j omega tau p by D p tau p. Now, it is a complex number, but think just think about it a little omega tau p is a dimensionless term and 1 plus j omega tau p is some complex number that has got no dimensions ok.

So, we will just say it is some complex number D p tau p is nothing, but your L p square ok. So, what you have here is delta P n star by L p square into a let us into this complex number 1 plus J omega tau p. Now, we will take these two terms a complex number divided by L p square.

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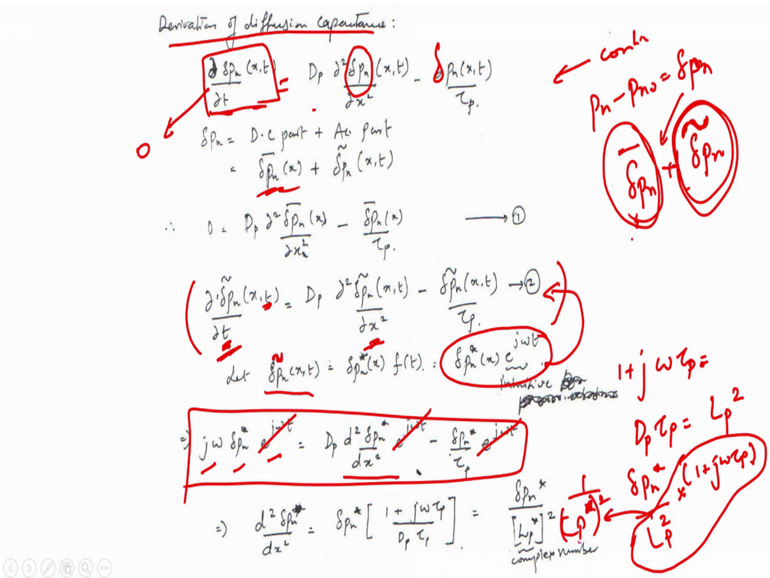

And we will say that this entire thing is equivalent to 1 by L p star square, where L p star is some kind of a complex diffusion length ok. So, just a mathematical construct here, it is a complex diffusion length; it is a L p square divided by 1 by divided by 1 plus j omega tau p ok. So, that is our mathematical construct and we are doing it in order to get a differential equation that is very familiar, which is d square delta P n by dx square is equal to delta P n by L p star square. So, delta P n star by L p star square, L p star is a complex number it is in fact, the square root of this complex number if you wish square root of this particular term.

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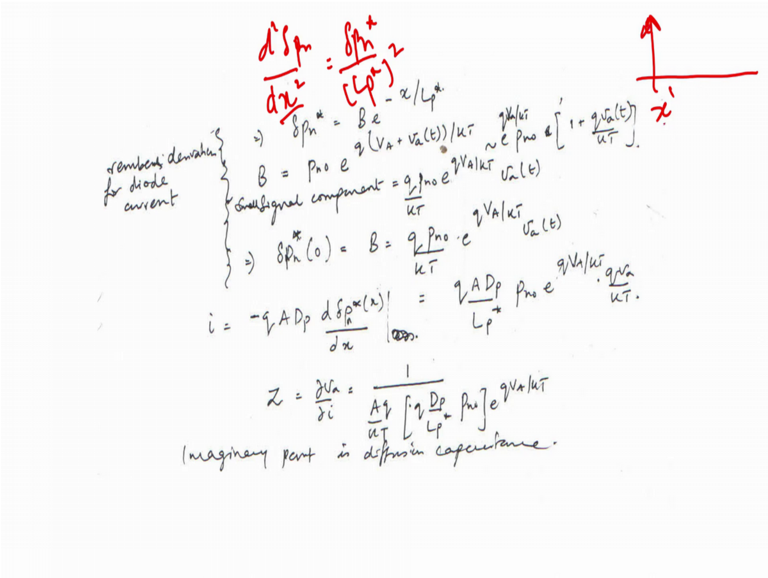

So, now we already know the solution to that equation, let me just rewrite that equation here so, you have your d square delta P n by dx square is equal to your delta P n star by L p star the whole square. Now, it is this come to my attention that, I have missed out mentioning something here ok.

So, throughout I have been using this variable x ok. So, please do remember this x corresponds to x dash ok, we are still interested in the translated coordinate system ok. So, if you have any query I am sure all of you understand this by now, but if you have any questions you can just get back to me on that.

So, x so we are talking about x dash ok. So, if you still use our old PN junction diode diagrams, but nevertheless I think you understand the context of what we are talking about. And the solution to this differential equation is again given by this particular term here, where B you know is your injected carrier minority carrier hole concentration at the depletion boundary on the n side. And that is equal to P n o e to the power in the previous case, we saw it is q V a by k T, but now it is going to be q V a plus the small signal V a by k T the small signal V a has got a time based component ok.

So, d turns out to be this particular term it is going to be P n o e to the power q into the DC voltage plus small signal voltage by k T minus 1 ok. So, that is the expression so, there is a minus 1 term here which is missing ok.

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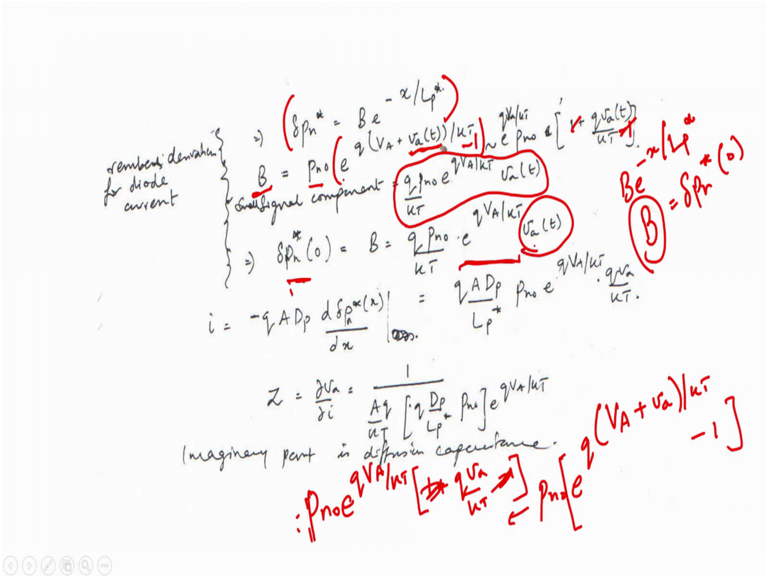

And we could now approximate this ok, we can approximate this and say that this particular term is nothing, but P n o e to the power will just take out the large signal or the DC components out k T and you leave behind only the small signal part, but we will approximate that small signal part ok. And we will say that it is we will use a Taylor expansion on that and say that it is 1 by q V a by k T minus 1 ok so, there is a minus 1 sorry about all these typos.

And therefore, the small signal component the small signal part alone becomes just this ok. So, all these cancels away and this is your small signal component. So, what we have done here is we have gotten rid of the exponential with regards to the small signal component; we have just trying to linearize this and say it is dependent on V a ok. So, that is the whole point of small signal analysis which is to which is the ability to use a Taylor expansion and linearize things and we have done that here.

And therefore, delta P n at x equal to 0 is going to be B e to the power e to the power minus x by L p star and at x equal to 0 it is simply equal to B. And therefore, delta P n at x equal to 0 delta P n star at x equal to 0 is simply given by this expression wherein, we have used Taylor expansion and linearized the V a term ok.

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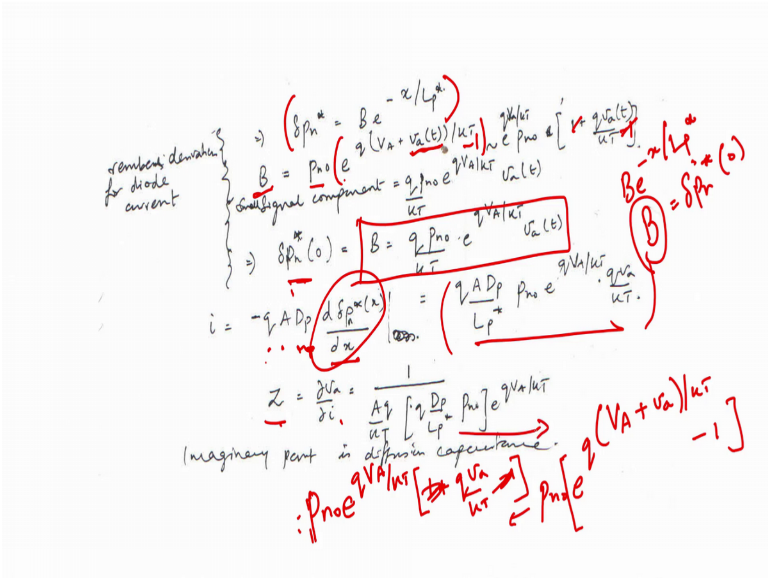

So, in summary this is this quite important this is the expression that we will use. And now what is the current component, the current is because of the diffusion term all right, it is q into a area of cross section into D p into D delta P n star by dx and that turns out to be this, because we now know what is B and we now know what is B into e to the power minus x by L p star. And therefore, we know what this derivative is and we can now find the current at x dash equal to 0 and you will end up with this expression here.

So, that is the current that is the small signal current due to the fluctuation, due to the voltage fluctuation the small signal voltage fluctuation, set at a DC bias of capital V a. And therefore, the impedance term a general complex impedance is d V a by d i because of the I being that small signal current and that will end up being a number like this ok. And since it is got this term L p star, this is definitely a complex number therefore, it is got a real term add a complex term and the imaginary part of that is the diffusion coefficient.

So, it is quite understandable if the last bit was quite confusing, but it is just a mathematical follow up of everything that we have done. So, once again if there is any questions on this feel free to send in a message ok.

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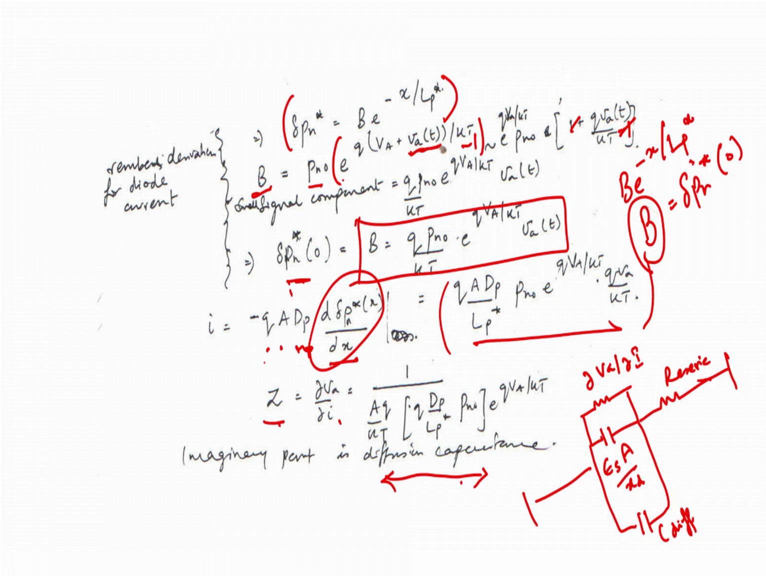

But that is a very intuitive analysis of what they how did the way to go about calculating the diffusion capacitance ok, it is because the key message here is, because it is because of the fluctuations and the injected minority carriers, which is quite significant in forward bias. And therefore, the diffusion capacitance is quite important when you are talking about a PN junction in forward bias.

So, therefore, to summarize the PN junction forward biased has got different terms, it is got it is own small signal resistance that is dau V a by dau I, it is got it is depletion capacitance which is say epsilon as A by xd. And then it is got its diffusion capacitance, which is defined by this analysis here and it is got a series resistance which is important because of the large currents. And therefore, more significant voltage drop since, it is not advisable to ignore the series resistance because, the doping profile is not defined the metal semiconductor junction is not defined etcetera.

So, if this is the equivalent circuit of the PN junction in forward bias. So, to summarize everything you have done so far we looked at what the PN junction is we looked at the electrostatics, we looked at the current voltage characteristics. And the key message was that the current voltage characteristics has got a rectifying nature to it and it is, because the diffusion of minority carriers that you have a current voltage characteristics, defined by an exponential function. And then finally, we can used all these information to calculate the impedance of the PN junction diode in forward and reverse bias.