Semiconductor Devices and Circuits
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Lecture - 26
PN Junctions: IV Characteristics

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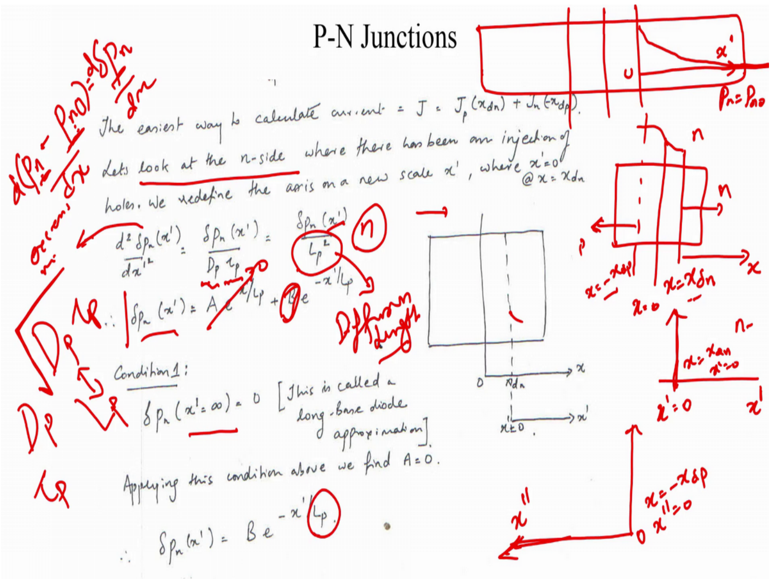

So, let us go ahead and start looking at the proper derivation of the current voltage characteristics of a P-N junction. But before we get to this point we need to understand a bit of what is going on either sides of the junction.

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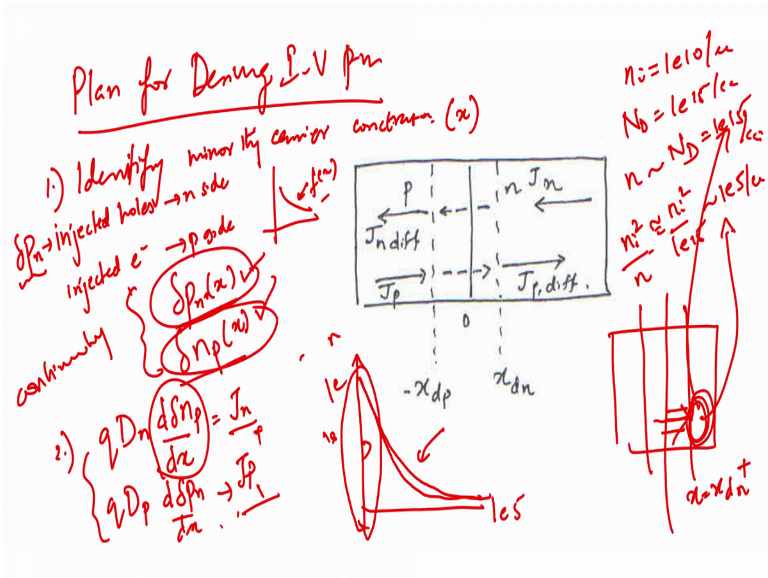

So, this is our P-N junction. So, let us let this picture be there, but I will I will use this. And we have a depletion region so, let us make all that as the x equal to minus x d p point and that is the x is equal to x d n point.

So, we have a P-N junction that is got there is got this depletion region in the middle. And let us say we have now forward bias to this P-N junction so, V a is going to be positive. So, as I mentioned a lot of holes are going to diffuse from the p to the n side, you are going to have lot of holes moving in there. You are going to have a lot of electrons diffusing from the this is the electron diffusion from the n to the p side that is the hole diffusion from the p to the n side. And you are going to have a small drift, which we are going to ignore it is a small drift of electrons from the p to the n side, and a small drift of holes from the n to the p side which we will ignore.

So, for all practical purposes the diffusion component is the current component. And since these holes are being injected so all this region is depleted. So, if you look at this boundary of the P-N junction so, at x equal to x d n if you just look at this interface here what is happening here? It is initially got a very small hole population at V a equal to 0 the whole population here is very, very small, right it is it is n i square by N D. So, just to give you an example if n i is say for silicon let us say it is about 1 e 10 per cc. Capital N D let us say we have doped it to 1 e 15 per cc. The n bulk region had about these many electrons if all the donors are ionized. And it has, because of the mass action law which is applicable at equilibrium, it has about n i square by 1 e 15 which is about 1 e 5 holes per cc.

So, that is the equilibrium condition right. So, if you were to not apply any voltage, and if I were to just deplete the semiconductor, then this region just after x equal to x d n; which is x equal to x d n plus. At that interface, we had these many holes and that many electrons. But the moment I apply forward bias, you are going to have a massive influx of holes from the p to the n side. They are going to move through the depletion region, and they are all going to appear here at this interface.

So, suddenly the whole population there which was initially 1 e 5 is going to spike, it is going to spike all the way up to, you know a very large number. Several orders of magnitude larger and that number depends on the bias voltage and we will see what that number is. And because of recombination, this whole population is going to immediately start declining in the semiconductor.

So, it is as though you see a huge injection of minority carriers, the holes of the minority carriers in the N type material. So, there is an injection of minority carriers. And this mind this minority carrier population is going to decrease with distance because of recombination. And we have already looked at this phenomenon, when we looked at the continuity equation and recombination generation mechanisms.

So, if you remember, if you look at the special case so, something called as a low level injection, the already looked at what is going to happen when you have a few minority carriers present in a material. And you know how the recombination controls the population density of these minority carriers with distance. We have already looked at that phenomenon, and we are going to be using that relation, here we are going to be using that special case the continuity equation are to analyze your P-N junction.

Now, this is going to be the overall idea, they are going to first. So, strategically I think it is useful to define the plan for deriving the current voltage characteristics. And I should have probably put a slide on that, but nevertheless I will write it out quite neatly. So, the plan for the current voltage characteristics for deriving the I-V characteristics of a P-N junction; so, what is the plan? The plan is first identify the minority carrier concentration on either sides; that means, it is the injected holes in the n side and injected electrons on the p side. We are going to identify the minority carrier concentration as a function of x as a function of distance. We are going to find out what is this function, what is f of x? The determines how the population of holes. So, let me call this as delta p on the n side which is the excess holes on the n side.

How does the excess holes on the n side vary with distance? We need to identify this point first. How do the excess electrons on the p side vary with distance? You are going to identify this. And now since they are going to vary with the distance, there is going to be a diffusion current, there is a diffusion current. So, how do we identify the diffusion current? It is proportional it is q D n d delta n p by dx. That is the diffusion of electrons and q D p d delta p n by dx is the diffusion of holes. We are going to then identify the diffusion current components and then add these 2 current components to get the total current. So, a very simple idea. First identify the minority carrier distribution n with the distance. And then use that function to differentiate that function and get the diffusion current of electrons and holes.

So, that is our strategy; that is our game plan to identify the current voltage characteristics. And we are going to do this by using continuity equation, and then identify the correct boundary conditions and identify the correct terms for these 2 expressions for these 2 and then use that to identify the current. So, the first part of it is to solve the continuity equation. Now, what we have to do is, we are going to shift the coordinate system a little. Because we are very comfortable is defining an x equal to 0 at the point of injection. So, what I am going to do is, our initial coordinate system we defined x equal to 0 here at the junction, and then we had at x equal to x d n we had the boundary of the depletion region on the n side, and at x equal to x at x equal to minus x d p, we had the boundary of the depletion region on the p side.

So, at x equal to x d n is the boundary the depletion region on the n side, and at x equal to minus x d p we have the boundary of the depletion region on the p side. So, this was the initial coordinate systems, that is my x. But now I want to change it, I am going to translate I am going to create a new coordinate system, just to make my mathematics a bit simpler.

And in that new coordinate system instead of x I am going to use x prime or x dash if you like. And I am going to mark the x x dash equal to 0 the origin point at x equal to x d

n. So, x equal to x d n corresponds to x equal x dash equal to 0. And this is the coordinate system are going to use only for the n side. I am not going to use this for the p side.

For the p side, I am going to use a different coordinate system; which is x equal to double prime. And it is heading off this way, it is heading off in the minus x direction. So, the positive values of x double prime are the negative values of x. And x double prime has got an origin located at your x equal to minus x d p. When x is equal to minus x d p x double prime is equal to 0.

And when x is equal to x d n x prime is equal to 0. So, I am going to define 2 new coordinate systems of this curve so that I can write my continuity equation in a manner that I am very familiar with and it is not going to change the physics that is just going to simplify the math a lot. Instead of using the x coordinate system and using a translation of coordinates I am just going to use, something I am going to use this transformation and we will end up the continuity equation that we are very, very familiar with.

So, you will find that the continuity equation for the, that describes the whole population, the injected whole population on the n side. Which is the delta p n on the n side is this. So, if you are wondering where this expression came from, I would strongly urge you to go back to the that the points where we discussed the continuity equation, and you will find that this was one of the special case is discussed which is the minority carrier low level injection of minority carriers. So, here you have your diffusion coefficient for holes. You have your minority carrier lifetime for holes, and delta p n is the excess minority carrier concentration. In some sense what does that mean? It means that let us say p n represents the number of holes on the n side per unit volume. P n o is the equilibrium number of holes on the n side per unit volume. Delta p, now we have shifted this device out of equilibrium and we now have p n minus p n o which is the excess minority carriers per unit volume on the n side being your delta p n. So, that is your delta p n.

And clearly that spatial derivative of this term is nothing but the spatial derivative of this term. And therefore, we can write everything every expression in terms of delta p n of x dash. So, this is the continuity equation, and we know that the solution to that is given by this particular expression here. Now, the key now is to identify the correct boundary conditions. Now the first boundary condition we will use is that x and x x dash is equal to infinite; that is, if I have an infinitely long P-N junction diode or if I have a very large P-N junction diode. And these are my bound depletion regions. So, my x dash coordinate system goes like this. At x star is equal to infinite, there are no minority carriers present; which means that all the minority carriers have recombined by then and nothing exists.

So, P n is equal to P n o; that means, these 2 values are exactly the same; these 2 values are exactly the same and delta p n delta p n is 0 that is very far from the junction. That is a very reasonable assumption because there is no band bending, the semiconductor is going to be like the bulk semiconductor has got it has got no idea as to what the junction is doing etcetera. And this assumption is something called is a long base diode approximation. And this term base comes and it will make a lots lot of sense when we discuss bats, but, but you can you can you can note this down as the long days diode approximation.

So, if you say that this is the first boundary condition and applied to that expression, you will find that the term a automatically has to be 0, because at x dash is equal to infinite this term blows up. It becomes infinite, but my delta p n is 0 and that can happen only if A is 0. So, I cannot allow this term to exist. So, this term has to disappear and that happens only when A is 0. So, remember we are using we are looking at the n side, we are looking at the n side this is not these expressions are not V a lid for the p side. We are looking at the holes injected into the n side and are using the x dash coordinate systems. We are only looking at the n side of the junction. So, that is written here, but just in case that is not clear.

So, what is the next boundary condition? I need to identify this coefficient B, and once I identify B I have my expression for delta p n, the next boundary condition is to identify the whole population at x dash is equal to 0.

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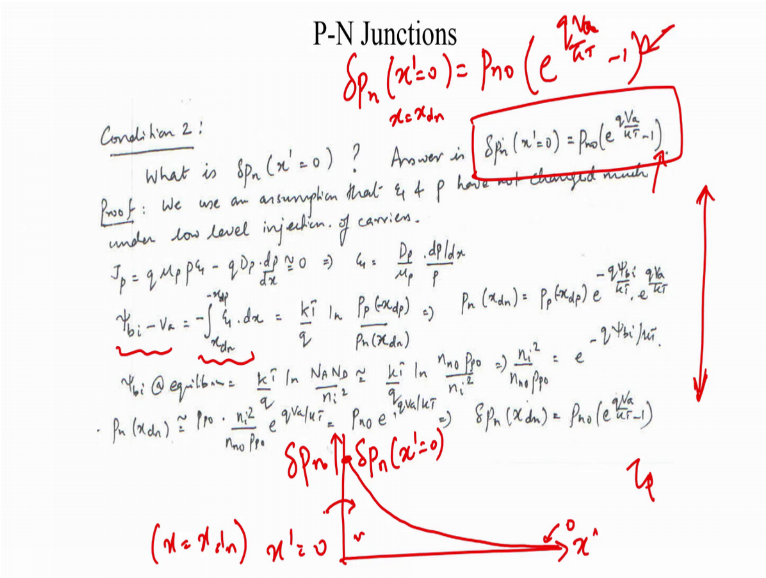

And here I have given you a short derivation, but the in short the answer is this. The whole population so, we have we have your x equal to x d n or x dash is equal to 0 coordinate system sitting here and that is my x dash. So, we have some excess holes injected there and it all tapers down to 0. So, let me just write this very clearly. So, this point here is my x dash is equal to 0, which corresponds to x equal to x d n. And on the y axis I am drawing sketching the delta p n; which is the excess holes injected into the n side because of the application of the forward bias. And these holes are recombining and they have got a minority carrier lifetime of tau p and therefore, you see this exponential.

Now, what is the whole population at this interface? What is delta p n at x dash is equal to 0. How many holes were injected into the n type semiconductor? Clearly they have to depend upon V a, because it is the applied voltage. And clearly they have to depend upon the exponential of q V a, because that is what the Boltzmann statistics tells you. But you will find that the expression is exactly given by this particular term here. And there is a short derivation it you end up with an expression by simply defining the built in potential as an integral of the electric field. That is of course, with some near equilibrium approximations. But what this expression tells you is that, my delta p n which is the excess electrons on the n side; excess holes injected into the n side at x dash equal to 0 which is at x equal to x d n is equal to p n o which is the equilibrium hole concentration on the n side into e to the power q V a by kT minus 1.

Which means at V a equal to 0 at equilibrium q V a by kT is 1, 1 minus 1 is 0 and therefore, there is no excess holes injected into the n side which makes a lot of sense. But then if I were to make my V a positive, I find that my excess electron, excess holes injected into the n side increases exponentially with the increase in applied voltage. So, this is the second boundary condition that we will apply.

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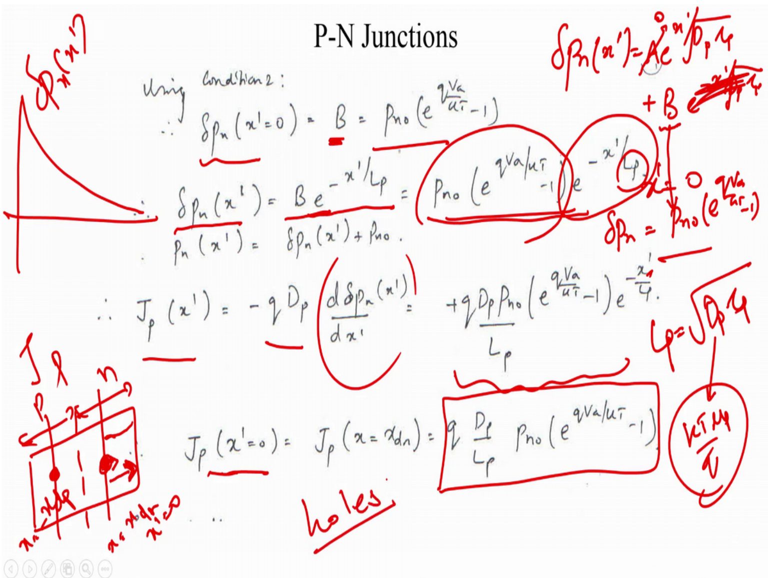

And that second boundary condition defines what my B is. So, if you remember my delta p n of x dash had 2 terms, it had an A e to the power x dash by D p tau p, and it had a B e to the power minus x dash by d p tau p and square root of D p tau p, sorry.

And we saw that A has to be 0, because of the long base approximation. But we need to find out what B is, and finding B we are going to use a condition, that an x equal to x x dash is equal to 0 my delta p n is equal to p n o into e to the power q V a by kT minus 1. So, that is my value. So, I substitute x dash equal to 0, that makes this term one and therefore, B is equal to this particular term here. That is exactly what is mentioned in that equation. And therefore, my complete expression for delta p n x dash is this particular term which is this.

Now, here I forgot to mention, I forgot to mention one key component. You will find that this D p tau p has become L p square. So, that is that is quite an important definition. D p is the diffusion coefficient of holes tau p is the minority carrier lifetime; which means it is the average time hole which is a minority carrier would last as a free carrier on and in an ns n doped semiconductor because it is going to recombine eventually. So, it is the lifetime of the minority carrier. In which case, in this case it is p hole. And this product in this product of D p tau p, it is quite significant, if you take the square root to this product. We define that square root as a term called as L P; which is the diffusion length of the holes.

So, this is something called as a diffusion length, and what it tells you is that it gives you an indicator. So, D p is telling you how quickly the species would diffuse and tau p is telling you for how much time they would diffuse. And therefore, in some sense the speed of diffusion and the time of diffusion is cut a length component to it. So, but since D p is not exactly velocity it is a D p tau p is exact is equal to L p square, it is a square of this diffusion length. And this diffusion length is an indicator to the distance through which if the just like you have an a time constant defining the, you know, the typical response or a measure of the response of a RC circuit, you have L p defining the measure of the length that t minority carriers are diffused into the material.

So, you have L p being a measure of the speed of diffusion and the minority carrier lifetime. So, I forgot to mention that explicitly. So, you find that this is the expression for the excess holes injected into the N type material and a P-N junction diode in forward bias. So, you have this term here which is your B and you have that term here which tells you an exponential dependence with distance. And L p is your diffusion length where L p is equal to square root of D p tau p. And what is D p? D p is approximately your kT mu by q from your Einstein's relation.

But of course, do you remember that we are not at equilibrium? We are just off equilibrium. So, you are fine with using these estimates, all right. So now, we have the concentration variation. So, that is what we have been looking for we have delta p n as a function of x dash. And therefore, since we know the concentration variation, we can now easily find the diffusion current. And the diffusion current is simply q D p into this derivative and that is a negative sign because we are talking about holes. And that answer turns out to be this. So, this is the diffusion current, you see that the diffusion current is dependent on x dash and rightly so, because the concentration profile is varying with x dash.

So, what we should be looking at is the easiest way to compute the current in the P-N junction. So, if that is the junction, that is the x d n and that is the minus x d p. So, this is the n side that is the p side that is x equal to minus x d p; and that is x equal to x d n or x dash equal to 0. We need to compute the diffusion current here at x dash equal to 0 because after this point, it becomes a little tricky to compute the total current. Because you have a diffusion component and you have the minority carriers moving in etcetera, etcetera.

So, we want to compute the diffusion current at this edge due to the holes. We want to compute the diffusion current at this edge due to the electrons, and sum these 2 diffusion current components to give you the total current. So, at x equal to x dash equal to 0, my diffusion current simply becomes this particular term. So, what have we done? Just to summarize the strategy, we have first identified so, because we applied a forward bias, we saw that holes were injected into the n side and electrons were injected into the p side.

And we firstly, calculated the distribution of this excess injected holes on the n side. And then we use that by solving the continuity equation, and we use that estimate to calculate the diffusion current due to holes in the n side. And in particular we are interested in the diffusion current value at x dash equal to 0 that is at the depletion boundary in the n side material. And with that we have completed all our analysis of the current due to holes. We have still not done anything with regards to electrons, but the electrons follow an exact you know process, and it is easy to calculate the electron current on the p side as well.

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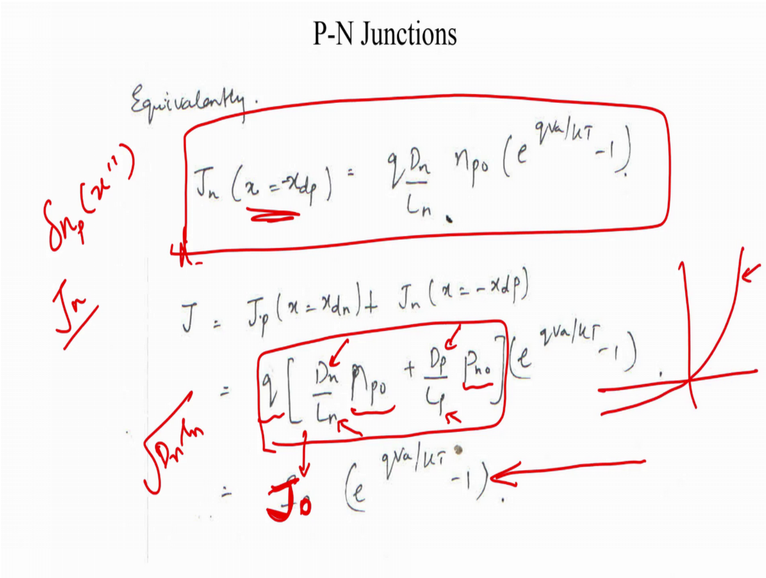

So, once again you solve the continuity equation to calculate the injected electrons into the p side. You find this profile, and then you find your diffusion current due to the electrons. And you will find the diffusion current due to the electrons at the depletion boundary or the p side is given by this expression, very symmetric solutions.

And therefore, the total current in the P-N junction is the summation of the diffusion current due to the electrons and the current due to the holes and that turns out to be this expression here; which is got an exponential dependence with the applied voltage. We saw that to be true even in the case of a metal semiconductor junction, but the key difference between a Schottky junction and the P-N junction; is that in a Schottky junction it was the majority carriers that were carrying the current, and in the P-N junction it is the minority carriers that are carrying the current. Quite a significant difference, which puts out the difference in the operating speeds of these 2 devices.

But both of these can operate as a diode or can have a rectifying nature in the current. So, this entire term can be treated as a constant right. So, this is what the charge in an electron; that is the diffusion coefficient of electrons, the diffusion coefficient of holes L n; which is basically square root of D n tau n and L p which is square root of t p tau p. And p n o which is the equilibrium concentration of holes in the n type material n p o which is the equilibrium concentration of N in the P type material; which is basically depend on the doping. And therefore, this entire term is treated as a constant, and we will denote that as J naught not I naught we are talking about current densities here J naught. So, that is a typo there it denotes. So, j is equal to J naught e to the power q V a by kT minus 1, and this is an expression that most of you will be very familiar with from your high school studies. So, this tells you that the current and the P-N junction varies like this. It is an exponential relation and this is the derivation of the current voltage relation.