

Semiconductor Devices and Circuits
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Lecture - 21
Schottky Contact : Electrostatics

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Metal-Semiconductor Junctions

Electrostatics: Schottky Contact

$$\frac{d\xi}{dx} = \frac{\rho}{\epsilon_s}$$

ρ = Charge concentration per unit volume
 ϵ_s = permittivity of the material

In the Depletion region,
 we make the depletion approximation: $\rho = qN_D$

$$\frac{d\xi}{dx} = \frac{qN_D}{\epsilon_s}$$

$$\xi = \frac{qN_D}{\epsilon_s} x + \text{const.}$$

$x = x_d, \xi = 0$

$$\xi = \frac{qN_D}{\epsilon_s} (x - x_d)$$

The diagram illustrates the structure and electrostatics of a Schottky contact. It shows a cross-section with a yellow Metal layer on the left, a blue Depletion Region in the middle, and a grey Bulk region on the right. The x-axis is labeled with $x=0$ at the metal-semiconductor interface and $x=x_d$ at the depletion region boundary. Below the structure, two plots are shown: the top one is the electric field E (in V/cm) versus x , which is zero in the metal, constant in the depletion region, and zero in the bulk; the bottom one is the potential ϕ (in V) versus x , which is constant in the metal, linear in the depletion region, and constant in the bulk. Handwritten red notes include: a bracket labeled 'Schottky' and 'Ohmic' pointing to the depletion region; 'Electrostatics' pointing to the field and potential plots; 'Current-Voltage' and 'Small Impedance' pointing to the depletion region.

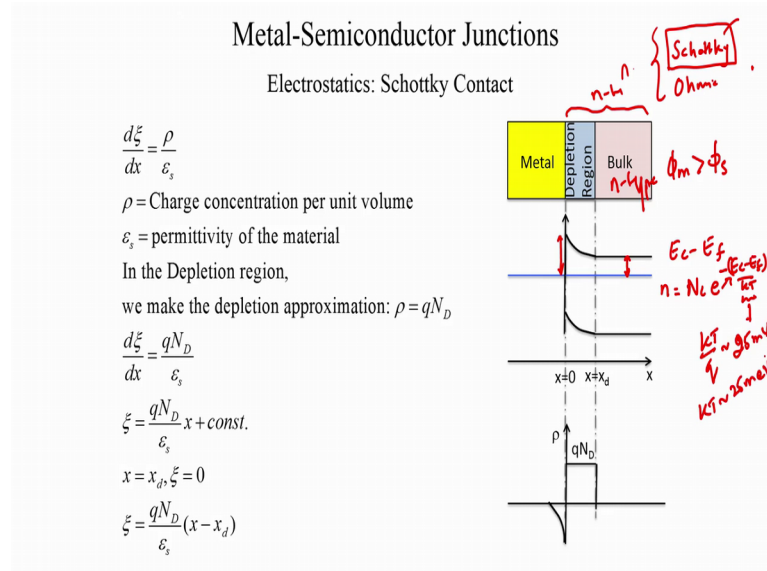
So, now let us continue on and look at the Schottky Contact. So, we saw that there are four there are two kinds of contacts that are formed in metal semiconductor junctions. The first is the Schottky contact, which is you know which we call the rectifying contact, but we really did not say why it is rectifying. And the second is the Ohmic contact. And what we will do now is first take the Schottky contact as our object of study ok. And we will always consider the Schottky and ohmic contacts for the n-type semiconductor.

And what we would like to do is firstly, study the electrostatics ok. So, we will study the electrostatics of the junction. And then look at the current voltage characteristics, we will then take the material out of equilibrium and study the current voltage characteristics. And from these two you will effectively try to develop a small signal impedance model for the device ok.

So, this is the first of the semiconductor devices and by going through this analysis we will be in a position to then repeat this analysis for other devices. So, in order to; so just

to revise you know just to look at the Schottky contact again, you have a metal and you have a n-type semiconductor so, this is your n-type semiconductor.

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And we already saw that the Fermi levels will align, the semiconductor will give electrons to the metal because your ϕ_m is greater than ϕ_s and this is an n-type semiconductor. So, the electrons will move into the metal, and the conduction band will bend away from the Fermi levels, and this will be the band bending diagram.

So, what we are trying to find out here is we are trying to solve out for the exact nature of the variation of this bending. You know how does the energy bands with in space? You know what are the electric fields in this region? What is the potential in this region? What is the charge in this region etcetera, etcetera? So, that is the objective of our study and that is what we mean by the study of electrostatics.

So, if you think of this region, we already discussed that you know the electrons are not encouraged to exist here, because you know this region has given up some electrons to the metal plus. And it has resulted in the bands bending in this particular manner which encourages electrons to roll downhill and stay away and keep away from this region.

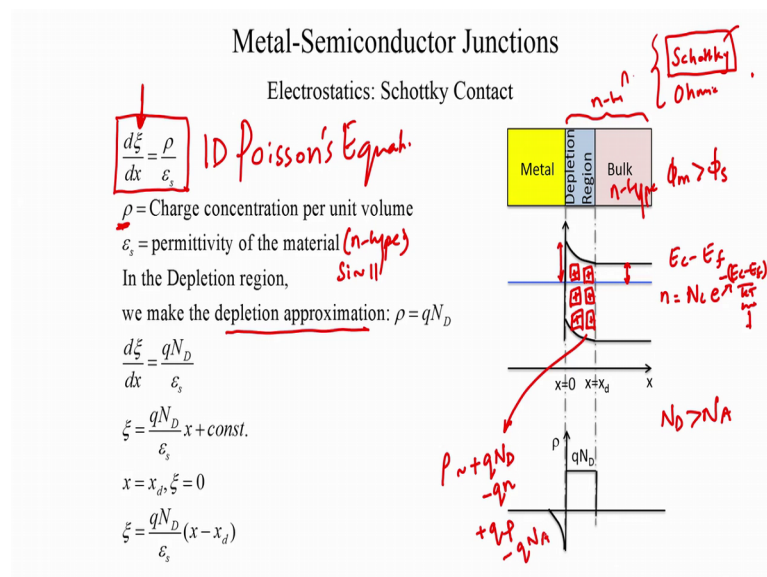
So, if you want to think about it in terms of looking at $E_c - E_f$ which is basically that difference. Then $E_c - E_f$ here is very large and therefore, the electron count which is n is equal to $N_c e^{-E_c - E_f / kT}$ is very small. And

therefore, the electron population in this region very close to the junction is quite small ok.

So since, this E_c minus E_f is greater than the E_c minus E_f in the bulk, the electron population here is much smaller than the electron population in the bulk. So, you should remember that the population scales exponentially with the energies. So, small differences in energies lead to large differences in carrier concentrations. And kT is not a very large number. So, I mean it is kT/q is of the order of 25 milli volts and kT is there for about 25 milli electron volts. So, it is not a very large number.

So, this region has got very few holes, because in a holes of the minority carriers in the n-type semiconductor. And it is gone now got very few electrons.

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And therefore, we said that this region is depleted of all mobile charges ok. But what are the charges present here. We discussed that it is all the fixed ions due to the donor doping. The donor dopants have all ionized and then they are all have their fixed positive charge that is sitting in here ok.

So, this is the condition and we want to look at the electrostatics of this problem. So, when we introduced the concept of junctions we said that in order to study the electrostatics. We will need to solve the differential form of Gauss's law, which is

something called as the Poisson's equation ok. And that is exactly what we are going to do here.

We are going to solve the Poisson's equation which have sort of rewritten out here. Which says that and this is a 1 dimensional Poisson equation which says that the $\frac{dE}{dx}$, where E is the electric field ok. So, I am calling E , but it is actually the Greek alphabet ψ .

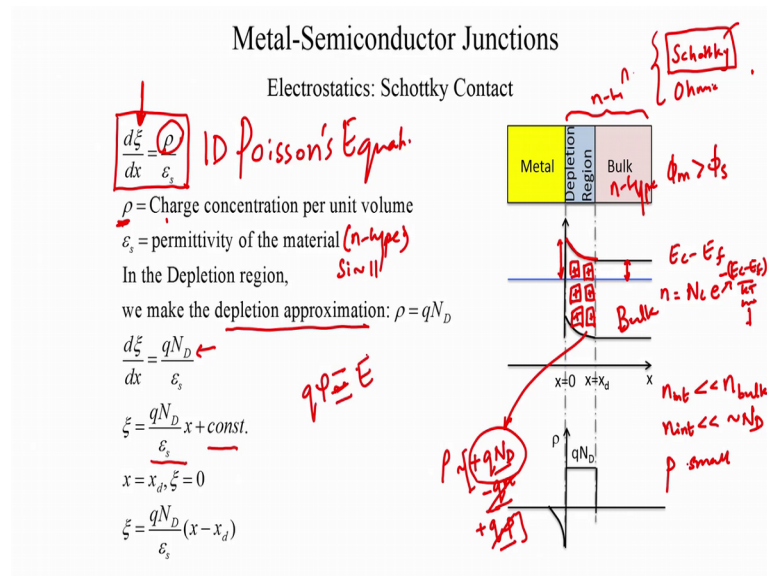
So, $\frac{dE}{dx}$ is equal to $\frac{\rho}{\epsilon}$, where ρ is the charge concentration per unit volume ok. In the space in the region of the semiconductor in which, we are trying to analyze electrostatics so that is the charge concentration per unit volume. ϵ is the permittivity of the material which in our case is the n-type semiconductor that we are looking at and for silicon and for silicon the relative permittivity is of the order of 11.

Now, in the depletion region we are going to make very some might say drastic, but let us say it is an approximation which is something called as a full depletion approximation ok. So, the depletion approximation basically says that the charge concentration here. So, what are the charges present in the depletion region in this region? You have the fixed donor ions you have the positive charge.

So, you have $+qN_D$ per unit volume so, these are all the ionized donor dopants. Then you have $-qn$ where n is the electron concentration and $+qp$ ok. So, that is the complete count of all the carriers or all the charges present in the in this region the depletion region. And if we have counter doped this material if you have added both N_D and N_A and kept N_D to be greater than N_A . We will also account for the ionized acceptors by writing $-qN_A$.

But here let us just keep it simple let us say there is no counter doping it is just these three components ok. You do have holes, you do have electrons and you do have donor dopants, but the electron count you see, the electron count at the interface is much smaller than the electron count in the bulks.

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So, if we had defined this region here which is very far from the junction as a bulk and the electron concentration of the bulk is approximately your N_D which is the ionized donor dopants. And therefore, your interface electron count is much less than N_D and that is why we said this region is depleted. And P in n-type semiconductor is again very very small so, this is also small.

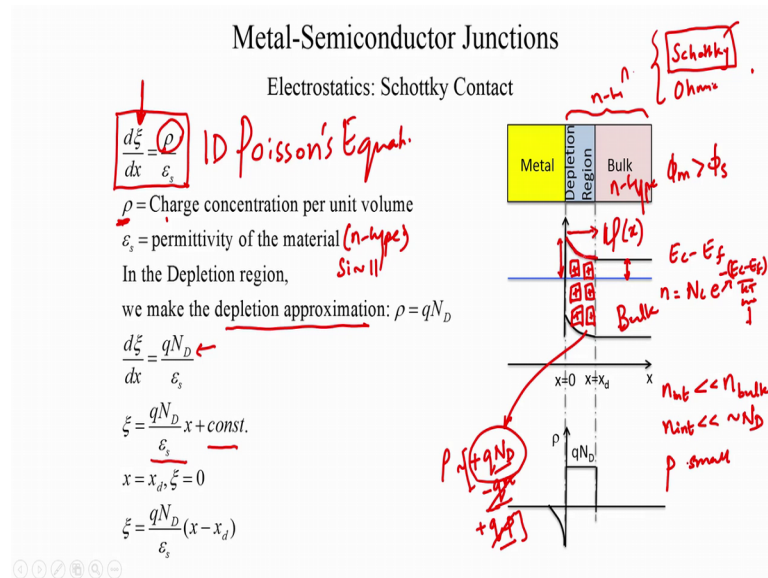
And therefore, we say that in the full depletion approximation we say that n and p are so small that they can be neglected ok. And therefore, the charge concentration per unit volume is simply $q N_D$. And therefore, we replace the ρ which is a charge concentration per unit volume with $q N_D$.

And we rewrite our Poisson's equation as dE by dx is equal to $q N_D$ by ϵ_s . Now you know now this is a very simple experiment this is simple differential equation to solve and your electric field turns out to be varying linearly with distance. So, that is what it tells us what is the solution of Poisson's equation under the full depletion approximation says that the electric field varies linearly with distance and there is an integration constant here ok.

Now, how do we determine that constant ok? We say that see what does electric field? The electric field is there because there is a bending, you see this bending is also the potential right. So, we are also we already defined we already said that your energies are essentially the q times the potential ok.

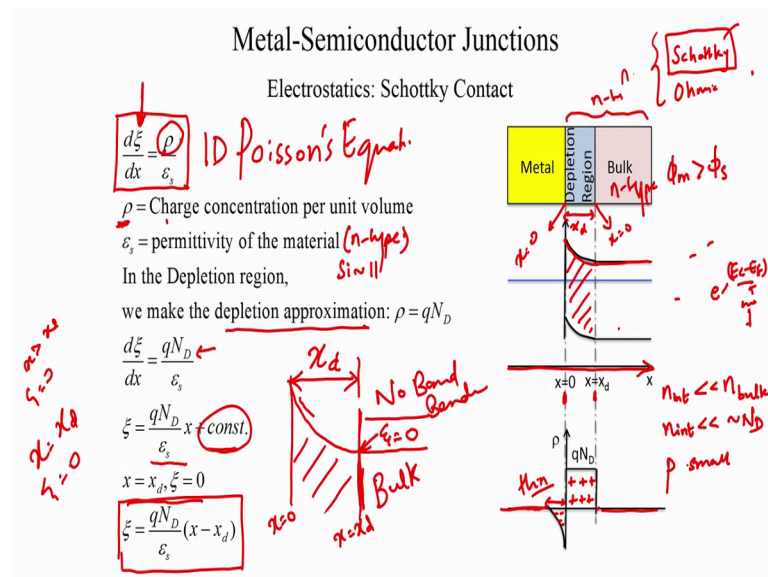
So, q times the potential is equivalent to your energies. So, the band bending in the energy band, the energy band bending ok, I am sorry the band bending of these energy levels is equivalent to the existence of a potential that varies with distance ok.

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So, the potential is varying with distance so, the potential is a function of x . And therefore, there is an electric field so, whenever you see a bent band it means that there is an electric field in that region. So, very far from the junction, you know there is once you cross this point there is no more band bending ok.

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So, you have you had band bending to a certain extent and then the band slowly flattened out and then it is completely flat. And beyond this region it looks like the semiconductor does not know about the existence of a junction at all ok. And everything here was the depletion region. Now, we define this width, as the width of the depletion region and give it a symbol x_d , the d stands for the depletion width. And beyond this point there is no band bending. So, any electric fields that exist in the material due to the formation of this junction it is all existing between x equal to 0 and x equal to x_d ok.

And therefore, the electric field from this point onward so, the electric field from this point onward is 0 all right. So, that is the boundary condition that we are going to use. So, we are going to say that at x equal to x_d , the electric field is 0 and of course, the electric field is 0 even for x greater than x_d .

Now, applying that boundary condition and using it. We can calculate this integration constant and we find that the electric field, the expression for the electric field is given by this term here ok. So, the electric field is $q N_D$ by epsilon into x minus x_d . So, let look at this picture a little bit more carefully ok, so, let us look at this map ok.

So, this I have drawn these two dotted lines to sort of mark out all the events in the depletion region. So, that is my x equal to 0 that is my x equal to x_d and I have shown the x axis here ok. So, that line there is x equal to x_0 and that is x equal to x_d . So, that is the width of the depletion region which is x_d . Now all the band bending happens within this region and beyond this the bands are all flat and this was my depletion region.

So, what are the charges? According to the full depletion approximation the charge concentration in the depletion region is simply q times N_D it is constant, because the doping is constant through the entire semiconductor. If the doping was varying with space then this would vary, but right now it is a constant. And we are not considering electrons and holes. And therefore, which is basically the full depletion approximation and therefore, that is not included into this picture.

So, you have all the positive charges here and these positive charges are balanced by negative charges in the metal which forms a very fine sheet. So, this picture is very exaggerated. So, this sheet is going to be very very thin ok, it is going to be a screen of electrons that balance out all the fields. And beyond this region there is no field in the metal and beyond this region the bands are all flat, and there is no field in the

semiconductor. So, that is the situation and we have calculated out the electric field to vary as $q N_D$ by ϵ_s into x minus x_d .

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Metal-Semiconductor Junctions
Electrostatics: Schottky Contact

$$\xi = -\frac{d\phi}{dx} = \frac{qN_D}{\epsilon_s}(x - x_d)$$

$$\phi(x) = -\frac{qN_D}{\epsilon_s} \left(\frac{x^2}{2} - x_d x \right) + \text{const.}$$

Define a convenient reference, say $\phi=0$ at $x=0$

$$\text{Built in potential} = \phi_{bi} = \phi(x_d) - \phi(0) = \frac{qN_D}{\epsilon_s} \frac{x_d^2}{2}$$

$$x_d = \sqrt{\frac{2\epsilon_s \phi_{bi}}{qN_D}}$$

The diagram illustrates the electrostatics of a Schottky contact. It shows a cross-section with a yellow Metal region on the left, a blue Depletion Region in the middle, and a pink Bulk region on the right. The x-axis is horizontal, with $x=0$ at the metal-semiconductor interface and $x=x_d$ at the edge of the depletion region. The electric field ξ is plotted above the x-axis, showing a linear decrease from $-\frac{qN_D x_d}{\epsilon_s}$ at $x=0$ to 0 at $x=x_d$. The potential ϕ is plotted below the x-axis, showing a parabolic increase from 0 at $x=0$ to ϕ_{bi} at $x=x_d$. Handwritten red annotations include: $\xi=0$ at $x=x_d$, $\xi = \frac{qN_D}{\epsilon_s}$ at $x=0$, $\phi=0$ at $x=0$, and 'Only 3-6' near the potential plot.

So, if you have to plot this electric field so, in the semiconductor, so again it is this plot is there only in the semiconductor. So, we are looking at only the semiconductor at x equal to 0. So, if you were to take this expression here at x equal to 0, the electric field is $q N_D$ by ϵ_s x_d with a negative sign.

So, that is the electric field at x equal to 0 and at x equal to x_d the electric field is 0, which was our boundary condition. And between these two points electric field varies linearly with x . So, now how do we calculate the potential so, what is the potential variation? The potential variation is essentially it is a measure of the band bending.

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Metal-Semiconductor Junctions

Electrostatics: Schottky Contact

$$\xi = -\frac{d\phi}{dx} = \frac{qN_D}{\epsilon_s}(x-x_d)$$

$$\phi(x) = -\frac{qN_D}{\epsilon_s} \left(\frac{x^2}{2} - x_d x \right) + const.$$

Define a convenient reference, say $\phi=0$ at $x=0$

Built in potential = $\phi_{bi} = \phi(x_d) - \phi(0) = \frac{qN_D}{\epsilon_s} \frac{x_d^2}{2}$

$$x_d = \sqrt{\frac{2\epsilon_s \phi_{bi}}{qN_D}}$$

So, if you can find the variation of the potential in the semiconductor with x you have essentially found the nature of the variation of the conduction band bending with x, the nature of the variation of the valence band bending with x etcetera. So, how do we calculate the potential variation? Now the electric field is nothing, but minus d phi by d x ok.

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Metal-Semiconductor Junctions

Electrostatics: Schottky Contact

$$\xi = -\frac{d\phi}{dx} = \frac{qN_D}{\epsilon_s}(x-x_d)$$

$$\phi(x) = -\frac{qN_D}{\epsilon_s} \left(\frac{x^2}{2} - x_d x \right) + const.$$

Define a convenient reference, say $\phi=0$ at $x=0$

Built in potential = $\phi_{bi} = \phi(x_d) - \phi(0) = \frac{qN_D}{\epsilon_s} \frac{x_d^2}{2}$

$$x_d = \sqrt{\frac{2\epsilon_s \phi_{bi}}{qN_D}}$$

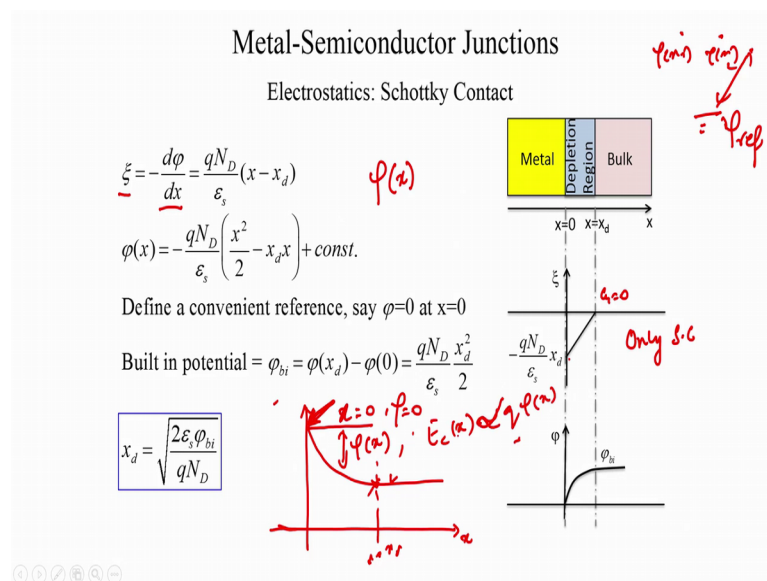
So, where phi is the potential and it is important so all though I am not shown it here, it is important to know how we have defined the potential. So, this is the band bending

right. So, let us look at this as the distance x that is the energies and that is my conduction band so, we just look at the conduction band.

Now, we have to keep a reference for potentials. Potential at any arbitrary point does not make any sense it is with respect to a certain reference. So, we need to have a reference potential and then measure the different locations in x . So, $\phi(x)$ $\phi(x_2)$ etcetera is all measured with respect to that reference. So, we will set that reference at this point we will say that at x equal to 0, my potential is 0. So, that is my ground ok, if you want to think of it that way that is my measurement ground.

And, as we go into the semiconductor, this gap starts increasing and therefore, my potential starts increasing so that is my potential variation. So, that is the way we are going to define this ϕ and the electric field is minus $d\phi$ by dx . Now it need not be I mean you are free to choose any reference you like ok.

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So, for example, an equally valid reference is you know choosing the potential in the bulk as a reference. So, if this is the band bending and say that is x equal to x_d . We could say that that is the reference potential that is ϕ equal to 0; no problem. As long as you apply the correct boundary conditions it is absolutely no problem as to what you take as your reference potential.

But now for this analysis, I have to choose one of these two and I have chosen this as my choice of reference. So, at x equal to 0 my ϕ is 0 and I want to know what is ϕ of x because, if I know what is ϕ of x I know the band bending; I know what is E of E_c of x because, E_c of x is going to scale or it is going to be proportional to q times ϕ of x plus some offsets. So, since the electric field is minus $d\phi$ by dx so minus $d\phi$ by dx is equal to this term here, which was the electric field which we obtained from solving Poisson's equation.

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Metal-Semiconductor Junctions

Electrostatics: Schottky Contact

$$\xi = -\frac{d\phi}{dx} = \left(\frac{qN_D}{\epsilon_s}(x-x_d)\right)$$

$$\phi(x) = -\frac{qN_D}{\epsilon_s} \left(\frac{x^2}{2} - x_d x\right) + \text{const.}$$

Define a convenient reference, say $\phi=0$ at $x=0$

$$\text{Built in potential} = \phi_{bi} = \phi(x_d) - \phi(0) = \frac{qN_D}{\epsilon_s} \frac{x_d^2}{2}$$

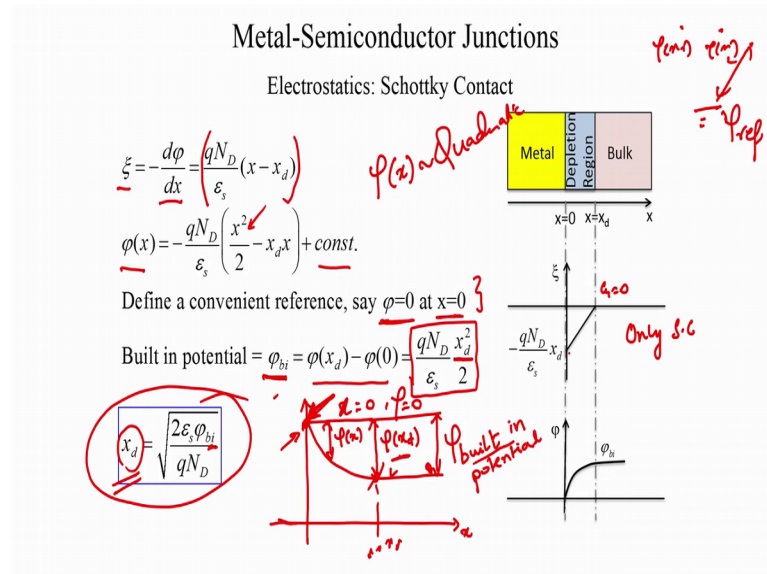
$x_d = \sqrt{\frac{2\epsilon_s \phi_{bi}}{qN_D}}$

$\phi(x)$ is Quadratic
 $\phi=0, \xi=0$
 $\xi = 0$ at $x=0$
 ϕ_{bi} at $x=x_d$
 Only s-c

And therefore, we solve this is the second differential equation and we find that ϕ of x varies as the square of the distance ok. So, it is a quadratic variation with distance plus an integration constant. Now, this integration constant really does not matter because we are going to measure ϕ of x with reference to some other location. But then we could use that reference point to identify that constant and we say that we have chosen our reference to be x equal to 0 is where ϕ is equal to 0.

So, if we had chosen the reference are to be different if we had chosen the reference to be ϕ equal to 0 at x equal to x_d . Then we will have a different integration constant, but now since we have chosen this as our reference.

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We find that the potential that is phi at x d ok. So, this is my potential variation in space ok. So, that is my phi of x and x keeps increasing and at this point we have your phi of x d. And beyond this the potential remains the same.

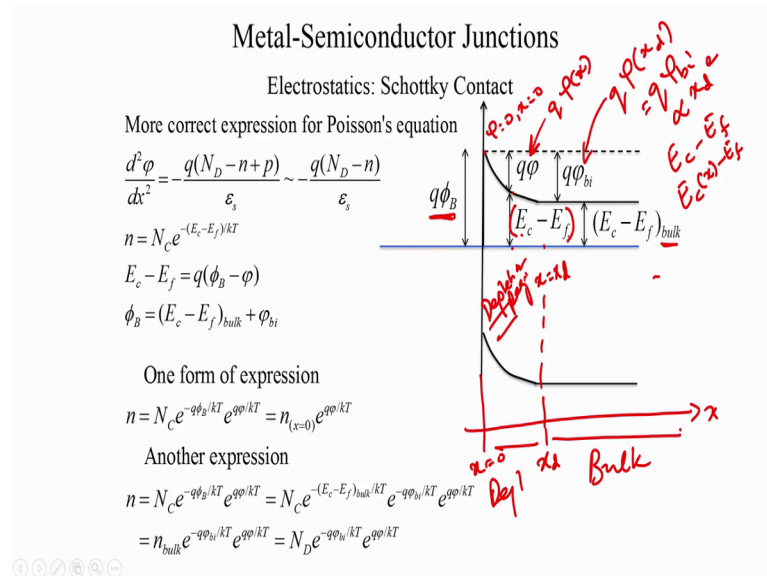
So, phi of x d minus this point that is phi that is the potential at this location minus this reference potential is given by this term here ok. And this has got a special meaning it is something called as the built in potential of your device and we will denote this with a special symbol phi b i. So, b i stands for the built-in potential, because from the edge of this junction till this point.

So, from this point till this location here we have a potential difference of phi b i ok. And phi b i is very nicely connected to x d and x d you know you can inverse this relation to obtain x d in terms of phi b i as the square root of 2 epsilon s phi bi by q N D. So, that is your depletion width. So, if you know the built in potential you can calculate a depletion width and on the other hand if you know you the depletion width you can calculate your built in potential.

So, that is the straightforward I mean that is the simplest analysis that one could do to determine the electrostatics of the Schottky contact ok. So, it started off with the full depletion approximation and we find that we get a reasonably good answer to what they are depletion width is and how it depends upon phi b i. Now all this is happening at thermal equilibrium, we have not applied any voltages. We have created the junction left

the device in the dark and we find that this is the electrostatics. So now let us say we decide not to make the full depletion approximation ok.

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So, we are not happy with the full depletion approximation we want to solve things a bit more accurately how do we solve it ok? So, let me just introduce it and I will introduce the tools and the techniques do it and this is something that will be helpful even in the future ok.

So, let us again come back to this you know a nice drawing here, I do not have to use my marker on this slide because it is all shown very neatly. And we have got a lot of labels on this drawing so, this is the band bending. So, that is your constant Fermi level, so that is a Schottky contact and you have your barrier here ok. So, the Schottky barrier formation and then that is your conduction band the conduction band bends and then flattens out after some x equal to x d.

So, that is the picture we are looking at so let us just mark that point x equal to x d or somewhere there and everything in the middle here is the depletion region. So, what are all these definitions so that is the barrier height which is given by q phi B ok, that is the potential that we had defined so that is our reference potential so, phi is 0 at x equal to 0. This is my x coordinate ok. So, that is x d that is x equal to 0 my phi is 0 at x equal to 0 and this is the potential that is varying with space because of the bending.

So, that is $q\phi$ of x which we had identified to be a quadratic term under the assumption of the full depletion approximation. And this term here is the built in potential. So, this is your $q\phi$ at $x = d$ which is your q times the built in potential, which we found you know to be various $x = d$ square. Now since E_c is varying with E_f , in this region this gap here that is between the conduction band and the Fermi level is essentially E_c minus E_f . However, to be more accurate you can say E_c as a function of x minus E_f .

So, that E_c minus E_f term varies so at this location what is the E_c minus E_f it is equal to $q\phi_B$. And at this location what is the E_c minus E_f it is equal to whatever it is in the bulk. So, we define $q\phi_B$ and the bulk E_c minus E_f so, this is what the semiconductor band before the creation of the junction ok. So, we give it a special symbol because everything in this region is the bulk region and this is all, the depletion region.

So, the E_c minus E_f in the bulk is given is clearly identified by defining the subscript bulk that and the E_c minus E_f here is varying with in space ok. So, once this picture is clear it helps with the lot of the analysis.

So, now let us say we want to do a more accurate study. So, we will say the Poisson's equation now I have written it directly in terms of the potential.

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Metal-Semiconductor Junctions

Electrostatics: Schottky Contact

More correct expression for Poisson's equation

$$\frac{d^2\phi}{dx^2} = -\frac{q(N_D - n + p)}{\epsilon_s} \approx -\frac{q(N_D - n)}{\epsilon_s}$$

$n = N_C e^{-(E_c - E_f)/kT}$

$E_c - E_f = q(\phi_B - \phi)$

$\phi_B = (E_c - E_f)_{bulk} + \phi_{bi}$

One form of expression

$$n = N_C e^{-q\phi_B/kT} e^{q\phi/kT} = n_{(x=0)} e^{q\phi/kT}$$

Another expression

$$n = N_C e^{-q\phi_B/kT} e^{q\phi/kT} = N_C e^{-(E_c - E_f)_{bulk}/kT} e^{-q\phi_{bi}/kT} e^{q\phi/kT}$$

$$= n_{bulk} e^{-q\phi_{bi}/kT} e^{q\phi/kT} = N_D e^{-q\phi_{bi}/kT} e^{q\phi/kT}$$

So, the way this comes about is let us say dE by dx is equal to ρ by ϵ_s where ρ is the charge concentration. Now clearly, since E is equal to minus $d\phi$ by dx you have $d^2\phi$ by dx^2 is equal to minus ρ by ϵ_s . So, you need to have a minus sign in place that.

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Metal-Semiconductor Junctions

Electrostatics: Schottky Contact

More correct expression for Poisson's equation

$$\frac{d^2\phi}{dx^2} = -\frac{q(N_D - n + p)}{\epsilon_s} \sim -\frac{q(N_D - n)}{\epsilon_s}$$

$n = N_c e^{-(E_c - E_f)/kT}$

$E_c - E_f = q(\phi_B - \phi)$

$\phi_B = (E_c - E_f)_{bulk} + \phi_{bi}$

One form of expression

$$n = N_c e^{-q\phi_B/kT} e^{q\phi/kT} = n_{(x=0)} e^{q\phi/kT}$$

Another expression

$$n = N_c e^{-q\phi_B/kT} e^{q\phi/kT} = N_c e^{-(E_c - E_f)_{bulk}/kT} e^{-q\phi_{bi}/kT} e^{q\phi/kT}$$

$$= n_{bulk} e^{-q\phi_{bi}/kT} e^{q\phi/kT} = N_D e^{-q\phi_{bi}/kT} e^{q\phi/kT}$$

So, in the full depletion approximation we had said that despite the charges being $q N_D$ minus $q n$ plus $q p$ in the full depletion approximation we had said that these two are negligible terms and we had we had gone ahead with just that ok. But now we want to do something a little bit more accurate so, we will say that ok.

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Metal-Semiconductor Junctions

Electrostatics: Schottky Contact

More correct expression for Poisson's equation

$$\frac{d^2\phi}{dx^2} = -\frac{q(N_D - n + p)}{\epsilon_s} \sim -\frac{q(N_D - n)}{\epsilon_s}$$

$$n = N_C e^{-(E_c - E_f)/kT}$$

$$E_c - E_f = q(\phi_B - \phi)$$

$$\phi_B = (E_c - E_f)_{\text{bulk}} + \phi_{bi}$$

One form of expression

$$n = N_C e^{-q\phi_B/kT} e^{q\phi/kT} = n_{(x=0)} e^{q\phi/kT}$$

Another expression

$$n = N_C e^{-q\phi_B/kT} e^{q\phi/kT} = N_C e^{-(E_c - E_f)_{\text{bulk}}/kT} e^{-q\phi_{bi}/kT} e^{q\phi/kT}$$

$$= n_{\text{bulk}} e^{-q\phi_{bi}/kT} e^{q\phi/kT} = N_D e^{-q\phi_{bi}/kT} e^{q\phi/kT}$$

We will say that you know getting rid of n and p is not really necessary because we really do not know the numbers particularly as you approach the bulk ok. So, over here it might be a great assumption to make, but as you start approaching the bulk it seems that it is quite logical to expect that the n the number of electrons keeps increasing. And therefore, if you are trying to perform an analysis in this region it is definitely not very advisable to get rid of the end ok.

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Metal-Semiconductor Junctions

Electrostatics: Schottky Contact

More correct expression for Poisson's equation

$$\frac{d^2\phi}{dx^2} = -\frac{q(N_D - n + p)}{\epsilon_s} \sim -\frac{q(N_D - n)}{\epsilon_s}$$

$$n = N_C e^{-(E_c - E_f)/kT}$$

$$E_c - E_f = q(\phi_B - \phi)$$

$$\phi_B = (E_c - E_f)_{\text{bulk}} + \phi_{bi}$$

One form of expression

$$n = N_C e^{-q\phi_B/kT} e^{q\phi/kT} = n_{(x=0)} e^{q\phi/kT}$$

Another expression

$$n = N_C e^{-q\phi_B/kT} e^{q\phi/kT} = N_C e^{-(E_c - E_f)_{\text{bulk}}/kT} e^{-q\phi_{bi}/kT} e^{q\phi/kT}$$

$$= n_{\text{bulk}} e^{-q\phi_{bi}/kT} e^{q\phi/kT} = N_D e^{-q\phi_{bi}/kT} e^{q\phi/kT}$$

So, what we are going to do? Is we are going to perform the study in a region of the depletion in the depletion region, but closer to the bulk as compared to the interface ok. So, we need a more accurate Poisson's equation and we say that although we could retain p we say that p is negligible, but there is no need because we have no information.

And in fact, it actually makes the mathematics a bit more symmetrical if you keep the p, but just for the sake of an example I have not retained the p. So, we say that the p is still negligible, because an n-type semi conductor. And in the depletion region as I approach the bulk I have this to be my charge carriers.

So, that is my Poisson's equation. So, you have a negative sign because of the potential you are using a potential term, but then you have q N D minus these are the positive donor ions minus the electrons in the bulk. So now, how do we go about solving this so, we need to identify. So, this is a constant it is not a problem.

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More correct expression for Poisson's equation

$$\frac{d^2\phi}{dx^2} = -\frac{q(N_D - n + p)}{\epsilon_s} \approx -\frac{q(N_D - n)}{\epsilon_s}$$

$n = N_C e^{-(E_c - E_f)/kT}$
 $E_c - E_f = q(\phi_B - \phi)$
 $\phi_B = (E_c - E_f)_{bulk} + \phi_{bi}$

One form of expression
 $n = N_C e^{-q\phi_B/kT} e^{q\phi/kT} = n_{(x=0)} e^{q\phi/kT}$

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 $= n_{bulk} e^{-q\phi_{bi}/kT} e^{q\phi/kT} = N_D e^{-q\phi_{bi}/kT} e^{q\phi/kT}$

But this is not a constant; it depends upon the potential because it depends upon E c minus E f ok. So, let us identify n bit more clearly. And I will just get rid of these marks here so, that slide is more clear. So, what is n? N is the effective density of states in the conduction band into e to the power minus E c minus E f by k T.

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More correct expression for Poisson's equation

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$$n = N_C e^{-(E_c - E_f)/kT}$$

$$E_c - E_f = q(\phi_B - \phi)$$

$$\phi_B = (E_c - E_f)_{bulk} + \phi_{bi}$$

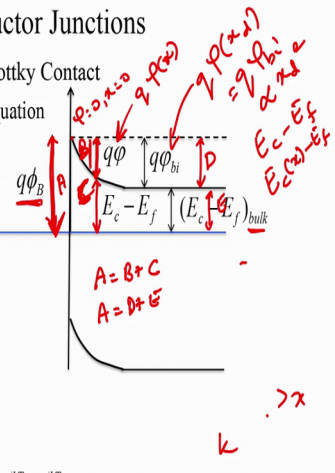
One form of expression

$$n = N_C e^{-q\phi_B/kT} e^{q\phi/kT} = n_{(x=0)} e^{q\phi/kT}$$

Another expression

$$n = N_C e^{-q\phi_B/kT} e^{q\phi/kT} = N_C e^{-(E_c - E_f)_{bulk}/kT} e^{-q\phi_{bi}/kT} e^{q\phi/kT}$$

$$= n_{bulk} e^{-q\phi_{bi}/kT} e^{q\phi/kT} = N_D e^{-q\phi_{bi}/kT} e^{q\phi/kT}$$



And we know that in the depletion region $E_c - E_f$ is a function of x ok. So, what is I need to represent $E_c - E_f$ as a function of the potential. I want to get everything in terms of the potential because the left hand side has got a potential term. I want to show I want to be able to construct a differential equation that I can solve ok. So, I do know, I can see that it is connected to the potential, but let me sort of make it very very clear.

So, what is $E_c - E_f$, $E_c - E_f$ is nothing, but q times the barrier height minus the potential so think about it. So, this is $E_c - E_f$ and that is the barrier height and that is the definition of the potential. So, this minus that so $1 - 2$ gives you 3 so that is exactly what we have written here. And what is the barrier height itself, the barrier height is connected to the bulk levels right the barrier height can be written as this plus this so let us call it 3 and 4 .

So, the barrier height so, 1 is equal to so repeated 3 . So, that is $1 - 2 = 3$ and let us call that 4 and $5 = 4 + 1$. So, 1 is equal to $2 + 3$ I mean please do not take it as numbers. So, maybe I should say $A = B + C$ do forgive me for D and E ok. So, A is equal to $B + C$ and A is also equal to $D + E$ you could use either one of these definitions now they just give you different table they give you a different picture. So, we could say that n is equal to $N_C e^{-(E_c - E_f)/kT}$ and we use this expression here and write enhance this term here.

So, what we are saying is the n at the interface. So, the expression for n at any location in the depletion region so, if you say this is the depletion region.

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Electrostatics: Schottky Contact

More correct expression for Poisson's equation

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$n = N_c e^{-(E_c - E_f)/kT}$
 $E_c - E_f = q(\phi_B - \phi)$
 $\phi_B = (E_c - E_f)_{bulk} + \phi_{bi}$

One form of expression

$$n = N_c e^{-q\phi_B/kT} e^{q\phi/kT} = n_{(x=0)} e^{q\phi/kT}$$

Another expression

$$n = N_c e^{-q\phi_B/kT} e^{q\phi/kT} = N_c e^{-(E_c - E_f)_{bulk}/kT} e^{-q\phi_{bi}/kT} e^{q\phi/kT} e^{q\phi_{bi}/kT}$$

$$= n_{bulk} e^{-q\phi_{bi}/kT} e^{q\phi/kT} = N_D e^{-q\phi_{bi}/kT} e^{q\phi/kT} = n(\phi)$$

And I want to get an expression for n at say some x. What we are saying is? It is the nth the interface that is n at x equal to 0 times e to the power q phi by k T; which means as phi increases n increases, which is expected because the interface has got very little n and as phi keeps increasing n increases till it approaches the bulk concentration.

So, now I can use this expression or I could write another expression, I could say that I could go one step further. I can say that I want to define the interface term more clearly. So, I say that n is equal to N c e to the power q phi B by k T into e to the power q phi by k T which is same as that. And then I can rewrite the barrier height phi B in terms of the bulk E c minus E f and phi b i using this expression here.

So, then if you look at this term N c into e to the power E c minus E f bulk by k t what is that? That is basically the electron count in the semiconductor before the creation of the junction. So, that is what is going on in the bulk. So, that is the electron count in the bulk which is approximately equal to your N D e ok. So, this is another expression for n, you could use either of these expressions.

Now, what we have done is we have taken this variable n and expressed it in terms of phi which is the potential and we have done that in order to solve this differential equation ok. So, we are just going through this example because it is quite. Now I think it is quite illustrative with regards to the techniques; we will use in future did it illustrate a lot of

these techniques. So, now you have your n in terms of phi, we will have n as a function of phi and we make that substitution.

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Electrostatics: Schottky Contact

Substitute for n

$$\frac{d^2\phi}{dx^2} = -\frac{q(N_D - n)}{\epsilon_s} = \frac{qN_D}{\epsilon_s} (e^{-q\phi_n/kT} e^{q\phi/kT} - 1)$$

To solve, multiply both sides by $2 \frac{d\phi}{dx}$

$$2 \frac{d\phi}{dx} \frac{d^2\phi}{dx^2} = \frac{qN_D}{\epsilon_s} (e^{-q\phi_n/kT} e^{q\phi/kT} - 1) 2 \frac{d\phi}{dx}$$

$$\frac{d\left(\frac{d\phi}{dx}\right)^2}{dx} = \frac{qN_D}{\epsilon_s} (e^{-q\phi_n/kT} e^{q\phi/kT} - 1) 2 \frac{d\phi}{dx}$$

$$d\xi^2 = \frac{2qN_D}{\epsilon_s} (e^{-q\phi_n/kT} e^{q\phi/kT} - 1) d\phi$$

This is now more easily solved

← Poisson's

$\frac{d^2y}{dx^2} = f(y)$

$2 \cdot \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} = f(y) \cdot \frac{dy}{dx}$

$\frac{d\left(\frac{dy}{dx}\right)^2}{dx} = 2 \cdot \frac{dy}{dx} \cdot \frac{d^2y}{dx^2}$

So that is my Poisson's equation so, that is the equation I want to solve to get a more accurate answer ok. And how do we solve this? Now this looks like a tricky equation right you have a d square phi by d x square term on the left hand side and you have got a function of phi on the right hand side. Essentially you have got an equation which says d square y by d x square is some function of y ok.

So, let us take a general x equation like this. So, here is one trick that you will find is very useful in order to solve an equation of this kind. And simply what you do is you simply multiply d y by d x on both sides ok. Now what is this term this is nothing, but the derivative of d y by d x whole square with respect to d x, because if you were to expand that you will first differentiate this term and then you will take the second order derivative that is exactly the same as this so that is your left hand side.

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Metal-Semiconductor Junctions
Electrostatics: Schottky Contact

Substitute for n

$$\frac{d^2\phi}{dx^2} = -\frac{q(N_D - n)}{\epsilon_s} = \frac{qN_D}{\epsilon_s} (e^{-q\phi_n/kT} e^{q\phi/kT} - 1)$$

To solve, multiply both sides by $2 \frac{d\phi}{dx}$

$$2 \frac{d\phi}{dx} \frac{d^2\phi}{dx^2} = \frac{qN_D}{\epsilon_s} (e^{-q\phi_n/kT} e^{q\phi/kT} - 1) 2 \frac{d\phi}{dx}$$

$$\frac{d}{dx} \left(\frac{d\phi}{dx} \right)^2 = \frac{qN_D}{\epsilon_s} (e^{-q\phi_n/kT} e^{q\phi/kT} - 1) 2 \frac{d\phi}{dx}$$

$$\int d\xi^2 = \frac{2qN_D}{\epsilon_s} (e^{-q\phi_n/kT} e^{q\phi/kT} - 1) d\phi$$

This is now more easily solved

← Poisson's

$\frac{d^2y}{dx^2} = f(y)$

$2 \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} = f(y) \cdot \frac{dy}{dx}$

$d \left(\frac{dy}{dx} \right)^2 = f(y) \cdot dy$

$\left(\frac{dy}{dx} \right)^2 = \int f(y) \cdot dy$

$\left(\frac{d\phi}{dx} \right)^2 = \xi^2$

$\xi = f(\phi)$

And your right hand side is simply d y by d x. So, you can get rid of these d x and now you have a very nice differential equation to solve. If you were to integrate both sides you will end up with d y by d x square being equal to the integral of f of y d y. So, that is one that is a technique to solve and that is what we will be using. So, what we do here is we multiply both sides by 2 d phi by d x. So, you have that term that essentially gives you this term here and that is equal to d phi by d x square is nothing, but the square of the electric field right.

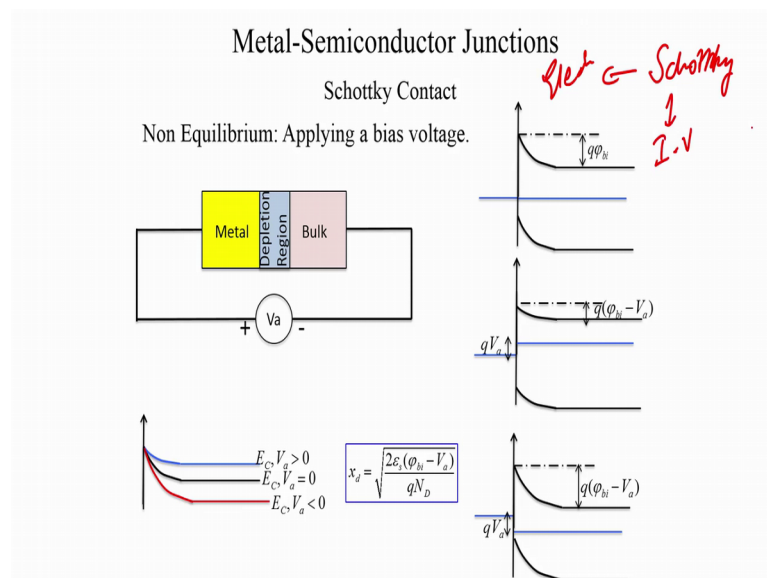
So, we now have an expression which says d electric field square is equal to all this term on the right hand side and you just integrate both sides, the left side will be the electric field square and after you integrate the right hand side term, you will get some function of phi and we now know the electric field to be the square root of that particular term.

So, this is very much solvable and in fact, we will go ahead and solve it in great detail later on. Of course, from this point on particularly this topic onwards the course does get a bit mathematical in nature. There is there is going to be there are going to be a lot of concepts to learn, but it is also important to learn how to calculate. So, we will go through a lot of derivations it might appear quite painful to some students, but nevertheless it is my duty to go through all these derivations ok. Because so, that you have an understanding as to where everything came from.

But having said that in your exams and in your examples we will not be quizzing you on how to derive a certain expressions so, that is not the point of the course. The point of this course is to you know in a sort of communicate certain concepts and quiz you on the concepts and yet show you methods and techniques to calculate and also show you how these tools came about ok. So, that is so that is why we need to go through all these this bit of mathematics, it is show you all the tools and the techniques and the methods to perform calculations ok.

So, do bear with me if the mathematics gets a bit too dull. So, that is the electrostatics of the Schottky contact we will not worry about the electrostatics anymore. And now we will; what we will do is; we will try to take the Schottky contact out of equilibrium.

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And apply a bias voltage and start observing whether there are any currents. So, we are all still working on the Schottky contact ok. So, it is still we look at the electrostatics and now we are going to look at the current voltage characteristics ok.