Semiconductor Devices and Circuits
Prof. Sanjiv Sambandan
Department of Instrumentation and Applied Physics
Indian Institute of Science, Bangalore

Lecture - 18
Continuity Equation

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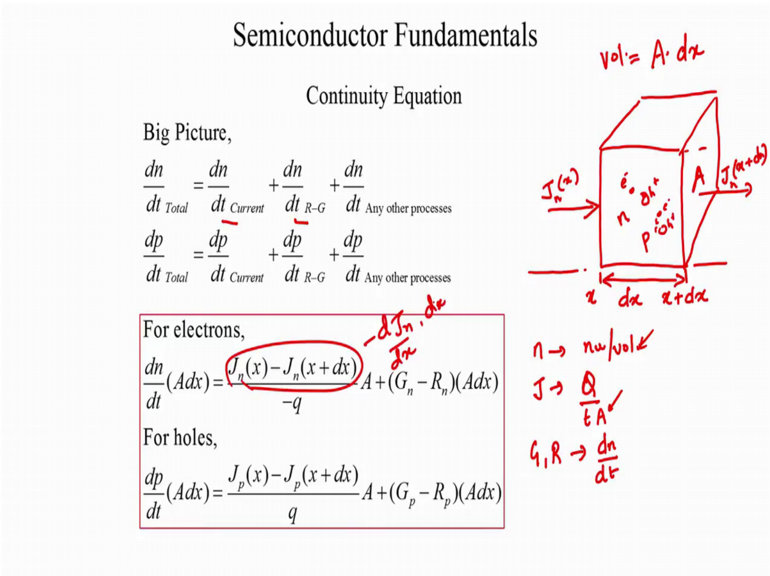

So, now we will begin our discussion on a very useful idea which is a something called as a Continuity Equation. And students are familiar in with other areas of engineering you know such a fluid mechanics etcetera, might have come across this term. And the idea is very similar which is to draw the complete budget or a complete balance of charge and mass and make sure that we consider all processes that influence charge in mass and establish some understanding of the behavior of the device.

So, to start with I think a figure here might be very useful, which is we always looked at this big picture wherein we considered a volume of a semiconductor. And we wanted to have a count or a measure of the number of electrons per unit volume inside this element of interest ok. So, let us say that the element of interest has a cross section A and that anything that is of interest is only happening in one direction, it means in one dimension which is the dimension x ok. And inside this volume had many mechanisms which could change the population of n and p.

So, we had processes that are called as generation mechanisms, which created electron hole pairs ok. We had a mechanisms of recombination which and highlighted electron hole pairs, and we discussed these mechanisms. And we also discussed mechanisms which result in current which is your drift and diffusion currents; so you have currents coming in to this region and currents going out of this region.

So, specifically we will define a little volume element to have these values let us say this boundary is x, this boundary is x plus d x ok. And it is got an area of cross section A and the current coming in is the J at x and the current that is going out is the J at x plus d x. So, these are all the processes that are going on. And therefore, you have currents coming in and going out you have R G mechanisms and any other processes which is affecting your total electron and hole count.

Now, if you consider only electrons ok, so let us not worry about holes first we will represent these currents as J n that is coming in and going out. You will find that your total so n is the number of carriers per unit volume; it is the count number per volume ok. J is the current density it is the rate of change of rate at which the charge is changing per unit area; so in some sense it is per unit area. Therefore, and your generation and recombination rates they are all talking about the rate of change of the number of electrons or holes per unit volume in this element. So, that is what your G and R.

So, let us just establish everything in terms of the total number ok, let us not worry about densities because here you have per unit area there you have per unit volume and therefore, things can get a bit confusing. So, we will just establish everything in terms of the total number. So, in this element of study so, this element has got a volume of A times d x so, that is the volume of this element. And the cross section area is A. So, d n by d t A d x is the total count of the total rate of change of electrons in this volume. That is equal to that is dependent on all the current that is coming in, minus the current that is going out, plus all the generation mechanisms minus the recombination mechanisms.

So, the current that is coming in brings in electrons in the current that is going out takes electrons out the generation mechanisms increases the electron population and the recombination mechanism destroys the electron population. So, since these are currents and n is a number, we are not talking about this charge it is a number right. And these are currents which I have already got the parameter charge embedded in it.

So, we are going to divide that by the charge of the electron which is minus q in order to get this in terms of a total number. So, this is the number of electrons that is what we are talking about here. And similarly we can write a similar expression for holes ok, but the only thing is the hole charge the holes is a positive quantity ok.

So, this is a very useful equation ok, it is a complete balance of everything that is going on and it is something called as your continuity equation. So now, we are going to work with these equations and bring them to a form that is much more usable ok. So, first let us look at what is J n minus J at x minus J at x plus d x.

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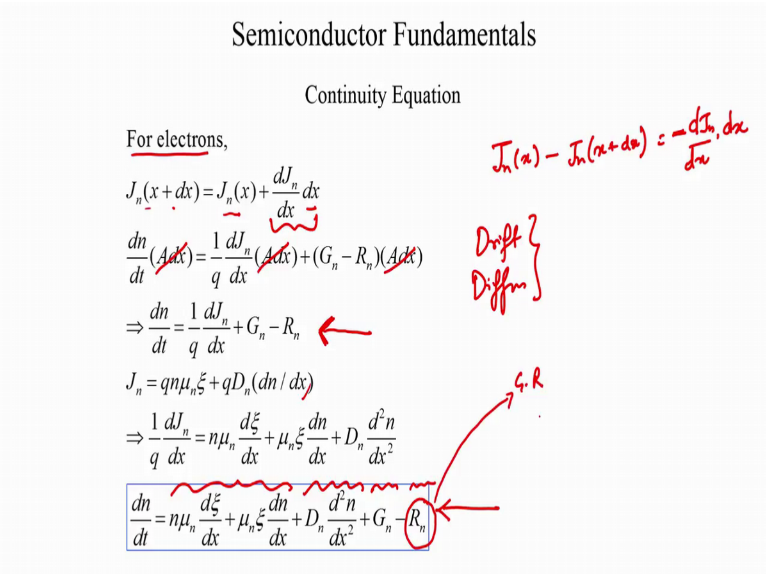

So, let us first only worry about the electrons. So if you write; if you think of your J at x plus d x has simply J n at x plus the slope into d x, then J n of x minus J n at x plus d x is simply equal to minus d J n by d x into d x ok. So therefore, this term this numerator is going to be minus d J n by d x into d x. And using that in that numerator your continuity equation now becomes d n by d t into A d x has got this term, which is based essentially this plus your generation recombination mechanisms.

Now, you will notice that this minus sign here has been cancelled using the minus sign at the electron charge, so there was a minus q because the charge is electron is negative. So, that is that minus sign has gone away and you have a plus in that location. So now, we can very easily clean out all these volume elements A d x can be cancelled everywhere and we have that equation to be appearing in a very simplified form as shown here.

Now, let us think about the currents a little bit more so, we had discussed earlier that you know the electron you have the drift currents and you have the diffusion currents and the in the case of electrons. The drift and diffusion currents were supporting each other, and therefore you have the drift current of electrons plus the diffusion current due to electrons contributing to the total current density ok.

So, we substitute for J n using this term, but what we need is the rate at the variation in J n with x so, d J n by d x into 1 by q. So, this term 1 by q d J n by d x is going to be is going to result in us taking a spatial derivative of the drift component and a spatial derivative of the diffusion component.

So, let us say that n so the spatial derivative the drift component is going to have two possible variation, two possible elements that could be varying in space that are that is basically your n and the electric field. So, you have d E by d x and you also have d n by d x so that is the drift contribution. And the diffusion contribution simply leads to your d square n by d x square appearing because of a second derivative on the concentration gradient.

So therefore, the continuity equation for electrons is given by this relation here. So, d n by d t has got components due to drift, components due to the diffusion currents, components due to the generation of electrons and components due to the recombination of electrons. And we have already looked at how to compute the recombination term for different cases, when we studied generation recombination mechanisms.

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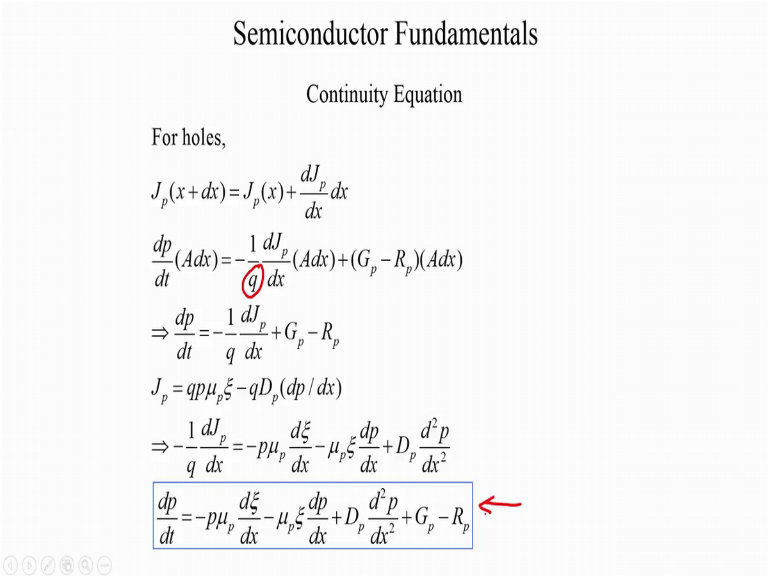

Now, the same exercise can be performed for holes as well except for a few minor differences. The first difference is that this charge is positive for holes and the second difference is that the drift and diffusion currents; you know as we discussed earlier are opposing each other in the case of holes so, you have a negative sign. So, you allow for these two corrections and you will find that the continuity equation for holes is given by this expression here.

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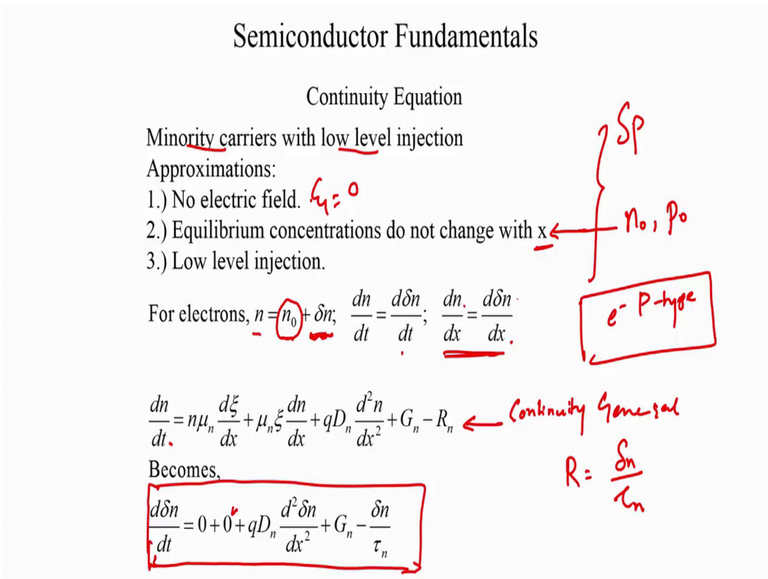

So, now we can run through some exercises ok. So, we will take the continuity equation it is a very useful expression, because its telling us the dynamics, it is giving us a description on the dynamics of the electron and hole population in any given volume. And we will use it for a special case and the special case is that. Let us only worry about the continuity equation for minority carriers with low level injection. And we are interested in the special case because this is something we will be using later down the line ok.

So, when we say; minority carriers with low level injection they are going to make a further approximations ok, we are interested in this very very special case, we are going to apply this general relation. So, this relation is general it is very useful and we are going to use this relation for this very very special condition which is minority carriers, low level injection, no electric fields so, your E is 0 ok. And the condition where your equilibrium concentrations do not change with space; that means, your n o and p o are independent of space ok, they do not change its space ok.

So, using these conditions we will first establish that n is your n o plus delta n. So, this is the low level injected carriers. And since these are minority carriers we are talking about the electrons in a p type material so, this is a p type material and we are talking about electrons in a p type material. So, d n by d t since, n naught is the equilibrium carrier concentration, d n naught by d t is a constant. Therefore, d n by d t is similar to d delta n by d t d n by d x is same as d delta n by d x because, n naught is not going to vary with x right.

And if you take your general continuity equations so, this is my continued equation with all the terms in it ok. That is a general equation and we are going to apply all these approximations and all these conditions that we have. So, my d n by d t becomes d delta n by d t, my electric field is 0. Therefore, I get a 0 here and I get a 0 here as well. My diffusion current component does exist, but the n can be replaced with delta n simply because of this relation.

And I have a some generation mechanism and my recombination, is the recombination of minority carriers which is given by this term. So, if you remember the R G mechanisms; we studied it, we solved for a very special case ok, which was the recombination rate for electrons in a p type material. And we had found that that recombination rate was delta n by tau n. So, that is the recombination rate that we are using so, this is the special case that we want to study.

So, now we have a new continuity equation, which is essentially got only these terms. So, there is no electric field and there is a it is we are looking at minority carriers with low level injection. So, let us see what this continuity equation tells us how do we solve for different conditions we just go through several examples.

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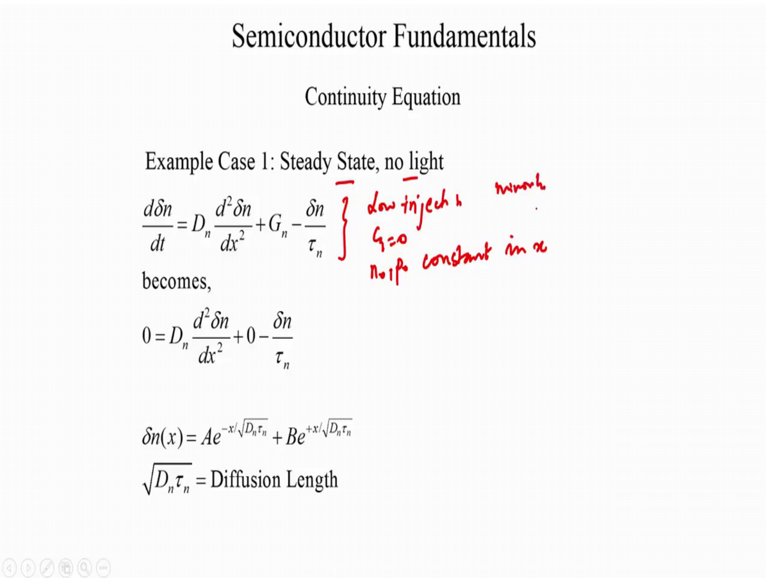

So, example one so, let us say we are at steady state and there is no light being thrown on the semiconductor. So, this is my continuity equation ok. This is all assuming everything if we discussed in the previous slide which is low level injection, no electric field n o p o are not varying with space. They are constant in x and with all these conditions and we are talking about minority carriers. So, it is be received we be obtained from the general continuity equation we obtained this new continuity equation under these conditions. So, that is what we are going to use now.

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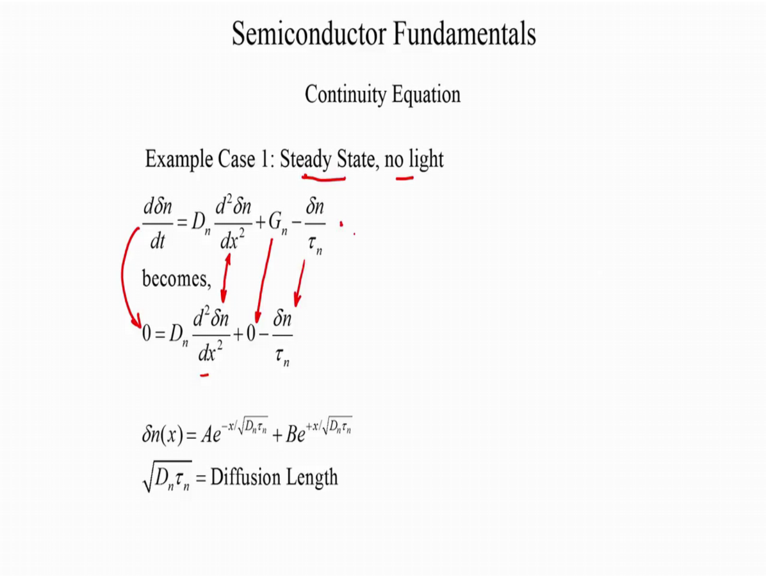

And we are going to use this under the condition where in the first example we are going to say we are at steady state and there is no light. So, steady state implies this derivative has to be 0. So, now we are looking at the total change in carrier concentration so steady state implies this derivative is 0.

And diffusion we have not made any comments on the diffusion current so, therefore, we will retain this term there is no light. So, therefore, they cannot be any generation mechanism and there does exist a recombination mechanism. So, this is the modified continuity equation for this particular example and you can solve this. So, this is a very useful relation because we will use it when we discuss our p n junction diodes ok.

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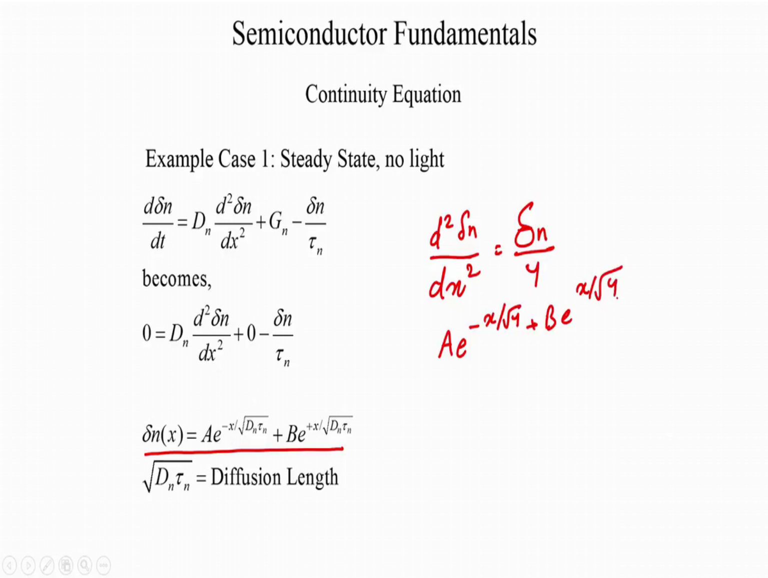

And the solution to this equation so what we have here is an equation that looks like this. And this is a differential equation that we have solved several times before you know we look at these differential equations when we looked at the case of particle in a box in quantum mechanics for example, ok. And the general solution takes up this form you got a you got A e to the power minus. So, suppose this was some parameter A say some parameter y, then it would the general solution remain x by square root of y plus B into e to the power x by square root of y ok.

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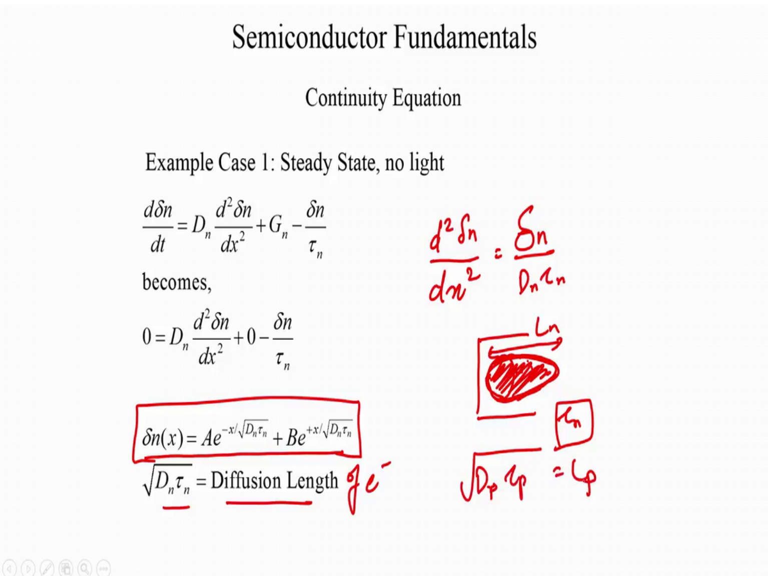

So, so here we have this product of D n and tau n and therefore, this happens to be the general solution for this differential equation. Now this product the square root of D n tau n is a very useful parameter, it is something called as a diffusion length. So, what is it saying? It is saying that the electrons which are the minority carriers and p type material can diffuse, they are diffusing you have some high concentration here they are trying to diffuse out ok.

And they are going to last for about tau n seconds, that is the time they are going to last before they recombine. So, it is telling us it is giving us a measure of the spread, diffusive spread of the electrons before the electrons have recombined. So, it is a length which is called as the diffusion length of the electrons. And similarly if you have dau p tau p it is the diffusion length of holes, when the holes are the minority carriers, which is holes in a p in a n type material.

So, this is the general equation and its it is worthwhile to remember this and by applying different boundary conditions based on your situation you can calculate what B and A are ok. So, what this equation tells us is for this particular example; delta n is spread out in space in this particular manner.

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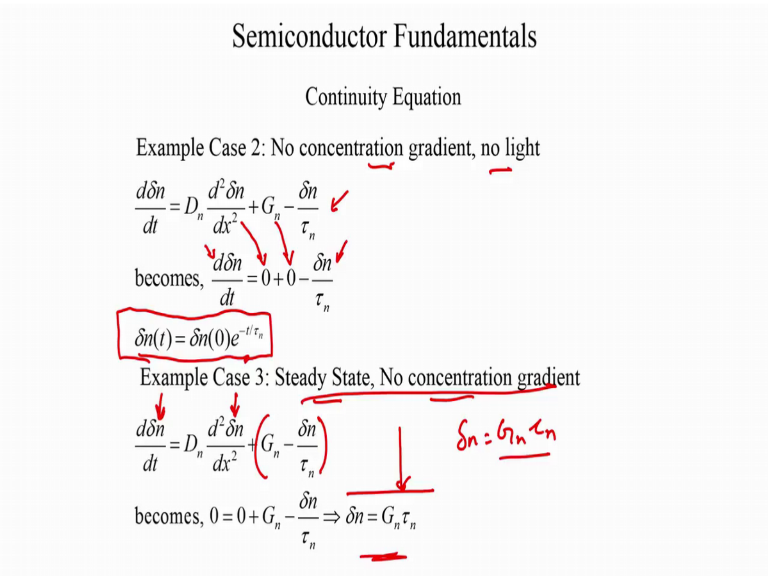

So, let us take another example, let us take the example of no concentration gradient and no light ok.

So, this is our general equation, if you have no concentration gradient here is no diffusion. So, that term is 0. If there is no light your G n is 0, but we have not made any conditions any assumptions on whether steady state or not. So, we will keep a time derivative there and that is your recombination rate. So, the solution to this is simply this so, you can see how the continuity equation can be used to obtain quantitative estimates of what is happening to the minority carriers in space and time for different conditions on the semiconductor.

So, just as another example, just to complete things we will assume a case of steady state and no concentration gradient. So, steady state implies the time derivative here is 0, concentration gradient there is no concentration gradient implies there is no diffusion. And therefore, you only have generation and recombination which means that such a situation is sustained only delta n is the generation rate into the lifetime of these carriers. So, that is essentially what so this, these words actually imply this quantitative result ok.

So, with that we will conclude our discussion on the continuity equation. And we are now in a position where and we can estimate the behavior of the semiconductor in equilibrium and out of equilibrium. And, we will now use this knowledge as the basis for all our discussions henceforth.