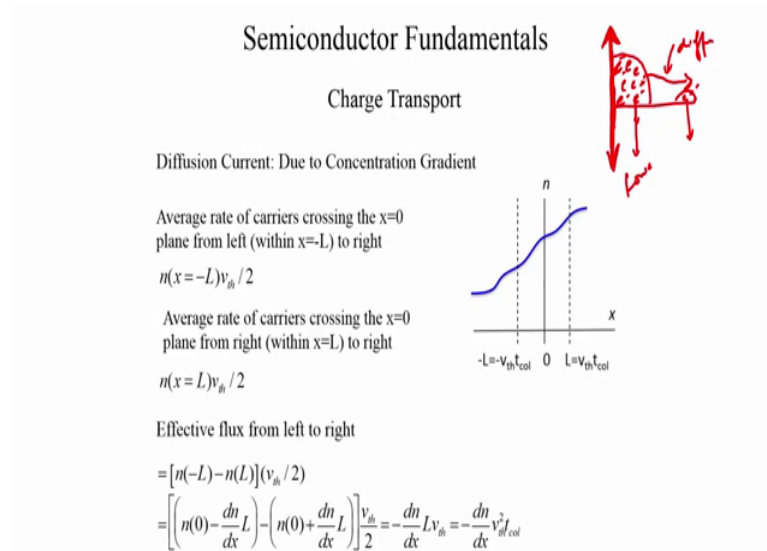


Semiconductor Devices and Circuits
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Lecture- 17
Charge Transport - Continued

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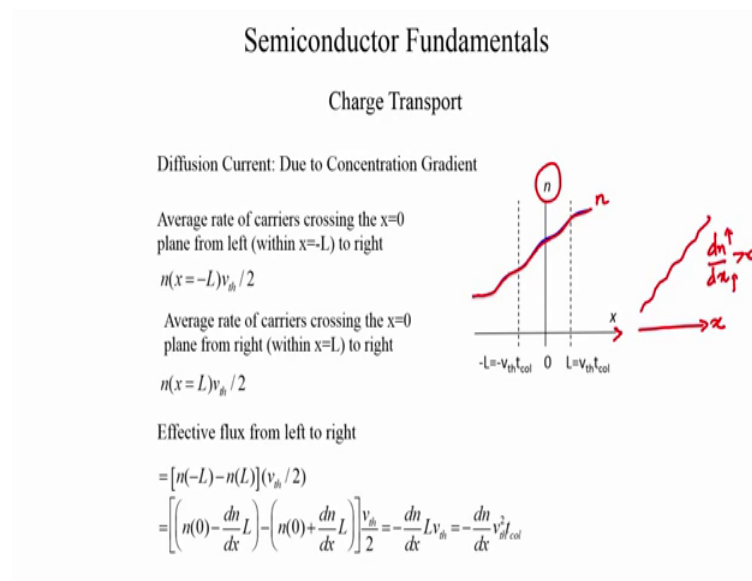
Now, the second mechanism of Charge Transport is something called as diffusion. Now, what do you mean by diffusion? When the while the driving force for what you said drift based conduction is the electric field a driving force for diffusion based conduction is the concentration gradient ok. So, if you were to pass by on a bakery for example, and if it is a really nice bakery and as a warm day probably you will find that you know you get a lot of sweet scents that are sort of in the even though you are not at the bakery, you do get a whiff of all the sweet scents in the air and that is got to do with all the molecules from all the sugars and everything else that is used in the bakery. So, diffusing through the atmosphere and ending up being sensed by a nose of course there is a big difference those molecules are all neutral species and their diffusion is defined by a slightly different parameters.

But what we are interested in is a diffusion of charged species which are electrons and holes and the diffusion of these species is got is basically driven or they know the flux of these the flux due to diffusion is because there exists a difference in concentration. So, if

you go back to your, if you go back to a drift, we saw that you know the drift is because of a potential difference right, but you can imagine the electrons ok. So, if you have an electron concentration which is very large and say this area, you have lots of electrons here, and you got very little electrons there. Now, it is very a there is a tendency for these electrons to move or migrate towards this low concentration region and that is what diffusion is.

But, if you think about it a presence of a lot of charged particles here implies this region has got low voltage, lower potential is compared to this region. And therefore, in the case of charged entities the diffusion can be thought of some kind of a self-induced drift ok, it might be a wrong way of thinking about it when it comes to other matters, but it does help the understanding if you if that in some sense ok. But essentially the driving force for diffusion is the concentration gradient. So, let us try to derive the diffusion current ok.

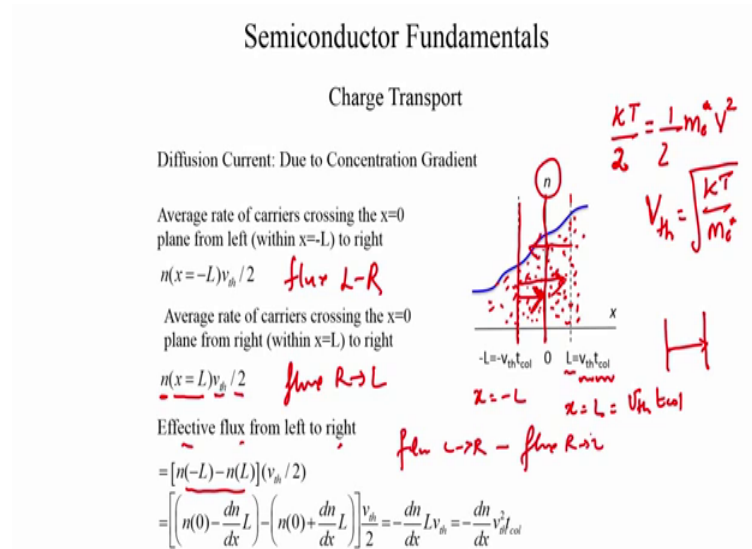
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The diffusion current so let us start off let us say we only talk about electrons a let us say with distance x ok, so that is my its just one-dimensional case that is x increasing. And you have a concentration of electrons that is varying its space you know we have just drawn this little squiggly line which is to indicate the electron concentration that is the number of electrons per unit volume that is varying with x . So, your dn/dx is greater than 0, which means as x increases my electron concentration is increasing,

because this is the direction of increasing x. Now, these electrons in the semiconductor, so let us say that does exists the situation of this kind.

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Now, if you look at all these electrons, here you got lots of electrons here, so that is basically all the electrons there and you have got few a electrons here ok. Now, these electrons have got some thermal velocity. We have not applied any voltage ok so, there is 0 voltage or 0 electric field being applied these electrons are simply have thermal velocity ok and what is thermal velocity in one dimension your thermal energy by 2 ok. So, it is only a 1D case. So, kT by 2 is equal to your half effective mass of the electron into V square and your V is your thermal velocity which is kT by m_e star square root. So, all the electrons have a velocity of this value.

And this velocity is in an arbitrary direction. So, if it is a one-dimensional case, some of the electrons are moving this way, some of these electrons are moving this way. And similarly on this side, you have few electrons moving this way and few electrons moving that way. So, this is you know this is the situation we are looking at.

So, our interest is to look at a very specific region and as these electrons are moving through they are colliding with the lattice. So, there is a mean collision time. And we had defined this mean collision time when we define mobility right. So, the electrons collide with the lattice, and therefore it is only within this mean collision time that the electrons actually see an acceleration ok. So, we think about the when we had looked at mobility

when we had looking mobility we defined the mobility in terms of the mean collision time.

So, this is the time between any scattering event ok, between any two consecutive scattering events statistically speaking. So, we are interested in a boundary that is defined by this value L . So, x equal to L which is equal to the thermal velocity the electrons into the mean collision. So, the electron moving with a certain thermal velocity travels this distance before it scatters ok, so that is the boundary we are looking at. And what we are interested in identifying is the total number of carriers that are going to effectively move from left to right and from right to left and cross the plane of x equal to 0. So, you want to make a get a count of these carriers ok.

So, since half the carriers are moving left. So, if we take the x equal to let us start with x equal to minus L , let us start with this boundary the leftmost boundary. So, if we get a count of how many carriers are moving in there on towards the right side that is towards x equal to 0, it is half the number ok. So, we will say that n by 2 carriers are moving towards the right and they all have a velocity of v_{th} , therefore the effective flux or the number of carriers per unit area that are per unit area that are moving in a given time interval from left to right is n into n at x equal to minus L into v_{th} by 2 ok, so that is the approximate average number of carriers moving from this boundary to the that.

And the since they are not going to have any scattering events, all these carriers are going to cross this x equal to 0 plane right. Now, what about the number of carriers moving from the right to the left ok, so that is going to be n the carrier concentration at x equal to L into v_{th} by 2 because half of them are going to be moving towards the left. And once again everything as starts moving towards the left is going to cross the x equal to 0 plane because there are no scattering events and that was the purpose of taking the mean collision time into this definition.

So, therefore, the effective flux from left to right is basically this particular term which is this flux which will call it flux left to right, and this is flux from right to left ok. So, the flux from left to right effectively is the flux from left to right minus the flux of carriers from right to left so that is the effective number of electrons moving from left to right.

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Semiconductor Fundamentals

Charge Transport

Diffusion Current: Due to Concentration Gradient

Average rate of carriers crossing the $x=0$ plane from left (within $x=-L$) to right

$$n(x=-L)v_{th}/2 \quad \text{flux L} \rightarrow \text{R}$$

Average rate of carriers crossing the $x=0$ plane from right (within $x=L$) to left

$$n(x=L)v_{th}/2 \quad \text{flux R} \rightarrow \text{L}$$

Effective flux from left to right

$$= [n(-L) - n(L)](v_{th}/2)$$

$$= \left[\left(n(0) - \frac{dn}{dx}L \right) - \left(n(0) + \frac{dn}{dx}L \right) \right] \frac{v_{th}}{2} = -\frac{dn}{dx}Lv_{th} = -\frac{dn}{dx}v_{th}^2 t_{col}$$

$\frac{kT}{2} = \frac{1}{2} m_0^* v^2$
 $v_{th} = \sqrt{\frac{kT}{m_0^*}}$

And that is given by n at minus L minus n of L into v_{th} by 2.

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Semiconductor Fundamentals

Charge Transport

Diffusion Current: Due to Concentration Gradient

Average rate of carriers crossing the $x=0$ plane from left (within $x=-L$) to right

$$n(x=-L)v_{th}/2 \quad \text{flux L} \rightarrow \text{R}$$

Average rate of carriers crossing the $x=0$ plane from right (within $x=L$) to left

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$\frac{kT}{2} = \frac{1}{2} m_0^* v^2$
 $v_{th} = \sqrt{\frac{kT}{m_0^*}}$

Now, if we know the carrier count at n at x equal to 0, so let us say so this is the carrier gradient right and these are two boundaries and that is your x equal to 0 plane. Now, if we know n at x equal to 0 which we call n of 0, the value of the carrier concentration at n at of n the carrier concentration n of L is simply n of x equal to 0 plus the slope into this distance L right. So, it is n of x equal to 0 by $d n$ by $d x$ into L or if you want you can think of it is a Taylor expansion.

Similarly, the carrier concentration here n of minus L is n at 0 minus $d n$ by $d x$ into L ok, because the way we have drawn at $d n$ by $d x$ is positive and therefore, these equations fit the physical picture. So, this is the carrier concentration at minus L n l written in terms of the carrier concentration at x equal to 0 . If you simply add these up these two terms disappear and you end up with this particular term defining the effective flux from left to right. And we will now replace this L with v th into t collision to get you this relation. So, this relation tells us that there is an effective flux from left to right, which is dependent on $d n$ by $d x$ and the square of the thermal velocity which is a constant and t the mean collision time which is also a constant, therefore, the driving force is $d n$ by $d x$.

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Semiconductor Fundamentals

Charge Transport

Effective current
(electrons flux from left to right is -ve \Rightarrow current is positive)

$$J_{q,n} = q \frac{dn}{dx} v_{th}^2 t_{col} = q \frac{dn}{dx} \frac{kT}{m^*} t_{col} = q \left(\frac{kT}{q} \right) \frac{dn}{dx}$$

$$= q D_n \frac{dn}{dx}$$

D_n = Diffusion coefficient for electrons

Similarly, for a similar profile of holes

$$J_{q,p} = -q D_p \frac{dp}{dx}$$

D_p = Diffusion coefficient for holes

Handwritten notes:

- $q \frac{dn}{dx} v_{th}^2 t_{col}$
- $\frac{1}{2} m^* v_{th}^2 = \frac{kT}{2}$
- $v_{th}^2 = \frac{kT}{m^*}$
- $-1.6 \times 10^{-19} C = -q$

Now, to understand this better, so that is the effective flux right, so that is the flux, what is the total that is a rate of change that is that is number of electrons moving from left to right to unit time because you taken into account v th, so it is a flux per unit time. So, the total current simply the charge transiting per unit time which is q times that flux ok. So, it is simply q times $d n$ by $d x$ into v th square into t collision.

Now, what is v th square, we had already defined v th square as since it is a since it is a or it is a 1D case your half mean cool $m e v$ th square was equal to $k T$ by 2 or v th square is equal to $k T$ by m , where the m is the effective mass of the electron or m star, where m star is effective mass of electron. So, we will replace v th square by this term k

T by m star. And therefore, we have the total current due to the concentration gradient from left to right being equal to this particular term.

Now, note what happened to the negative sign, the negative sign, so if you look at the previous page there was a negative sign here that was to do with the effective flux, because there are few electrons here and more electrons there, therefore it is understandable that the effective flux was in this direction. But then the electron charge is also negative q, it is minus 1.6 e minus 19 coulombs which is equal to minus q ok. So, we have removed the negative sign by having a minus q into this negative flux which is basically telling you that the current is positive. So, you have all the electrons drifting this way which means that there is a positive current going on from left to right.

Now, to this term we will add in we will try to replace these two terms of m and t collision by the parameter mu. So, if we remember you know how does mu connect to t collision and the effective mass of the electron ok.

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Semiconductor Fundamentals

Charge Transport

Effective current
(electrons flux from left to right is -ve => current is positive)

$$J_{q,n} = q \frac{dn}{dx} v_{d,eff} = q \frac{dn}{dx} \frac{kT}{m^*} \tau_{eff} = q \left(\frac{kT}{q} \right) \frac{dn}{dx}$$

$$= q D_n \frac{dn}{dx}$$

D_n = Diffusion coefficient for electrons

Similarly, for a similar profile of holes

$$J_{q,p} = -q D_p \frac{dp}{dx}$$

D_p = Diffusion coefficient for holes

Handwritten notes:

- $q \frac{dn}{dx} v_{th}^2 \tau_{col}$
- $\frac{1}{2} m^* v_{th}^2 = \frac{kT}{2}$
- $v_d = \mu E$

So, if we just step back ok, and look at what is mu, mu is nothing but the relay it is the constant the proportionality for the drift velocity and the electric field. So, the drift velocity is mu times the electric field right. So, all the way discussing diffusion we need to step back and understand what mu is an how it is connected to m star and t collision. So, it does not mean that this process is dependent on an applied electric field, there is no

need for an applied electric field, it is only to understand knew better that we need to sort of step back.

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Semiconductor Fundamentals

Charge Transport

Effective current
(electrons flux from left to right is -ve => current is positive)

$$J_{q,n} = q \frac{dn}{dx} v_{drift} = q \frac{dn}{dx} \frac{kT}{m} \tau_{col} = q \left(\frac{kT}{m} \tau_{col} \right) \frac{dn}{dx}$$

$$= q D_n \frac{dn}{dx}$$

D_n = Diffusion coefficient for electrons

Similarly, for a similar profile of holes

$$J_{q,p} = -q D_p \frac{dp}{dx}$$

D_p = Diffusion coefficient for holes

Handwritten red notes:

- $q \frac{dn}{dx} v_n^2 t_{col}$
- $q \tau = \frac{m^* v_d}{t_{col}}$
- $v_d = \frac{q t_{col}}{m^*}$
- $\frac{t_{col}}{m^*} = \frac{\mu}{q}$

And when if you apply an electric field the total force experienced by the electron is this which you can say is the rate of change of momentum of the electron. So, it is m^* into v_{drift} by $t_{collision}$ and therefore, v_{drift} was $q \tau_{collision}$ by m^* into electric field where this term is the mobility ok, so that is the definition of μ . So, if I were to apply a field to the semiconductor the carriers would move with this drift velocity, but right now we are not applying a field, but nevertheless this relation still holds in the sense that $t_{collision}$ by m^* is nothing but μ time μ divided by q ok, so that is what we are going to use that relation and replace this ratio of $t_{collision}$ by m^* with μ by q . So, we replace $t_{collision}$ by m^* with relation μ by q in order to get this particular expression.

Now, this term in bracket has got a very special significance and it is called the diffusion coefficient and it is denoted by the symbol capital D . And since we are talking about electrons here this is the diffusion coefficient of electrons ok we will call it D_n . So, you could also call that μ_n . So, the so therefore, the diffusion current can be written as $q D_n \frac{dn}{dx}$ ok. So, this is D_n is nothing but the capital D subscript n , which is that diffusion coefficient for electrons. Similarly, we could define a diffusion coefficient for holes and therefore, define a diffusion current for holes.

Now, if I were to have a similar profile if I had to have a picture of P, and if I have a flux if I have a concentration gradient you will find that all the holes are moving from right to left because that is because they have got a higher concentration on the right and lower concentration on the left. And therefore, the current from left to right will be negative ok. Now the now the electron current the negative sign disappeared because the charge was negative, but in the case of holes you have a positive charge, and therefore the if the flux is in this way the current from left to right is a negative current which is essentially telling you that the current is in the same direction as the migration of holes.

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Semiconductor Fundamentals

Charge Transport

Effective current
(electrons flux from left to right is -ve \Rightarrow current is positive)

$$J_{eff,n} = q \frac{dn}{dx} v_{drift} = q \frac{dn}{dx} \frac{kT}{m} \tau_{col} = q \left(\frac{kT \mu}{q} \right) \frac{dn}{dx}$$

$$= q D_n \frac{dn}{dx}$$

D_n = Diffusion coefficient for electrons

Similarly, for a similar profile of holes

$$J_{eff,p} = -q D_p \frac{dp}{dx}$$

D_p = Diffusion coefficient for holes

Handwritten notes:

- $q \frac{dn}{dx} v_{th}^2 \tau_{col}$
- $q \tau_{col} = \frac{m^* v_{th}^2}{kT}$
- $D_n = \frac{q k T \mu}{q}$
- Arrows pointing to $J_{df,n}$ and D_n
- Arrows pointing to $J_{df,p}$ and D_p

So, this is the diffusion current density for electrons. And we define the diffusion current density as $J_{df,n}$ for electrons and $J_{df,p}$ for holes. And D_n is the diffusion coefficient for electrons, and D_p is the diffusion coefficient for holes. And we will and it is useful to note this relation and we can sort of derive this, and I will show that to you in a little while. So, D_n is $k T \mu$ by q ok. So, this relation is something called as the Einstein's relation. In order to understand how Einstein's relation comes about, all you need to do is take any situation it is not just limited to carriers in a semi conductor in any situation where the carrier count follows a Boltzmann distribution.

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Semiconductor Fundamentals

Charge Transport

Effective current
(electrons flux from left to right is -ve => current is positive)

$$J_{q,n} = q \frac{dn}{dx} v_{drift} = q \frac{dn}{dx} \frac{kT}{m} \tau_{col} = q \left(\frac{kT}{q} \right) \frac{dn}{dx}$$


$$= q D_n \frac{dn}{dx}$$

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Similarly, for a similar profile of holes

$$J_{q,p} = -q D_p \frac{dp}{dx}$$

D_p = Diffusion coefficient for holes

potential
 $-\phi/\phi_0$
 $n = n_0 e^{-\phi/\phi_0}$

 $q n \mu \epsilon = q D_n \frac{dn}{dx}$
 $n = n_0 e^{-\phi/\phi_0}$
 $\frac{dn}{dx} = (n_0 e^{-\phi/\phi_0}) \left(-\frac{d\phi}{dx} \right)$
 $\epsilon = -d\phi/dx$

In the sense that n scales is proportional to e to the power minus a potential, so let us say $q \phi$ by $k t$ or let us say it is a minus ϕ by ϕ naught. So, if you we have a Boltzmann slight distribution where n say some n naught e to the power minus ϕ by ϕ naught, where ϕ is a potential you will end up with the Einstein relation being valid just to derive it. So, if you take if we take on charge carriers itself or carriers the electrons and holes in a semiconductor.

The when left alone at thermal equilibrium, you need to have if there exists any drift current it must be completely balanced out by the diffusion current ok. So, at thermal equilibrium, you cannot have currents. And if there is a drift current it must be balance start by diffusion current. And using that as a starting point you need to have your q , let us only take electrons is an example let us say electrons the only carriers. So, you need to have $q n \mu$ electric field matching your diffusion current which is $q D n \frac{dn}{dx}$. Now, if n follows a Boltzmann relation say n is equal to some n naught e to the power minus ϕ by let us say instead of thermal voltage we will just call it some ϕ naught which is a constant your $\frac{dn}{dx}$ is equal to n naught e to the power minus ϕ by ϕ naught all right into minus $d\phi$ by dx ok. And your electric field is equal to minus $d\phi$ by dx . So, you substitute these two in this particular expression here.

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Semiconductor Fundamentals

Charge Transport

Effective current
(electrons flux from left to right is -ve => current is positive)

$$J_{q,n} = q \frac{dn}{dx} v_{drift} = q \frac{dn}{dx} \frac{kT}{m} \tau_{col} = q \left(\frac{kT \mu}{q} \right) \frac{dn}{dx}$$

$$= q D_n \frac{dn}{dx}$$

D_n = Diffusion coefficient for electrons

Similarly, for a similar profile of holes

$$J_{q,p} = -q D_p \frac{dp}{dx}$$

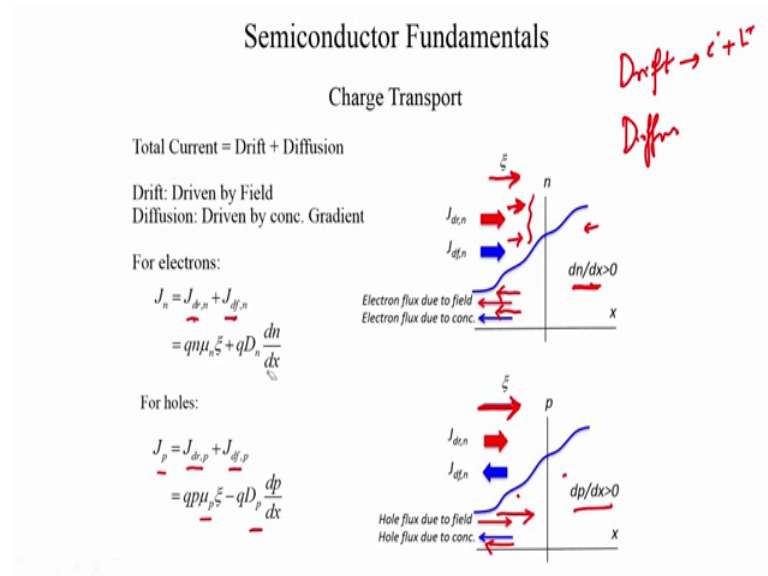
D_p = Diffusion coefficient for holes

Handwritten notes:

potential \downarrow
 $-\phi/\phi_0$
 $n = n_0 e^{-\phi/\phi_0}$
 $q \mu \frac{d\phi}{dx} = q \frac{D_n}{\phi_0} \frac{d\phi}{dx}$
 $D_n = \mu \phi_0 \cdot \frac{kT}{q}$
 $\boxed{D_n = \frac{\mu kT}{q}}$
 $q n \mu \phi_0 =$
 $n = n_0 e^{-\phi/\phi_0}$
 $\frac{dn}{dx} = (n_0 e^{-\phi/\phi_0}) \cdot \frac{d\phi}{dx} \cdot \frac{1}{\phi_0}$
 $\left\{ \begin{array}{l} \frac{dn}{dx} \\ \phi_0 = -d\phi/dx \end{array} \right.$

And you will end up with relation $q n \mu d\phi$ by dx ok, let us get it to the negative sign is equal to $q D n n_0 e^{-\phi/\phi_0}$ to the power minus ϕ by ϕ_0 is again n , so $n d\phi$ by dx . So, the term n and $d\phi$ by dx cancel out ok, and sorry I missed out a 1 by ϕ_0 term here ok and so you take into account the 1 by ϕ_0 ok. So, you end up with $D n$ being equal to let us just bring that n right here being equal to μ into ϕ_0 . Now, a ϕ_0 is a thermal voltage which is kT by q , we will end up with your $D n \mu kT$ by q which is your Einstein's relation, so that is the way you know this relation comes about ok. So, the diffusion coefficient is very much connected to the mobility of carriers.

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So, now let us look at the total current ok. So, we have identified the two mechanisms you have drift and you have diffusion. In the case of drift, we found that the electron and hole currents add up ok, the electron currents and the hole currents add up to give you the total current, but in the case of diffusion for the same profile the effective current was in the opposite direction. So, if we will if we want to define the total electron current ok, it is the current due to drift plus diffusion of electrons the current due to so let us take this profile here ok.

So, you have $dn/dx > 0$ and let us say we apply an electric field in this direction ok. So, all the electrons will move from right to left because of the applied electric field ok, and all the electrons will move from right to left because of the concentration gradient, so which means that the drift current where the driving force is the electric field and the diffusion current where the driving force or the concentration gradient both exist in the same direction and they both contribute to the curve. So, J_{drift} plus $J_{diffusion}$ for the electrons is going to be the summation of $qn\mu_n$ the electric field plus $qD_n dn/dx$ to give you the total electron current density. But for holes if you have the same situation you have $dp/dx > 0$, you have the electric field which is the driving force for drift pointing in the same direction.

But what happens is that the whole flux due to the concentration gradient is towards the left, because there are more holes here as compared to here. And the whole flux due to

the field is going to be along the direction of the field. So, it is going to move from right to left and therefore, these two are against each other. And therefore, the total hole current is going to be the total drift current due to holes plus the diffusion current due to holes, but the diffusion current has got a negative sign. And therefore, you have $q p \mu_p$ into electric field minus $q D_p \frac{dp}{dx}$ with the total effective current due to holes, so that is a quick summary of the different current mechanisms.