Semiconductor Devices and Circuits
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Lecture - 15
Recombination and Generation – Continued

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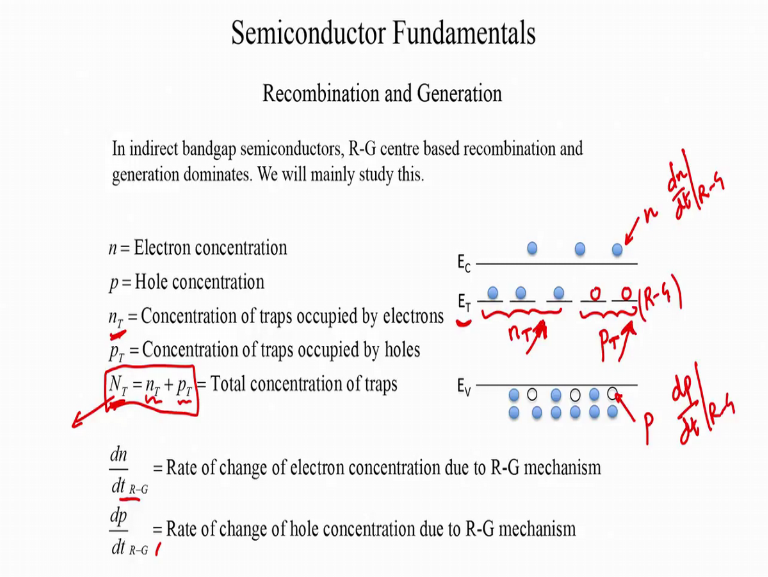

Semiconductor Fundamentals

Recombination and Generation

In indirect bandgap semiconductors, R-G centre based recombination and generation dominates. We will mainly study this.

n = Electron concentration

p = Hole concentration

n_T = Concentration of traps occupied by electrons

p_T = Concentration of traps occupied by holes

$\boxed{N_T = n_T + p_T}$ = Total concentration of traps

$\frac{dn}{dt}_{R\text{-}G}$ = Rate of change of electron concentration due to R-G mechanism

$\frac{dp}{dt}_{R\text{-}G}$ = Rate of change of hole concentration due to R-G mechanism

In order to establish the analysis, we will set up a few definitions ok. So, we are interested in R G center based recombination generation mechanisms. So, the first definition is let us say that the electron concentration, the free electron concentration is n and the hole concentration is p and therefore, d n by d t due to R G mechanisms is given by the time derivative here and the rate of change of the hole concentration due to R G mechanisms alone is given by the rate term here, the now we have we are only looking at R G center based recombination generation.

So, let us say at energy level E T we have a we have a certain density of states, density of trap states and let us say that the total number of trap states per unit volume is given by capital N T. So, that is a total number of trap states per unit volume that is the trap concentration. Now, of this capital N T some of the traps are occupied by electrons and some of the traps are empty. The traps that are occupied by electrons, we will say that it has got a concentration of small n T ok.

So, this, these are traps that are occupied by electrons. It is a number of traps per unit volume located an E T that are occupied by electrons and the number of empty states or you can say that the empty traps or traps that are occupied by holes are P T, ok.

So, there are P T traps per unit volume that are empty and since n T and p T have to sum to capital N T we have a fixed condition here, where in capital N T is a constant while whereas, n T and p T are more dynamic ok. So, small n T and p T, the small these counts could change with time ok. So, for example, if you are not in steady state these concentrations could change in time, but the sum of these two cannot change time capital N T is fixed ok. So, these are our little definitions and you will use these definitions to define the dynamics of R G mechanisms.

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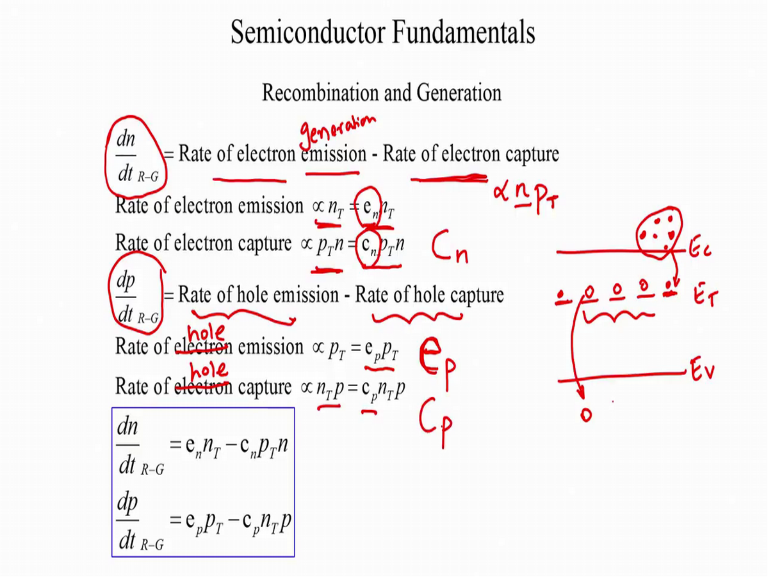

Semiconductor Fundamentals

Recombination and Generation

$\frac{dn}{dt}_{R-G}$ = Rate of electron emission - Rate of electron capture

generation

$\propto n p_T$

Rate of electron emission $\propto n_T = \mathrm{e}_n n_T$

Rate of electron capture $\propto p_T n = \mathrm{c}_n p_T n$

C_n

$\frac{dp}{dt}_{R-G}$ = Rate of hole emission - Rate of hole capture

Rate of ~~electron~~ hole emission $\propto p_T = \mathrm{e}_p p_T$

e_p

Rate of ~~electron~~ hole capture $\propto n_T p = \mathrm{c}_p n_T p$

C_p

$$\frac{dn}{dt}_{R-G} = \mathrm{e}_n n_T - \mathrm{c}_n p_T n$$

$$\frac{dp}{dt}_{R-G} = \mathrm{e}_p p_T - \mathrm{c}_p n_T p$$

So, what is the rate of change of electrons due to R G processes? Any change would depend upon the rate at which electrons are emitted minus the rate at which the electrons are captured. So, you have, you have your conduction band edge, you have your valence band edge and you have some trap states here. Some of these trap states are occupied by electrons and you have some free electrons there.

Now, what is the rate at which electrons are emitted. The rate at which electrons are emitted, the emission can only happen from the trap states ok. We are not talking about band to band generation here. So, emission implies generation. So, this by this we actually implied generation, and use the word emitted because these electrons are going to be generated from the trap states which is equivalent to saying that the electrons are emitted from the trap states.

So, what is the rate at which the electrons are emitted, they depend upon the number of electrons present in the trap states. If none of the trap states have any electrons the rate at which electrons are emitted is 0 because there are no electrons to be emitted ok.

Therefore, it is only when the electrons are present in these trap states that an emission is possible and therefore, the rate at which electrons are emitted is proportional to n T because n T is the concentration of trap states that are filled with electrons. And you will define a proportionality constant called e n which is to say the emission of n type or the emission of electrons and say that the rate of electron emission is equal to e n into n T.

Now, what is the rate at which electrons are captured? The electron capture involves a process wherein the electron that is sitting in the conduction band and which was free is now going to be captured by a trap state ok. So, there is an empty state here and this free electron is going to drop down and it is going to occupy this empty state. So, that is the capture of an electron. So, what does that mean, what does that depend upon. Firstly, it has to depend upon the number of electrons or the concentration of electrons in the conduction band. If there are no electrons in the conduction band then the rate of electron capture 0 because there are no electrons to be captured.

So, definitely the rate of electron capture has to be proportional to n. So, let us say there are electrons to be captured. What else is it proportional to? These electrons can be captured and for them to capture, they need to find an empty state ok. If there are no empty states at E T, they the electron cannot be captured because there is no state for the electron to occupy. So, remember the states are nothing, but solutions to Schrodinger’s equation.

So, you need to have 2 conditions, you need to have electrons that are there to be captured and you need to have empty states for these captured electrons to occupy. So, it is only when you have these 2 together that you can have a rate of electron capture. So, therefore, the rate of electron captures proportional to n and it is also proportional to the concentration of empty states and therefore, it is proportional to P T into n where P T is the concentration of empty states or the number of empty states per unit volume are located at E T.

So, rate of an electron capture can be defined to be equal to C n P T n where this term C n is a proportionality constant which relates to capture of electrons, ok. So, we have defined two constants of proportionality e n and c n. So, now, let us talk about the holes. Just like in the case of electrons, what is the rate of change of the whole population? The whole population depends upon the rate at which the elect the holes are emitted from the trap, trap states and the rate at which the holes are captured into the trap states.

Just like in the case of electrons, so, all the empty states are essentially holes right. So, just like in the case of electrons, the rate of hole emission. So, I am sorry this is a type on this should be holes, the rate of hole emission, has to be proportional to P T because if there are no holes in these states (Refer Time: 0:8:12) all the states are filled with electrons there can be no holes emitted. So, it is only when you have holes located in these states that you can have an emission of a hole to the valence band.

So, therefore, the rate of hole emission is equal to e p into p T where e p is again another proportionality constant relating to the emission of holes. And the rate of hole capture by the same arguments be used for electrons is proportional to n T into p with c p being a constant proportionality which is the capture of holes. So, we now have 4 rates, the rate at which electrons are generated and the rates rate at which electrons are captured, the rate at which holes are emitted and the rate at which holes are captured.

So, based on that we can set up 2 need differential equations ok, one pertaining to the electrons and one pertaining to the holes and this basically tells you that the rate at which the electron population changes because of R G mechanisms.

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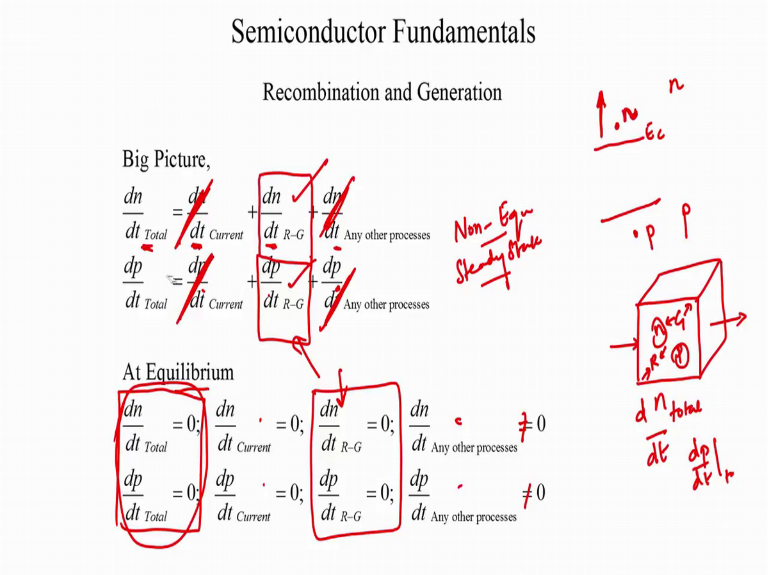

Semiconductor Fundamentals

Recombination and Generation

Big Picture,

$$\frac{dn}{dt}\Big|_{Total} = \cancel{\frac{dn}{dt}\Big|_{Current}} + \frac{dn}{dt}\Big|_{R-G} + \cancel{\frac{dn}{dt}\Big|_{\text{Any other processes}}}$$

$$\frac{dp}{dt}\Big|_{Total} = \cancel{\frac{dp}{dt}\Big|_{Current}} + \frac{dp}{dt}\Big|_{R-G} + \cancel{\frac{dp}{dt}\Big|_{\text{Any other processes}}}$$

At Equilibrium

$$\frac{dn}{dt}\Big|_{Total} = 0; \quad \frac{dn}{dt}\Big|_{Current} = 0; \quad \frac{dn}{dt}\Big|_{R-G} = 0; \quad \frac{dn}{dt}\Big|_{\text{Any other processes}} \neq 0$$

$$\frac{dp}{dt}\Big|_{Total} = 0; \quad \frac{dp}{dt}\Big|_{Current} = 0; \quad \frac{dp}{dt}\Big|_{R-G} = 0; \quad \frac{dp}{dt}\Big|_{\text{Any other processes}} \neq 0$$

So, remember we are not talking about the total. If you go back a few slides, this is the total change of the electron population in any given volume. We are only talking about the R G mechanisms. So, the rate at which the electron and hole count changes because of R G mechanisms is the rate at which electrons are emitted minus rate at which electrons are captured and the rate at which holes are affected due to R G processes is due to the rate at which is dependent upon the rate at which holes are emitted minus rate at which holes are captured. So, this is our starting point for any analysis.

So, at non equilibrium so we are now out of equilibrium because we are having a lot of dynamics and there are there is generation and recombination mechanisms going on.

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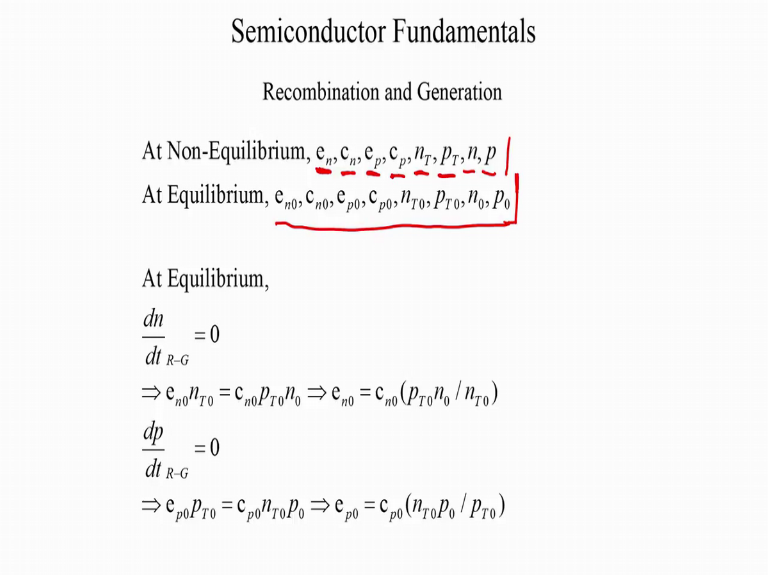

Semiconductor Fundamentals

Recombination and Generation

At Non-Equilibrium, $e_n, c_n, e_p, c_p, n_T, p_T, n, p$

At Equilibrium, $e_{n0}, c_{n0}, e_{p0}, c_{p0}, n_{T0}, p_{T0}, n_0, p_0$

At Equilibrium,

$$\frac{dn}{dt}\Big|_{R-G} = 0$$

$$\Rightarrow e_{n0} n_{T0} = c_{n0} p_{T0} n_0 \Rightarrow e_{n0} = c_{n0}(p_{T0} n_0 / n_{T0})$$

$$\frac{dp}{dt}\Big|_{R-G} = 0$$

$$\Rightarrow e_{p0} p_{T0} = c_{p0} n_{T0} p_0 \Rightarrow e_{p0} = c_{p0}(n_{T0} p_0 / p_{T0})$$

At non equilibrium we have been using these variables, e n for the proportionality constant determining the emission of electrons, c n for the capture of electrons, e p for the emission of holes, c p for the capture of holes, n T which is for the number of occupied trap states. P T for the number of unoccupied trap states and all per unit volume and for the free electron concentration and P for the hole concentration.

Now, this is at non equilibrium. Now at equilibrium denote these variables with a special symbol and we want to say that at equilibrium these values have got a special value which is given by the subscript O attached to all these parameters. So, at equilibrium all these parameters take these values and this is a special condition because everything is at equilibrium.

Now, why do we want to consider equilibrium condition because we can make a lot of calculations at equilibrium conditions, we can easily calculate electron and hole counts at equilibrium and we want to take advantage of that to see what happens when we push the material slightly out of equilibrium and into the non equilibrium condition.

So, at equilibrium you have d n by d t R G has to be equal to 0 because everything is balanced out. So, which means that if you go back to these 2 equations and you rename or you know change these variables to their equilibrium values. So, e n becomes e n o, n T becomes n T o minus c n o p T o n o that is this equation at equilibrium.

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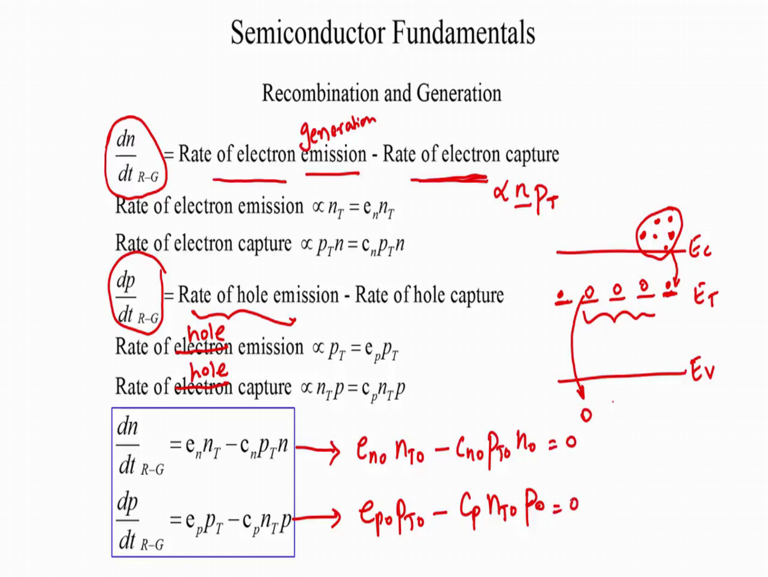

Semiconductor Fundamentals

Recombination and Generation

$\frac{dn}{dt}\Big|_{R-G}$ = Rate of electron emission - Rate of electron capture

generation

$\propto \underline{n} p_T$

Rate of electron emission $\propto n_T = e_n n_T$

Rate of electron capture $\propto p_T n = c_n p_T n$

$\frac{dp}{dt}\Big|_{R-G}$ = Rate of hole emission - Rate of hole capture

Rate of ~~electron~~ hole emission $\propto p_T = e_p p_T$

Rate of ~~electron~~ hole capture $\propto n_T p = c_p n_T p$

$$\boxed{\begin{aligned} \frac{dn}{dt}\Big|_{R-G} &= e_n n_T - c_n p_T n \\ \frac{dp}{dt}\Big|_{R-G} &= e_p p_T - c_p n_T p \end{aligned}}$$

$\longrightarrow e_{no} n_{To} - c_{no} p_{To} n_o = 0$

$\longrightarrow e_{po} p_{To} - c_p n_{To} p_o = 0$

And this equation becomes e p o P T o minus C p n T o P o. So, if you were to consider the equilibrium condition we know that these rates have to be 0 and using that we can develop a relation between e n o and C n o. So, were trying to get rid of this variable e n o we will not just represent all our equations with C n o and C p o.

So, for electrons your e n o is nothing but this term here and your e p o is nothing but this term here. So, this is all at equilibrium.

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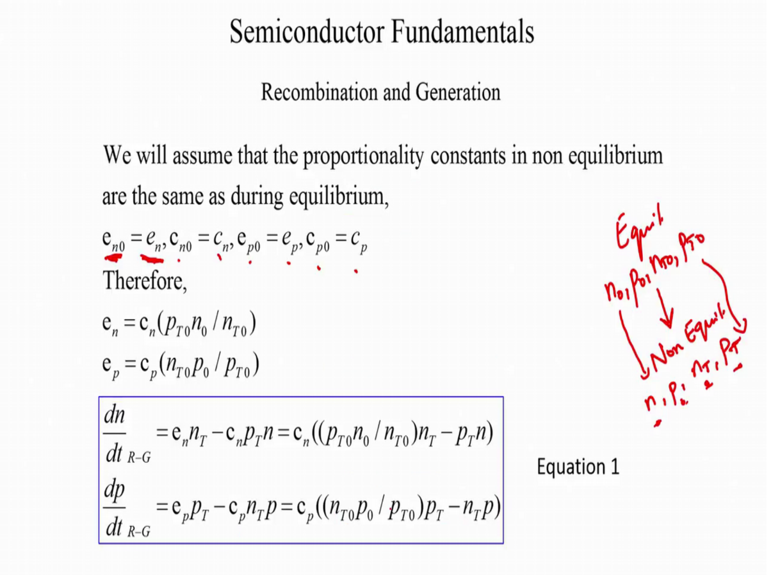

Semiconductor Fundamentals

Recombination and Generation

We will assume that the proportionality constants in non equilibrium are the same as during equilibrium,

$e_{n0} = e_n, c_{n0} = c_n, e_{p0} = e_p, c_{p0} = c_p$

Therefore,

$e_n = c_n(p_{T0}n_0 / n_{T0})$

$e_p = c_p(n_{T0}p_0 / p_{T0})$

$$\boxed{\begin{aligned} \left.\frac{dn}{dt}\right|_{R\text{-}G} &= e_n n_T - c_n p_T n = c_n((p_{T0}n_0 / n_{T0})n_T - p_T n) \\ \left.\frac{dp}{dt}\right|_{R\text{-}G} &= e_p p_T - c_p n_T p = c_p((n_{T0}p_0 / p_{T0})p_T - n_T p) \end{aligned}}$$

Equation 1

Now we make a small assumption. We will say that as we move the semiconductor, so, this is my equilibrium condition. So, I had all these parameters n, p, n T, p T this is my sorry we had a this is my equilibrium condition where we had the parameters n o, P o, n T o and P T o and we moved this semiconductors very gently out of equilibrium to the non equilibrium condition and here my parameters became n p n T and P T.

Now, we will say that, yes these parameters, these 4 parameters are different but the rates or the proportionality constants are not changing too much. So, we are going to make their assumption ok. So, we will say that and in the non equilibrium condition, the proportionality constants that relates these rates have not changed too much, ok.

So, therefore, e n is the same as e n o, C n is the same as C n o, e p is the same as e p o and c p is a same as c p o, but although the proportionality constants are being assumed to be the same, we are not assuming that n p n T and p T are not have not changed because a that is a bad assumption because then there is no difference between non equilibrium and equilibrium condition.

So, yes n, p, n T and p T have changed from their equilibrium positions have changed from their equilibrium values but the rates, the proportionality rates the dictate electron hole capture an electron and hole emission have not changed. Therefore, if you make that assumption based on this connection or this relation between the emission rates and the capture rates they can say that e n can be written as C n times this little parameter here and e p can be written as C p times this parameter here ok. So, essentially what we have done is say that e n is equal to e n o and C n is equal to C n o.

Now, if we make this, if you take advantage of this assumption the differential equations that we had for electron and hole change for R through R G mechanisms is now given by these 2 terms, which is now simplified to writing everything in terms of the capture rates. So, we write e n in terms of c n and e p in terms of c p and say that these differential equations are now essentially the same as these 2 terms here, and we call this very important equation, we give it a label and call it equation 1.

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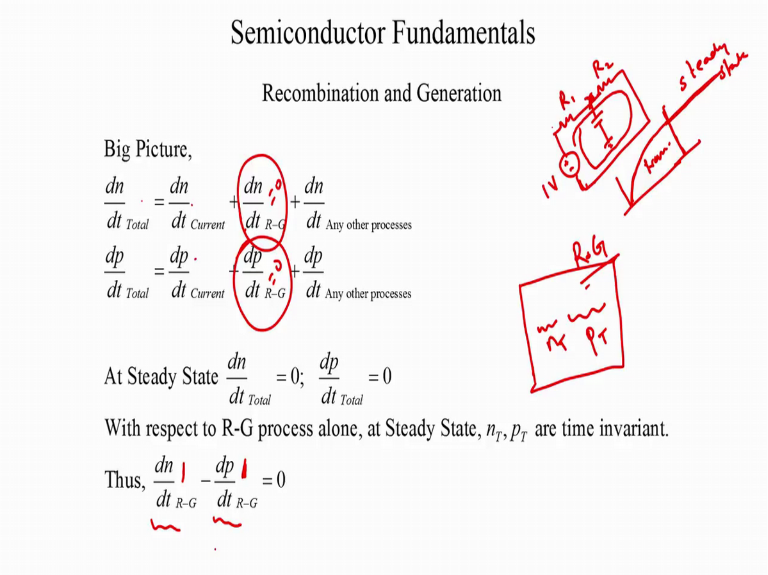

Semiconductor Fundamentals

Recombination and Generation

Big Picture,

$$\frac{dn}{dt}\Big|_{Total} = \frac{dn}{dt}\Big|_{Current} + \frac{dn}{dt}\Big|_{R-G} + \frac{dn}{dt}\Big|_{Any\ other\ processes}$$

$$\frac{dp}{dt}\Big|_{Total} = \frac{dp}{dt}\Big|_{Current} + \frac{dp}{dt}\Big|_{R-G} + \frac{dp}{dt}\Big|_{Any\ other\ processes}$$

At Steady State $\frac{dn}{dt}\Big|_{Total} = 0; \quad \frac{dp}{dt}\Big|_{Total} = 0$

With respect to R-G process alone, at Steady State, n_T, p_T are time invariant.

Thus, $\frac{dn}{dt}\Big|_{R-G} - \frac{dp}{dt}\Big|_{R-G} = 0$

So, now let us just go back to the big picture ok. So, this is the total change in electron and hole count in our volume of interest and this is the change in electron hole count due to R G mechanisms and that is what we have been discussing so far. Now, at equilibrium these parameters were both equal to 0 but at steady state, these 2 need not be at, at sorry, at non equilibrium these 2 parameters need not be equal to 0.

However, at steady state condition, so, we are now out of equilibrium but we have established a steady state condition. So, what does that mean? So, let me just give you an example in terms of electrical circuits. So, let us say you have a, say a resistor capacitor circuit, say something of something like this and you have a voltage source that is driving a current through this resistor and capacitor.

Equilibrium condition means that this voltage source is 0 which means there is no voltage there is no current through the circuits everything is balanced out. In fact, there are there is a lot of movement of electrons inside your resistor, you can see all the electrons moving in both directions, but the movement the flux is so well balanced out that there are no currents in any of these elements.

So, that is the equilibrium condition. So, what steadies, what non equilibrium implies is I have now applied a voltage ok. So, let us say I have applied 1 volt, now the moment I apply 1 volt because there is a certain time constant to the circuit it is going to take some time for all these voltages at all these nodes to establish. So, you will see some dynamics there ok.

So, that is the non equilibrium transient state ok, everything is moving with time. So, all these voltages are all shifting with time, they are all changing, the currents are all changing with time. And then after some time these transients will all die out and the system will achieve something called a steady state. So, these voltages would now have settled or there would be very little variation.

In fact, ideally this busy there would be let us say 0 variation in steady state ok. So, in steady state these transients would all die out and you will have no variation and you will establish a fixed current, a time invariant current through the circuit and so on so forth, ok. So, this condition is called as being a non equilibrium but being in steady state and whereas, this condition is called being a non equilibrium, but in transient ok. So, that is the, that is the best analogy I can give to this (Refer Time: 20:16) to what we are looking at.

So, in non equilibrium these rates need not be 0, they are not 0 and in the non equilibrium transient condition all these parameters need not be 0 but at steady state and non equilibrium, these total effective variations in n and p have to be equal to 0 whereas, these individual, you know rates need not be 0 ok. So, now, let us consider the case where we are at non equilibrium, we are looking at R G mechanisms, we are at non equilibrium and we are at steady state ok.

So, which means that these parameters are not 0 but there should be something within the perspective of the R G process that has achieved steady state condition and that is basically your count of say the trap states that are occupied or the count of the trap states that are unoccupied. So, these could be assumed to be fixed then time in variant at steady state within the perspective, within the study of your R G mechanisms.

So, therefore, with the R G processes alone, we can say that both d n by d t and d p by d t are not 0 individually when it non equilibrium steady state but due to the fact that it is steady state this is balanced out by any changes in the whole concentration and therefore, d n by d t is equal to d p by d t in non equilibrium steady state condition.

So, we will now take this general equation which we have labeled as equation 1 and try to look at these steady state implications on this equation and the steady state implications are that this rate must be equal to this rate.

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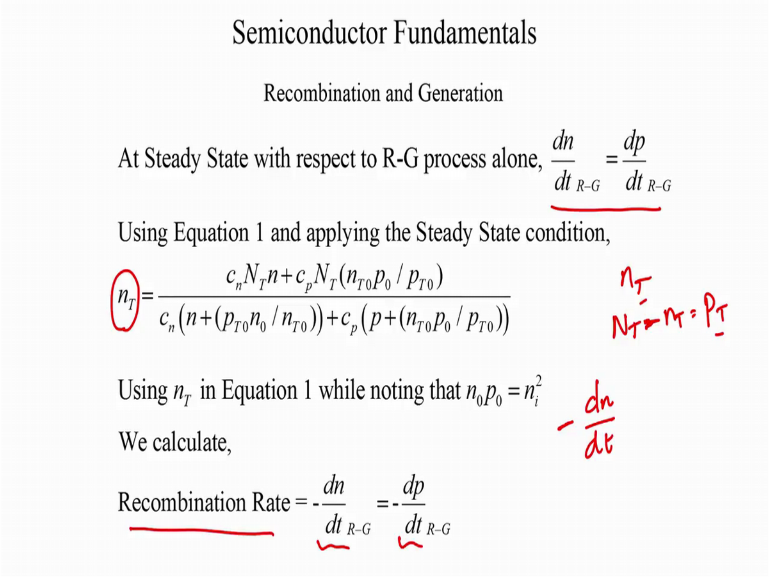

Semiconductor Fundamentals

Recombination and Generation

At Steady State with respect to R-G process alone, $\underline{\frac{dn}{dt}_{R-G} = \frac{dp}{dt}_{R-G}}$

Using Equation 1 and applying the Steady State condition,

$$\boxed{n_T} = \frac{c_n N_T n + c_p N_T (n_{T0} p_0 / p_{T0})}{c_n \left(n + (p_{T0} n_0 / n_{T0})\right) + c_p \left(p + (n_{T0} p_0 / p_{T0})\right)}$$

n_T

$N_T - n_T = p_T$

Using n_T in Equation 1 while noting that $n_0 p_0 = n_i^2$

$-\frac{dn}{dt}$

We calculate,

$\underline{\text{Recombination Rate}} = -\frac{dn}{dt}_{R-G} = -\frac{dp}{dt}_{R-G}$

So, using equation 1 and establishing this condition we can now solve the equation and estimate what n T is. So, that is the number of occupied states it is just a bit of Algebra that is wherein you equate these 2 terms. And using that value of n t we can also get P T because we know n T we can calculate p T as n T minus, capital N T minus small n T which is your small p T and using n T and p T and in plugging it into any 1 of these equations into any one of these, we can get the recombination rate which is basically your d n by d t. The rate at which we are losing electrons in the rate at which we are losing holes so we can calculate our recombination rate.

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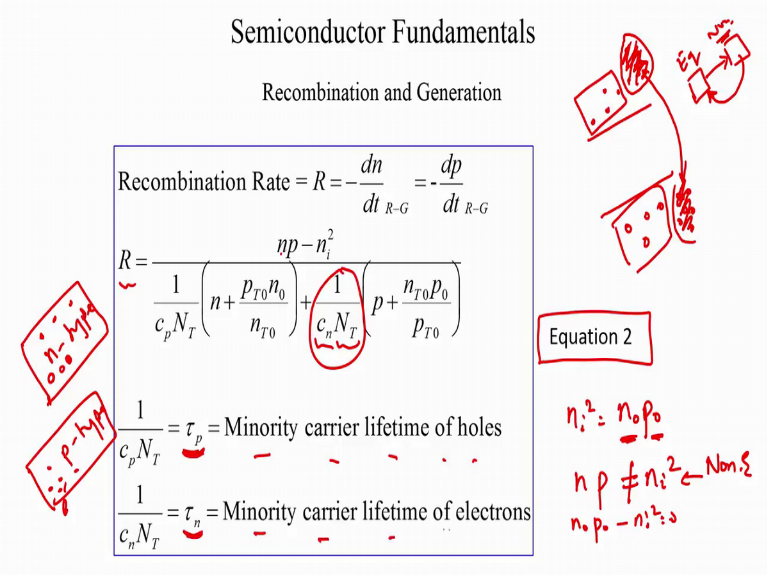

Semiconductor Fundamentals

Recombination and Generation

$$\text{Recombination Rate} = R = -\frac{dn}{dt}_{R-G} = -\frac{dp}{dt}_{R-G}$$

$$R = \frac{np - n_i^2}{\frac{1}{c_p N_T}\left(n + \frac{p_{T0} n_0}{n_{T0}}\right) + \frac{1}{c_n N_T}\left(p + \frac{n_{T0} p_0}{p_{T0}}\right)}$$

Equation 2

$$\frac{1}{c_p N_T} = \tau_p = \text{Minority carrier lifetime of holes}$$

$$\frac{1}{c_n N_T} = \tau_n = \text{Minority carrier lifetime of electrons}$$

So, in steady state we find that the recombination rate is given by this equation here and this is a very- very useful equation, we will give it a special label and call it equation 2. So, what it tells you is that the rate at which the carriers, any free carriers recombine. So, which means what, which means at equilibrium I had some carriers, I had some electrons and I had some holes now when I took it out of equilibrium in the non equilibrium condition I added some more electrons. So, these are suddenly appeared into the material and we have added some more holes.

So, this is a non equilibrium condition and with time these if the non equilibrium condition is not forced, if it is not maintained, if the semiconductor is simply taken out of non out of equal equilibrium and left alone it will try to find its way back to equilibrium, and the rate at which these carriers would recombine would be given by this rate here, ok.

So, if there are no more mechanisms to generate carriers and there is no other process to take it keep it in, non in the non equilibrium condition, the semiconductor will head back to equilibrium and it will head back to equilibrium by sort of an highlighting these carriers, these excess carriers because this is the equilibrium carrier concentration, ok.

So, in order to for the semiconductor to get back to equilibrium these carriers will have to recombine, this accessory carriers will have to recombine and that rate can be calculated by calculating this recombination rate. Although we will be looking at a more general case wherein there is no necessity for the semiconductor to stop being forced in non equilibrium. So, this is just an example what I am stating here, all right.

So, in this equation what do we see we see that n i is the intrinsic carrier concentration which is so, n i square is nothing but your n o p o because your n o was your equilibrium electron concentration p o is the equilibrium hole concentration and n o p is n i square, n and p are the non equilibrium electron and hole concentrations it is a product of the non equilibrium electron hole concentration. And n p is not equal to n i square because they are in non equilibrium condition and this rate depends upon how much we have shifted away from the mass action law.

So, if we wear an equilibrium, if we wear in equilibrium n p would have been equal to n o p o and n o p o minus n i square would have been 0, all this numerator would have been 0 and there is no need for any recombination rate. On the other hand if you are a non equilibrium n p is not equal to n i square and that difference that deviation from n i square is what is determining the recombination rate.

The next two points are, look at these two terms. So, capital N T is the total number of trap states per unit volume which is related to R G centres and is c p’s are the capture coefficients for I am sorry the proportionality constants for the capture of holes in the capture of electrons and this term 1 by c p n T has got a very special significance. It is given a symbol tau p and it is called the minority carrier lifetime of holes. What does that mean? It means that if you take a semiconductor that is doped. So, let us say a semiconductors n doped. So, it is an n type semiconductor.

So, it is an n type semiconductor, you have a large number of electrons and a small number of holes, the whole population is n i square by n in equilibrium. Now, you add some excess you take this in the semiconductor out of equilibrium and you add some excess holes inside the semiconductor ok.

Now there are many electrons and you have just injected, as you have injected some extra holes inside the semiconductor you have shift into semiconductor out of equilibrium. Now the typical time a measure of the time these holes would last in this n type semiconductor is the minority carrier lifetime of holes, why minority carrier because when n type semiconductor the holes are the minority they are the minority carriers and the electrons are in majority they are the majority carriers.

So, when I inject holes into an n type semiconductor, they would last for typically tau p seconds before they recombine. So, that is the, that is the lifetime. Similarly, if we inject electrons into a P type semiconductor. So, in a P type semiconductor holes are the majority carriers electrons are the minority carriers. If we inject electrons into a P type semiconductor then they would last for a life for a life time for about tau n seconds before they recombine.

So, that is called as the minority carrier lifetime of electrons. So, there are, so, it is important that we are not interested in the lifetime of holes in a P type material. So, that is not the minority carrier lifetime, we are interested in the lifetime of holes in an n type material and the lifetime of electrons in a P type material.

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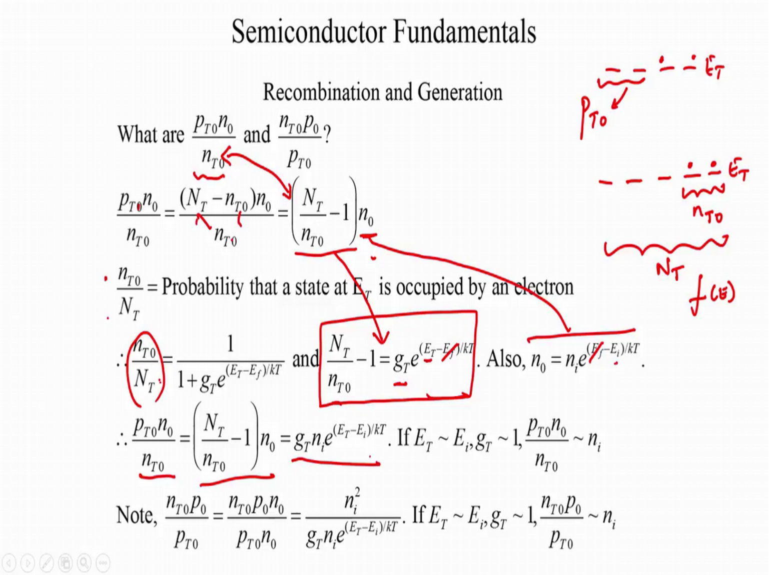

Semiconductor Fundamentals

Recombination and Generation

What are $\frac{p_{T0}n_0}{n_{T0}}$ and $\frac{n_{T0}p_0}{p_{T0}}$?

$$\frac{p_{T0}n_0}{n_{T0}} = \frac{(N_T - n_{T0})n_0}{n_{T0}} = \left(\frac{N_T}{n_{T0}} - 1\right)n_0$$

$\frac{n_{T0}}{N_T}$ = Probability that a state at E_T is occupied by an electron

$$\therefore \frac{n_{T0}}{N_T} = \frac{1}{1+g_T e^{(E_T-E_f)/kT}} \text{ and } \boxed{\frac{N_T}{n_{T0}} - 1 = g_T e^{(E_T-E_f)/kT}}. \text{ Also, } n_0 = n_i e^{(E_f-E_i)/kT}.$$

$$\therefore \frac{p_{T0}n_0}{n_{T0}} = \left(\frac{N_T}{n_{T0}} - 1\right)n_0 = g_T n_i e^{(E_T-E_i)/kT}. \text{ If } E_T \sim E_i, g_T \sim 1, \frac{p_{T0}n_0}{n_{T0}} \sim n_i$$

$$\text{Note, } \frac{n_{T0}p_0}{p_{T0}} = \frac{n_{T0}p_0n_0}{p_{T0}n_0} = \frac{n_i^2}{g_T n_i e^{(E_T-E_i)/kT}}. \text{ If } E_T \sim E_i, g_T \sim 1, \frac{n_{T0}p_0}{p_{T0}} \sim n_i$$

So, we will now replace these 2 terms with tau p and tau n, but before that let us take a look at what these 2 terms are, ok. So, what are that? What are those terms, what are p T o n o by n T o and n T o p o by p T o, ok, so will get a good gauge on this. So, p T o n o by n T o are all equilibrium values, they all got the subscript o around it but P T o is the number of trap states per unit volume that was empty ok. So, this is P T o and therefore, P T o nothing but capital N T minus n T o, it is a total number minus those that are filled and you just divide n T o here and you end up with this expression.

So, this expression is the same as that. Now let us take the reciprocal of this term which is n T o by capital N T. What is n T o by capital N T? So, you have the trap states here, you have some of them that are filled with electrons and this concentration is n T o at equilibrium and this total number is capital N.

So, what is n T o by N T, it is the probability that the states at E T are filled with electrons ok, it is a measure of that probability which basically implies that we are talking about the Fermi function, right. So, Fermi function is the probability, gives you the probability that the energy states at energy level E are occupied by electrons.

So, f of E which we discussed earlier is that probability. So, we are talking about the same case and we had looked at this condition even in the case of doping, when we looked at the number of donor sites or the dopants that are ionized as compared to the total dopant concentration. So, it is exactly the same argument.

So, n T o by N T is essentially the Fermi function with the small correction factor g T and therefore, this term which is the reciprocal of this Fermi function. So, you take the reciprocal of this which is 1 plus g T to the power E T minus E f by k T minus 1 is essentially this. So, this term here is that term within the brackets and n o is the equilibrium hole concentration which is nothing, but n i e to the power E f minus E i by k T.

So, the product of these 2 will result in the E f disappearing ok. So, P T o n o by n T o is nothing, but this term and this product is nothing but g T e to the power E T minus E i by k T. Now, if these trap locations are close to mid gap. So, we have not said anything about where E T is.

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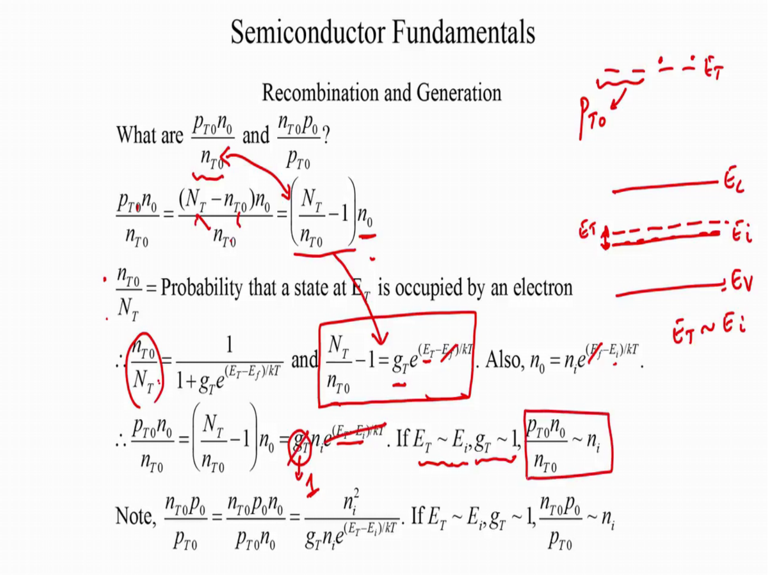

Semiconductor Fundamentals

Recombination and Generation

What are $\frac{p_{T0}n_0}{n_{T0}}$ and $\frac{n_{T0}p_0}{p_{T0}}$?

$$\frac{p_{T0}n_0}{n_{T0}} = \frac{(N_T - n_{T0})n_0}{n_{T0}} = \left(\frac{N_T}{n_{T0}} - 1\right)n_0$$

$\frac{n_{T0}}{N_T}$ = Probability that a state at E_T is occupied by an electron

$\therefore \frac{n_{T0}}{N_T} = \frac{1}{1+g_T e^{(E_T-E_f)/kT}}$ and $\frac{N_T}{n_{T0}} - 1 = g_T e^{(E_T-E_f)/kT}$. Also, $n_0 = n_i e^{(E_f-E_i)/kT}$.

$\therefore \frac{p_{T0}n_0}{n_{T0}} = \left(\frac{N_T}{n_{T0}} - 1\right)n_0 = g_T n_i e^{(E_T-E_i)/kT}$. If $E_T \sim E_i, g_T \sim 1, \frac{p_{T0}n_0}{n_{T0}} \sim n_i$

Note, $\frac{n_{T0}p_0}{p_{T0}} = \frac{n_{T0}p_0n_0}{p_{T0}n_0} = \frac{n_i^2}{g_T n_i e^{(E_T-E_i)/kT}}$. If $E_T \sim E_i, g_T \sim 1, \frac{n_{T0}p_0}{p_{T0}} \sim n_i$

So, that is E c that is E v, let us say this is my intrinsic Fermi level position. If E T, if these traps the R G centres are all located close to mid gap. So, let us say E T is approaching E i, E i is slightly off mid gap by a factor of k T by 2 logarithm of v n v and n c ratio and if ET approaches E I, E T minus E i will approach 0. So, this term will become 0 and this g T can nicely be approximated as 1 ok. So, g T will take a value of 1.

So, therefore, in the condition that E T lies in the mid gap area, we can safely say that g T is 1 and therefore, the exponent disappears because E T minus E i is 0, g T disappears and therefore, this entire term is approximately n i ok. So, that is the order of magnitude of that term.

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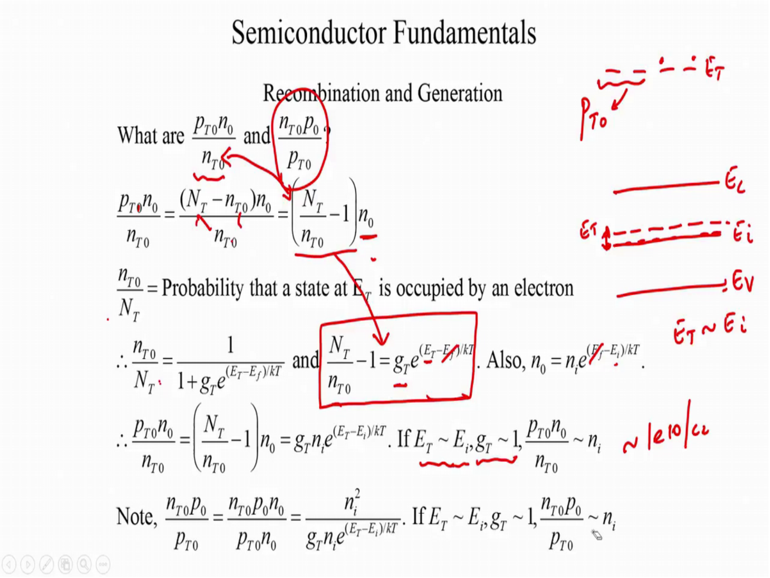

Semiconductor Fundamentals

Recombination and Generation

What are $\frac{p_{T0}n_0}{n_{T0}}$ and $\frac{n_{T0}p_0}{p_{T0}}$?

$$\frac{p_{T0}n_0}{n_{T0}} = \frac{(N_T - n_{T0})n_0}{n_{T0}} = \left(\frac{N_T}{n_{T0}} - 1\right)n_0$$

$\frac{n_{T0}}{N_T}$ = Probability that a state at E_T is occupied by an electron

$$\therefore \frac{n_{T0}}{N_T} = \frac{1}{1+g_T e^{(E_T-E_f)/kT}} \text{ and } \boxed{\frac{N_T}{n_{T0}} - 1 = g_T e^{(E_T-E_f)/kT}}. \text{ Also, } n_0 = n_i e^{(E_f-E_i)/kT}.$$

$$\therefore \frac{p_{T0}n_0}{n_{T0}} = \left(\frac{N_T}{n_{T0}} - 1\right)n_0 = g_T n_i e^{(E_T-E_i)/kT}. \text{ If } E_T \sim E_i, g_T \sim 1, \frac{p_{T0}n_0}{n_{T0}} \sim n_i$$

$$\text{Note, } \frac{n_{T0}p_0}{p_{T0}} = \frac{n_{T0}p_0 n_0}{p_{T0}n_0} = \frac{n_i^2}{g_T n_i e^{(E_T-E_i)/kT}}. \text{ If } E_T \sim E_i, g_T \sim 1, \frac{n_{T0}p_0}{p_{T0}} \sim n_i$$

So, that term is approximately equal to 1 e 10 per c c. And similarly n T o p o by P T o by same arguments, can be shown to be approximately n i if E i is approximately close to E T. So, we now got some simplified results to plug into this big recombination rate expression here. So, we know that this is we have given this is a separate symbol Tau p and Tau n and we know that these two are approximately n i provided E T lies close to v i.

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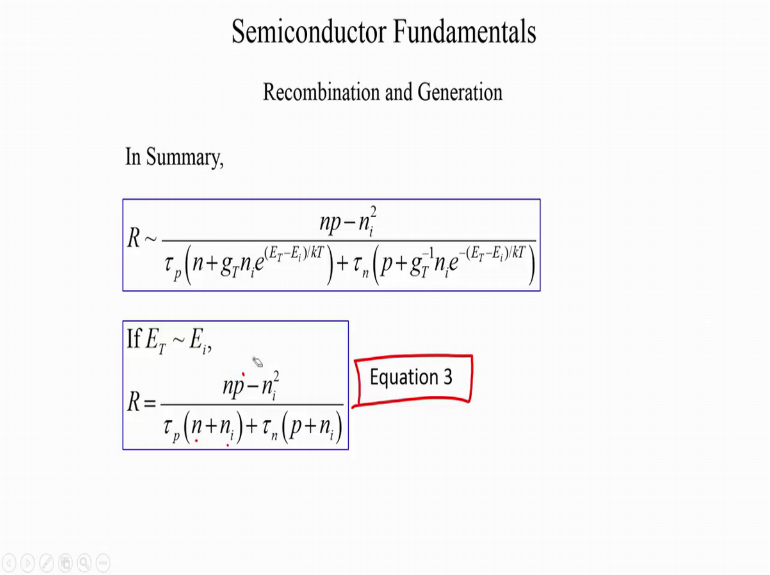

Semiconductor Fundamentals

Recombination and Generation

In Summary,

$$\boxed{R \sim \frac{np - n_i^2}{\tau_p\left(n + g_T n_i e^{(E_T-E_i)/kT}\right) + \tau_n\left(p + g_T^{-1} n_i e^{-(E_T-E_i)/kT}\right)}}$$

$$\boxed{\text{If } E_T \sim E_i, \quad R = \frac{np - n_i^2}{\tau_p(n+n_i) + \tau_n(p+n_i)}}$$

And therefore, we can write a much simpler expression for R which is n p minus n i square by tau p into n plus n i plus tau n into p plus n i and we call this as equation 3 and it is something that is extremely useful in making a quick computation.

So, what this expression tells you is that gives you a gauge on the recombination rate that is the rate at which the electrons and holes are being an highlighted in non equilibrium ok. So, n and p are non equilibrium values we have taken a semiconductor out of equilibrium what is the rate at which the n and p are being annihilated or R recombining. So, that is what R is. So, let us solve a few examples.

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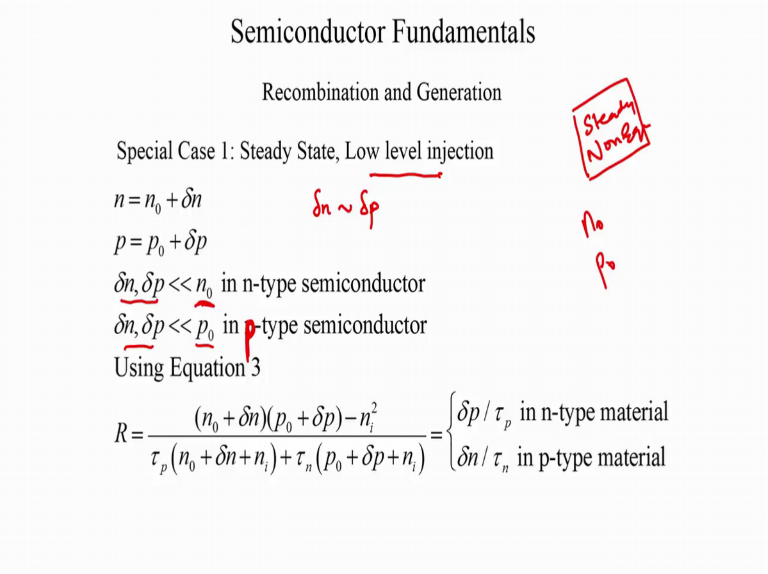

Semiconductor Fundamentals

Recombination and Generation

Special Case 1: Steady State, Low level injection

Steady NonEq

$n = n_0 + \delta n$

$\delta n \sim \delta p$

n_0

$p = p_0 + \delta p$

p_0

$\delta n, \delta p << n_0$ in n-type semiconductor

$\delta n, \delta p << p_0$ in p-type semiconductor

Using Equation 3

$$R = \frac{(n_0 + \delta n)(p_0 + \delta p) - n_i^2}{\tau_p (n_0 + \delta n + n_i) + \tau_n (p_0 + \delta p + n_i)} = \begin{cases} \delta p / \tau_p \text{ in n-type material} \\ \delta n / \tau_n \text{ in p-type material} \end{cases}$$

So, we will make use of equation 3 and let us solve a few examples. So, we will take 2 conditions, two special cases because these are going to be useful to us later down the road and we can simply refer back to these arguments.

So, the first special case is something called as Low Level injection. So, we are in non equilibrium and we are in steady state and we have a condition called as Low Level injection. So, what does low level injection mean it means that the excess carriers added. So, let us say n o and p o are the equilibrium values of the electrons and holes and in non equilibrium we have added a small number of electrons and a small number of holes delta n and delta p and the key to low level injection is a these values are really small.

So, what do you mean by small? So, firstly delta n is the order of delta p and delta n delta p is less than the majority carriers, is much smaller than the majority carriers in a semi conductor. So, delta n delta p is much smaller than n o in n type semiconductors and delta and delta p is much smaller than p o in p type semiconductors, again sorry another typo here it is in p type semiconductors.

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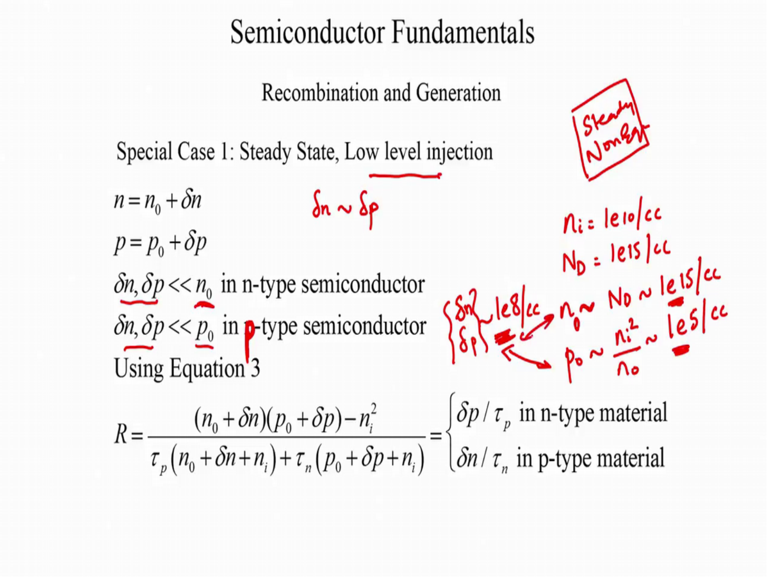

Semiconductor Fundamentals

Recombination and Generation

Special Case 1: Steady State, Low level injection

Steady Non-Eq

$n = n_0 + \delta n$

$\delta n \sim \delta p$

$p = p_0 + \delta p$

$\delta n, \delta p << n_0$ in n-type semiconductor

$\delta n, \delta p << p_0$ in p-type semiconductor

Using Equation 3

$n_i = 1e10/cc$

$N_D = 1e15/cc$

$\left.\begin{matrix}\delta n\\ \delta p\end{matrix}\right\} \sim 1e8/cc$

$n_0 \sim N_D \sim 1e15/cc$

$p_0 \sim \frac{n_i^2}{n_0} \sim 1e5/cc$

$$R = \frac{(n_0 + \delta n)(p_0 + \delta p) - n_i^2}{\tau_p (n_0 + \delta n + n_i) + \tau_n (p_0 + \delta p + n_i)} = \begin{cases} \delta p / \tau_p \text{ in n-type material} \\ \delta n / \tau_n \text{ in p-type material} \end{cases}$$

So, just to give you some numbers ok. So, let us say, let us say we have silicon. So, n i is 1 e 10 per c c and we dope the silicon with donor dopants of 1 e 15 per c c, which means that the doped semiconductor at equilibrium will have let us say it is purely extrinsic which means all the dopants are ionized which are. So, in equilibrium my n o will be close to n d which is approximately 1 e 15 per c c, my p o is n i square by n o which is approximately 1 e 5 per c c and my delta n and delta p are let us say about 1 e 8 per c c.

So, 1 e 8 is definitely much smaller than 1 e 15 and these arguments could be applied to that condition note that 1 e 8 may not be smaller than 1 e 5, that is we are not talking about we are not comparing the low level injection to the minority carriers, we are comparing to the majority carriers. So, that is the only condition we need. So, that is what we mean by Low Level injection.

So, now, under these conditions let us go ahead and use our equation 3. So, just for the sake of clarity, I will write it down again it was it said that, it said that R is equal to n p minus n i square by Tau p into n plus n i plus Tau n into p plus n i. So, that was equation 3.

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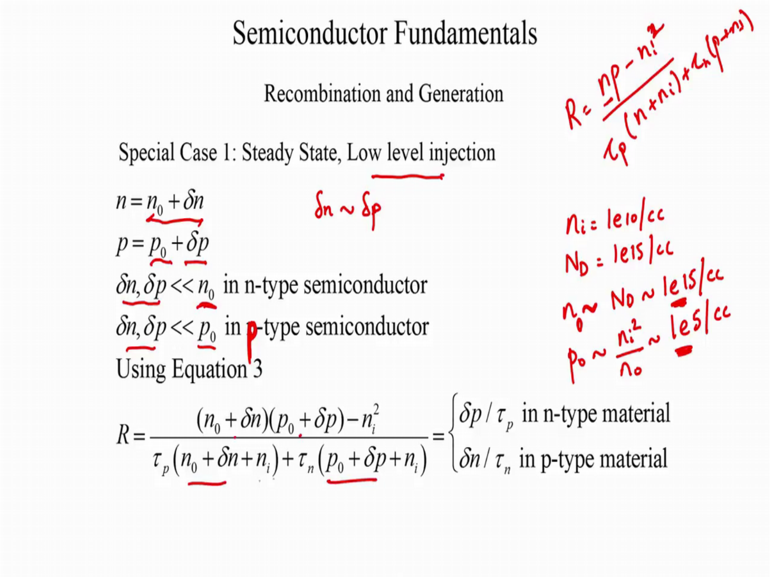

Semiconductor Fundamentals

Recombination and Generation

Special Case 1: Steady State, Low level injection

$R = \dfrac{np - n_i^2}{\tau_p(n+n_i)+\tau_n(p+n_i)}$

$n = n_0 + \delta n$

$\delta n \sim \delta p$

$p = p_0 + \delta p$

$\delta n, \delta p << n_0$ in n-type semiconductor

$\delta n, \delta p << p_0$ in p-type semiconductor

Using Equation 3

$$R = \frac{(n_0+\delta n)(p_0+\delta p)-n_i^2}{\tau_p\left(n_0+\delta n+n_i\right)+\tau_n\left(p_0+\delta p+n_i\right)} = \begin{cases} \delta p/\tau_p & \text{in n-type material} \\ \delta n/\tau_n & \text{in p-type material} \end{cases}$$

$n_i = 1e10/cc$

$N_D = 1e15/cc$

$n_0 \sim N_D \sim 1e15/cc$

$p_0 \sim \dfrac{n_i^2}{n_0} \sim 1e5/cc$

So, now what does my n, n is nothing, but n the equilibrium value plus delta n and p is nothing, but p naught plus delta p. So, we plug in those values for n n p everywhere and we open out this term ok.

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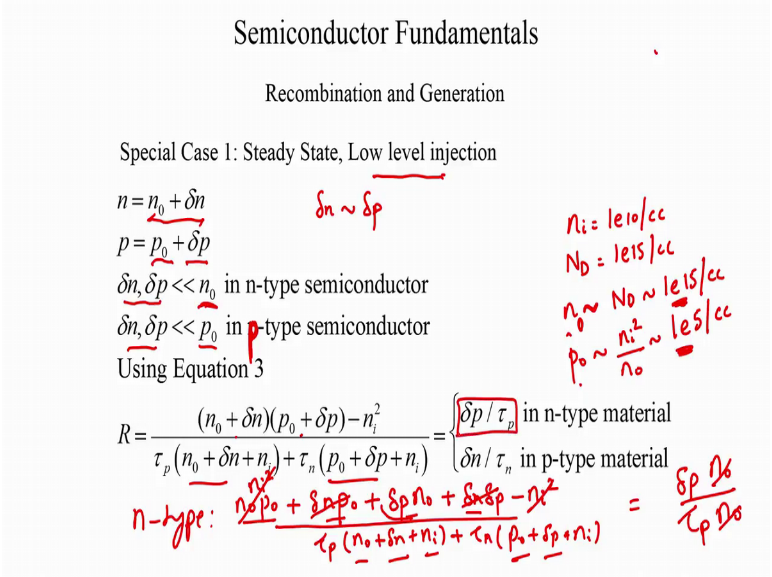

Semiconductor Fundamentals

Recombination and Generation

Special Case 1: Steady State, Low level injection

$n = n_0 + \delta n$

$\delta n \sim \delta p$

$p = p_0 + \delta p$

$\delta n, \delta p << n_0$ in n-type semiconductor

$\delta n, \delta p << p_0$ in p-type semiconductor

Using Equation 3

$$R = \frac{(n_0+\delta n)(p_0+\delta p)-n_i^2}{\tau_p\left(n_0+\delta n+n_i\right)+\tau_n\left(p_0+\delta p+n_i\right)} = \begin{cases} \boxed{\delta p/\tau_p} & \text{in n-type material} \\ \delta n/\tau_n & \text{in p-type material} \end{cases}$$

$n_i = 1e10/cc$

$N_D = 1e15/cc$

$n_0 \sim N_D \sim 1e15/cc$

$p_0 \sim \dfrac{n_i^2}{n_0} \sim 1e5/cc$

n-type: $\dfrac{\overset{n_i^2}{\cancel{n_0 p_0}} + \cancel{\delta n p_0} + \delta p n_0 + \cancel{\delta n \delta p} - \cancel{n_i^2}}{\tau_p(n_0+\delta n+n_i)+\tau_n(p_0+\delta p+n_i)} = \dfrac{\delta p \cancel{n_0}}{\tau_p \cancel{n_0}}$

So, what happens when we open out this term. So, you will end up with n o p o, the numerator will be n o p o plus delta n p o plus delta p n o plus delta n delta p minus n i square the whole divided by Tau p into n o plus delta n plus n i plus Tau p Tau n into p o plus delta p plus n i.

Now, n o p o is nothing, but n i square. So, this is n i square and these 2 terms can cancel. Now, between these 3 terms since delta n and delta p are much smaller than n o and since my n o is much greater than p o in an n type semiconductor. So, let us say we are looking at n type semiconductors. So, in an n type semiconductor this term dominates and the other terms are all negligible and the numerator will simply be delta p n o.

And in the denominator n o is going to be much greater than n I, it is going to be much greater than delta n, it is going to be much greater than p o, it is going to be much greater than delta p and therefore, only this term dominates in the denominator and therefore, my recombination rate for low level injection at steady state for an n type material is going to be the excess minority carriers injected divided by the minority carrier lifetime. So, in this case it is going to be the excess holes injected divided by the minority carrier lifetime of holes.

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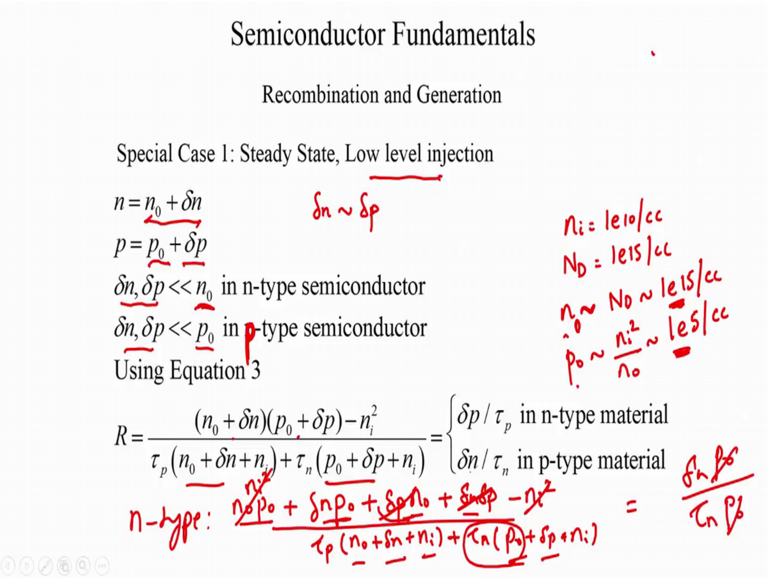

Semiconductor Fundamentals

Recombination and Generation

Special Case 1: Steady State, Low level injection

$n = n_0 + \delta n$

$p = p_0 + \delta p$

$\delta n, \delta p << n_0$ in n-type semiconductor

$\delta n, \delta p << p_0$ in p-type semiconductor

Using Equation 3

$$R = \frac{(n_0+\delta n)(p_0+\delta p) - n_i^2}{\tau_p(n_0+\delta n+n_i)+\tau_n(p_0+\delta p+n_i)} = \begin{cases} \delta p/\tau_p & \text{in n-type material} \\ \delta n/\tau_n & \text{in p-type material} \end{cases}$$

$\delta n \sim \delta p$

$n_i = 1e10/cc$

$N_D = 1e15/cc$

$n_0 \sim N_D \sim 1e15/cc$

$p_0 \sim \frac{n_i^2}{n_0} \sim 1e5/cc$

n-type: $$\frac{\cancel{n_0 p_0} + \delta n p_0 + \delta p n_0 + \cancel{\delta n \delta p} - \cancel{n_i^2}}{\tau_p(n_0+\delta n+n_i)+\boxed{\tau_n(p_0}+\delta p+n_i)} = \frac{\delta n \cancel{p_0}}{\tau_n \cancel{p_0}}$$

Similarly in the case of p type semiconductor, so if you retain these expressions, p o is much greater than n o and delta n p. So, these 2 terms disappear and the numerator becomes delta p, I am sorry delta n into p o and from the same arguments this term is what dominates and your denominator will be Tau n p o and the recombination rate is the is delta n by Tau n, that is the excess electrons injected in the p type material divided by the minority carrier lifetime of electrons.

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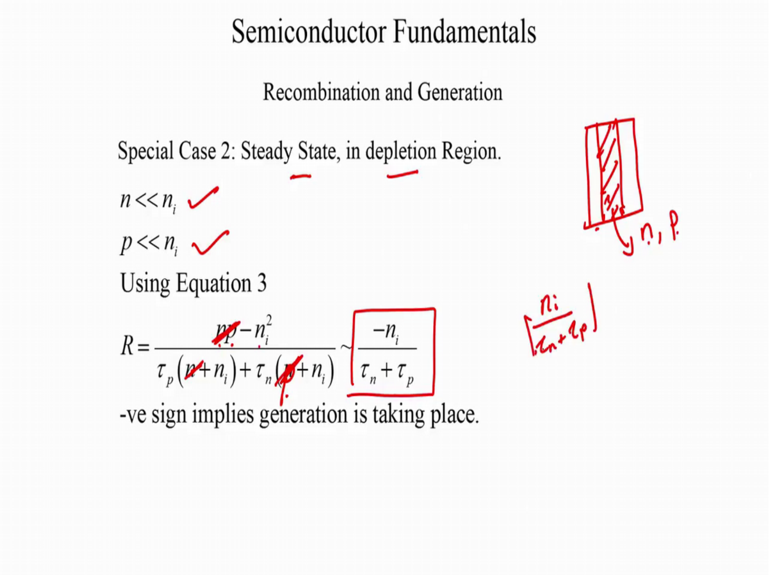

Semiconductor Fundamentals

Recombination and Generation

Special Case 2: Steady State, in depletion Region.

$n << n_i$ ✓

$p << n_i$ ✓

Using Equation 3

$$R = \frac{\cancel{np} - n_i^2}{\tau_p(\cancel{n}+n_i)+\tau_n(\cancel{p}+n_i)} \sim \boxed{\frac{-n_i}{\tau_n+\tau_p}}$$

-ve sign implies generation is taking place.

So, the second example and the final example, we will look at is the steady state, the recon trying to calculate the recombination rate in steady state in a depletion region. So, we have not yet discussed what a depletion region is, we will do so very quickly but a depletion region in a semiconductor is essentially a region and a semiconductor which is sort of engineered to have very little free carriers. So, that the n and p count are going to be extremely low, it is depleted of free carriers.

So, which basically means that n is much less than n i p is much less than n i. So, if you use these 2 conditions in this equation your n p is going to be much less than n i square. So, the numerator simply n i square, your denominator the n is not important compared to compared to n i. So, here this is a typo again it should have been p, but that is not important compared to n i and therefore, you end up with the recombination rate being minus n i by Tau n by Tau n plus Tau p and this negative sign tells you that the recombination rate is negative which means you are actually having generation.

So, in a depletion region there is carriers are being generated, there is an encouragement towards generating carriers and that generation rate is n i by Tau n plus Tau p.