Semiconductor Devices and Circuits
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Semiconductor Fundamentals

Lecture – 12
Doping

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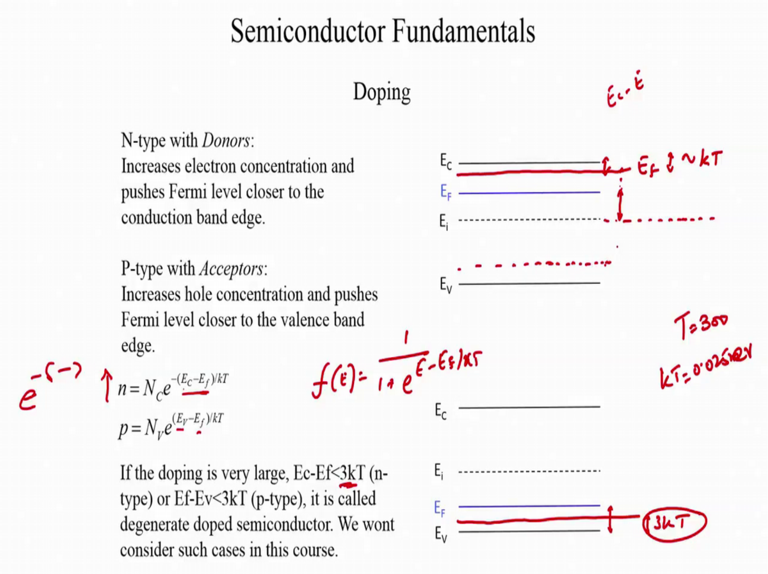

Semiconductor Fundamentals

Doping

N-type with *Donors*:
Increases electron concentration and pushes Fermi level closer to the conduction band edge.

P-type with *Acceptors*:
Increases hole concentration and pushes Fermi level closer to the valence band edge.

$n = N_C e^{-(E_C - E_f)/kT}$

$p = N_V e^{(E_V - E_f)/kT}$

If the doping is very large, Ec-Ef<3kT (n-type) or Ef-Ev<3kT (p-type), it is called degenerate doped semiconductor. We wont consider such cases in this course.

So, here is a better a clearer picture of what I was talking about which is, when you dope the semiconductor n type, you find that the fermi level E f moves above and towards the conduction band. So, this is the imaginary intrinsic fermi level position which is no longer valid because, we have we no longer have a pure semiconductor.

So, this was the fermi level location when the semiconductor was pure and we now doped with n type; which means, we have increased the electron concentration, which means we need to now decrease the fermi level, has to pick a place where E c minus E f has to be smaller. Because, the electron concentration varies as e to the power minus this gap; there E c minus E f gap.

So, the fermi level will have to move closer to the conduction band edge. Similarly, for p type doping, we have to increased the whole count which means, we need to reduce the gap between E v and E f. And therefore, what you say the hole, the fermi level moves closer to the valence band edge.

So now, these equations if you remember, well these all based on the Boltzmann's approximation. So, we started off with the fermi function which is f of E is equal to 1 by 1 plus e to the power e minus E f by k T and we assumed that e minus E f is much larger than k T,.

So, if you keep doping the semiconductor more and more and let us say the fermi level is pushed closer and closer towards the fermi level, you will we will end up with the situation where E f is so close to the conduction band edge that E c minus E f is no longer greater than k T. Let us say of the order of k T. It is no longer much greater than your k T. In those cases, we call the semiconductor to be degenerately doped.

And that is a technical term and by definition, the point at which we start calling the semiconductor degenerately doped is the point at which E c minus E f ok. E c minus E f this gap the fermi level has gone so, close to the conduction band, that this gap has become less than 3 times k T ok.

So, at T equal to 300 Kelvin you know, k T is equal to about 0.025 million electron volt which is about 25 milli electron volts. So, 3 times that is about 0.075 electron volt and when E c minus E f is less than that value, we say that the semiconductor which is n type doped is not degenerately doped and a lot of the calculations have to be done more carefully and we cannot make many of the assumptions we have made so far.

Similarly, if you are going to dope the semiconductor p type and you start pushing the fermi level closer and closer to the valence band, we will reach a point where E F is within 3 k T of the valence band edge and once again, we say that the semiconductors degenerately doped p type and once again we cannot make many of the assumptions we have made.

So, as far as this course is concerned, we will not look at degenerately doped semiconductors, we will be very much within this 3 k T threshold and the fermi level location will be well below these borders. So, if this is the border force, this is say 3 k T, that is called these 2 energy levels are just to be very clear.

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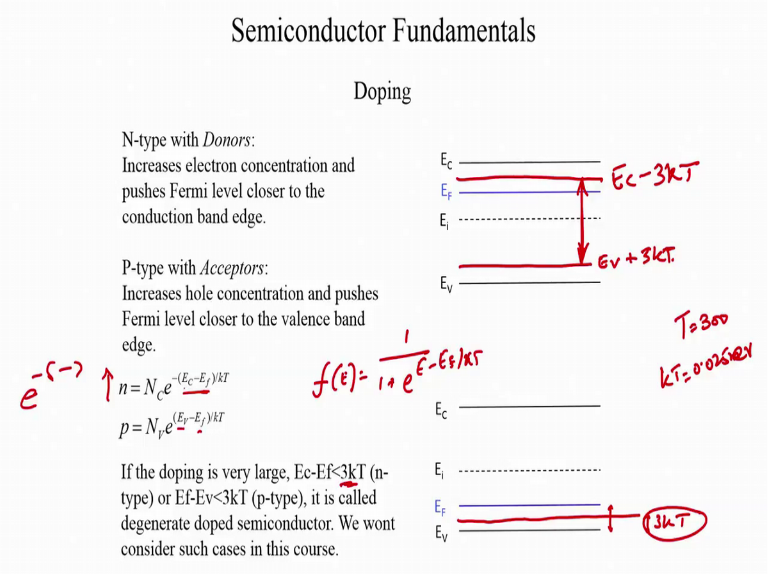

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As far as the scores and the tests and the assignments are concerned, if this is my E V and we say this is E V plus 3 k T and that is E C minus 3 k T. We will only consider cases where the fermi level despite any amount of doping lies in these regions in within this band ok. So, that we can still comfortably go ahead and use all these simple calculation techniques to calculate your electron and hole count.

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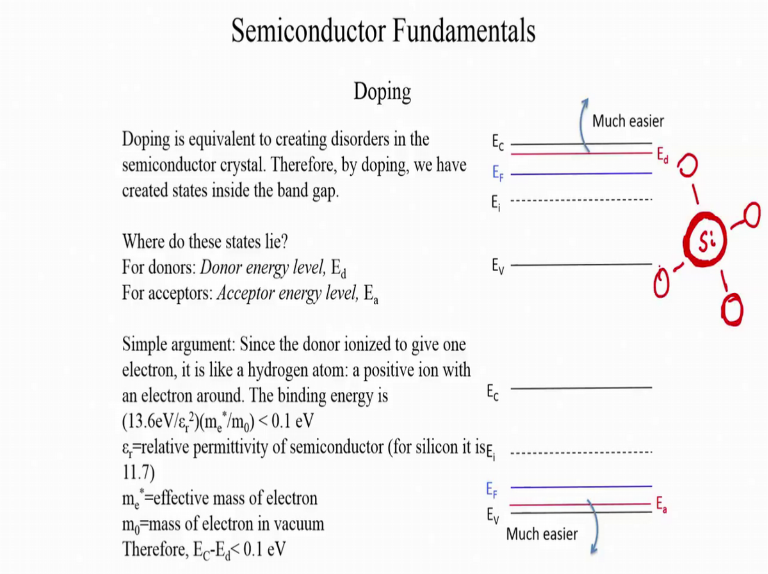

Semiconductor Fundamentals

Doping

Doping is equivalent to creating disorders in the semiconductor crystal. Therefore, by doping, we have created states inside the band gap.

Where do these states lie?
For donors: *Donor energy level,* E_d
For acceptors: *Acceptor energy level,* E_a

Simple argument: Since the donor ionized to give one electron, it is like a hydrogen atom: a positive ion with an electron around. The binding energy is $(13.6\text{eV}/\varepsilon_r^2)(m_e^*/m_0) < 0.1$ eV
ε_r=relative permittivity of semiconductor (for silicon it is 11.7)
m_e^*=effective mass of electron
m_0=mass of electron in vacuum
Therefore, E_C-E_d< 0.1 eV

So now, what does doping actually do? So, why is it you know what is the physical intuition behind the fact that doping significantly improves the electron and hole count? And what do you mean by doping? Let us look at this with a different perspective.

We already looked at it. As a as a first glance, we know what it does. You know how the dopant atom bonds with the silicon lattice and contributes an electron and or contributes a hole. But, let us look at it more closely. So, when you dope, let us take an example of an n type semiconductor, ok. So, we have a silicon lattice, ok. So, silicon has been a role model. And therefore, I keep using the word silicon for semiconductor, but that does not mean that these concepts are not valid for other semiconductors, all right.

So, let us say you had your silicon lattice ok. It had these 4 would we say bonds to the other silicon atoms and we replaced this silicon with your N type dopant which is a donor doping which is a species that had 5 electrons.

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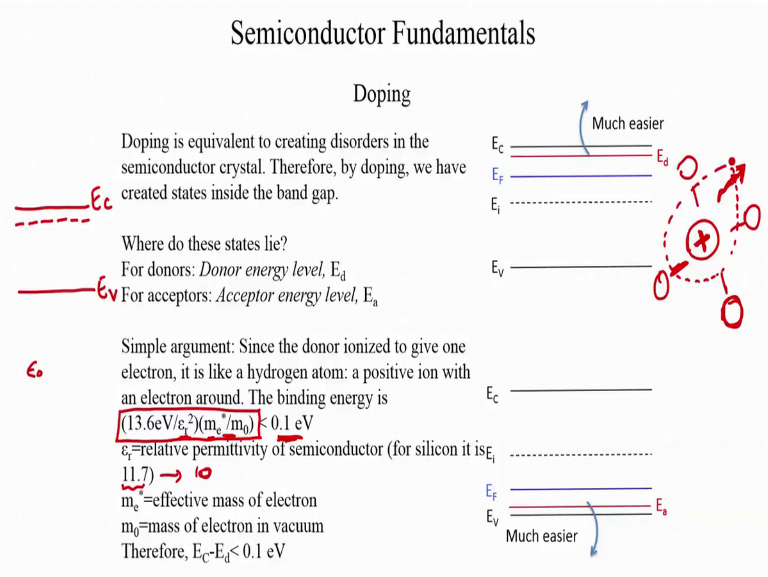

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So, 4 of them participated in bonding and the 5th one was free to move about in the lattice. And therefore, it left the donor with a single positive charge because there is 1 electron more that should have been with the donor for charge neutrality.

But is no longer at the donor and therefore, there is a single positive charge. So, if you think about this species here ok, this 1 electron and 1 positive charge ok, what is the binding energy? You know what does the energy that is held these 2 species together ok.

One can imagine this to be something like a hydrogen atom. It is not exactly a hydrogen atom because, there are other electrons present in the donor.

But considering the fact that it is a single positive charge and a single electron that was initially bound to this donor and which has now moved away from the donor because of ionization, the energy needed to pull that electron away is the energy required to do. So, in a hydrogen atom; so, that gives you that gives you a by comparing it to a hydrogen atom, it gives you a good estimate as to the ballpark number for that energy for that energy.

So, that is what we are trying to do. We are trying to estimate what does that energy needed for me to take the electron away from the donor and push it into the conduction band and by calculating that energy; we can actually calculate the energy level created by the impurity.

So, what does that mean? So, in a pure crystalline silicon atom, a silicon lattice, sorry you have your valence band edge you have your conduction band edge and if your crystalline lattice is perfect, it is perfectly crystalline, no defects, no impurities. Then, you do not have any states in the band gap. But, the moment you start creating defects, you start adding impurities, you will start seeing states appear inside the band gap.

And as we move towards more and more disordered materials say you ultimately take it towards amorphous silicon or a glass, you will find that there is a density of states inside the gap itself. So, the gap is no longer clean, but we will not go to such extremes at this point. All you have done now is, we have added a donor, we have added an impurity into the silicon.

Therefore, there are some traps that have appeared, there have been some states that have been created inside this gap. And the purpose of this study is to locate those states where are at what energy are those states sitting. And we are going to do that by identifying the energy required to take the electron out from this little entity, which is a positively charged core and electron around it.

Now, if you compare it to a hydrogen atom, the energy required would be this with some corrections. It would be 13.6 electron volts, but we need to correct for certain elements which is the permittivity of the material is different ok. So, we will we need to bring in a relative permittivity. You cannot it can no longer be epsilon naught which is 8.85 e to the power minus 12. You cannot have that value. You need to have a relative permittivity correction, ok.

So, you bring in an epsilon r square term and you also need to account for the effective mass of the electron. You can no longer use the mass of the electron as is used in vacuum because, you have the electron propagating through the lattice and you have this interaction with the lattice which results in an effective mass of m e star.

So, by taking into account these 2 corrections ok. So, we multiplied by a factor of m e star by m naught and divided by epsilon r which is the relative permittivity of the semiconductor epsilon r square which is a epsilon r is the relative permittivity of the semiconductor and continue using a 13.6 electron volt energy. If you make these corrections taking account, these corrections and plug in numbers for silicon.

So, any styles will be same order as m naught ok. It is not orders of magnitude different. It could be slightly more or slightly less depending on the difference semiconductors. And for silicon, epsilon r is the order of 11.7. Say, let us let us call it 10 ok. Just to give you an easy calculation.

So, the binding energy is about 13.6 electron volt into something which is the order of 1 which is your m e by m naught divided by 100 which is about 0.1 give or take 0.1 electron volt. In fact, it is less than 0.1 electron volts. Therefore, this trap energy is actually look at the moment you add a donor, what this is saying is that, the moment you add a donor, you are creating a trap state or you are creating energy states due to this donor which is located energy level E d, ok.

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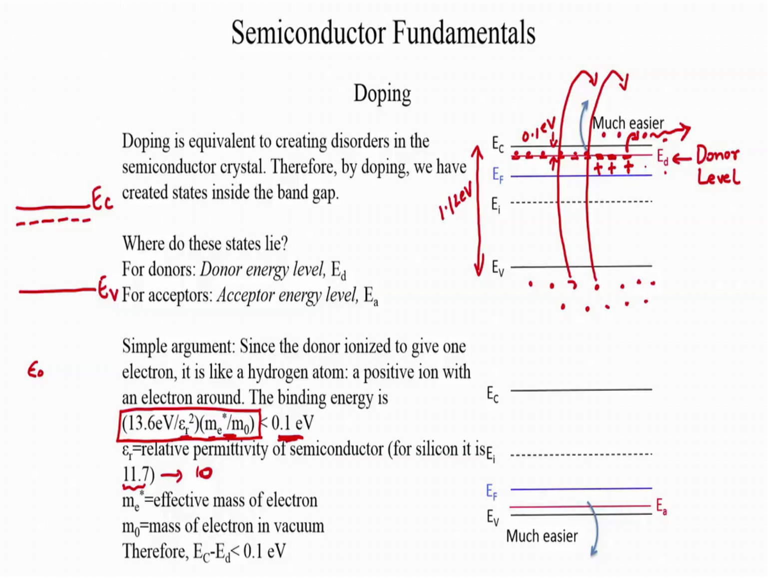

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So, this is something called as the donor energy level or the donor level and this E d is located at around 0.1 electron volt from the conduction band edge. So, in the case of silicon your band gap this is about 1.12 electron volts and this donor dopant has led to the creation of states inside the band gap and those states are located at around 0.1 or in fact, less than 0.1 electron volt, that is about 10 percent of the band the band gap, 0.1 electron volt from the conduction band edge.

So, what does this mean? It means that, this donor which was initially charged neutral had a positively charged core and had all it is 5 electrons ok. So, this was the charge neutral species and this was charge neutral. Let us say temperature at t equal to 0, but the moment the temperature increased a little these electrons gained enough energy. They gained this 0.1 electron volt energy to snap away from the donor and to get into the conduction band of the lattice.

Therefore, these donors that lost their fifth electron all became positively charged. So, in other words, they were ionized and these electrons which left the donor and moved into the conduction band were now free to conduct, ok. So, we have increased electron count in the lattice the free electron count in the lattice. So, we are not when I say electron count, you know just as a means of speaking; I mean, it is just the it is understood that it is the free electron count ok.

So, we are really not interested in the electrons in the valence band there are plenty of those. So, the addition of the donor has increased the electron count the free electron count in the conduction band and it has now become positively charged because of it getting ionized and since the donor level is located. So, close to the conduction band edge it really does not take too much of temperature to activate these electrons to ionize these donors.

So, at 300 Kelvin, in the case of silicon, we are able to promote the electron sitting in the valence band. If you add it, consider your pure intrinsic semiconductors, pure intrinsic silicon, our 300 Kelvin we can promote about 1 e 10 per cc electrons from the valence band to the conduction band and that is at an energy gap of 1.12 electron volt.

Here, I have a gap which is 10 percent of the original energy gap which is 0.1 electron volts because of these donors. And therefore, it is much easier to ionize these donors and this can happen at a much lower temperature. Again, we can ionize these donors and promote all these electrons into the conduction band. And therefore, the addition of a donor like dopant can significantly improve the electron population,.

The same argument can be made for holes. So, that is that is what the doping, that is what doping is doing. If you look at it from the perspective of the energy levels and the creation of these traps inside the gap.

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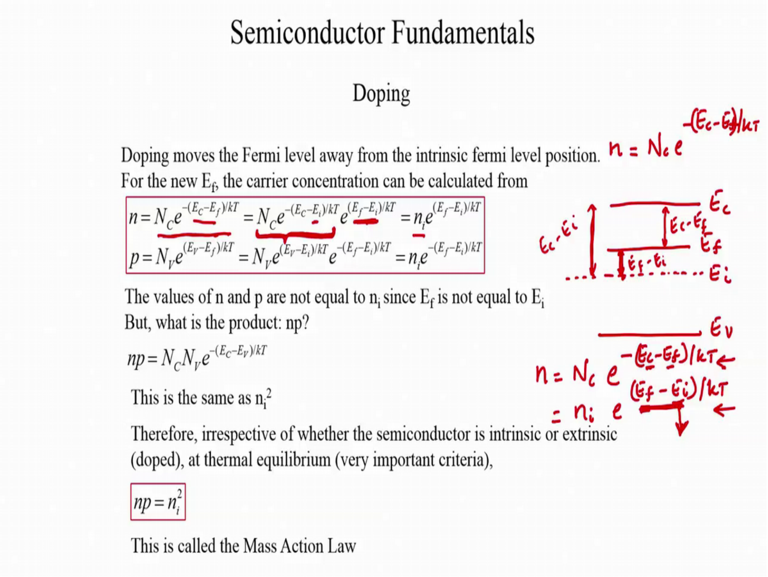

Semiconductor Fundamentals

Doping

Doping moves the Fermi level away from the intrinsic fermi level position.

For the new E_f, the carrier concentration can be calculated from

$$\boxed{\begin{aligned} n &= N_C e^{-(E_C-E_f)/kT} = N_C e^{-(E_C-E_i)/kT} e^{(E_f-E_i)/kT} = n_i e^{(E_f-E_i)/kT} \\ p &= N_V e^{(E_V-E_f)/kT} = N_V e^{(E_V-E_i)/kT} e^{-(E_f-E_i)/kT} = n_i e^{-(E_f-E_i)/kT} \end{aligned}}$$

The values of n and p are not equal to n_i since E_f is not equal to E_i

But, what is the product: np?

$$np = N_C N_V e^{-(E_C-E_V)/kT}$$

This is the same as n_i^2

Therefore, irrespective of whether the semiconductor is intrinsic or extrinsic (doped), at thermal equilibrium (very important criteria),

$$\boxed{np = n_i^2}$$

This is called the Mass Action Law

$n = N_c e^{-(E_c-E_f)/kT}$

$n = N_c e^{-(E_c-E_f)/kT}$

$= n_i e^{(E_f-E_i)/kT}$

So now, let us calculate again ok. So, we know we want to calculate the free electron concentration and the hole concentration in the valence band after doping right.

So, this calculation is straightforward. It is just N c e to the power minus E c minus E f by k T. So, that is the calculation, but then there is a much nicer or a much more intuitive or much more useful relation that you can get now since. So, let us let us draw this that is E c that is E v and now you have some E f which is different from your E i. E i is just offset from mid gap. It is close to mid gap. So, I will just locate it there.

So, that was the fermi level for a pure semiconductor. But now, after doping, the fermi level has shifted to E f and if it is n type doping, it is E f is going to be closer to E c. Now this expression tells you that the electron count is N c e which is the effective density of conduction band states k, effective density of states in the conduction band into e to the power minus E c minus E f by k T, but E c minus E f is also equal to E c minus E i ok. So, that is E c minus E i minus E f minus E i.

So, this term is essentially that minus this and we can sort of split that expression by bringing in the E i term here ok. So, why am I why are we splitting this? Because, this coefficient here is nothing but the electron concentration in the intrinsic semiconducto because, your E f has now become E i and for a doped semiconductor this expression this electron count is essentially this term into the exponent having an energy gap which is the difference between the new location of the fermi level and the intrinsic fermi level position

So, this expression is essentially n i e to the power E f minus E i by k T. So, these 2 expressions are identical. So, you could count your electrons by saying it is N c e to the power minus E c minus E f by k T or we could say that, it is n i e to the power E f minus E i by k T. So, this expression relates both of these expressions are useful ok. It depends on the situation.

This expression relates the carrier count by noting down the energy difference between the conduction band edge and the fermi level. Whereas, this expression relate makes a carrier count by noting down the energy difference between the new fermi level position and the intrinsic fermi level position. So, this is the fermi level position for an intrinsic or pure semiconductor and after doping, it is relocated to E f. And, in this expression can tell you what this new fermi level. You know, how far this new Fermi level moved away from E i ok.

So, both these expressions are useful and we could you know create a similar expression for the holes which is the whole population in the valence band is basically N v into e to the power E v minus E f by k T.

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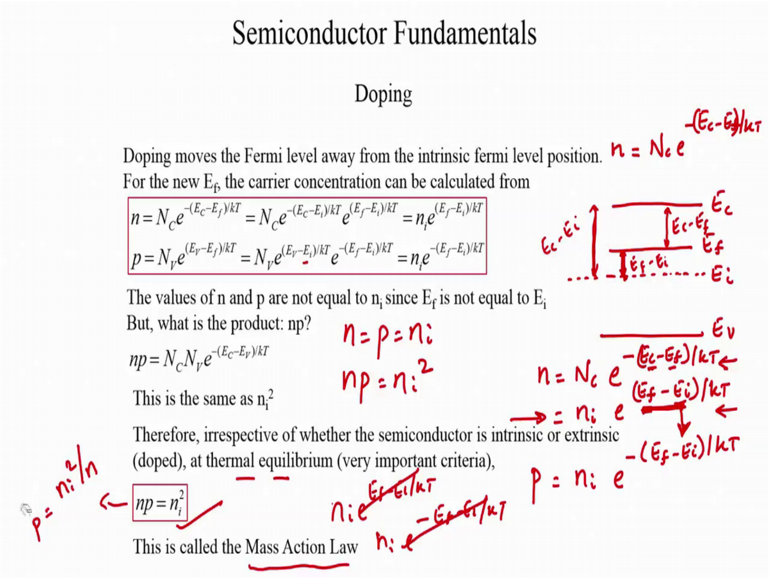

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The values of n and p are not equal to n_i since E_f is not equal to E_i

But, what is the product: np?

$$np = N_C N_V e^{-(E_C-E_V)/kT}$$

This is the same as n_i^2

Therefore, irrespective of whether the semiconductor is intrinsic or extrinsic (doped), at thermal equilibrium (very important criteria),

$$\boxed{np = n_i^2}$$

This is called the Mass Action Law

We again bring in the E i and you find that the whole population can also be represented as n i e to the power minus E f minus E i by k T. So, this expression; So, which is this expression here for the electron count and this expression here for the hole count looks more or less similar except for the fact that the exponential here has got a positive sign and the exponential here has got a negative sign, ok.

So, that is the that is the major difference. So now, what is n into p in the case of an intrinsic semiconductor which can be what was the electron and hole count in an intrinsic semiconductor my E f is equal to E i. Therefore, this experiment is 1 and therefore, the electron and hole count were both equal to n i. So, in an intrinsic semiconductor we had n is equal to p is equal to n i and the product n p was equal to n i square.

But, what about the doped semiconductor r? What is this product? N p, is now equal to n i e to the power E f minus E i by k T into n i e to the power minus E f minus E i by k T these 2 terms cancel. And therefore, this product is still equal to n i square. So, irrespective of whether the semiconductors doped or intrinsic the electron count into the whole count is always equal to n i square, provided the semiconductor is at thermal equilibrium which means that you have taken the semiconductor.

You have either captured intrinsic or doped it you have taken the semiconductor and you kept it in the dark. You have not applied any light. You are not thrown applied any voltage. You kept them in the dark as it is with the only influence on the semiconductor being due to temperature. So, that was thermally that is thermal equilibrium. So, every process that is going on inside the semiconductor is very nicely balanced by a reverse process.

So, there is nothing changing. I mean, there is there are many things going on you are creating a lot of free electrons. You are losing a lot of free electrons. All this is going on, but it is all balanced in a very nice way, that statistically speaking, then the intrinsic carrier concentration is fixed.

And, you know the other properties and semiconductor is not really waiting with time ok. So, that is what you mean by thermal equilibrium and this property that n p is equal to n i square is very useful. Because, if I know my electron count and I want to find my whole count, I just need to do n i square by n, ok. So, this gives me my whole population. So, this relation that the n p is equals to n i square something called as a mass action law.

And this might appear to be a very odd looking term. You know what is the meaning behind mass action law? But, it essentially comes from looking at chemical reaction rates, ok. So, you can think of it as a chemical reaction that is going on. And what is the reaction it is got to do with the generation and the recombination or the loss of these free carriers again? We will talk about that later. So, that is the that is the process that is going on and just like how you sort of consider reactants in a reaction for a chemical reaction.

They consider the electron and hole count in this process of thermal generation of carriers and thermal recombination of carriers which we even talk about soon.

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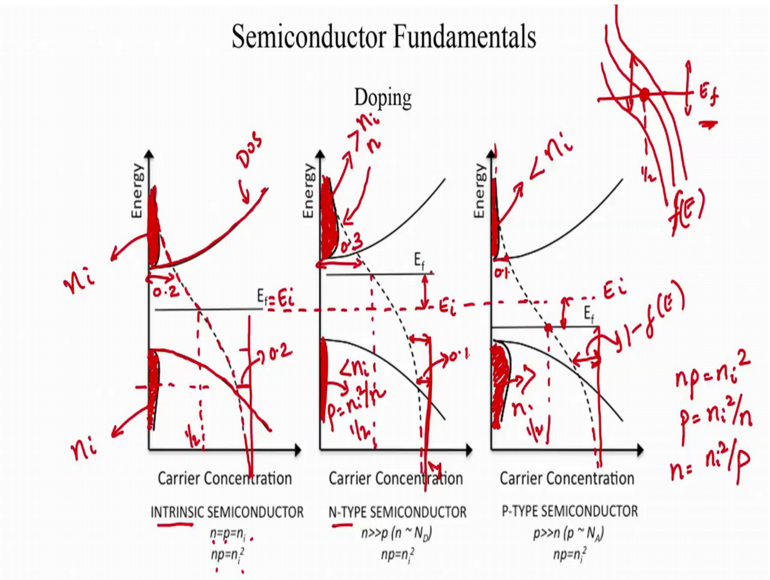

Semiconductor Fundamentals

Doping

INTRINSIC SEMICONDUCTOR
$n=p=n_i$
$np=n_i^2$

N-TYPE SEMICONDUCTOR
$n>>p$ $(n \sim N_D)$
$np=n_i^2$

P-TYPE SEMICONDUCTOR
$p>>n$ $(p \sim N_A)$
$np=n_i^2$

So, what does doping do. So, we looked at the we now have a good understanding of what doping does in terms of equations, but it is always nice look at this graphical picture, ok. So, let us go back to our picture for intrinsic semiconductors, ok. So, this was my density of states there are many plots drawn on this.

So, please do not get confused. So, that is the density of states. So, that is density of states versus energy this dotted line is the fermi function. It is f of e versus energy and as we notice, it takes a value of half when it crosses e is equal to E f and since this is an intrinsic semiconductor, this E f is equal to E i.

So, that is my intrinsic semiconductor position and we already saw that the electron count was basically the product of the density of states in the conduction band and the probability of finding carriers in the conduction in these states. So, that was my electron count which is this product and that is the whole count.

Now, during by doping, what we have done is if you dope it, n type, ok. So, this is the intrinsic case where n is equal to p is equal to n i n p is equal to n i square. If you dope it, n type we have added a donor dopant and let us say this is my imaginary E i, this was E i in the intrinsic semiconductor. By adding a donor doping, we have increased electron population and we have effectively moved E f closer to the conduction band edge and it is located above E i.

Now, since the fermi level is located at a different location from E i, the fermi function shifts ok. It is no longer going to remain the same v. It is now going to adjust itself. So, as to take a value of half in this point, earlier the fermi function was taking a value of half at E i. But now, it is going to take a value of half at E f. Because, earlier E f was equal to E i. So, and now E f is not equal to E i.

So, if the fermi level if you if you can imagine the fermi function as moving having moved up a little ok, along with E f ok. Then, it is very clear to see that the probabilities here are now suddenly increased the values which are initially very small. So, let us say, initially the value here was less than half and let us say it was 0.2. But now, since we have moved the entire f of e curve upward, this value is now much greater than 0.2 ok.

At here and here it was half which is 0.5. So, here it could now become 0.3 that is the value of f of e which means the probability of me finding an electron in these states has increased the density of states hasn't changed ok. It is not going to change unless you dope it very strongly. We are not talking about those levels of dope.

If the density of states hasn’t changed, but the probability of finding an electron in those states has increased in the case of n type doping [noise.] The probably defining electrons in the states above E f have increased in the case of n type doping and it is not only that the electron population is increased. So now, the electron population is increased significantly, ok. So, you can see that this shaded region has got a larger area as compared to this region here which was the intrinsic case.

It is not only that the electron population is increased the whole population has also decreased ok. Why is that because, this is the fermi function. Let us say, this was this is the value of 1. It is asymptotically heading towards 1. As you go to minus infinity and the probability of finding hole depends upon this difference, it is 1 minus f of e.

Now, initially the probability of m e finding a hole at say some energy level here was 1 minus f of e. Let us call that to say 0.2, but since we have moved shifted the entire fermi f of e curve upward because E f has gone up. These values are not suddenly decreased. So, what was 0.2? At some energy level there has now become a much smaller number.

Let us say it is 0.1. These are just numbers under stuff, these are not calculated values, these are just numbers I am putting out in order to get the message across. So, what was the 1 minus f of e was much larger than intrinsic case. But the moment we doped it n type, 1 minus f of e for energy levels e less than E f have suddenly gone down which means, the probability of m e finding a hole in these states has suddenly gone down. And therefore, while the electron count has increased the hole concentration has decreased.

The area under this curve which is symbolizing the whole population is now become less than the case of the intrinsic semiconductor. So, in an intrinsic semiconductor, the number of holes were n i, the hole concentration was n i, the electron concentration was n i in an n type semiconductor. The electron concentration is greater than n i, but the hole concentration has gone less than n i and how much less.

If n is the electron concentration the hole concentration has to be n i square by n. In the p type semiconductor, the opposite happens if you have doped semiconductor p type, we have moved the fermi level down. So, this is my intrinsic fermi level position; that is, E i. We have now moved the fermi level below E i,.

And, f of e has to shift downward in order to take a value of half. So, you can think of f of e as you know sort of pinned to this fermi level position, right. So, if this is the E f imagine a sort of a pin right there because, it always has to take a value of half. So, as we move this up and down, this entire curve also has to move up and down it has to move either up or it has to come down. So, that is f of e responding to the relocation of E f.

So, by now, moving E f downward, f of e is also shifted downward which means that the probability of m e finding an electron has now gone down drastically. So, it was 0.2. There it had increased to 0.3 for n type doping. But now, it has become 0.1, that is a 1 the probability of m e finding a electron in this stage above E f has gone down.

So, although the density of states hasn't changed, the number of electrons in the conduction band has decreased and it will decrease to a value less than n i which was the intrinsic case for a p type dopant. For n type doping, the electron count has increased electron concentration is increased and similarly, if you look at 1 minus f of e, that is this gap, since the fermi level has now moved down.

Since the fermi level is moved down and it is brought f of e along with it. This value of 1 minus f of e has now increased which means the probability of finding a hole in the states below E f has now gone up. And therefore, my whole population has now suddenly shot up the hole count in the valence band states has now gone up. So, here in case of p type doping my electron count is going to be less than n i and the hole count is going to be greater than n i.

And in all cases, n into p is equal to n i squared. So, if I know my electron count, I can find my hole count as n i square by n or if I know my hole count, I can find my electron count as n i square by p.