Semiconductor Devices and Circuits
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Lecture – 10
Density of States - Continued, Fermi Function

Now, what about a 1D case?

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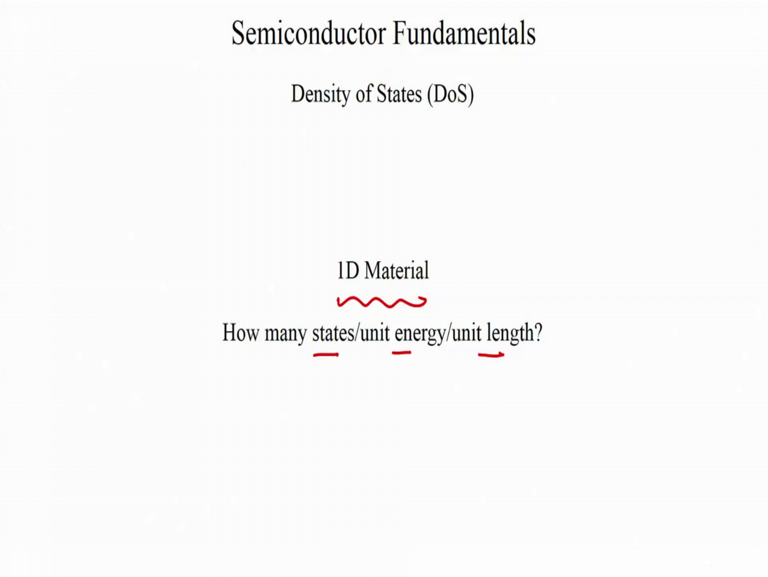

Semiconductor Fundamentals

Density of States (DoS)

1D Material

How many states/unit energy/unit length?

We will quickly run through the same procedure. But, now we need to ask us a question how many states per unit energy per unit length, in a 1D material, ok. The 1D material does not have volume or area it is just got a length.

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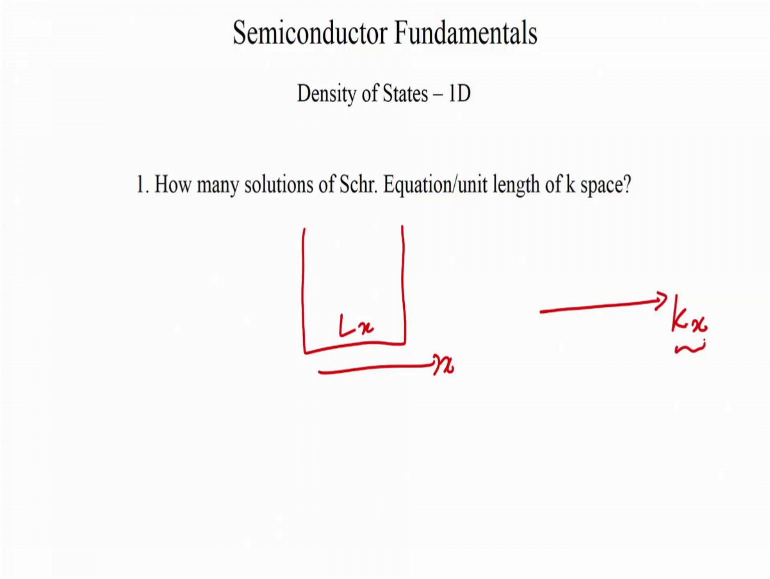
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Density of States – 1D

1. How many solutions of Schr. Equation/unit length of k space?

And we follow the same set of questions how many solutions are Schrodinger's equation per unit length in k space, ok. And now, we have only a 1 D box. We are only interested in a 1 D box and we only have.

So, let us say this is the x direction. The box has got length L x and we are only have got a k x space. There is no k y, there is no k z and there is only a k x space.

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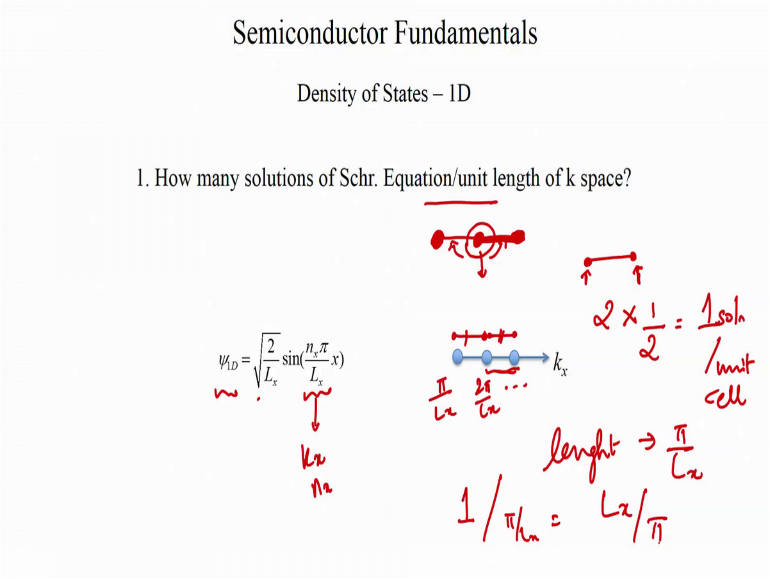
Semiconductor Fundamentals

Density of States – 1D

1. How many solutions of Schr. Equation/unit length of k space?

$\psi_{1D} = \sqrt{\frac{2}{L_x}}\sin(\frac{n_x \pi}{L_x}x)$

And the wave function for a 1D box is simply this and that is my k x term which is quantized with the quantum number n x. And therefore, in this space, I have you know you have your pi by L x 2 pi by L x and so on, ok.

And if you look at the number of solutions per unit length of k space first so, these are my unit cells. Now, since that is 1 unit cell, that is the second unit cell, ok. If you look at the number of solutions per unit cell, each cell has got 2 solutions. There is one solution or this end and one solution on that end. There are 2 solutions, but each solution is shared by 2 unit cells.

So, if you look at this solution, for example, it is shared by this cell as well as this cell, right. So, each cell is just a line this line segment, ok connecting these 2 discrete points and these discrete points of the solutions to Schrodinger's equation.

So, each solution is shared by 2 cells and therefore, there is one solution per unit cell and what is the length of the unit cell? The length of 1 unit cell is pi by L x, ok. So, the length of this unit cell is this length here which is pi by L x. So, the number of solutions per unit length is going to be one solution divided by the length of the unit cell which is L x by pi. But now, once again we need to make our correction, ok.

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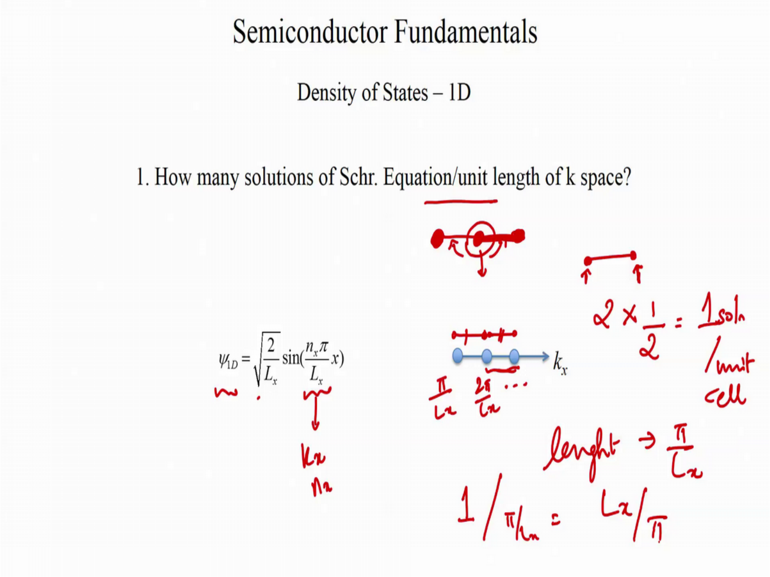

Semiconductor Fundamentals

Density of States – 1D

1. How many solutions of Schr. Equation/unit length of k space?

$$\psi_{1D} = \sqrt{\frac{2}{L_x}}\sin\left(\frac{n_x \pi}{L_x}x\right)$$

k_x

n_x

$\frac{\pi}{L_x}$ $\frac{2\pi}{L_x}$ ...

k_x

$2 \times \frac{1}{2} = 1$ soln / unit cell

lenght → $\frac{\pi}{L_x}$

$1/\frac{\pi}{L_x} = L_x/\pi$

So, let us deploy our correction terms. You know we have these correction terms in the case of 3D and 2D. So, L x by pi, but then we are double counting for n ok. For n x could take plus and minus values.

So, we need to divide by half but then there are also 2 spins, ok snd therefore, this is the correction term. So, the correction term is simply 1 and therefore, you your initial guess on the answer was right. So, there are these many solutions to Schrodinger's equation per unit length in k space. And now, the next question how many solutions of Schrodinger's equation between k and k plus dk.

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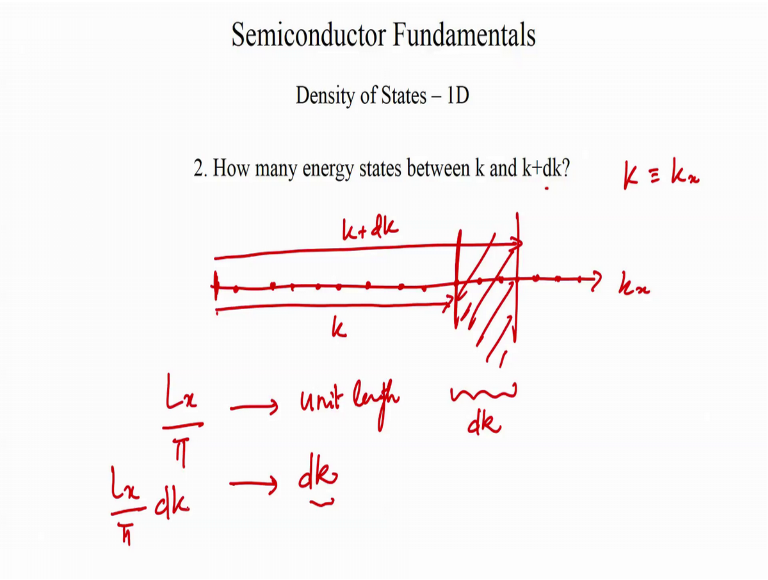

Semiconductor Fundamentals

Density of States – 1D

2. How many energy states between k and k+dk?

So now, let us draw extend our k x, ok. You have this massive grid of points sitting on this line it is a 1D case. So, we only have 1 dimension. You have so many points and then I. So, what is the where is my k; my k is going to be this long and my K plus dx is there ok.

So, what I am interested in is the number of solutions in this region and this region has got a length of dk. So, how many solutions exist in this region between k and k plus dk? So, that is my question, ok. So, k now is simply k x, there is no k x the k y or k z. So, k is simply k x and that is k plus dx a dk and this is K and the region between them has got a length of dk and we are trying to find out how many states exist between k and k plus dk.

Now, the number of states per unit cell was already this right. It was L x by pi per unit length. So, there are this many these many solutions per unit length and we want to know how many solutions in this region of dk, for a length of dk how many solutions exist and the answer is L x by pi dk.

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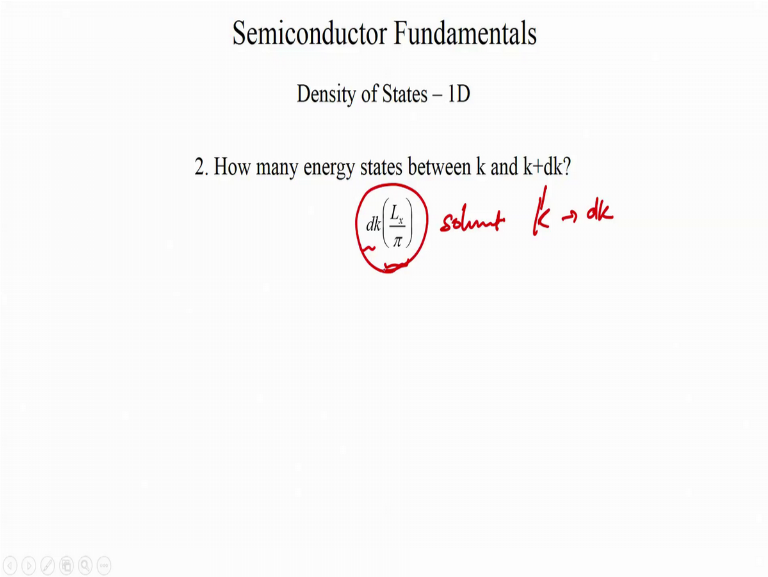

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Density of States – 1D

2. How many energy states between k and k+dk?

$$\boxed{dk\left(\frac{L_x}{\pi}\right)} \quad \text{solut} \quad k \rightarrow dk$$

So, there are decay into L x by pi states or solutions of Schrodinger's equation solutions in the region between k and k plus dk. So, that is the answer we are looking for. And now, the next question is of course, convert everything to energy, ok.

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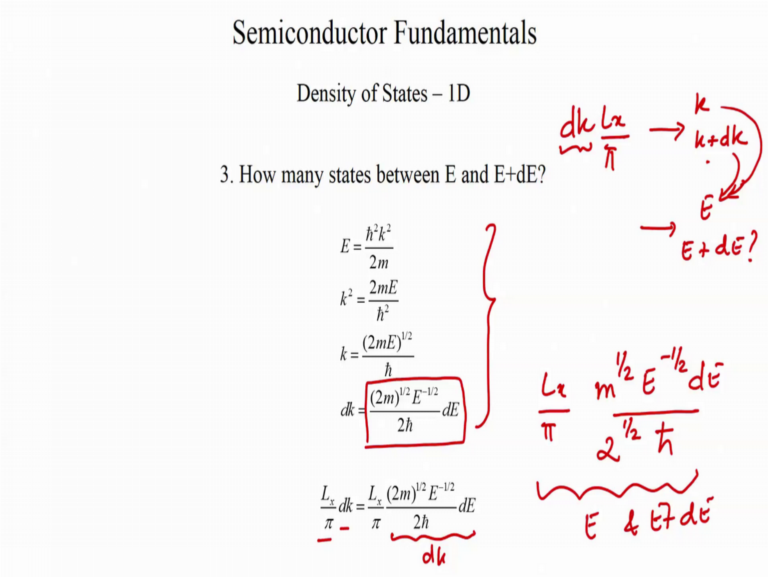

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Density of States – 1D

3. How many states between E and E+dE?

$$dk\frac{L_x}{\pi} \rightarrow \begin{matrix} k \\ k+dk \end{matrix}$$

$$\rightarrow \begin{matrix} E \\ E+dE? \end{matrix}$$

$$E = \frac{\hbar^2 k^2}{2m}$$

$$k^2 = \frac{2mE}{\hbar^2}$$

$$k = \frac{(2mE)^{1/2}}{\hbar}$$

$$dk = \boxed{\frac{(2m)^{1/2}E^{-1/2}}{2\hbar}dE}$$

$$\frac{L_x}{\pi}dk = \frac{L_x}{\pi}\underbrace{\frac{(2m)^{1/2}E^{-1/2}}{2\hbar}dE}_{dk}$$

$$\underbrace{\frac{L_x}{\pi}\frac{m^{1/2}E^{-1/2}dE}{2^{1/2}\hbar}}_{E \; \& \; E+dE}$$

So now, we know that there are d k L x by pi solutions between k and k plus dk. Now, how many solutions exist between E and E plus d E. So, we convert all the terms from k space to E space to energy E and we use the same set of relations ok. The only thing is this time we only have a decay term ok. That is, so, dk is related to E in this manner.

So, L x by pi dk is simply L x by pi into this particular term here which is your dk. And therefore, there are these many solutions, ok. So, there are L x by pi into we can cancel some terms of. So, you have m power half E to the power minus half d E by 2 power half h bar. So, you have these many solutions present between E and E plus d.

Now, of course, the next question is, we want to get to per unit length therefore, how many solutions per unit length ok.

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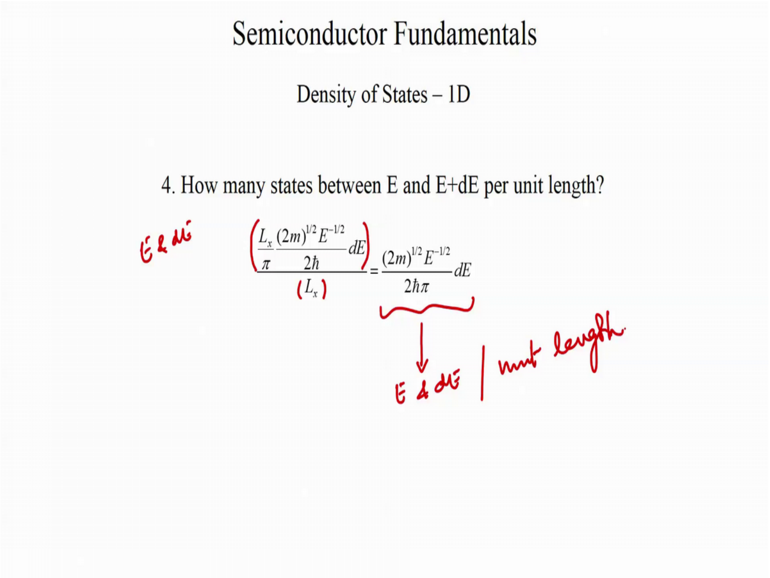

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Density of States – 1D

4. How many states between E and E+dE per unit length?

E & dE

$$\frac{\left(\frac{L_x}{\pi}\frac{(2m)^{1/2}E^{-1/2}}{2\hbar}dE\right)}{(L_x)}=\underbrace{\frac{(2m)^{1/2}E^{-1/2}}{2\hbar\pi}dE}$$

E & dE | unit length

So, we have these many solutions between E and E plus d E and the pull and the length of the box is L x. Therefore, that divided by this is the number of solutions between E and E plus d E per unit length of your semiconductor and finally, to get to the density of states, we need to get a per energy count, ok.

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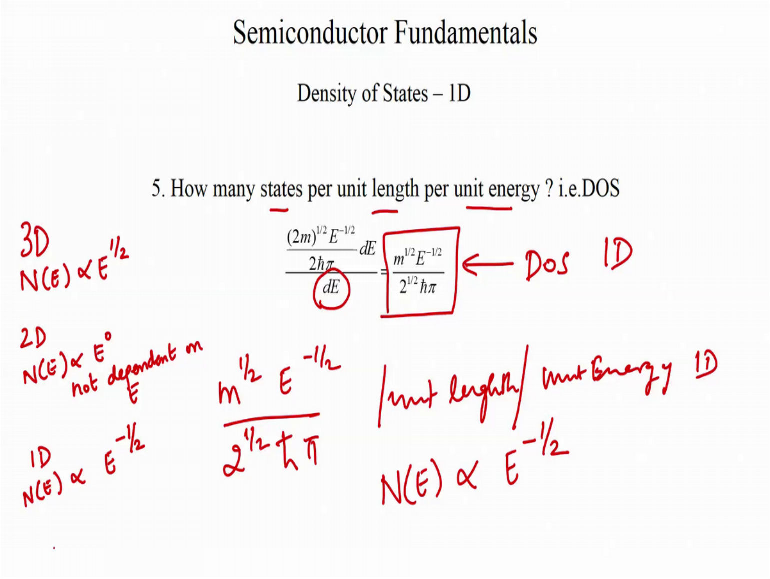

Semiconductor Fundamentals

Density of States – 1D

5. How many states per unit length per unit energy ? i.e.DOS

$$\frac{\frac{(2m)^{1/2}E^{-1/2}}{2\hbar\pi}dE}{dE} = \boxed{\frac{m^{1/2}E^{-1/2}}{2^{1/2}\hbar\pi}} \leftarrow \text{DOS 1D}$$

3D

$N(E) \propto E^{1/2}$

2D

$N(E) \propto E^{0}$ not dependent on E

1D

$N(E) \propto E^{-1/2}$

$$\frac{m^{1/2}E^{-1/2}}{2^{1/2}\hbar\pi}$$

/unit length/ unit Energy 1D

$N(E) \propto E^{-1/2}$

So, how many states are present per unit length per unit energy? So, we once again divided by d E, we get rid of the d E term and you have this little answer here which is the density of states for a 1D material. So, there are m power half E to the power minus half 2 power half, there is a square root of 2 h bar pi states per unit length per unit energy in a 1D material.

So, how does the density of states vary with energy for a 1D material? It varies as E to the power minus half. So, you see a nice trend here. So, for a 3D material you saw that the density of states was proportional to E to the power half for a 2D material the density of states was proportional to E to the power 0 or in another sense, in other words, it was not dependent on E. And for a 1D material, the density of states is proportional to E to the power minus half, so, that is how it varies from 3D to 2D to 1D.

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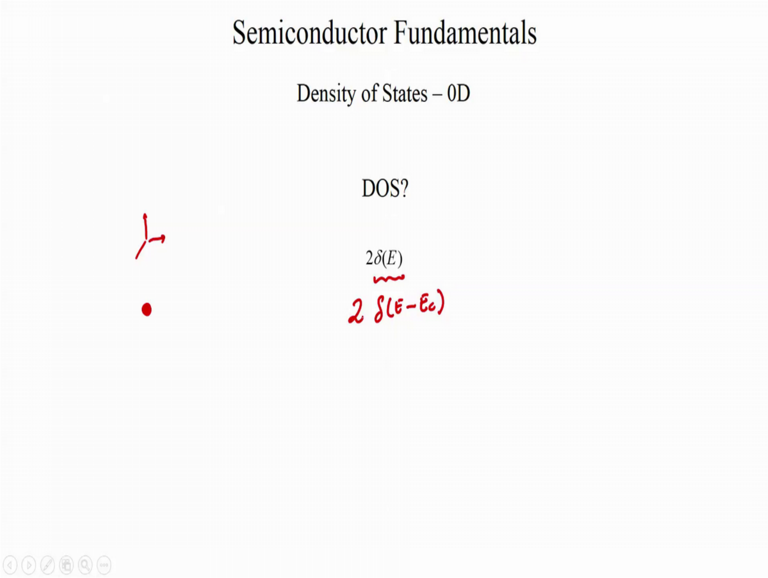

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Density of States – 0D

DOS?

$2\delta(E)$

$2\delta(E-E_c)$

So, what is the density of states for a 0D material. For a 0D material, you have neither a set of points in k x nor k y nor k z. You only have one solution, one probably just 1 point in k space. And therefore, your density of states is simply a direct function, ok. You have states at one particular energy. So, if you wish this could be written as at one particular energy and you have say 2 spins and therefore, that is permitted, ok.

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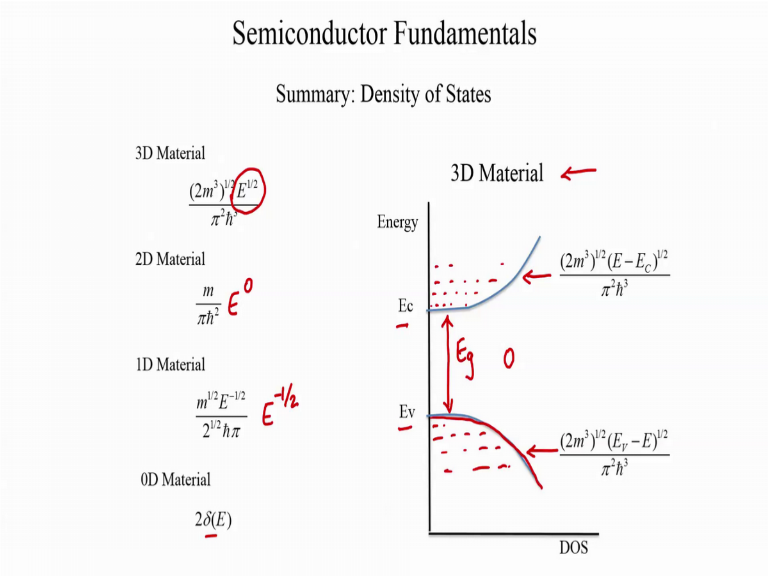

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Summary: Density of States

3D Material

$$\frac{(2m^3)^{1/2}E^{1/2}}{\pi^2\hbar^3}$$

2D Material

$$\frac{m}{\pi\hbar^2} \quad E^0$$

1D Material

$$\frac{m^{1/2}E^{-1/2}}{2^{1/2}\hbar\pi} \quad E^{-1/2}$$

0D Material

$$2\delta(E)$$

This is finally, summarize the you know the concepts on the density of states and we looked at 3D materials; wherein we found that the density of states depended on the E to the power half, that is, it varied with the square root of energy for 2D materials. The density of states did not vary with energy at all. So, you could call it as it varying as E to the power 0. For 1D materials it varied with E to the power minus half and for 0 D materials which are you know similar to quantum dots it is just a delta function of the energy. There is just 1 or 2 energy states and the 2 is (Refer Time: 10:18) for the 2 spins.

Now, in this course, we will focus mostly on 3D materials, ok. So, and therefore, this is the density of states picture that is of great use to us. So, we have our semiconductor which has got a valence band and it is got a conduction band. There is an energy gap, ok. And that that is called E g and then you have states in the conduction band above the conduction band edge and you have states below the valence band edge, ok. And the density of states which is the number of states per unit volume per unit energy varies as E minus E c to the power half; so, this is like a square root behavior.

Similarly, here too you have a square root behavior with the density of state distribution and inside the gap as long as the semiconductor is very clean, as long as it is the perfect crystal, you have no states. So, if you look at this density of states here, it is 0. And you have a density of states varying as the square root of energy above and below these conduction band edges. So, this pictures of great use to us throughout this course and it would sort of help gather some amount of intuition, you know it is a it is helpful to you know visualize this very clearly.

So moving on so, what we have counted for now is the number of states available and the electrons can now start occupying these states, ok. And, what we are trying to get at ultimately is the number of electrons that are available for conduction because, ultimately we want to measure the current voltage characteristics. Now, we know the number of states, but we now need to find out as to what the probability of occupancy of those states are.

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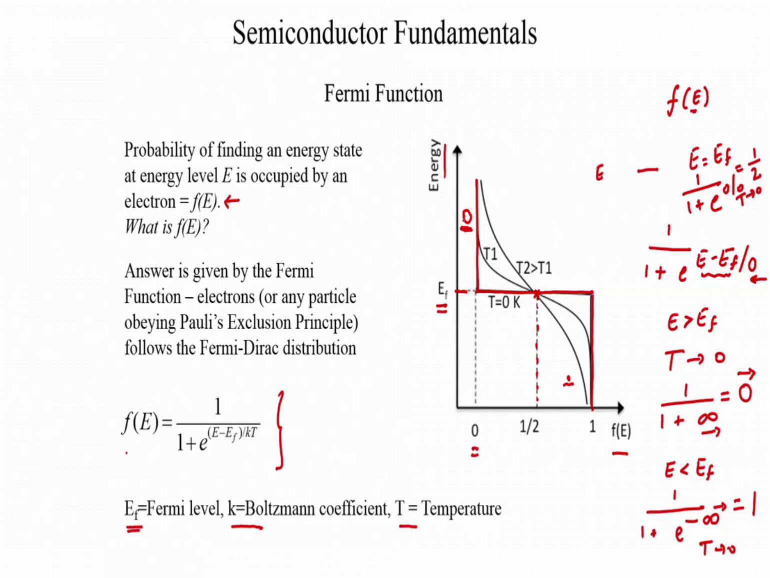

Semiconductor Fundamentals

Fermi Function

Probability of finding an energy state at energy level *E* is occupied by an electron = *f(E)*. ←
What is f(E)?

Answer is given by the Fermi Function – electrons (or any particle obeying Pauli's Exclusion Principle) follows the Fermi-Dirac distribution

$$f(E) = \frac{1}{1+e^{(E-E_f)/kT}}$$

E_f=Fermi level, k=Boltzmann coefficient, T = Temperature

$f(E)$

E — $E = E_f$ $\frac{1}{1+e^{0/0}} = \frac{1}{2}$ $T \to 0$

$\frac{1}{1+e^{E-E_f/0}}$ ←

$E > E_f$

$T \to 0$

$\frac{1}{1+\infty} = \vec{0}$

$E < E_f$

$\frac{1}{1+e^{-\infty}} \to = 1$

$T \to 0$

So, the probability of finding an energy state, so, which is basically our solutions to Schrodinger a solution Schrodinger's equation. So, if you have a state at some energy E, there is a certain probability that this state is occupied with an electron and that probability is defined by this variable; I mean or the symbol f of E. It depends it is a function of energy. So, it depends upon the energy and we call it f of E.

So, what is f of E? So, since electrons are fermions which means that, these are particles that obey the Pauli's exclusion principle you find that f of E is defined by something called as a Fermi Dirac distribution, ok. And it turns out that f of E is defined by this function here which is 1 by 1 plus E to the power E minus E f by K T in this function that you see your K is your Boltzmann's coefficient, T is the temperature and E f is something called as the fermi level.

So, it is an energy level which has got a special implication in semiconductors and it is called as the fermi level. So, it is a technical term and we will continue using this and you will typically use the symbol E f to denote the fermi level. Now, what do you mean by the fermi level? So, if you were to take this function ok, f of E and plot it here, ok. So, I have energy on the y axis and I have f of E on the x axis.

So, at T equal to 0, what is this function it is going to be 1 by 1 plus E to the power E minus E f by 0 ok. So, this means this is going to be a very large number, but depending on whether E is greater than E f or E is less than E f. This number could be very largely positive or it could be very largely negative, ok. So, suppose E is greater than E f ok. So, E f is some energy level. So, for now, let us just mark it somewhere let us just call that E f. So, this is the energy scale and let us say E is greater than E f which means that this quantity is positive and as the temperature approaches 0. This number is a very large positive number which starts heading towards infinity.

So, as T approaches 0, you have 1 by 1 plus a term approaching infinity ok. If E is greater than E f; so which means that this value is going to be heading towards 0 therefore, for all energies at T equal to 0, for all energies above E f, the value of the fermi function or f of E is 0. So, you can see that this is going to be the fermi function it is got a value of 0.

Now, the moment E becomes less than E f ok, you have 1 by 1 plus E to the power minus a very large number, ok. So, this number starts heading towards infinity as T tends to 0. So, this value now starts heading towards 1 because E to the power minus infinity is going to be 0 and you have 1 by 1 which is 1.

So, therefore, for all values of energy less than E f at T equal to 0 kelvin, the function takes a value f of E is equal to 1 and at E equal to E f. So, when you have E is equal to E f, ok. So, when you have E equal to E f, the fermi function has got a value of 1 by 1 plus E to the power 0 by 0. So, in some sense it is undefined so, we need to take limits. So, let us say T is heading toward 0.

So, you have 1 by 1 plus E to the power 0 which is 1 and therefore, this function tends to 1 by 2, ok. So, at E equal to E f at T equal to 0, you have the function actually somehow making the switch from the value of 1 to the value of 0. And therefore, you can say in the limit the value of this function is half, ok. So, this is the fermi function, this is the behavior of the fermi function at T equal to 0.

So, what does this say? Since f of E is the probability as to whether an energy state is occupied by an electron on f of E equal to 1 implies that the electron definitely occupies that state and f of E equals to 0 implies that the electron definitely does not occupy that state. So, what this picture tells you is that, at T equal to 0, any state that is below E f has got a guaranteed chance that an electron will occupy it. So, if an electron if a state exists here, it is definitely going to be occupied by an electron and at T equal to 0.

What f of E tells you is that if a state exists above E f, it is definitely not going to be occupied by an electron which means that the state is vacant or in other words that state is definitely occupied by a hole ok. So, this is the meaning of f of E. Now, as temperature increases, ok, so, this is what happens at 0, as T approaches 0 Kelvin, right. So, T approaching 0 Kelvin is T, T equal to 0 Kelvin is not possible, ok. That is, it is against the laws of thermodynamics.

So, let us say, let us take a more realistic case where you have a temperature T 1 that is greater than 0 ok.

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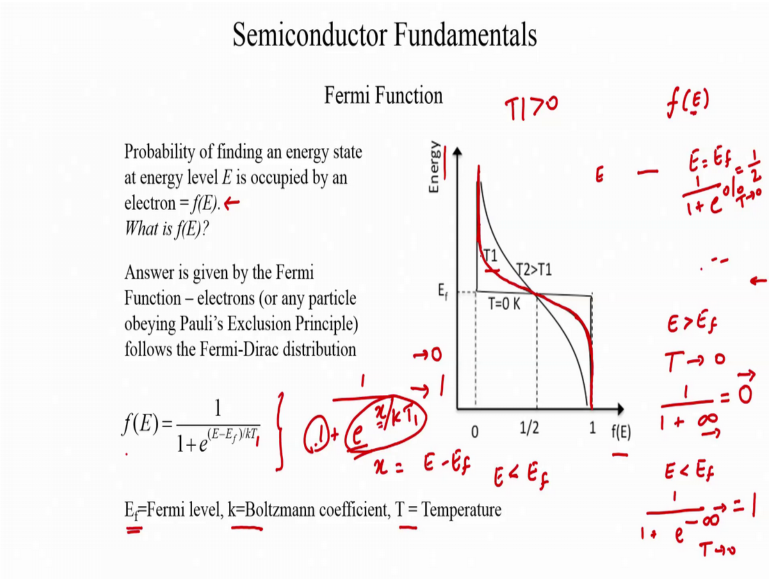

Semiconductor Fundamentals

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$$f(E) = \frac{1}{1+e^{(E-E_f)/kT_1}}$$

E_f=Fermi level, k=Boltzmann coefficient, T = Temperature

So now, what happens to f of E. So, you will find that if you were to plot for some temperature T 1, if you were to plot this function, you will find that it is got this particular shape. So, it is deviated from it is T equal to 0 shape and it is got a shape that looks like this.

So, what does this say? As E goes much less than E f ok so, 1 by 1 plus E to the power E minus E f becomes a significantly negative number, ok. So, I let us call it let us call it x for all practical purposes, just for the sake of for the sake of arguments here. So, as x tends to so, x is your E minus E f. So, as E becomes. So, as E becomes less than E f ok. If E becomes less than E f x becomes a negative number.

And if you have an exponent of a very largely negative number, it is going to be much less than 1 and therefore, this function tends to 1. So, at where as you head very far away from E f and below E f, the firm value of the function heads towards 1 asymptotically and equivalently as you had above E f and very far away from E f, ok. So, as you had go to higher and higher energies, x is a positive number that is continuously increasing and it this term dominates this term and you will find that 1 by a very large number tends to 0 and therefore, this function here starts heading towards 0.

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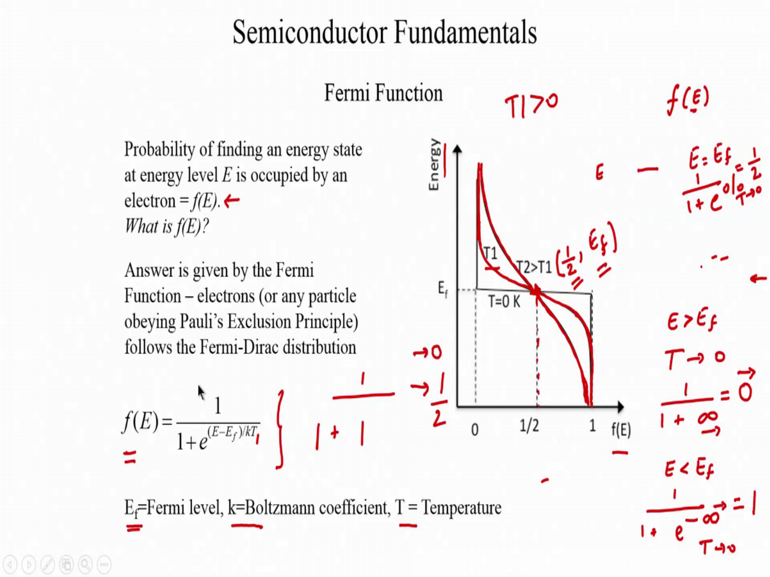

Semiconductor Fundamentals

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E_f=Fermi level, k=Boltzmann coefficient, T = Temperature

But at E equal to E f at E equal to E f, your x is equal to 0 ok. So, x is equal to 0 and which means that this term here is equal to 1 and therefore, at equal to E f the value of the function is 1 by 2. So, irrespective of the temperature, you will find that when my E is equal to E f the function takes a value of half or in other words the function crosses this point this coordinate of the x coordinate being half this plot.

If you plot energy versus f of E, you will find that for any temperature you will always the function will always cross this coordinate of half comma E f ok, that is E f as being the energy and half being the probability. So, at temperature T 2, the function deviates further, ok. It moves further away from the T equal to 0 condition and it sort of start begins to smoothen out.

So, you will see that the occupancy probability above E f begins to increase and the probability of non-occupancy below E f also begins to increase, ok. So, that is the nature of f of E and this nature is very important ok. So, we have identified the number of states defined by the density of states and what we are doing here is we are trying to find out what the probability is that those states are occupied by an electron, ok.

So, let us look at some of the key properties of this f of this probability, ok ao, which is called as the probability of occupancy of the electron defined by f of E.

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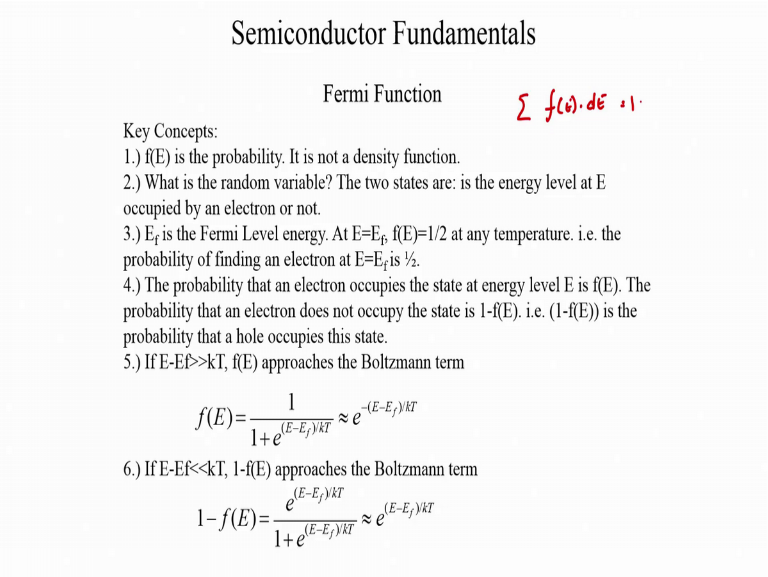

Semiconductor Fundamentals

Fermi Function

$\Sigma f(E) \cdot dE = 1.$

Key Concepts:

1.) f(E) is the probability. It is not a density function.

2.) What is the random variable? The two states are: is the energy level at E occupied by an electron or not.

3.) E_f is the Fermi Level energy. At $E=E_f$, f(E)=1/2 at any temperature. i.e. the probability of finding an electron at $E=E_f$ is ½.

4.) The probability that an electron occupies the state at energy level E is f(E). The probability that an electron does not occupy the state is 1-f(E). i.e. (1-f(E)) is the probability that a hole occupies this state.

5.) If E-Ef>>kT, f(E) approaches the Boltzmann term

$$f(E)=\frac{1}{1+e^{(E-E_f)/kT}}\approx e^{-(E-E_f)/kT}$$

6.) If E-Ef<<kT, 1-f(E) approaches the Boltzmann term

$$1-f(E)=\frac{e^{(E-E_f)/kT}}{1+e^{(E-E_f)/kT}}\approx e^{(E-E_f)/kT}$$

So, what are the key points, ok. So, these are some key elements to remember. So, f of E is the probability ok. Please do not mistake it for a density function. It is not a density function, ok.

So, if it were to be a density function then f of E d E would have to be the probability and which means that this summed over all the states should be equal to 1 which is clearly not the case. So, please do not mistake it to be a density function f of E is the probability.

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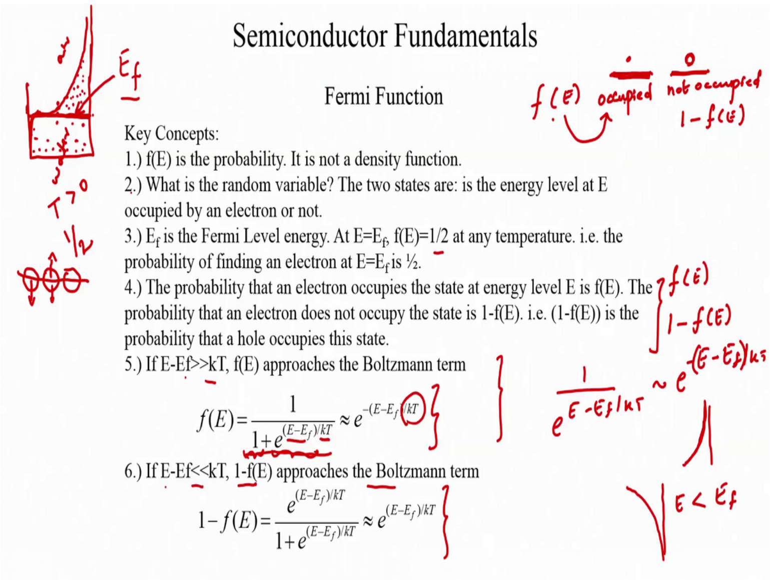

Semiconductor Fundamentals

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$$f(E)=\frac{1}{1+e^{(E-E_f)/kT}}\approx e^{-(E-E_f)/kT}$$

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$$1-f(E)=\frac{e^{(E-E_f)/kT}}{1+e^{(E-E_f)/kT}}\approx e^{(E-E_f)/kT}$$

And, if it is a probability, what is the random variable? The random variable is as to are the 2 conditions or the 2 states right, given an energy level is it occupied by an electron or is it not occupied by an electron, ok.

So, these are the 2 possibilities it is like a coin toss with a head and tail being the 2 outcomes. So, here you have 2 outcomes which is given an energy level is that energy level occupied by an electron. So, it is let us say occupied is one state and not occupied is the other state and the probability that it is occupied is given by f of E.

So, which implies that if f of E is the probability that it is occupied, what is the probability that is not occupied? It has to be 1 minus f of E ok. Because the probability of the summation of the probabilities of these 2 states occurring has to be equal to 1 it has to be either this or that they cannot be a third option. So, so that is the second point which is the random variable is the occupancy or non-occupancy.

Now, what is E f? E f is the fermi energy level as is the fermi level ok. So, it defines an energy at which f of E takes a value of half and we already discussed that. So, if you look at the value of this probability of occupancy, it takes a value of half at E equal to E f.

And we already saw that, the probability of non-occupancy that is given an energy state at a level E, the probability that is occupied by an electron is f of E. And therefore, the probability is not occupied by an electron is 1 minus f of E which is what which is simply saying that this is the probability that it is occupied by a hole ok, which is a vacancy.

So, f of E is the probability of probability that a state is occupied by an electron and 1 minus f of E is the probability that that state and at energy level e is occupied by a hole. And then finally, if E minus E f ok. So, if this term is much larger than k T ok. So, I will not say much larger. Let us say it is greater than 3 k T which is basically implying that this value is much greater than 1.

So, if the sum of the denominator can be approximated to 1 by e to the power E minus E f by K T, then this becomes the Boltzmann distribution ok. So, this is your Boltzmann distribution which is simply saying that, as the energy goes up, the probability that you will find an electron goes down exponentially. Now, if E minus E f is approaching values of K T, you cannot what he say simplify this denominator and you need to take into account the entire fermi function value.

Similarly if E minus E f is much less than k T ok so, this is this was the case when E minus E f is much greater than k T if E minus E f is much less than K T, 1 minus f of E which is basically the probability of occupancy by a whole approach approaches the Boltzmann distribution which says that the probability that a hole occupies an energy level E goes down exponentially as E goes much less than E f ok. So, that is basically these approximations.

And we will use these approximations quite a lot because it helps simplify a lot of the mathematics and the calculation now. So, this is all we have to say about the fermi function and the fermi level and the fermi distribution, but the. So, let me tell you a little bit more, let me give you some intuition about this. So, very nice way to imagine it ok it may not be the perfect way, but it is an intuitive way to imagine, it is to consider a bucket of water, ok.

So, let us say you have a bucket and you have got some water you fill these water molecules 1 by 1. All these water molecules are filled up and you have a water level that is still this point. Now, suppose I ask you the question so let us say this is all the air and that is the water as you see it. Now, suppose I ask you this question as to where is the air water interface, very intuitively somebody might point out to this layer here.

But does this mean that there are no water molecules above this. What is the probability that I will find a water molecule below this level at this level and above this level ok. Suppose, I ask you this question if the temperature T is greater than 0, you will definitely find some water molecules above this level. It is just that they are going to be much sparser as compared to the number of water molecules I will find below this level, ok.

So, this level can be imagined to be what is called as the fermi level ok. It is the in some sense, the highest energy level that could be occupied by all these species at T equal to 0 ok, where the probability of occupancy is still not 0 ok it is equal to half at equal to 0. Above this level at some temperature T greater than 0, you will find that the distribution of water molecules that is a concentration of water goes down exponentially. It is like the Boltzmann's distribution ok.

And below this, you have a much larger probability of finding water molecules and if you imagine the surface ok, let us say these are the water molecules in the surface. You can see that, it is you know it is shared between the air and water boundaries. So, you will sort of imagine that to be your you know probability being half, ok. So, it is a very hand wavy, but very useful analogy to think of what the fermi level is ok.

And we will come back to this bucket of water examples every once in a while. So now, if this is the fermi level, then what do you mean by the density of states with regards to this water bucket analogy? So, the best the best parallel to the density of states is the shape of the bucket, ok.

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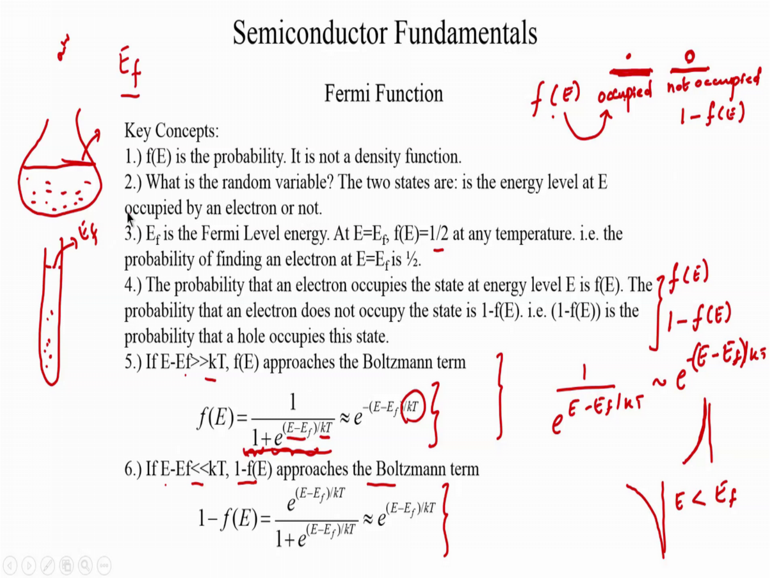

Semiconductor Fundamentals

Fermi Function

Key Concepts:

1.) f(E) is the probability. It is not a density function.

2.) What is the random variable? The two states are: is the energy level at E occupied by an electron or not.

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So, if you have a bucket that is shaped like that, you have a different density of states distribution and the water molecules need to fill all these states and the fermi level set sets itself at some particular point, ok.

On the other hand, if we have a bucket like this, it is got a very different density of states and for the same number of molecules the fermi level will be located elsewhere. So, this analogy cannot be used throughout I mean, it is just a very hand wavy visualization of what we are talking about, but nevertheless, it is useful. So, that is the key element about the fermi; fermi function.

Now, some of you might wonder as to where this f of E came about, ok.

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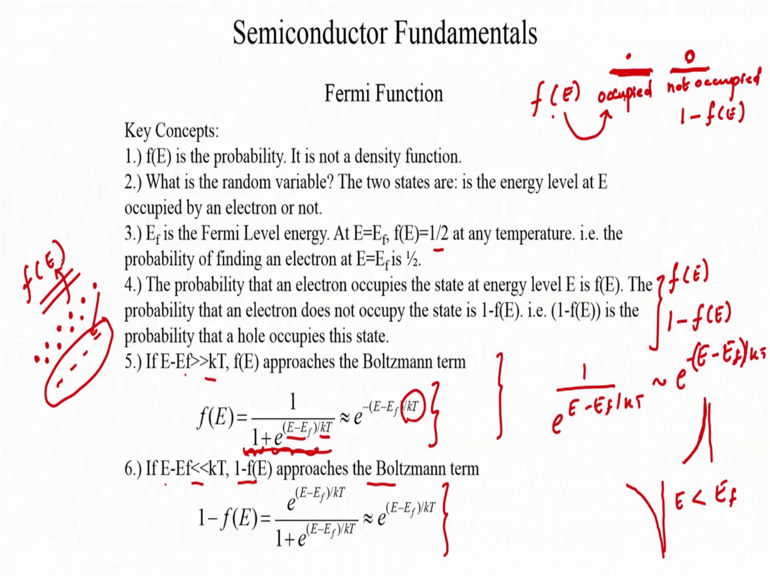

Semiconductor Fundamentals

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So, although it is not a part of this course, I can post some notes online, probably I will give you a link to where you can find these notes and essentially f of E comes about by finding out the best possible way to distribute say n electrons among s different states. So, these are n identical particles distributed in say a set of empty say states or seats, ok. So, that is, if you find the many methods to do it, the best possible way where you can maximize or you know find the best possible options is what defines your f of E ok. So, it is a bit of statistical mechanics, all right.

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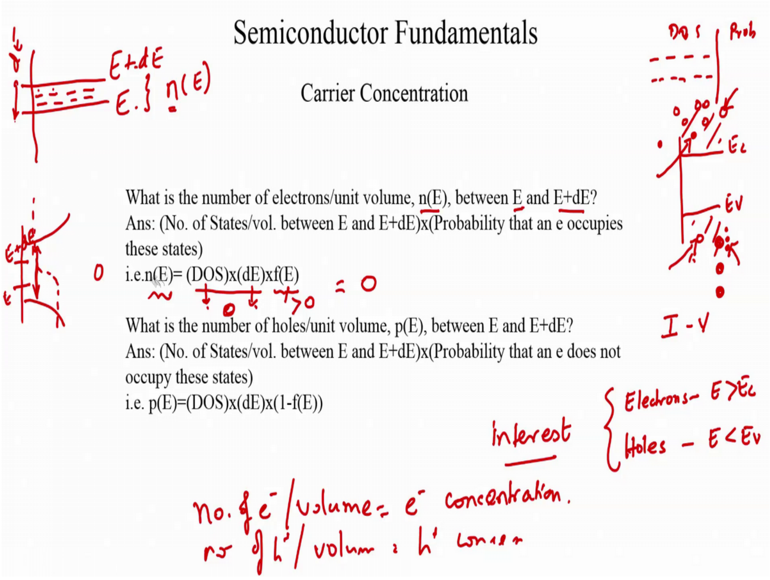

Semiconductor Fundamentals

Carrier Concentration

What is the number of electrons/unit volume, n(E), between E and E+dE?

Ans: (No. of States/vol. between E and E+dE)x(Probability that an e occupies these states)

i.e. n(E)= (DOS)x(dE)xf(E)

What is the number of holes/unit volume, p(E), between E and E+dE?

Ans: (No. of States/vol. between E and E+dE)x(Probability that an e does not occupy these states)

i.e. p(E)=(DOS)x(dE)x(1-f(E))

So, what do we have? We have the number o-f states which is defined by the density of states. So, if the density of states tells me how many states are there per unit energy per unit volume and we have the probability that those states are occupied by an electron. So, using these 2, we can now begin to count the carriers, ok. So, we are ultimately interested in counting the number of electrons available for conduction.

Most particularly, if this is the conduction band edge and this is the valence band edge, we are interested in the electrons in the conduction band and the holes in the valence band. We are not so interested in the electrons in the valence band. There are many of those because these are all trapped to these are all bound to all the different silicon atoms, ok. There are many of many electrons in the valence band and equivalently, there are many holes in the conduction band it is largely empty.

So, we are not interested in the holes in the conduction band and the electrons in the valence band. But, we are very interested in the electrons in the conduction band and the holes in the valence band because, these are the species that are going to you know constitute my current through these devices and ultimately, I want to get to a point where I can start to define my current-voltage characteristics and all these devices.

So, always remember, we are interested in the electrons in the conduction band for E greater than E c and holes in the valence band; that is E less than E v. This is of interest to us and we want to establish a method to count these, ok. So, how do we count, how do we identify the number of electrons? So more particularly, the number of electrons per unit volume so, number of electrons per unit volume which we will call as the electron concentration, ok. It is not per unit area right now, it is per unit volume and the number of holes per unit volume that we will call as the hole concentration.

So, how do we identify these 2 ok. So, if you ask yourself this question, what is the number of electrons per unit volume which we will define by the symbol n of E lying between E and E plus d E. So, I have 2 energy levels ok and this energy level is E that energy level is E plus d E and we want to know how many electrons are present between these 2 and we will call that as n of E. It is a function of energy. If this E changes, the n will might change.

So, the answer is very clear. So, you need to first know how many states are there for the electrons to occupy ok. If there are no states, there can be no electrons because the electrons have to have a state to occupy because these states are essentially the solutions of Schrodinger's equation. So, you need to identify the number of states per unit volume per unit energy or let us say the number of states per unit volume between n E E plus d E, which is nothing part the density of states in to the density of states is the number of states per unit volume per unit energy.

But, since we are interested in E between the number of states between E and E plus d E, so, this energy gap is has got a value d E. So, it is density of states into d E which the number of states between E and E plus d E per unit volume. So, I know the count of the number of states, but it does not mean that all these states are occupied by electrons. Because, there is a certain probability that the electron will occupy these states and that probability as we saw is defined by f of E.

So, f of E into the number of states between E and E plus d E is my electron count, that is n of E. So, if you were to take a semiconductor, that is extremely clean ok which means that there are no states in the band gap ok. there are 0 states in the band gap. And let us say, I pick an energy level between E and E plus d E somewhere there at a temperature T greater than 0.

And we ask the question as to how many electrons are present between E and E plus d E. Well, the first answer is, the number of states that is the density of states into d E is 0 f of E is not 0 because f of E could be taking a value that is you know nonzero. So, that is a probability that an electron will occupy that energy level provided. There is a state for the electron to occupy it, ok.

But, since ours band gap is very clean and there are 0 states, even though f of E is not 0, the number of electrons to be found is 0. On the other hand, if let us say we want to find the number of electrons in some energy level lying in the conduction band, ok.

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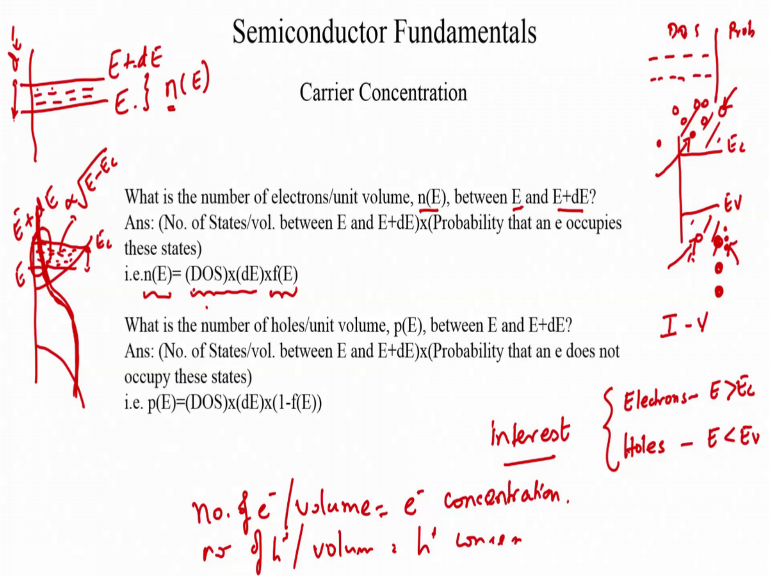

Semiconductor Fundamentals

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i.e.n(E)= (DOS)x(dE)xf(E)

What is the number of holes/unit volume, p(E), between E and E+dE?
Ans: (No. of States/vol. between E and E+dE)x(Probability that an e does not occupy these states)
i.e. p(E)=(DOS)x(dE)x(1-f(E))

So, let us say that is E c so, that is my E c and I want to know the number of electrons between E and E plus d E. So, what do I do? I need to identify the density of states into d E. So, here I do have a density of state distribution and I know that it varies as the square root of E minus E c is proportional to this.

So, we do have states. So, E the density of states into d E tells you the number of states per unit volume in this region and f of E is the probability that these states are occupied the probability is going to be low. But, it may not be 0 at T greater than 0 ok. At T equal to 0, it is definitely 0 but at T greater than 0 this probability is non zero.

Therefore, you do have some probability a very small probability that these states are occupied. And therefore, you can establish a carry account and it is these electrons that are going to help you in providing a current through the device, all right. So, next, if you ask the question what is the number of holes between E and E plus d E. So, which means, I need to identify p of E which is the number of holes per unit volume between E and E plus d E.

Then again, I get the density of states. I need to find the number of states available, that is the density of states into d E into the probability that these states are not occupied, f of E is the probability that states are occupied. But, the probability that the states are not occupied is 1 minus f of E. And therefore, this is my count of the number of holes per unit volume between E and E plus d E. So, this is a very useful relation to remember. It is again, it is good to develop intuition with regards to these concepts, all right.

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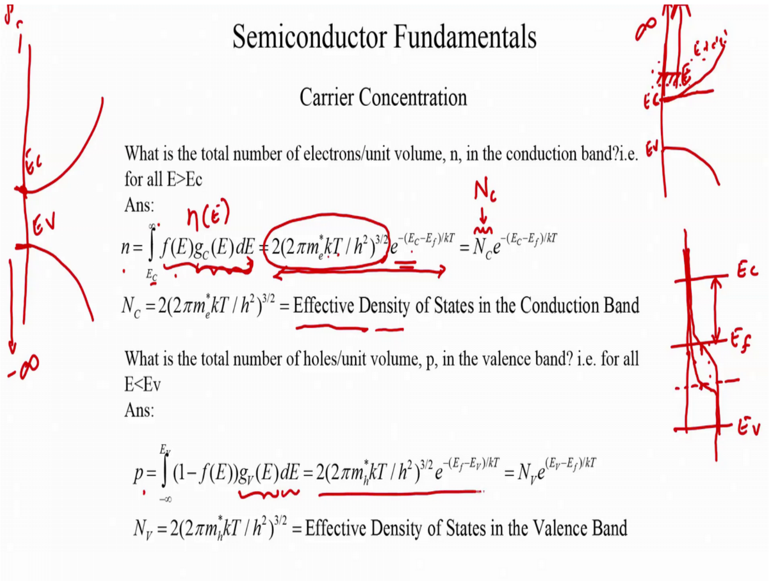

Semiconductor Fundamentals

Carrier Concentration

What is the total number of electrons/unit volume, n, in the conduction band?i.e. for all E>Ec

Ans:

$$n = \int_{E_C}^{\infty} f(E)g_C(E)dE = 2(2\pi m_e^* kT / h^2)^{3/2} e^{-(E_C - E_f)/kT} = N_C e^{-(E_C - E_f)/kT}$$

$N_C = 2(2\pi m_e^* kT / h^2)^{3/2}$ = Effective Density of States in the Conduction Band

What is the total number of holes/unit volume, p, in the valence band? i.e. for all E<Ev

Ans:

$$p = \int_{-\infty}^{E_V} (1 - f(E))g_V(E)dE = 2(2\pi m_h^* kT / h^2)^{3/2} e^{-(E_f - E_V)/kT} = N_V e^{(E_V - E_f)/kT}$$

$N_V = 2(2\pi m_h^* kT / h^2)^{3/2}$ = Effective Density of States in the Valence Band

So now, we are interested to find the current ok. And it is all the electrons above the conduction band edge that are going to contribute to the current and all the holes below the valence band edge that are going to contribute to the current. So, we are not only interested in the number of electrons between some energy level E and E plus d E ok.

But instead, we are interested in the number of electrons from E c that is the band edge, the conduction band edge, all the way up till infinite, ok. That is all the electrons present in the conduction band, because, all these are going to aid my conduction.

So, how do I collect or count all the electrons above the conduction band edge? It is straightforward. It is going to be the answer if obtained the previous example which is n of E, but summed over all the different energy levels, right. So, this is n of E and I am going to take a summation which is basically, I am going to take an integral from E c till infinite of f of E into the number of states between E and E plus d E.

So, if I were to sum all these electrons present in all these little bands from E to E plus d E, then, I will get my total number of electrons in the conduction band and if I make use of the Boltzmann approximation and I perform this integral, you will end up with a term that looks like this. After you apply the limits, we will find that the number of electrons from E c till infinite turns out to be this value in the case of a 3D semiconductor.

So, you see firstly, it depends on E to the power minus E c minus E f ok. So, if I have my E c here and if I have my E f here and let us say I have my E v here. This distance is important and the further the fermi level moves away from E c, the smaller the number of free electrons and that is clearly understood because, this is the probability ok. The probability varies like this and if I were to move the fermi level away, the probability is going to vary.

Because it is going to take a value of half at the fermi level so, my half will appear here. Instead of a taking a value half there, it will take a value of half at a much lower value of energy and it is going to take a much smaller probability of occupancy at the energy levels above E c. So, as E c minus E f increases, if that has that gap increases, my electron count is going to start coming down ok. The free electron count is going to start coming down.

Now, this term here is a constant, ok. Now, this pre factor to this exponential is a constant m of m E star is the effective mass of the electron, k is the Boltzmann's coefficient, T is the temperature, h is the Planck's constant and that particular term is typically denoted by a constant symbol ok. It is called N c which is the effective density of states in the conduction band.

So, this is not the density of states, it is not the number of states per unit volume per unit energy. It is the effective density of states which is basically the number of states per unit volume in all energy is from E c to infinite, ok. So, it is analog goes to that measure.

Now, similarly, if you want to find the total number of holes in the valence band because that is what is going to help me carry current, we find that the number of holes in the valence band is the summation of all the holes from E between E and E plus d E. But going from minus infinite to E v so, that is my E v that is my E c. To find the number of electrons, I needed to find all the electrons sitting from here all the way till infinite and to find the number of holes in the valence band, I am going to find all the hole sitting between E v and minus infinite.

So, I am going to integrate from minus infinite to E v 1 minus f of E. The density of states in the valence band times dE is going to give me a measure that looks like this. So, this is the total hole count in the valence band and what is that?

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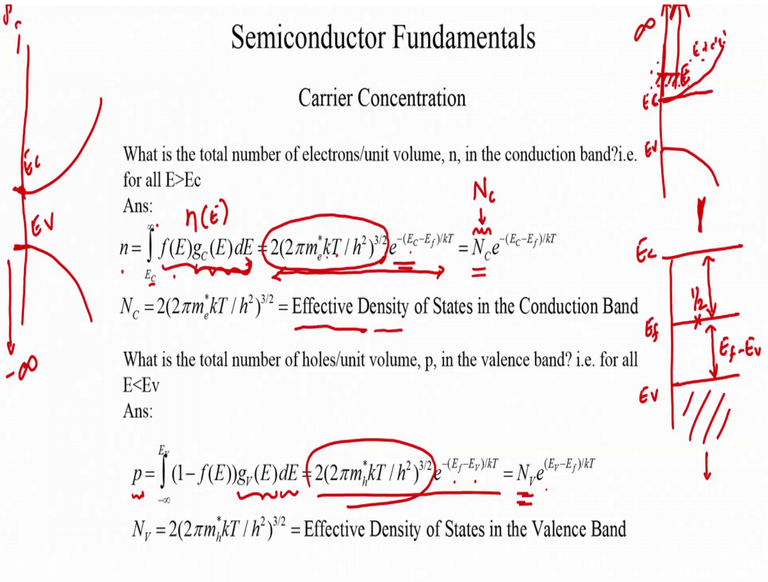

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$N_V = 2(2\pi m_h^* kT/h^2)^{3/2}$ = Effective Density of States in the Valence Band

So, just like in the case of electrons, I have my E v and I have my E f and I have my E c. We found that as this gap increases the number of electrons in the conduction band decreases.

Because the probability is got a lot lower. Similarly, if this gap E f minus E v ok so, this is E f minus E v begins to increase, the number of holes available in the valence band will start to decrease once again; why because, 1 minus f of E will start to get a lot smaller if this gap starts increasing. So, and all this happens because f of E has to take a value of half at E f. So, if I move E f around, I change the probability of occupancies in the states above E c and below E v.

Now, just like we defined an effective density of states in the conduction band, we can define an effective density of states in the valence band which is called as N v. So, the number of holes available for conduction are the number of holes between minus infinite to E v which is the effective density of states in the valence band which is essentially this constant here times E to the power E v minus E f by K T. So, that is the way you count the number of carriers available for conduction.