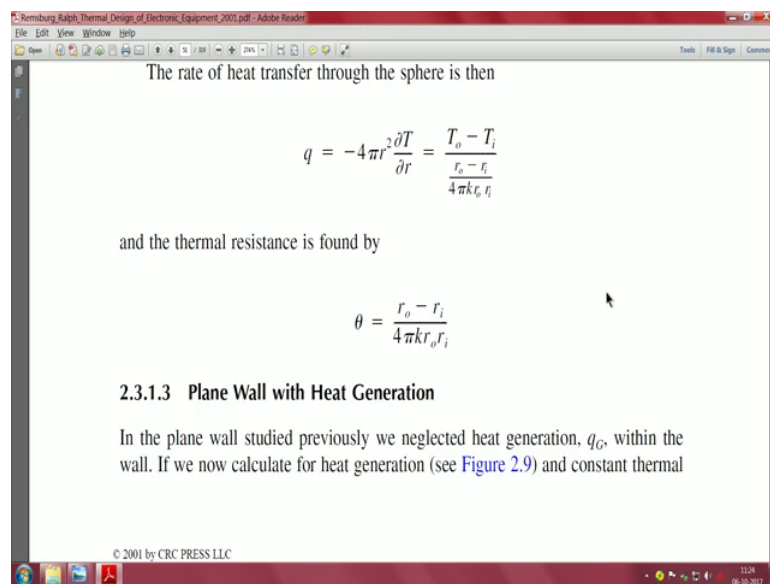


**Electronics Enclosures Thermal Issues**  
**Prof. N. V. Chalapathi Rao**  
**Department of Electronic Systems Engineering**  
**Indian Institute of Science, Bangalore**

**Lecture – 10**  
**Parallel combined effects**

Hello, me to start from where I what you call rush through yesterday. So, you see my again a copy from the book.

(Refer Slide Time: 00:27)



The rate of heat transfer through the sphere is then

$$q = -4\pi r^2 \frac{\partial T}{\partial r} = \frac{T_o - T_i}{\frac{r_o - r_i}{4\pi k_o r_i}}$$

and the thermal resistance is found by

$$\theta = \frac{r_o - r_i}{4\pi k_o r_i}$$

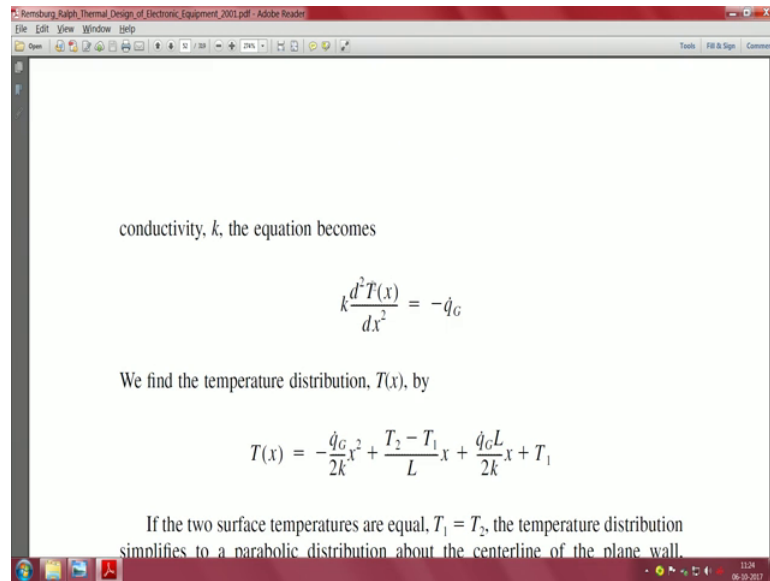
**2.3.1.3 Plane Wall with Heat Generation**

In the plane wall studied previously we neglected heat generation,  $q_G$ , within the wall. If we now calculate for heat generation (see [Figure 2.9](#)) and constant thermal

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In the plane wall we neglected heat generation within the wall, if we now calculate the heat generation the constant thermal conductivity.

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conductivity,  $k$ , the equation becomes

$$k \frac{d^2 T(x)}{dx^2} = -\dot{q}_G$$

We find the temperature distribution,  $T(x)$ , by

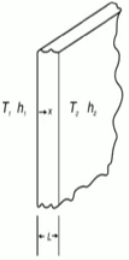
$$T(x) = -\frac{\dot{q}_G}{2k}x^2 + \frac{T_2 - T_1}{L}x + \frac{\dot{q}_G L}{2k}x + T_1$$

If the two surface temperatures are equal,  $T_1 = T_2$ , the temperature distribution simplifies to a parabolic distribution about the centerline of the plane wall.

The equation becomes and so on here you see here already know the second differential of the temperature with respect to the distance has come about..

So, the temperature distribution instantaneously can be given by big equation here. Now, do we need to what you call learn this as an equipment designer you probably do not need to know this; however, if you have configuration already in place like I have shown the drive which is taken from one of the commercial manufacturers, which was used in a small, what you call electric vehicle we have made such places the total amount of space available for us is seriously limited in those places maybe we need to consider these things here.

(Refer Slide Time: 01:47)

TABLE 2.1 Conduction in Plates and Walls <sup>2</sup>	
Description	Equations
Convectively heated and cooled plate 	Convectively heated and cooled plate $q'' = \frac{h_i \Delta T_{i-o}}{Bi_i + 1 + \frac{h_i}{h_o}}$ $\frac{\Delta T_{T_o - T_i}}{\Delta T_{T_2 - T_1}} = \frac{\frac{h_o L}{k} + 1}{\frac{h_i L}{k} + 1 + \frac{h_i}{h_o}}$

So, if you go back to my, what you call this slides here which are again as I said acknowledge schramberg from where they have taken, for different idealized situations things have already been solved. The subsequent section will talk about instead of just the properties of the heat sink where the heat is generated inside the equipment, typically on one end the cooling you know, let us just has been going on and then some amount of optimization has been done.

And the other end the circulatory also there improving and having understood these various mechanisms they are reducing the losses, especially heat losses. If you increase efficiency; obviously, the inefficiency which is a heat will be easily be reduced, say at one time if you have to take a let us say even today if you have to take a 500 watts bench supply linear bench supply its likely to be minimum of 3 u, that is height into approximately take it as 50 mm about to 150 mm. Actually it comes to 132 mm and the width is a 19 inch rack width which is typically I mean for 28 is outside typically around 400.

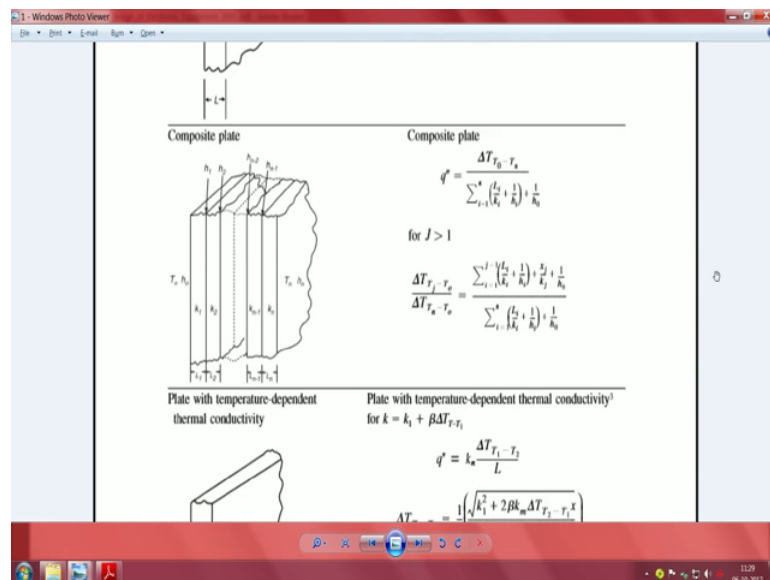
So, most of this bench supply is will come in a width of a brown 200 and a height about 132, earlier they were giving a maximum of maybe 200 or 100 watts, but the thing is it is a pure sine wave and inside we have a lot of heat dissipation and then since they were not understood just after the second world war, things were made big because it helped cooling.

Right now, if you are looking for a 100 or 200 va or 200 watts for supply has become very compact. In fact, to some of them come in a single u, how they achieved it probably faster switching speeds and they have filtering techniques and so on and so on. Net result is heat loss is minimized, somewhere else in the next chapter we will talk about how to calculate these things.

But again allow me to remind you that you better go through the book and try to solve the problems, otherwise we have enough individual items are already covered in other heat transfer lectures. You can find them directly in neither in our NPTEL in the regular lectures online or; obviously, other lectures and since this is only a general lecture and not intended for you to take any examination, I am just exposing you to these things so that you will know what it is.

Now, if you look at it individually every condition we know the correlations have already been made, the rate of what you call heat depends and how you are seen this now here, how heat takes place is already available in either text books or this sort of industry what you call manuals.

(Refer Slide Time: 05:41)



So, in the case of a composite plate same thing this whole thing has been given here saying in the case if you have plates with different thermal conductivity. So, you have a huge what you call equations just need to know that it is possible for you to make use of these things.

(Refer Slide Time: 06:01)

Plate with temperature-dependent thermal conductivity

Plate with temperature-dependent thermal conductivity<sup>3</sup>  
for  $k = k_1 + \beta \Delta T_{T-r_1}$

$$q'' = k_m \frac{\Delta T_{T_1-T_2}}{L}$$

$$\Delta T_{T_1-T_2} = \frac{1}{\beta} \left( \frac{\sqrt{k_1^2 + 2\beta k_m \Delta T_{T_1-T_2}}}{L - k_1} \right)$$

where

$$k_m = \frac{k_1 + k_2}{2}$$

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Similarly, yesterday we are showing you this.

(Refer Slide Time: 06:04)

Description	Equations
Thin rectangular plate on the surface of a semi-infinite solid	Thin rectangular plate on the surface of a semi-infinite solid <sup>4</sup> $q = \frac{kw \pi \Delta T_{T_1-T_2}}{\ln\left(\frac{b}{a}\right)}$
Infinite thin plate with heated circular hole	Infinite thin plate with heated circular hole for $T = T_1$ at $r = r_1$ and $r > r_1$ $\frac{\Delta T_{T_2-r_2}}{\Delta T_{T_1-r_1}} = \frac{K\left(\frac{b}{r_2}\right)}{K\left(\frac{a}{r_1}\right)}$

So, very simple equations about how, what is almost semi infinite solid, when this is semi infinite solid meaning after a little while there is no further temperature gradient. You understand the subsequent term when it becomes longer and longer and the radius becomes very high and the circumference is more you have very little temperature what you call variation. So, that is where you know they call it a semi infinite solid.

So, the heat if you input all these parameters you will get the directly the total amount of watts that can go inside. So, these are all you know derived via geometry. So, it is possible for you to evaluate similarly this is a semi infinite solid and this one is a heated circular hole, as I have pointed out earlier something called the stud mounted transistors. If any of you remember old Arab stir transistors typically which is 3, 3, 7, 5 and those things they approximate to this.

And why that I make those things because it is easier to mount and drilling a hole is easy and often it is tapped, tapped means internal threads are cut that ensures there is a whole surface here gets the temperature and since the start itself is made out of high conductivity copper, most of the thing will be available here same thing it is with diodes. Also when you have high power diodes this annular ring will be almost at the same temperature, you understood know, infinite thin plate with the heated circular hole same thing is above.

(Refer Slide Time: 08:42)

Infinite thin plate with heated circular hole

Infinite thin plate with heated circular hole for  $q$  at  $r = r_1$ , and  $r > r_1$

$$\frac{k \delta \Delta T_T - r_w}{q} = \frac{K_0 \left( \frac{Bi}{\delta} \right)}{2 \pi \left( \frac{Bi_1}{\delta} \right) K_1 \left( \frac{Bi_1}{\delta} \right)}$$

where:

$$B = \sqrt{Bi_1 + Bi_2}$$

$$T_w = \frac{T_1 + HT_2}{1 + H}$$

$$H = \frac{Bi_1}{Bi_2}$$

So, you see here all the terms that are covered there have been given here, saying at a given what you call radius here can you see here at a given radius here what is the temperature.

(Refer Slide Time: 08:58)

where:

$$B = \sqrt{Bi_1 + Bi_2}$$

$$T_m = \frac{T_1 + HT_2}{1 + H}$$

$$H = \frac{Bi_1}{Bi_2}$$


---

Finite plate with centered hole

Finite plate with centered hole<sup>5</sup>

$$q' = \frac{k\Delta T_{T_1 - T_2} \left[ \frac{wd}{2w} + \ln\left(\frac{w}{ar}\right) \right]}{2\pi}$$

(Continued)

As you keep going down you notice that the moment you make it a finite plate and dimensions are known various parameters change, these are useful for especially know people who are making a writing code themselves or individual cases.

(Refer Slide Time: 09:16)

**TABLE 2.1 (continued)**  
Conduction in Plates and Walls<sup>2</sup>

Description	Equations																		
Tube centered in a finite plate 	Tube centered in a finite plate <sup>4</sup> for $r < \frac{d}{10}$ $q' = \frac{2\pi k\Delta T_{T_1 - T_2}}{\ln\left(\frac{d}{2r}\right) - c}$ <table border="1"> <thead> <tr> <th><math>w/d</math></th> <th><math>c</math></th> </tr> </thead> <tbody> <tr> <td>1.00</td> <td>0.1658</td> </tr> <tr> <td>1.25</td> <td>0.0793</td> </tr> <tr> <td>1.50</td> <td>0.0356</td> </tr> <tr> <td>2.00</td> <td>0.0075</td> </tr> <tr> <td>2.50</td> <td>0.0016</td> </tr> <tr> <td>3.00</td> <td>0.0003</td> </tr> <tr> <td>4.00</td> <td><math>1.4 \times 10^{-5}</math></td> </tr> <tr> <td><math>\infty</math></td> <td>0.0</td> </tr> </tbody> </table>	$w/d$	$c$	1.00	0.1658	1.25	0.0793	1.50	0.0356	2.00	0.0075	2.50	0.0016	3.00	0.0003	4.00	$1.4 \times 10^{-5}$	$\infty$	0.0
$w/d$	$c$																		
1.00	0.1658																		
1.25	0.0793																		
1.50	0.0356																		
2.00	0.0075																		
2.50	0.0016																		
3.00	0.0003																		
4.00	$1.4 \times 10^{-5}$																		
$\infty$	0.0																		
Infinite plate with internal	Infinite plate with internal heat generation <sup>2</sup>																		

But right now, since this book itself as I said was probably around 30 years old 90 2000, 2000 yeah approximately 30 years old at that time this would useful information even today people can see it.

Tube centered in a finest plate. So, as things you know move when what you call the shape factor changes here, saying when it is a square how these things behave when it is thin and long how does this things behave. So, as we keep going down.

(Refer Slide Time: 10:03)

3.00	0.0003
4.00	$1.4 \times 10^{-5}$
$\infty$	0.0

**Infinite plate with internal heat generation**

**Infinite plate with internal heat generation<sup>2</sup>**

$$T = T_1, \quad x = 0$$

$$T = T_2, \quad x = L$$

$$\frac{\Delta T_{T_2 - T_1}}{\Delta T_{T_2 - T_1}} = X + \frac{PoX(1-X)}{2}$$

where  $X = \frac{D}{L}$

**Infinite plate with convection boundaries and internal heat**

**Infinite plate with convection boundaries and internal heat generation<sup>3</sup>**

You have things like with internal heat generation again same now where is the x y l.

(Refer Slide Time: 10:08)

**Infinite plate with convection boundaries and internal heat generation**

**Infinite plate with convection boundaries and internal heat generation<sup>3</sup>**

$$\frac{\Delta T_{T_1 - T_2}}{\Delta T_{T_1 - T_2}} = \frac{1 - Po\left(\frac{1}{Bi_2} + 1\right)}{1 + Bi_2 + H} + \frac{Po}{Bi_2} + \frac{Po}{2}(1-X^2)$$

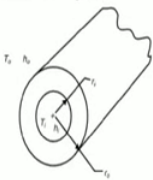

$$+ \frac{Bi_1 \left[ 1 + Po\left(\frac{1}{Bi_2} + 0.5\right) \right] (1-X)}{1 + Bi_1 + H}$$

where  $H = \frac{h_2}{h_1}$

Infinite plate with convention convection boundaries it internal so on.

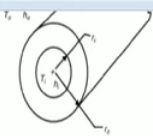
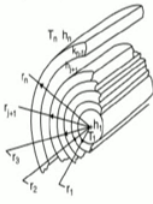



(Refer Slide Time: 10:21)

TABLE 2.2 Rods, Tubes, Cylinders, Disks, Pipes, and Wires <sup>2</sup>	
Description	Equations
Infinite hollow cylinder 	Infinite hollow cylinder $q' = \frac{2\pi k \Delta T_{r_o - r_i}}{\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{Bi_o}}$ $\frac{\Delta T_{r_o - r_i}}{\Delta T_{r_i - r_o}} = \frac{\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{Bi_o}}{\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{Bi_o}}$ where $Bi_o = \frac{h_o r_o}{k}$
Composite cylinder 	Composite cylinder $q' = \frac{2\pi \Delta T_{r_o - r_i}}{\sum_{j=1}^{n-1} \left(\frac{1}{k_j}\right) \ln\left(\frac{r_{j+1}}{r_j}\right) + \sum_{j=1}^n \frac{1}{r_j h_j}}$ for $j > 1$

The handbook contains most of the conditions and the necessary equations for you to find out the rate of heat or the heat flux, which is at any given point how to calculate if you want it on the surface and so on.

(Refer Slide Time: 10:43)

	$\frac{\Delta T_{r_o - r_i}}{\Delta T_{r_i - r_o}} = \frac{\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{Bi_o}}{\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{Bi_o}}$ where $Bi_o = \frac{h_o r_o}{k}$
Composite cylinder 	Composite cylinder $q' = \frac{2\pi \Delta T_{r_o - r_i}}{\sum_{j=1}^{n-1} \left(\frac{1}{k_j}\right) \ln\left(\frac{r_{j+1}}{r_j}\right) + \sum_{j=1}^n \frac{1}{r_j h_j}}$ for $j > 1$ $\frac{\Delta T_{r_j - r_{j+1}}}{\Delta T_{r_o - r_i}} = \frac{\sum_{j=1}^j \left[ \frac{1}{k_j} \ln\left(\frac{r_{j+1}}{r_j}\right) + \frac{1}{r_j h_j} \right] + \frac{1}{k_{j+1}} \ln\left(\frac{r_o}{r_{j+1}}\right) + \frac{1}{r_o h_o}}{\sum_{j=1}^{n-1} \left[ \frac{1}{k_j} \ln\left(\frac{r_{j+1}}{r_j}\right) + \frac{1}{r_j h_j} \right] + \frac{1}{r_n h_n}}$ where $T_j$ is the temperature in the $j$ th layer
Insulated tube 	Insulated tube $q' = \frac{2\pi k \Delta T_{r_i - r_f}}{\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{Bi_o}}$ where

And then in the case of composite cylinders subsequently in the what you call description, what I spoke to you about wires and insulated devices and all that these things is you know they were given here where  $t_j$  is the temperature in the  $j$ th layer.

(Refer Slide Time: 11:04)

The slide contains the following content:

- Insulated tube (top left):** A diagram of a tube with multiple concentric insulation layers. The inner radius is  $r_0$ , the outer radius is  $r_n$ , and the thickness of the  $j$ -th layer is  $t_j$ . The temperature in the  $j$ -th layer is  $T_j$ .
- Equation for  $j > 1$ :**

$$\frac{\Delta T_{T_j - r_1}}{\Delta T_{r_n - r_1}} = \frac{\sum_{i=1}^{j-1} \left[ \frac{1}{k_i} \left( \frac{r_{i+1}}{r_i} \right) + \frac{1}{h_i} \right] + \frac{1}{k_j} \left( \frac{r_j}{r_i} \right) + \frac{1}{h_j}}{\sum_{i=1}^{n-1} \left[ \frac{1}{k_i} \left( \frac{r_{i+1}}{r_i} \right) + \frac{1}{h_i} \right] + \sum_{i=1}^n \frac{1}{h_i}}$$
- Insulated tube (middle left):** A diagram of a single-layer insulated tube with inner radius  $r_0$ , outer radius  $r_1$ , and insulation thickness  $t_1$ . The thermal conductivity of the insulation is  $k$  and the convective heat transfer coefficient is  $h$ .
- Equation for insulated tube:**

$$q' = \frac{2\pi k \Delta T_{T_1 - r_1}}{h \left( \frac{r_1^2}{r_0} \right) + \frac{1}{h_0}}$$
- where:**

$$k \ll k_{tube} \text{ and } Bi_k = \frac{h r_1}{k}$$
- Maximum heat loss occurs when  $r_0 = \frac{k}{h}$**
- Infinite cylinder with temperature-dependent thermal conductivity (bottom left):** A diagram of a cylinder with a temperature profile  $T(r)$  across its radius  $r$ .
- Infinite cylinder with temperature-dependent thermal conductivity<sup>3</sup> with:**

$$k = k_0 + \beta \Delta T_{T_1 - r_1}$$

$$k = k_0 \text{ at } r_0$$

$$k = k_1 \text{ at } r_1$$

$$2\pi k_0 \Delta T_{T_1 - r_1}$$

In the case of insulated tube, maximum heat loss occurs when  $r_0$  is equal to  $k$  by  $h$  and so on and so on like that infinite cylinder with temperature dependence because occasionally things change.

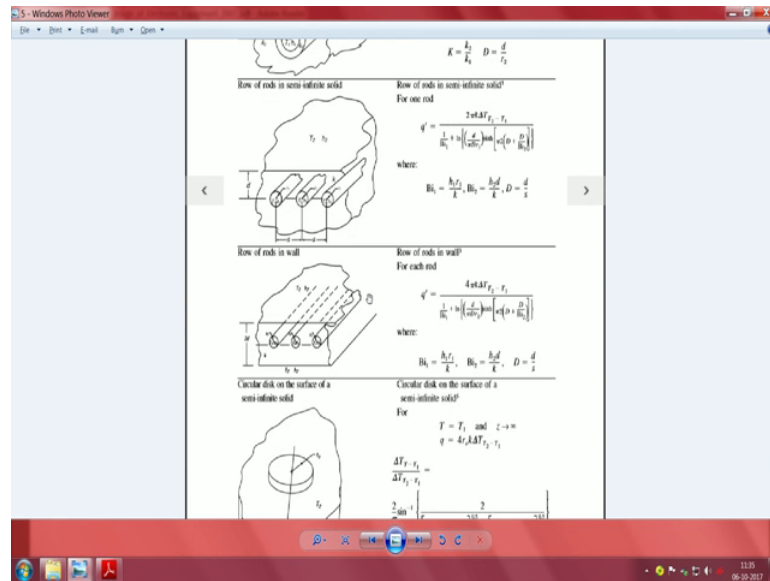
(Refer Slide Time: 11:25)

The slide shows TABLE 2.2 (continued) with the following entries:

Description	Equations
Pipe in semi-infinite solid	$q' = \frac{2\pi k_s \Delta T_{T_1 - r_1}}{Bi_1 \left[ \frac{1}{2} + \frac{1}{2} \left( \frac{r_1}{r_0} \right) \left( \frac{1}{2} + \frac{Bi_1}{2} \right) \right]}$ <p>where:</p> $Bi_1 = \frac{h_1 r_1}{k_s} \quad Bi_2 = \frac{h_2 r_1}{k_s}$ $K = \frac{k_s}{k_s} \quad D = \frac{d}{r_1}$
Row of rods in semi-infinite solid	<p>Row of rods in semi-infinite solid<sup>2</sup></p> <p>For one rod</p> $q' = \frac{2\pi k_s \Delta T_{T_1 - r_1}}{Bi_1 \left[ \frac{1}{2} + \frac{1}{2} \left( \frac{D}{2\pi r_1} \right) \ln \left( \frac{2r_1}{D} \right) \right]}$ <p>where:</p> $Bi_1 = \frac{h_1 r_1}{k_s} \quad Bi_2 = \frac{h_2 r_1}{k_s} \quad D = \frac{d}{r_1}$
Row of rods in wall	<p>Row of rods in wall<sup>3</sup></p> <p>For each rod</p> $q' = \frac{4\pi k_s \Delta T_{T_1 - r_1}}{\dots}$

So, we have here you know in a semi infinite solids if they pipe is embedded as in the case of fluid cooling or alternatively, we have series of rods which are mounted row for rods in semi infinite solids in a wall.

(Refer Slide Time: 11:48)



So, best example I can give you in your common experience is I do not know if I have told you already saying where does such a thing occurred. Long ago if you remember refrigerators had a condenser not attach to the body, it was behind at the back and all the refrigerant after expansion used to go to these that condenser at the back because it was considered at that point of time that is the way to do it and so on, maybe manufacturing into maybe everything.

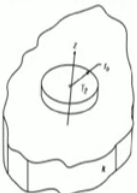

But right now, if you see things are different you will not you are not likely to find externally mounted condenser, often the condenser is built directly into the walls either the sidewalls may or may not include the rare wall because the space is you know made separately and the evaporator is probably built at the back and air is blown through that, in such conditions you end up with things like how to analyze row of rods in a wall.

So, even in the conventional what you call old refrigerators where you have the highest just or the what you call evaporator which is on the top thing which is called direct cooling. So, you have a box which is made out of sheet metal and these what is equivalent to these rods that small pipes are embedded in it sometimes they emboss the pattern and then the whole thing is pot weld it and then it forms a narrow channel, for only convenience sake I am taking it is a little like a rod, it is not a rod this is actually a tube in which fluid is passing and in only in the case of the what you call sidewalls even

there also it is a tube, but the outside surface of a tube can be treated as a rod, in those cases these it can be used effectively.



So, we have a circular disc on the surface of a semi infinite solid.

(Refer Slide Time: 14:24)

TABLE 2.2 (continued) Rods, Tubes, Cylinders, Disks, Pipes, and Wires <sup>2</sup>	
Description	Equations
Circular disk in an infinite solid 	Circular disk in an infinite solid <sup>1</sup> $q = 8r_0 k \Delta T r_1^{-1}$
Infinite hollow square rod 	Infinite hollow square rod <sup>1</sup> $q' = \frac{2\pi k \Delta T r_1^{-1} r_2}{\frac{k}{h_1 r_0} + \ln\left(\frac{1.08w}{2r_0}\right) + \frac{\pi k}{2h_2 w}}$

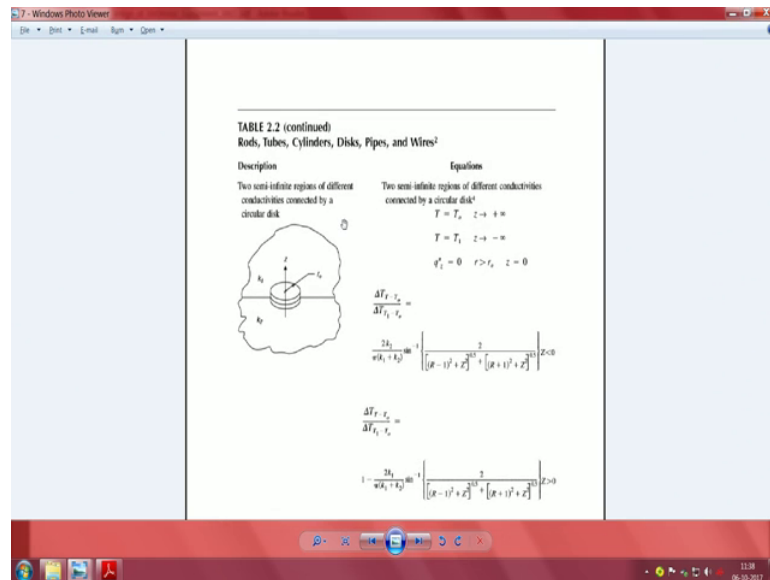
So, various correlations have been worked out. So, we have infinite hollow square roiled means just to make things you know as if things are easy to do, hollow square pipe vertical cylinder in a semi infinite solid.

(Refer Slide Time: 14:36)

	Infinite hollow square pipe <sup>1</sup> $q' = \begin{cases} \frac{2\pi k \Delta T r_1^{-1} r_2}{0.785 \ln\left(\frac{w}{d}\right)} & \frac{w}{d} < 1.4 \\ \frac{2\pi k \Delta T r_1^{-1} r_2}{0.93 \ln\left(\frac{w}{d}\right) - 0.0502} & \frac{w}{d} > 1.4 \end{cases}$
Vertical cylinder in a semi-infinite solid 	Vertical cylinder in a semi-infinite solid <sup>1</sup> $q = \left[ \frac{2D}{\ln\left[2D\left(1 + \frac{1}{Bi_0}\right)\right] + \frac{Bi_0}{D}} \right] \pi r_0 k \Delta T r_1^{-1} r_0$ <p>where <math>Bi_0 = \frac{hd}{k}</math>, <math>D = \frac{d}{r_0}</math></p>

Why are all these mentioned as somebody has said each case is unique and there is always this delta increment in knowledge. So, since this information is available, if you know the geometry of the existing materials it is very much possible for us to go through the all these equations.

(Refer Slide Time: 15:05)

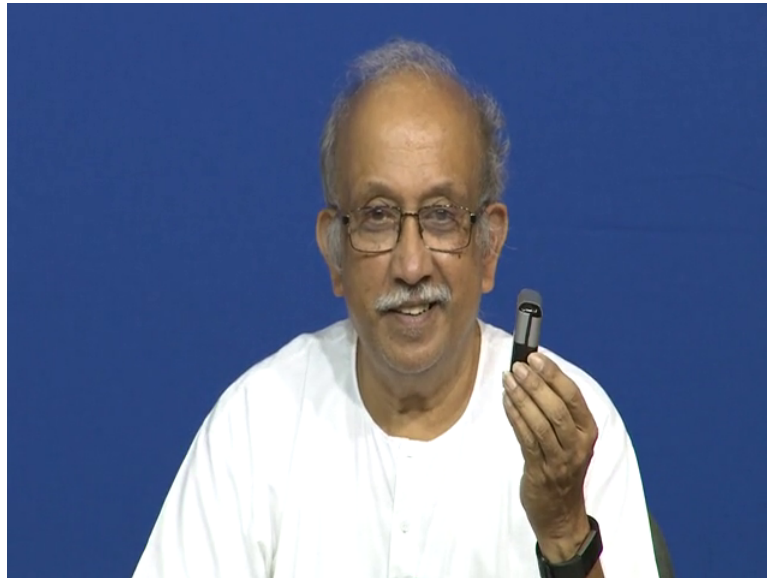


Semi infinite regions of different conductivity is connected and so on know..

All most known things heat flow between 2 rods in an insulated infinite 5 plate, where it is likely to face these conditions. One of the things I can think of is probably when you are talking about the peltier junction, if you remember in the first introductory lecture I showed you the peltier what you call a small heater come cooler, what we are doing there. You will see peltier is usually a 30 mm by 30 mm square understand, know the peltier device is only a 30 mm square and that is attached to a plate and one side it is cold and one side it is hot. Relative of course, instead of saying cold at the higher temperature and lower temperature and the reverse a current, there things like this are very useful contrary to intuitive reasoning it is just a stick a peltier on something it does not cool by itself, in spite of your giving all the current as specified by it because one other fine print is it is with respect to one surface to the other surface and. Secondly, there should not be any heat transfer between the cold and hotter surfaces low and higher temperature.

Only when it is probably insulated and so on and so on, it is possible for us to transfer these things across, I am just for convenience sake I am showing you a box which is actually it belongs to my hearing aid.

(Refer Slide Time: 17:17)



So, if this were to be the peltier device they create a bridge, understand you know a bridge which goes a little and then is a typically like a prism since slightly different, this bridge inside here holds the device to be cooled. So, in case you have a low noise amplifier or you need in mic in our case the biomedical thing will be part of the colder side or one of the load side I will call it.

This side this bridge forms part of the so called heat sink. So, you might have fins, I will put it here fins that are extended in this direction, now comes to the recritical thing this thing is hot, this thing is cold there should not be any short circuiting between these 2. So, a lot of insulation has to be provided.

Typically after the more the module is I am sorry after the prototype is built, the orifices are the spaces formed using polyurethane foam and if you want to make it even better it is usually split into heat sink is side a separate, load side is separate and now decide what to do with it.

So, in some places the load is kept in the middle and you have heat dissipater this side, heat dissipater this side seen that from both the think and then you have a back to back

peltier. So, this can probably contain peltier with hot this side, cold this surface again peltier here hot the side cold surface. So, at the bottom since this is cold you can now add whatever the load you want here, you seen that you know a back to back peltier.

In understanding or modeling these devices this heat transfer equations which are there can be used, I suggest you go back to any of these suppliers catalogs and see how the peltier modules are and how best to make them and then. In fact, I have seen some where they 4 peltier you know modules are fit in all the directions and you have the load inside, not a novelty depends on the criticality.

But overall the what you say efficiency of the system is poor, it is very poor compared to any phase change, typically if you have heat pipes a much better solution can be made heat pipes are again passive little orientation issue is there, but basically their passive devices though inside there is a phase change and all that you do not need to do anything in it work seems to work naturally.

So, depending on every ambient is what you call cool and then you have heat generated it is easier to take away only problem is we cannot go easily below ambient and all that depending on the working fluid. So, in spite of the best choice of working fluid and all that unless you keep the other surface cooled by probably refrigerant or something or even spraying nitrogen, liquid nitrogen not easy to use them, but for a large extent in case you want a simple passive cooling probably a heat pipe that is better than trying to use this peltier of things.

But all of them have one thing in common this maps. So, if you see here again once again coming back to n this point the issue being different conductivity is connected by a circular disks, where do you have this conductivity problem because in between you may end up with small thin washers, long long ago we were using. So, called mica washer with or without heat sink compound later on we have come to polyester or some other thing and where possible, now silicon compounds are used in place of these wet devices.

(Refer Slide Time: 22:14)

The slide displays the following content:

$$1 - \frac{2k_1}{\pi(k_1 + k_2)} \sin^{-1} \left\{ \frac{2}{[(R-1)^2 + Z^2]^{0.5} + [(R+1)^2 + Z^2]^{0.5}} \right\} \quad Z > 0$$

$$q = \left( \frac{4r_1 k_1 k_2}{k_1 + k_2} \right) \Delta T_{r_2 - r_1}$$

where  $Z = \frac{z}{r_o}$ ,  $R = \frac{r}{r_o}$

---

Heat flow between two rods in an insulated infinite plate

Heat flow between two rods in an insulated infinite plate<sup>3</sup>

$$q' = \frac{2\pi k \Delta T_{r_1, r_2}}{\frac{\pi}{w} + \ln\left(\frac{w}{\pi r}\right)}$$

So, the what you call sometimes we have this, this is typically you know the case where I was talking to you about in case you have a hot and cold thing, how the short circuit work works and so on. Infinite cylinder with convection boundary and internal heat generation, probably a heater like what you have seen your soldering iron.

(Refer Slide Time: 22:27)

**TABLE 2.2 (continued)**  
Rods, Tubes, Cylinders, Disks, Pipes, and Wires<sup>2</sup>

Description	Equations
Infinite cylinder with convection boundary and internal heat generation	Infinite cylinder with convection boundary and internal heat generation <sup>3</sup> $\frac{\Delta T_{r_1 - r_2} k}{q_o r_o} = 0.25 \left( \frac{2}{Bi} + 1 - R^2 \right)$ where $Bi = \frac{hr_o}{k}$ , $R = \frac{r}{r_o}$
Hollow infinite cylinder with convection boundary on outside surface and internal heat generation	Hollow infinite cylinder with convection boundary on outside surface and internal heat generation <sup>3</sup> with $q_r^* = 0$ and $r = r_1$ $\frac{\Delta T_{r_1 - r_2} k}{q_o r_o} = 0.25 \left[ \frac{2}{Bi} (1 - R^2) + 1 - R^2 + 2R_o^2 \ln R \right]$ where $R = \frac{r}{r_o}$ , $Bi = \frac{hr_o}{k}$ , $R_o = \frac{r_o}{r_o}$

Hollow infinite cylinder with convection and outside surface and internal heat generation all possible variants here.



(Refer Slide Time: 22:44)

The slide displays four cases of heat transfer in hollow cylinders with internal heat generation:

- Case 1:** Hollow infinite cylinder with convection boundary on outside surface and internal heat generation. Conditions:  $q_r^* = 0$  and  $r = r_1$ . Equation:  $\frac{\Delta T_{r_1 - r_2}}{q_0 r_1^2} = 0.25 \left[ \frac{2}{Bi} (1 - R_1^2) + 1 - R^2 + 2R_1^2 \ln R \right]$ . Where  $R = \frac{r_2}{r_1}$ ,  $Bi = \frac{hr_2}{k}$ ,  $R_1 = \frac{r_1}{r_2}$ .
- Case 2:** Hollow infinite cylinder with convection-cooled inside surface and internal heat generation. Conditions:  $q_r^* = 0$  and  $r = r_2$ . Equation:  $\frac{\Delta T_{r_1 - r_2}}{q_0 r_1^2} = 0.25 \left[ \frac{2}{Bi} (R_1^2 - 1) + 1 - R^2 + 2R_1^2 \ln R \right]$ . Where  $R = \frac{r_2}{r_1}$ ,  $Bi = \frac{hr_1}{k}$ ,  $R_1 = \frac{r_1}{r_2}$ .

You have the solution already by which you know it is possible for you to calculate the actual wattage, the rate of change of watts with the respective position.

(Refer Slide Time: 23:15)

The slide describes an electrically heated wire with temperature-dependent thermal and electrical conductivities:

- Description:** Electrically heated wire with temperature-dependent thermal and electrical conductivities.
- Equations:**
  - Electrically heated wire with temperature-dependent thermal and electrical conductivities<sup>17</sup> with  $T = T_0$ ,  $R = r_0$ .
  - $\frac{k_1}{k_0} = 1 + \beta \Delta T_{r_1 - r_2}$
  - $\frac{k_2}{k_0} = 1 + \beta_s \Delta T_{r_1 - r_2}$
  - $\frac{\Delta T_{r_1 - r_2}}{B} = R \left\{ 1 + \beta \left[ \frac{R}{8} - \frac{\beta_s r (2+R)}{16} \right] \right\}$
  - where  $B = \frac{k_0 r_0^2 \beta_0}{k_0 r_0^2}$ ,  $R = 1 - \frac{r^2}{r_0^2}$
  - $k_0$  = thermal conductivity
- Notes:**
  - Bi = Biot Number,  $AL/R$
  - $d$  = diameter, m
  - $h$  = heat transfer coefficient,  $W/m^2 K$
  - $k$  = thermal conductivity,  $W/m K$
  - $L$  = length, m
  - $q$  = rate of heat flow, W
  - $q'$  = linear heat flux,  $W/m$
  - $q''$  = rate of heat flux,  $W/m^2$
  - $q_0$  = volumetric heat flux,  $W/m^3$
  - $r$  = radius, m
  - $s$  = spacing, m
  - $w$  = width, m
  - $\beta$  = coefficient of thermal expansion ( $^{\circ}C^{-1}$ )

A more accurate equation accounts for the variable effect of  $r_0$  on the heat transfer coefficient  $h_{c,r}$ .

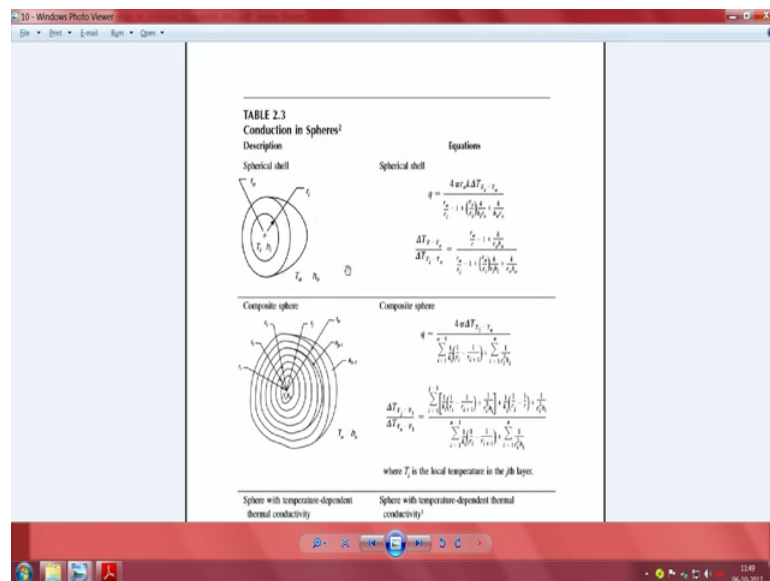
So, you can see how thing and then if you know that the isothermals can easily be plotted from here, we have this beautiful electrically heated wire with temperature dependent thermal and electrical conductivity you have seen this. We have this beautiful, all the  $q$  and  $q$  square and all that you know volumetric heat flux and area heat flux, then normal

linear heat flux all of them and then there is also a quotient of things are all have been solved well formulated and solved well.

Now, we go into this thing. So, at this point as an equipment designer because it was made for the equipment designers I feel it is enough for you to know that these are the equations that are used in your packages. So, if you take flotherm or so many if the other thing and then occasionally if you already have a pcb layout package, it is very much possible for you to use some of those correlations and then try to module your walls of the your what you call boxes and all that.

If time permits and if my students or colleagues are available I will see if one small demonstration can be made. So, let me move on to the next slide.

(Refer Slide Time: 25:00)



So, all sorts of, think about a spherical shell composite sphere where are we likely to find this composite sphere, anywhere imagine you would like to know how to keep something warm inside or how to keep something cool inside, both ways easiest thing to we have is what you call is spherical surfaces. So, you may be working with a device which is underwater, a typically a lamp. So, when you have this lamp and all these the electronics will be giving you heat.

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$\sum_{i=1}^n \frac{r_i - r_{i-1}}{k_i} + \frac{r_n}{h_1}$

where  $T_j$  is the local temperature in the  $j$ th layer.

---

**Sphere with temperature-dependent thermal conductivity**

**Sphere with temperature-dependent thermal conductivity<sup>3</sup>**

with

$$T = T_i \quad r = r_i$$

$$T = T_o \quad r = r_o$$

$$k = k_o + \beta \Delta T_{T-T_o}$$

$$q = \frac{4\pi r_o k_m \Delta T_{T_i-T_o}}{\frac{r_o}{r_i} - 1}$$

where  $k_m = \frac{k_o + k_i}{2}$

So, these are idealized surfaces where if you have a sphere and then if you have a composite sphere as you come down temperature dependent thermal conductivity which rare, but it does what you call happen.

(Refer Slide Time: 26:29)

**Sphere in an infinite medium**

**Sphere in an infinite medium<sup>3</sup>**

with  $T = T_i$  at  $r \rightarrow \infty$

$$q = 4\pi r_o k \Delta T_{T_i-T_o}$$

---

**Two spheres separated by a large difference in an infinite medium**

**Two spheres separated by a large difference in an infinite medium<sup>4</sup>**

$$q = \frac{4\pi r k \Delta T_{T_1-T_2}}{2\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}$$

for  $s > 2r$ , error  $\approx 1\%$

---

**Spherical shell with specified inside surface heat flux and internal heat generation**

**Spherical shell with specified inside surface heat flux and internal heat generation**

with  $T = T_o$  at  $r = r_o$

$$q_o'' = q_i'' \quad r = r_i$$

$$\frac{\Delta T_{T_o-T_i} k}{q_i'' r_i} = \frac{q_o'' r_o}{6q_i''} \left[ \frac{2(R-R_o)}{RR_o} + R_o^2 - R^2 \right] + \frac{R_o - R}{RR_o}$$

So, we have sphere in a semi infinite solid, sphere in a infinite medium it goes on like 2 spheres separated by large difference in a infinite medium, such things make sense probably at a very very small scale. When you are talking about chips or when we are talking about heat generating what you call configurations and whenever you have a

thermocouple and if you have self heating thermistors and so on. In those places we have a medium and then you have 2 devices.

So, all sorts what you call configurations spell specified with infinite I mean specified inside surface heat flux internal heat generation, infinite medium all these correlations have already been made.

(Refer Slide Time: 27:31)

**TABLE 2.3 (continued)**  
Conduction in Spheres<sup>2</sup>

Description	Equations
	$\frac{\Delta T r_o r_i k}{q_o r_o^2} = \frac{1}{3R} \quad r > r_o$
	where $Bi = \frac{hr_o}{k}$ , $R = \frac{r}{r_o}$

**Notes:**

$A$ = area, m <sup>2</sup>	$q$ = rate of heat flow, W
$Bi$ = Biot number, $hL/k$	$q''$ = rate of heat flux, W/m <sup>2</sup>
$d$ = diameter, m	$q_o$ = volumetric heat flux, W/m <sup>3</sup>
$h$ = heat transfer coefficient, W/m <sup>2</sup> K	$r$ = radius, m
$k$ = thermal conductivity, W/m K	$s$ = spacing, m
$L$ = length, m	$\beta$ = coefficient of thermal expansion (°C <sup>-1</sup> )

$q'' = q''_o + hq''_s$

And then, all the what you call parameters that have been used have been mentioned here.

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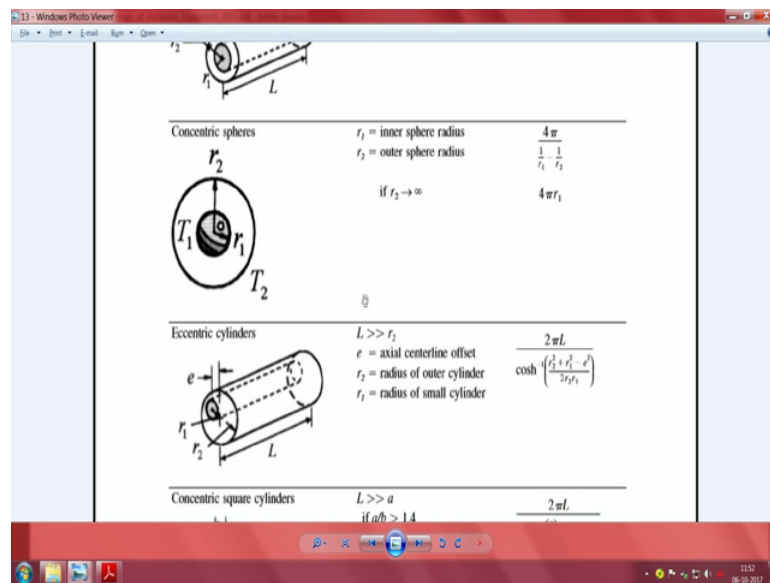
**TABLE 2.4**  
Shape Factors for Steady-State Conduction

Configuration	Restrictions	Conduction Shape Factor
Plane wall	$A$ = area $L$ = wall thickness	$\frac{A}{L}$
Concentric cylinders	$L \gg r_1$ $r_1$ = inner cylinder radius $r_2$ = outer cylinder radius	$\frac{2\pi L}{\ln(r_2/r_1)}$
Concentric spheres	$r_1$ = inner sphere radius $r_2$ = outer sphere radius	$\frac{4\pi r_1 r_2}{r_2 - r_1}$

So, we again come back to interesting things saying while that is all used for it one particular what you call way of doing it, everywhere the word infinite finite is used here things have been try to be made saying can you give a simple conduction shape factor.

So, then the shape factor can itself be used in your simple conduction formula and then you deal with things like this typically is a part of a heat sink, same thing you can have.

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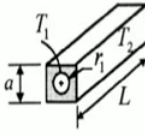
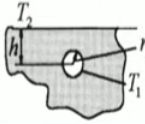


If you have concentric cylinders you say have concentric spheres, eccentric cylinders if initially the first lecture I showed you we made a temperature compensated pressure sensor. So, I have a pressure sensor at one point and we have a, the outside thing is kept and to compensate for it we have a small, what you call pn junction sensor in it all of them mounted in a thermal block such places you require these things.

So, typically you have a heater and then typically you have a sensor and then it is possible for you to maintain something at the correct temperature which even today is used in places like a, when controlled devices. If you see those I mean large transmitters and all that you do have specially cut crystals which have a low drift with respect to temperature, in spite of a there are enclosed in a often in a space and maintained at an elevated temperature because cooling is not easy and maintaining something at around seventy degrees does not degrade the crystal.

So, you have a small oven and then you have a crystal inside the oven and you use all this way of analyzing those things, to see while making it what should be the optimum mass and volume of the device.

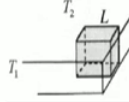


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Configuration	Restrictions	Conduction Shape Factor
Circular cylinder in a square cylinder, concentric	$a > 2r$ $a =$ side of square	$\frac{2\pi L}{\ln\left(\frac{a^2}{0.25r^2}\right)}$
		
Buried sphere	$h =$ distance below surface $h > r_1$ if $h \rightarrow \infty$	$\frac{4\pi r_1}{1 + \frac{r_1}{2h}}$ $4\pi r_1$
		

So, by playing around with this radii, I playing around it you can optimize and minimize unwanted loads on the whole thing.

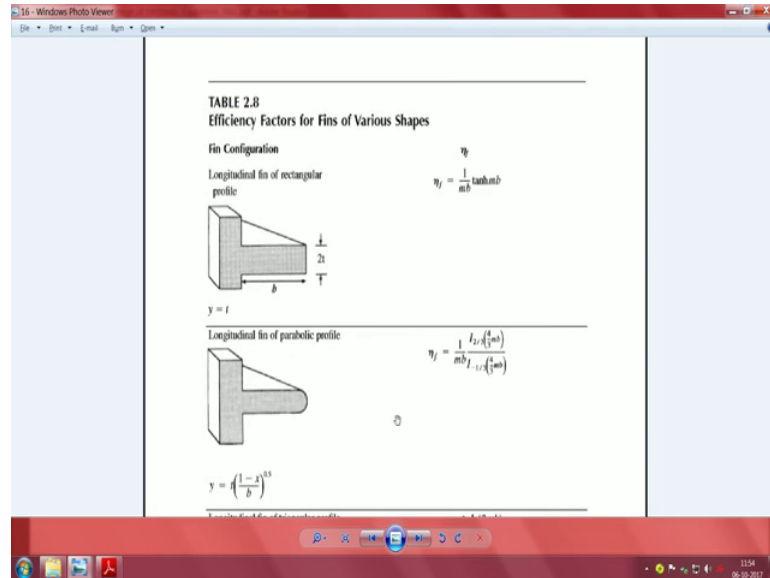
So, you have here every known thing, shape factors for steady state conduction.

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Configuration	Restrictions	Conduction Shape Factor
Corner of three adjoining walls	$W > 1/5$	$0.15L$
		
Disk on semi-infinite medium	$r =$ radius of disk	$4r$
		
Vertical cylinder in semi-infinite medium	$L \gg D$	$\frac{2\pi L}{\ln\left(\frac{4L}{D}\right)}$
		
Conduction between two parallel cylinders	$L \gg D_1, D_2$	

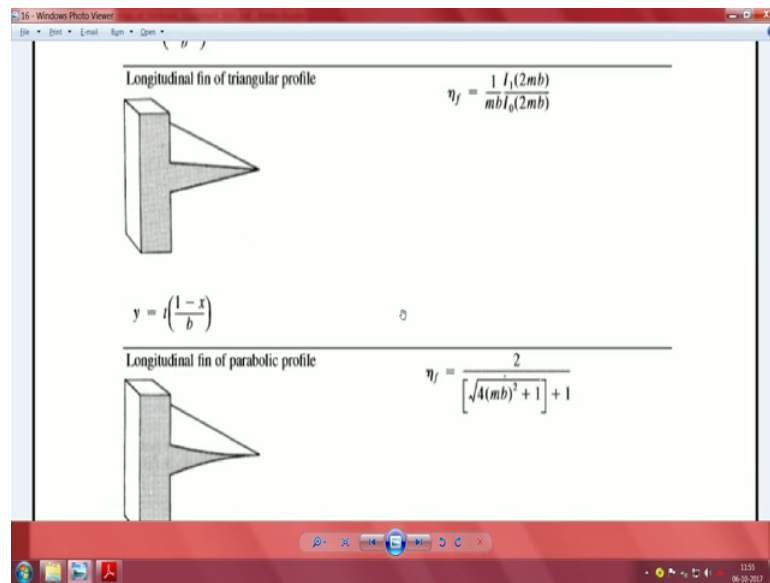
And then for a buried sphere, for a buried cylinder buried rectangular box steady trade conduction all the known combinations have all been given here.

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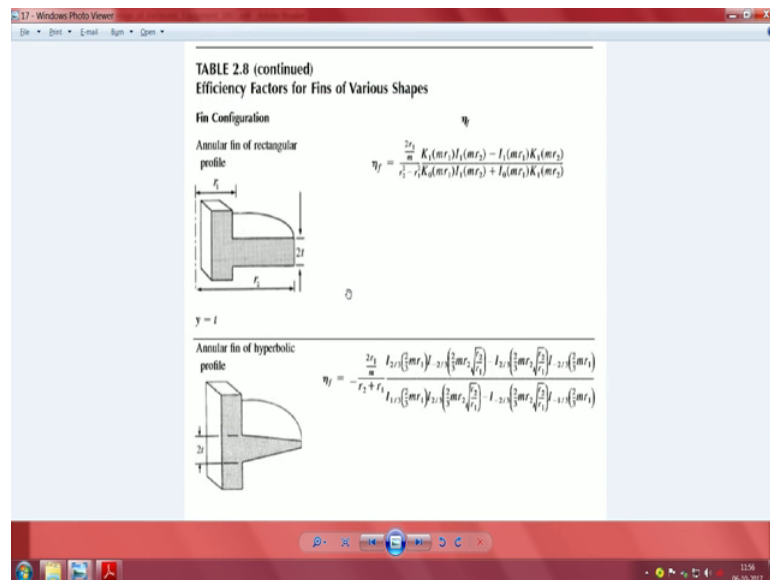
Now, we come to very interesting thing which I have told you earlier about saying why we end up with fins, a various geometries because as the as you come to the tip of the fin convection takes away the heat. So, they are generally tapered because in the beginning you need to have more conduction and as you go further and further the temperature drop is not you know affected very much by having an unwanted thing because there is no, not enough area like that. So, the area gets tapered in tapered.

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So, lot of devices about simple rectangular profile and I think this is a picture you know this is a parabola we are expected to consider this as a parabola and then triangular profile and longitudinal fin of parabola parabolic profile. So, a lot of these in our relations have been made here given a fin configuration.

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If it is annular fin with the hyperbolic profile meaning, suppose you have a disc all around if some of you recollect, I have shown you an electric pole on which around what



you call cap like thing is that inside we have the various devices inside and imagine it is a stack of CDs and it is heated you do find these things occasionally.

And the thing is formulations for these are available. So, if you have a rectangular profile hyperbolic profile circular profile and so on.

(Refer Slide Time: 32:38)

The screenshot shows a presentation slide with the following content:

- Spine fin of circular profile:**
  - Diagram: A circular fin with thickness  $2t$  and length  $b$ .
  - Equation:  $y = t \left( \frac{r}{r_0} \right)$
  - Efficiency formula:  $\eta_f = \frac{1}{\sqrt{2}} \frac{\tanh(\sqrt{2}mb)}{\sqrt{2}mb}$
- Spine fin of parabolic profile:**
  - Diagram: A parabolic fin with thickness  $2t$  and length  $b$ .
  - Equation:  $y = t$
  - Efficiency formula:  $\eta_f = \frac{2}{(\sqrt{2}mb)^2} \frac{I_0(\sqrt{2}mb)}{I_1(\sqrt{2}mb)}$

We have including triangular profile the difference is earlier they were all.

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The screenshot shows a presentation slide titled "TABLE 2.8 (continued) Efficiency Factors for Fins of Various Shapes". The content is as follows:

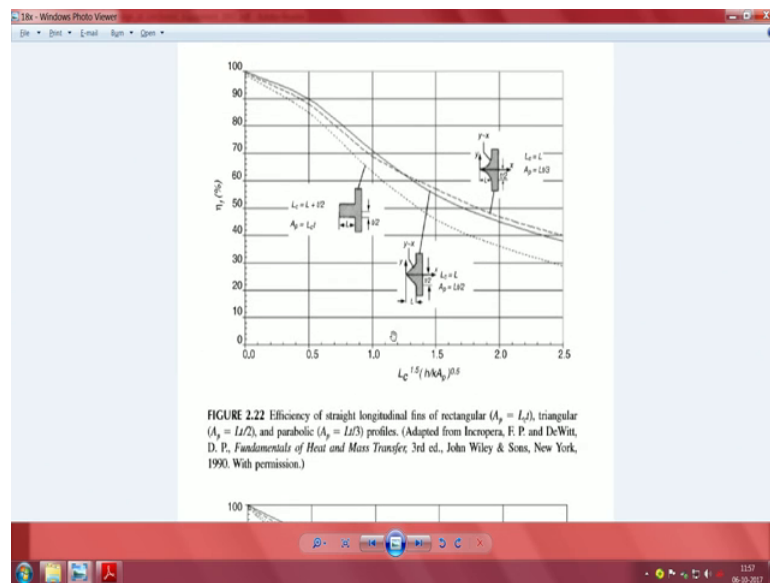
- TABLE 2.8 (continued)**
- Efficiency Factors for Fins of Various Shapes**
- Fin Configuration**
- Spine fin of triangular profile:**
  - Diagram: A triangular fin with thickness  $2t$  and length  $b$ .
  - Equation:  $y = t \left( \frac{1-x}{b} \right)$
  - Efficiency formula:  $\eta_f = \frac{4}{2(\sqrt{2}mb)^2} \frac{I_0(\sqrt{2}mb)}{I_1(\sqrt{2}mb)}$
- Spine fin of parabolic profile:**
  - Diagram: A parabolic fin with thickness  $2t$  and length  $b$ .
  - Equation:  $y = t \left( \frac{1-x^2}{b} \right)$
  - Efficiency formula:  $\eta_f = \frac{2}{\left[ \frac{b}{t} (mb)^2 + 1 \right] + 1}$

You have seen this annular fin meaning the thing is going on radially and this is typically what you are likely to find as pin fin heat sinks, but because of the manufacturing what you call difficulty, usually they continue to be straight fins and plus they have a fan on top of it.

So, go and check out for a fan with pentium cooler and in case you do not have a what you call such a device and you want it natural cooling probably you can have a heat sink is extruded with tapered fins at least in one of the direction you can have there may not be a spine fin its a tapered fin and then other direction you make spaces and sometimes it is done at a you know angular thing.

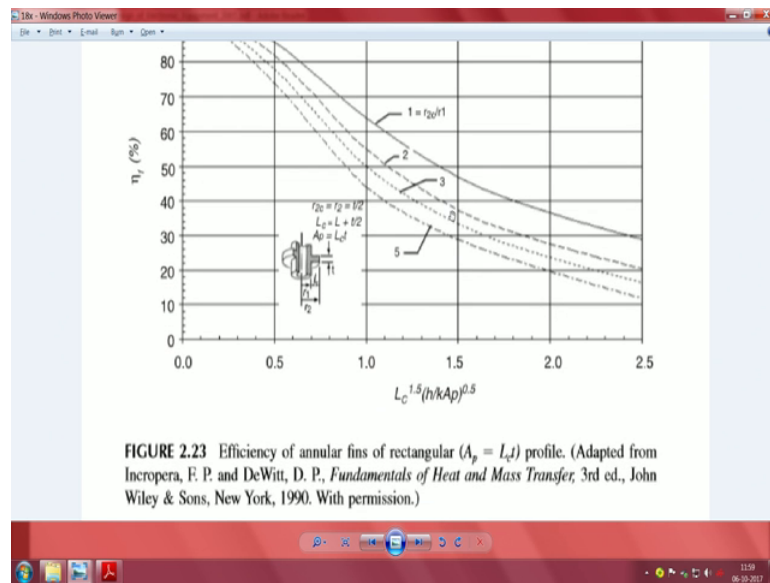
We still have the advantage of having lot of these things.

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So, this is what I was talking about efficiency of a straight longitudinal fins of rectangular triangular and so on. So, as the what you call length increases how well those things operate. So, this is sort of looks a little what you call obvious you see here the moment, moment you go here because of this cross section already the efficiency has gone up if you have it only a simple thing like this the efficiency comes down when length becomes long there even if a length, you know becomes long it does not matter. It is still better you seen that almost 10 percent increasing efficiency is there which we will come to from the base figure compared to this it is 25 percent better than this that is reason why these things continue to be used same thing about circular devices.

(Refer Slide Time: 35:03)



So, you have in case you know these things vary and they are tapered and all that stuff you know what can happen like that at this point I think I will stop, next time when I start if you come towards the end finally, it is a combined effect of simultaneous radiation and convection together.

(Refer Slide Time: 35:47)

parallel path for radiation.

### 7.4 RADIATION AND CONVECTION IN PARALLEL

Many problems in electronic cooling involve the combined effect of simultaneous radiation and convection heat transfer. The total rate of heat transfer for the system shown in Figure 7.5 is the sum of the radiation and convection effects,  $q = q_r + q_c$ , which we can also write as

$$q = h_r A (T_1 - T_2) + \bar{h}_c A (T_1 - T_2) = (h_r + \bar{h}_c) A (T_1 - T_2)$$

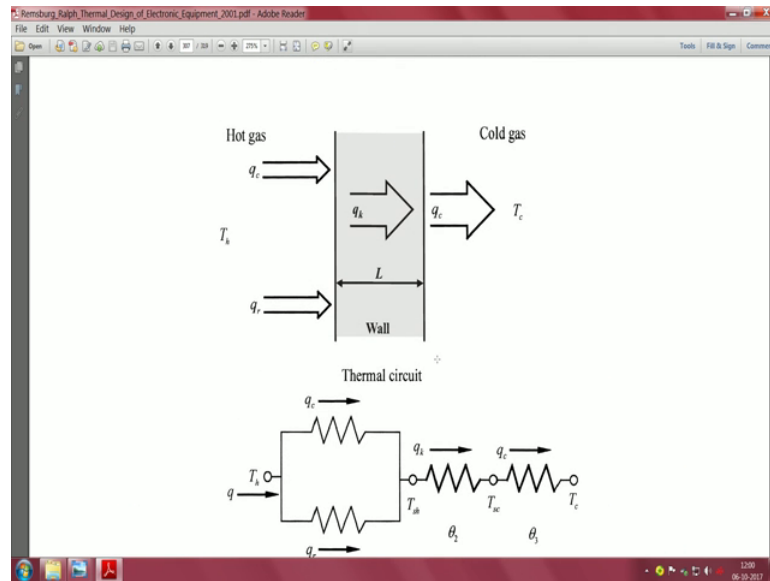
The radiation heat transfer coefficient is the equation

$$h_r = \frac{q_r}{A_1 T_1 - T_2} = \delta_{1,2} \left[ \frac{\sigma(T_1^4 - T_2^4)}{T_1 - T_2} \right]$$

In most systems, determining the radiant heat transfer coefficient directly is very difficult. Since the temperature factor  $\delta_{1,2}$  contains the temperatures of the emitter and the receiver, we can evaluate it only when we know both. In electronics cooling, the temperature of the emitter usually varies with power; therefore, we must estimate

So, you have several of these, what you call way of things which are operated directly hot gas versus cold gas.

(Refer Slide Time: 35:55)



So, I will try to continue with this in the next lecture, come back a little start and make it in an organized way. So, far once that I have shown you saying they are very useful for people who write their own code saying the equations are available and they can be solved and we have computers now which I do. Both the things that is you can use finite element analysis or cfd one is the what you call subset of the other using this complex problems of heat flow can be solved I just wanted to expose you saying most of these correlations and all aware. Over the years they have been brought about, but the initial is 200 years back for year sort of made the saying there is a directly proportionality and then if you can do things later on, if we can some of earned the heat transfer coefficient and the various physical parameters you can always you know what you call predict and by in the complimentary way if you know the parameters you can determine the heat transfer coefficient.

So, thank you let us get back little differently in the next class.

Thank you.