Mathematical Methods and Techniques in Signal Processing - I
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Lecture – 08
Introduction to vector space

Welcome all, to mathematical methods and techniques in signal processing. This is the start of the module for this course.

In the previous modules we just recapped the basics of discrete time models, basically reviewing what signals are, what systems are and then leading to the state space representation and we got a feel for what it is as a signal is going through tab delay line that it is a vector. And this formulation of the transformation of a signal to a vector is very important because we can invoke the techniques in linear algebra towards a very precise mathematical formulation for our signal processing problems. So, this is a very important step. So, the moment we transform a signal to a vector now it is in an abstract space and all the mathematical techniques that we are used to in linear algebra we can now apply to the signals and that is a very important steps.

So, let us recall some of our foundations in a linear algebra will not be go extensively into linear algebra, but we will be touching upon the essential concepts which are required for this course.

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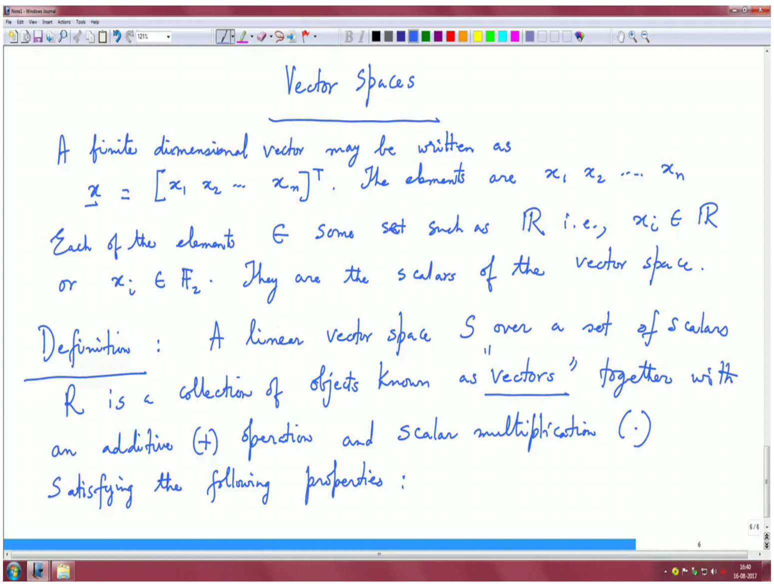

So, we will start with matrix spaces. A finite dimensional vector may be written as some vector x having this coordinates x 1, x 2 so on till x n. So, we can say this is the column vector and elements of this vector are x 1, x 2 so on till x n. And each of the elements they belong to some set such as may be the real line that is x i belongs to R or perhaps x i can belong to a binary field.

Now, they are the scalars of the vector space. Now, we will define what this linear vector spaces. So, like we can imagine a finite dimensional vector comprising of some elements and this elements can belong to some real field or say some other finite field and then also be a scalars for this vector space. So, with this we are in a position to start with the definition of a vector space a linear vector space S over a set of scalars R is a collection of objects known as vectors together with an additive operation and scalar multiplication satisfying the following properties. So, this is the a linear vector space S over a set of scalars are is the collection of objects known as vectors together with an additive operation and a multiplicative I mean an scalar multiplicative operations satisfying the following properties.

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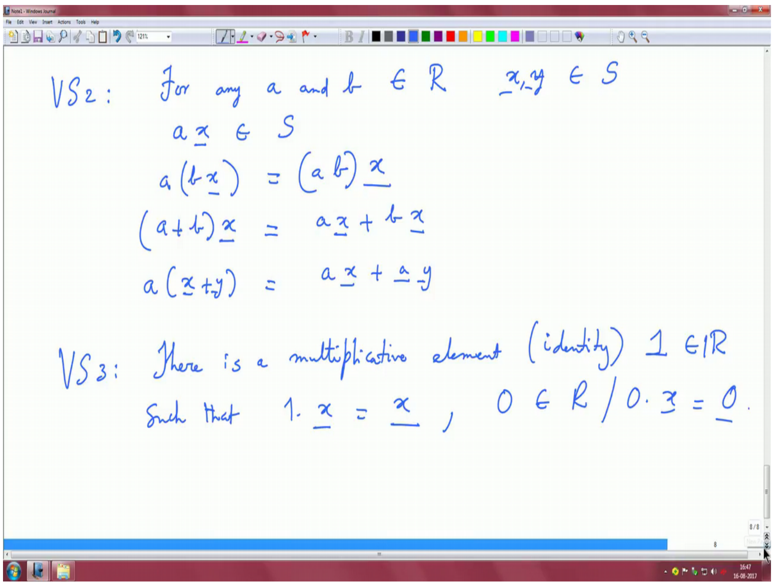

So, let us see what the properties are S forms a group under addition. Now, what are the properties for a group? We have closure, we have associativity then we have identity existence with the identity element and then existence of the inverse right these four properties. So, we will just enumerate and list all this. So, for any x and y belonging to S x plus y belongs to S. Taken x it is a vector x belonging to S take another vector y belong belonging to x S, x plus y should belong to S right this is closure; b, there is an identity element in S denoted by this null vector such that x plus 0 is a 0 plus x which is x this is basically the identity element 0 is basically the identity element for this. Then for every element x belonging to S there is another element y belonging to S such that x plus y is equal to 0. (Refer Time: 08:25) vector x belonging to S there exists some y belonging to S such that x plus y is 0 and y which is basically minus x is the additive inverse.

Addition operation is associative this means for any x y z belonging to S, x plus y plus z depend it does not depend on the group a. You can group x and y first add then up then add z, you can group y and z and then add x and that result is the same. So, this is one set of properties, S form a group under addition. For any a and b belonging to R that they are scalars and x comma y belonging to this set S a x belongs to S, window scalar multiplied this with this vector that belongs to the space. Now, a times b x is basically a b times x and then if you take a plus b times x which is basically x plus b x this is basically it is distribution and then you take a times x plus y this is also distributes it is a x plus a y.

The other property is there is multiplicative element or also multiplicative identity element precisely which is 1 belonging to R, such that 1 times x is you multiply the vector by the identity element you get back your original vector and there exists 0 belonging to R subspace 0 with x is basically in your null vector. Now, these are basically the properties of a vector space.

So, if I give you a collection of vectors and possibly some scalars on some set and then if I ask you if it, if this is indeed forms a vector space you have to verify all these properties are true.

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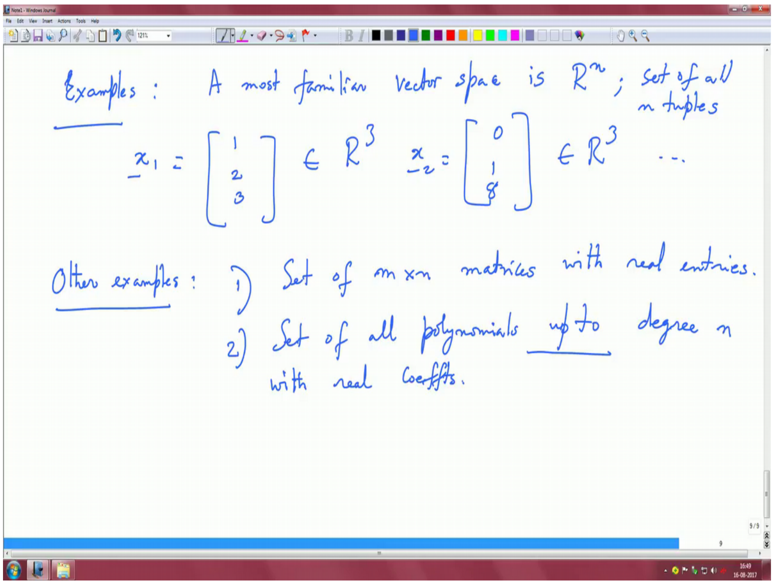

Then you will see some examples of vector space. A most familiar vector space is basically R n that is a set of all n tuples where each of the elements are basically reits right this is a very familiar vector space.

So, if say n equals 3 you can think about x 1 he say some 1 2 3 that is belonging to R 3 yeah, similarly you have 0 1 8 is belongs to R 3, so on. And there are other examples slightly nontrivial, but it is conceptually very easy to imagine. Set of n by n matrices with real entries, can you think about this as a vector space; set of all n by n matrices with real entries. Then is a set of all polynomials this is very important up to degree n with real coefficients. So, set up all polynomials up to degree n with real coefficients this also forms a vector space right. So, imagine a parabola right the degrees 2 maximum degrees 2; that means, I can have ax square plus bx plus c right a b c are all reals right. So, now, do you think the form a vector space? Take one parabola add another do it take one polynomial that is second degree polynomial add another polynomial to this of up to that degree, verify if you are going to have the space of all, it belongs to space all polynomial up to degree 2 that gives you sort of an idea.

Scale of parabola you can stretch it right or it since it is up to degree n a line is part of that be express is part of it, a constant c is part of it as well. So, it covers the entire space. So, therefore, this is indeed a vector space, but I will leave this as an excise to you to sort of formally verify if all the axioms or satisfied.

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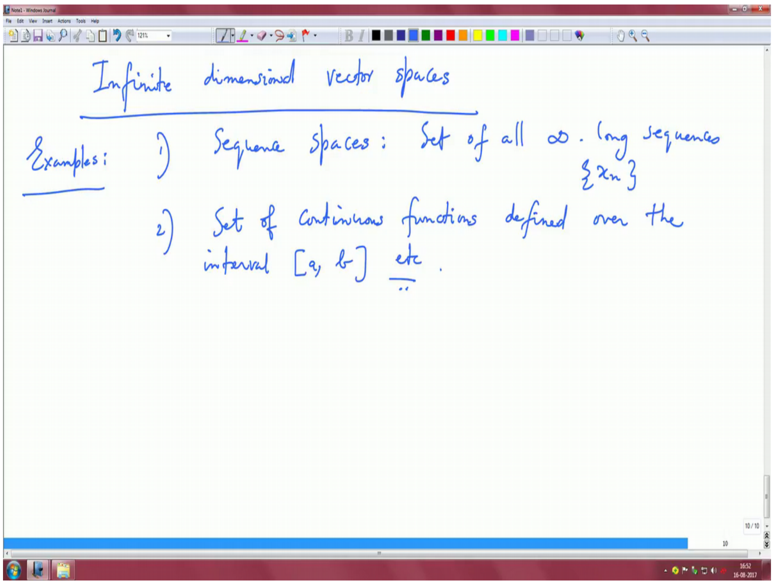

Now, this idea can be sort of thought about in infinite dimensional vector spaces. So, this examples include sequence spaces where in we have a set of all infinitely long sequences x n. The other possibility is the set of continuous functions defined over the interval a comma b. So, these are all infinite dimensional spaces a good example when you have the think about expanding this continuous functions is four year representation right. If you think about a signal and its 4 year representation and each coordinate of the signal representation is basically point in this space, in the signal space and that is basically infinite dimensional vector space. So, we will get into that either that is where we are leading towards.

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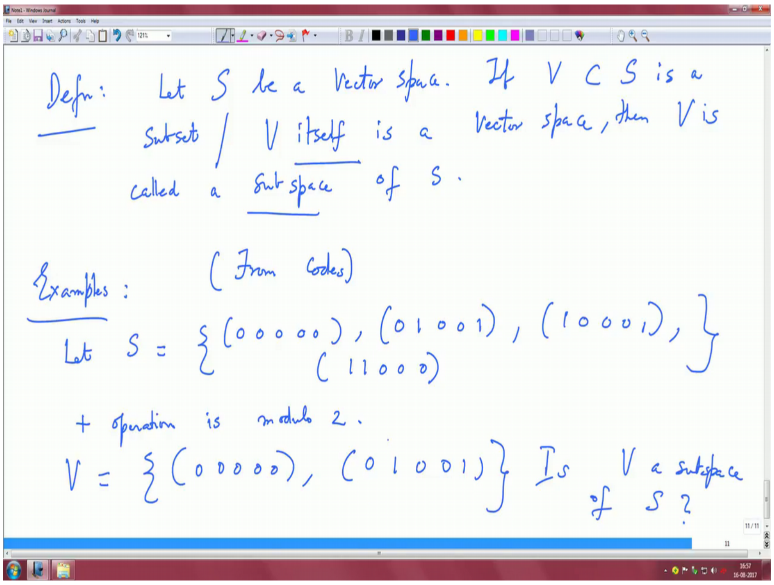

So, we have another notion of a subspace. Once you have defined a vector space we are something called a subspace. Let S be a vector space if V is part of this space is a subset such that V itself is a vector space then V is called a subspace of S and we will see this through some examples. So, we will take the example of codes binary codes. Let S be equal to this set and the addition operation is basically modulo 2. Can we think of this as a vector space first right? To take 0 and add it with any of these 3 elements you get back the elementary right. So, if you take 0 1 0 0 1 you add it with this you will get this back. So, this is basically like an identity right.

So, think about what 2 elements you can add to get this 0 right for example, if I were to add 1 1 0 0 0 with say 1 0 0 0 1 right, suppose I added these 2 elements. So, basically this you will get this element, closure is verified. Now, it is if we add the same thing same vector with itself under modulo 2 to you get the null vector right 0 1 0 0 1 plus 0 1 0 0 1 under modulo 2 it is basically 0 it is trivial right. So, we can think about is a self inverse, satisfies all the properties. And over this binary field basically you can only think about 0s and 1s as you scalars, 0 times anything you are going to get back the 0 vector, 1 times this is vector you are going to get back the same vector itself and you can verify the distributive properties etcetera. So, this indeed forms a vector space right.

Now, I choose this set V to be 0 0 0 0 0 and 0 1 0 0 1 can you verify is V a subspace of S; that means, we should be contained in S the elements are definitely contained in S and just quickly check through the axioms that we did earlier that it has to be a group under addition and a distributive property and the multiplicative identity 1 right. So, 1 times that vector is basically you get back to a vector 0 times vector is you have the 0 vector here distribution distributive property is carried out and it is a group under addition right. So, this is trivially verified. So, it is indeed a subspace of S.

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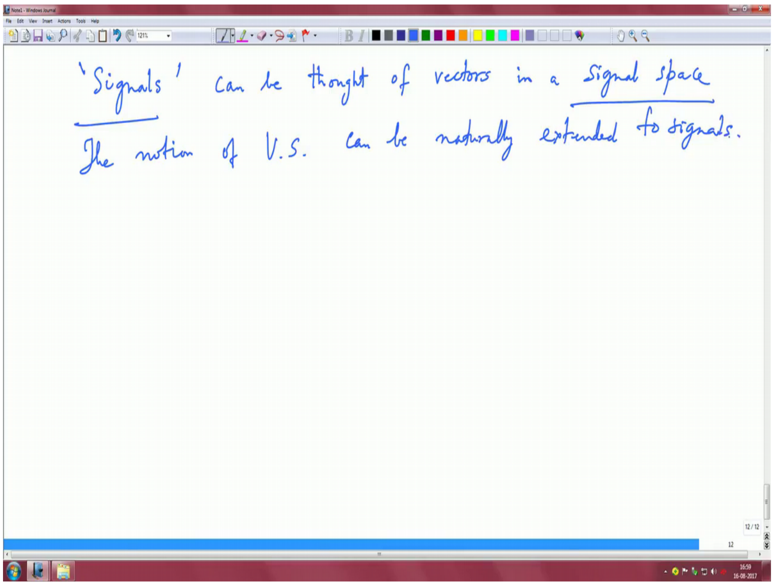

Now, signals, this is very important for us because is a signal processing course not a linear algebra course. So, we have to bridge the connection. Signals can be thought of vectors in a signal space and the notion of vector space can be naturally extended to signals. So, this is a very important point that we should always be thinking through and this is what we are going to get towards the end of the first module basically which is review of vector spaces, inner product space, a signal spaces eventually towards signal geometric this is where we are getting towards. So, clear so for with basic definitions of vector space and subspace.