Mathematical Methods and Techniques in Signal Processing - I
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Lecture – 77
Demo on KL Transform

So, let us have some interactive problem solving sessions by my students have taken this course. So, you will see some illustrations and examples in the problem solving which is useful to understand and digest the concepts learnt during the lectures.

Hello all I am Amrita a PhD student at ISE. I will be showing you a demo on implementation of KL transform. First let us look at the code for implementing KL Transform.

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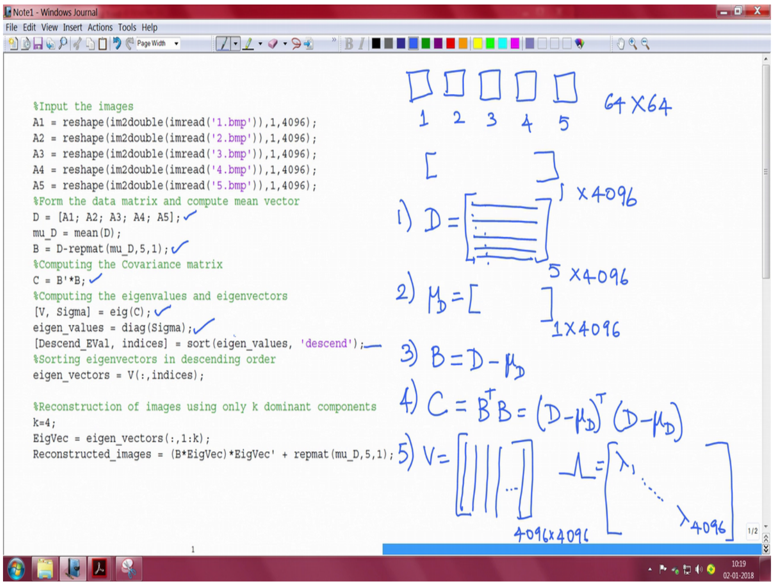

Let us consider 5 input images. So, here I have 5 images, and each image is of size 64 by 64. For KL transform we have studied that the input to the transform should be in the form of a row vector, for this purpose we convert our images into a row vector of size 1 by 4096.

The first step in KL transform is to create the data matrix D. So, in the data matrix D we have each row corresponding to one of the input image so the dimension of the data matrix D will be 5 by 4096 that is performed here.

The second step is to compute mu D, that is the mean vector along each of the dimension present in D. So, mu D is a row vector of dimension 1 by 4096, the next step we perform here is to make the data matrix 0 mean for this purpose we create matrix B, where we subtract the data matrix and the mean that we computed mu D.

So, the data matrix B now has 0 mean, the next step is to compute the covariance matrix C which is given by B transpose B that is actually nothing, but D minus mu D transpose D minus mu D. So, we compute the covariance matrix in this step. Once the covariance matrix is computed we use the Eigen value decomposition to extract the Eigen values, and Eigen vectors of the covariance matrix. So, here we obtain V which is a square matrix of size 4096 by 4096, where each column corresponds to 1 Eigenvector of the matrix C.

Similarly, we get sigma matrix which is a diagonal matrix where we have the Eigen values along the diagonal elements. So, once we obtain the Eigen values and eigenvectors we extract the Eigen values from the diagonal elements and store them in Eigen values. For KL transform we need to sort the Eigen values in the descending order. So, once this is performed we rearrange the Eigen vectors based on the order obtained. So, let us see how the Eigen values are obtained after making them in the descending order.

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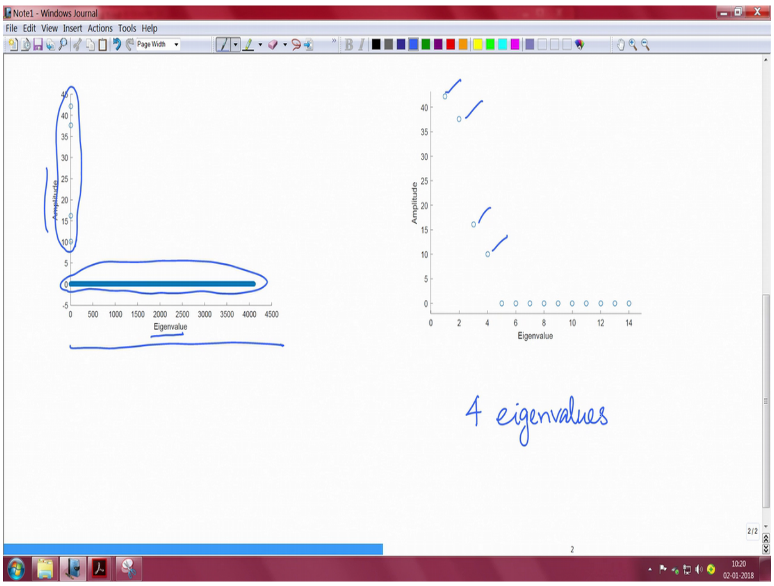

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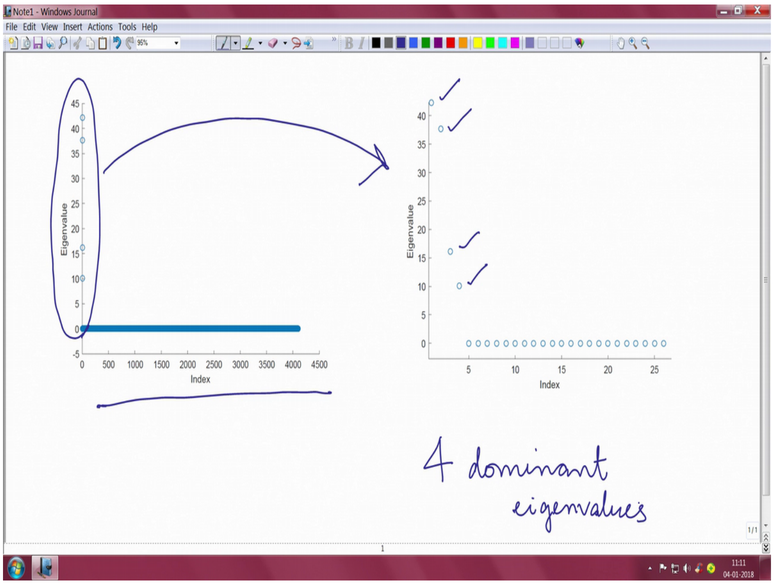

Here I have plotted the Eigen values along the y-axis, and the corresponding indices along the x-axis. Most of the Eigen values here are closed to 0, except a few that can be seen here. Let us look at the dominant Eigen values that are present here we see that only 1, 2, 3, and 4 Eigen values are dominant.

Let us see if we can reconstruct the images using only 4 Eigen values. So, here I have plotted on image here along the x axis we have all the Eigen values, and along the y axis we have their corresponding amplitude. We see that most of the Eigen values are close to the range of 0, and there are a few values that have a value much greater than 0. So, for this purpose let me zoom into the initial location and see how many Eigen values have value much greater than 0. We see that we get 1, 2, 3, and 4; 4 Eigen values which have amplitude or value much higher than 0. So, using this information let us see if we can reconstruct the images.

So, here we choose k that is we choose the first k dominant components for the reconstruction part. So, based on the value of k we choose k Eigen vectors corresponding to the k dominant Eigen values and use those Eigen vectors for reconstructing the images.

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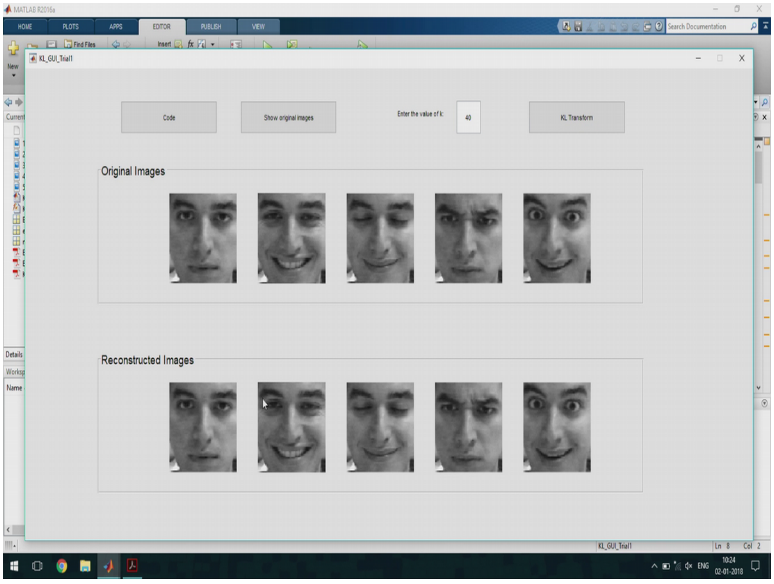

So, let us use a MATLAB GUI, to see how this performs. So, these are the set of 5 images that I have used for the KL transform. Let us choose k equal to 1 and see what happens. So, when I perform KL transform these are the images I get after reconstruction. Visually, we can see that the images are not as expected they are not close to the original images. Let us increase the value of k and see if there is any improvement we see that the fifth image is corresponding to the original image visually, but the other images are still not close to the original ones.

So, let us try increasing the value again, we see that almost all the images are close to the original images except a few. So, the last try we see is by taking k equal to 4, now we see that all the images 1, 2, 3, 4, and 5 are close to the original images this is because we saw that only 4 Eigen values are having value much greater than 0 that is they are the dominant Eigen values for these 5 images.

So, let us see what happens if we increase the value of k beyond 4. Let me use say 40 we see that even after taking 40 dominant Eigen values and vectors into consideration there is not much difference in the reconstructed images that we obtained. So, this shows that by using only the dominant k Eigen values we will be able to reconstruct the images.

Thank you.