Mathematical Methods and Techniques in Signal Processing - I
Prof. Shayan Srinivasa Garani
Department of Electronic Systems Engineering
Indian Institute of Science, Bangalore

Lecture – 76
Applications of KL transform

So, let us start with some applications of KL since we learnt the derivation of the KL transformation, we will see the applications. One of the applications is dimensionality reduction often these days you have this initiative of big data you have sensing sensors and you have you can collect data and the data would often be in higher dimensions and each dimension can be attributed to an independent coordinate.

And when you consider such high dimensional data sometimes it is difficult to manage the data; you might want to store probably low dimensional data right you might want to see which of the coordinates are really significant and you want to store low dimensional data. But in the process of sensing in the collection process you sample from high dimensional space, but when you want to store and do some processing you might want to work with low dimensional data.

So, if we were to do dimensional reduction from higher dimensions to lower dimensions how can we go about it? So, fortunately one of them can one of the techniques would be using the KL transformation and we will see this carefully here ok.

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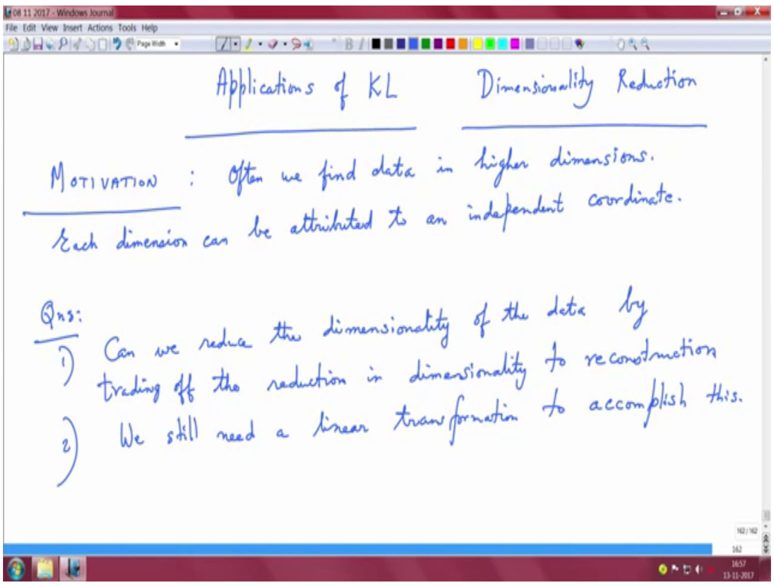

So, motivation often we find data in higher dimensions and this can be because of the sensing process. Each dimension can be attributed to an independent coordinate and some of the questions that we need to ask is one can we reduce the dimensionality of the data by trading off the reduction in dimensionality to reconstruction error right there is always a trade off between reconstruction error and reducing the dimensions.

And of course, you still need a linear transformation to accomplish this. So, let us see what we mean.

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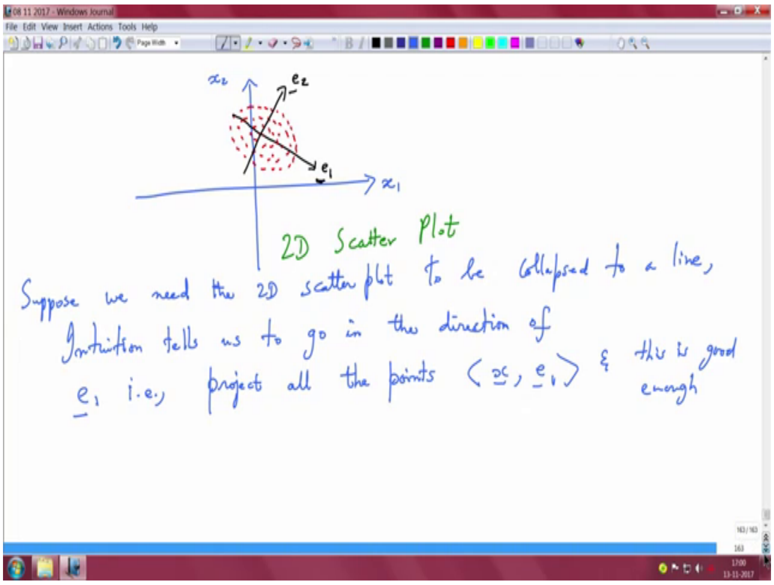

Suppose I give you a scatter plot the data is lying around in the space like; this is a 2D scatter plot. So, if you need to represent 2D points to just a point on a line what is the best way to go about this? If we ask ourselves an intuitive question that I just want I have a 2D scatter plot well it is difficult for me to have the data in two dimensions, I want just points on a line. So, your intuition should tell you that maybe I go in this direction.

So, two directions like this let me say this is e 1 let us say this is e 2 is vector right. So, I say if I take all these points and I just project them onto this coordinate system. And if I choose the line e 1 that would be the best line in some sense that would represent just points on a line if I were to collapse it into a line probably not just x 1 or x 2 right.

So, intuition tells us to go in the direction of e 1 that is project all the points in the direction of e 1 and this is good enough right. So, I think I have to pose the question here and write this question here suppose we need 2D scatter plot to be collapsed to a line; suppose we wanted this then intuition will tell us that we need to go this way.

Now, think about this sort of extensions in higher dimensions and we will come up with a procedure for dimensionality reduction and I will outline this procedure.

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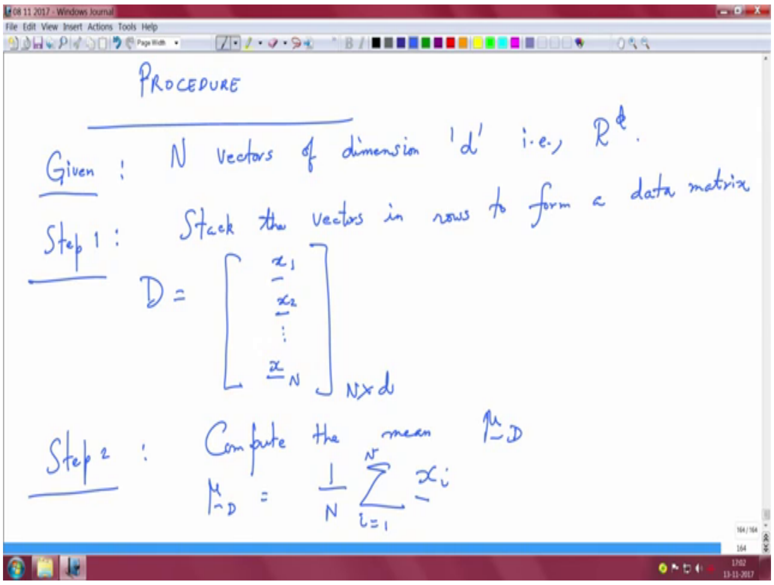

So, what we are given we are given N vectors of dimension d that is R d right. A first step called step 1 is stack the vectors in rows to form a data matrix right; I formulate a data matrix by stacking all the vectors x 1, x 2 dot dot dot x N and this is my N by d matrix because I have N such vectors and d such coordinates that form in the columns.

Now, step 2 is I need to compute the mean empirical mean compute the mean which is mu D this is given by 1 upon N basically you average all the vectors. And here you have to be careful if you cannot you do not have to do it like this if you know the distribution you have to do the averaging over the distribution right its little detail, but I think is I am sure you can sense it you can take the expectation mathematically expectation.

Now, in each coordinate each dimension you have a mean right.

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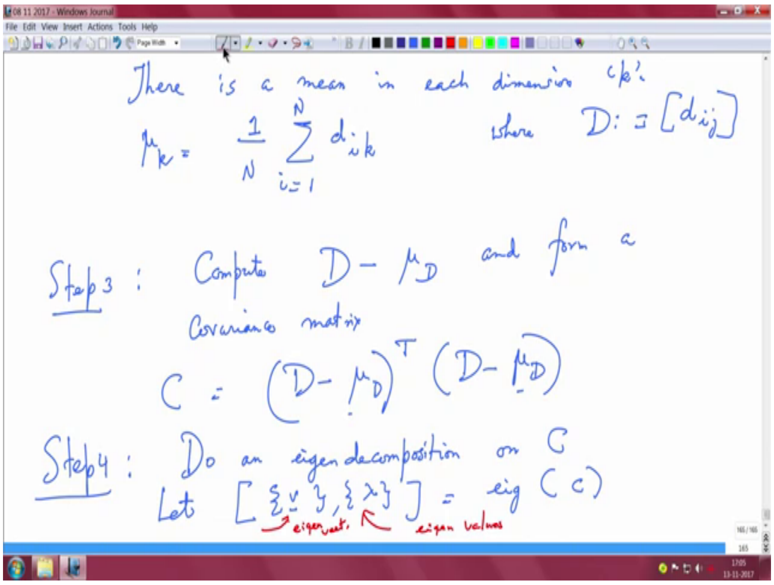

There is a mean in each dimension k. So, therefore, mu k is 1 upon N summation i equals 1 to N d i k where I can write this data matrix is basically some entries d i j right; if I just stack them as row vectors and then each element will be d i j in the matrix and I just have to do this straightforward.

Step 3 compute D minus mu D that is you have to remove the bias from your data and form a covariance matrix. And that is basically given by D minus mu D transpose times D minus mu D. And if you require expectation, if you know the distribution you have to take the expectation over the distribution often when you just collect data you may not get an exact loss form expression for the distribution right in such a case you can do it like this.

Step 4 do an Eigen decomposition on C. So, in your MATLAB form let v and lambda this set be the Eigen decomposition of the covariance matrix. Now, these are your Eigen values and these are your Eigenvectors ok.

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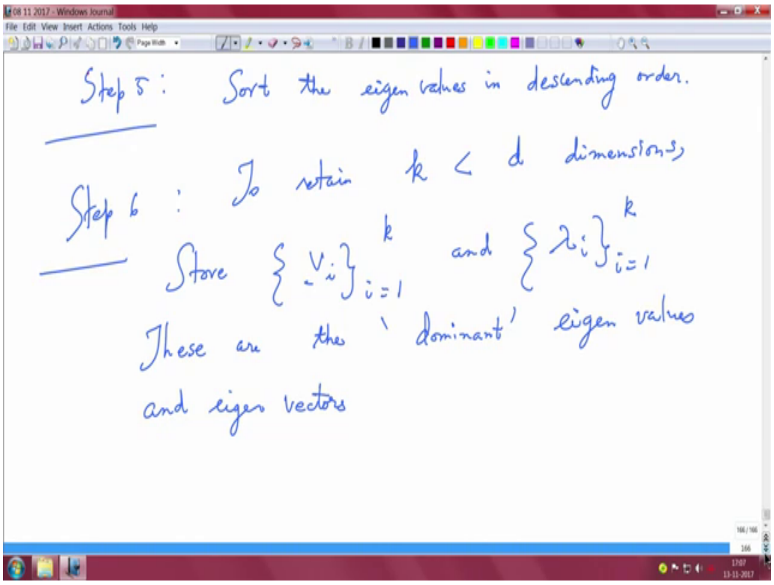

Then we go to step 5. So, we have to sort the Eigen values in descending order right we need to do this. Step 6 since we want k less than d dimensions right to retain k less than d dimensions that is we want to retain the first k restore some v i; i equals 1 to k and some lambda i equals 1 to k.

We choose those Eigen values and Eigen values and those Eigen vectors corresponding Eigen vectors right. And we call them the dominant Eigen values and Eigen vectors from the list these are the dominant Eigen values and Eigenvectors.

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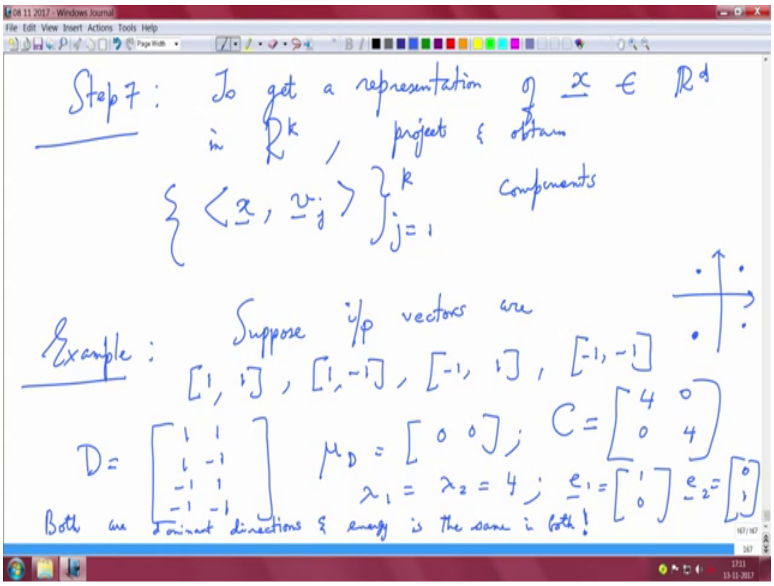

Next step 7 to get a representation of x belonging to R d in R k project x to v j for all j equals to 1 to k components right. I take the vector x project it on to the Eigenvectors v j j equals 1 to k and I get some projection and those are the coordinates of the vector upon dimensionality reduction right this completes the algorithm.

So, if you want to do it, program it you can you can essentially follow these steps. So, I think in will give you a simple example to sort of get you a field suppose input vectors are 1 1 1 minus 1 minus 1 1 and then minus 1 minus 1 right. If I do my data matrix I will get 1 1 1 minus 1 minus 1 1 minus 1 minus 1.

And; obviously, you can compute the mean and that is 0 and 0 right you just average it out you get 0 and 0. You formulate your covariance matrix you land up with something like this I would want you to verify this by hand you can do can do these calculations offline.

So, but what is important here is once I get this covariance matrix just observe this covariance matrix. Lambda 1 equals lambda 2 equals 4 and one of your Eigen vectors e 1 is 1 0 the other e 2 is 0 1 and it is not so, surprising because you plot these things right one of them is 1 comma 1 here 1 comma minus 1 is here. So, basically they are four corners of the square and; obviously, you can expect that these two are the axis that you would want to consider.

And if I work to ask you which is dominant which dimension has dominant energy as you mean these vectors are equally likely right this probability 1 by 4, 1 by 4, 1 by 4 all of them you can choose either e 1 or e 2 right. So, both are dominant directions and energy is the same in both right. So, if you want to take this and project it take this and project it onto 1 0 right you will get this value, here take this and project it you get here you get 1 you just get 1 1 and minus 1; similarly, 1 and minus in the other direction ok.

So, I think that gives us sort of a picture to sort of wrap up the KL transformation; I think is a very useful technique to do dimensionality reduction and it has very interesting properties, it satisfies energy compaction, it is a linear transformation it is unitary right. And this gives us a kind of perspective that is also a basis for representation and the basis happens to be the Eigen basis and we will talk about Eigen basis we are talking about the KL transformation.