**Mathematical Methods and Techniques in Signal Processing – I**
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**Lecture – 73**
**Problem on limits of integration of periodic functions**

So let us have some interactive problem solving sessions by students who have taken this course. So you will see some illustrations and examples into problem solving, which is useful to understand and digest the concepts learnt during the lectures.

My name is Payak PhD student at IAC; I am going to solve a problem which is actually a final exam problem in the MMTSB course offered at IAC in the year 2017. So the problem statement is as follows.

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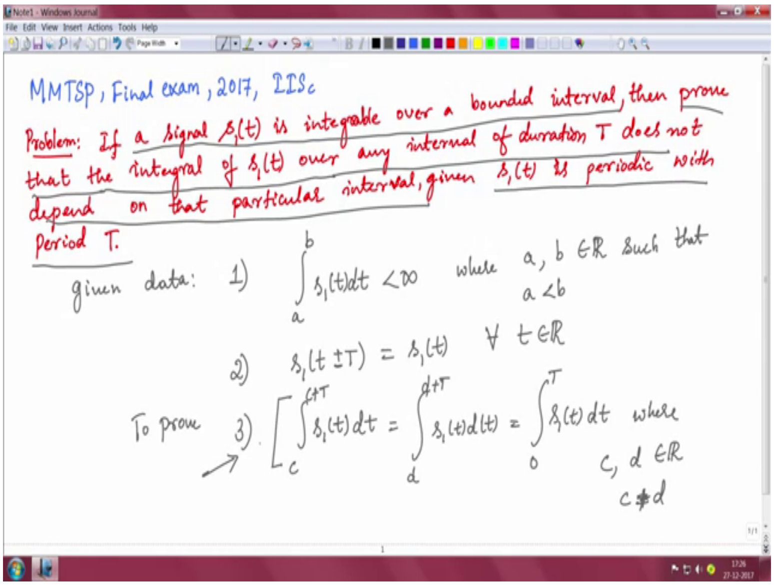


If a signal s 1 of t is integrable over a bounded interval, then through that the integral of s 1 of t over any interval of duration T does not depend on that particular interval, given the signal s 1 of T is periodic with period T. So let us go over this problem statement again and extract all the given data first point. So the problem says a signal s 1 of t is integrable over a bounded interval.

So mathematically this statement is asked is the following. Signal s 1 of t is integrable over the interval a b that means, the integral s 1 of t from a to b is less than infinity.

Where a and b are two real numbers such that, a is less than b that is the meaning of this statement. Second point is that, signal s 1 of t is periodic with period t that means; s 1 of t plus or minus capital T is s 1 of t for all t ok. So let us look at what we have to prove. So we have to prove that the signal the integral of s 1 of t over any interval of duration T does not depend on that particular interval. So mathematically, so this is what we have to prove, mathematically what it means is the following. Integral of s 1 of t over any interval of duration capital T.

So the duration of this interval is capital T should not depend on the location of this interval on the real line that means, this integral should be equal to the integral of s 1 of t from d to t plus t should be same as integral s 1 of t from 0 to t. Where c and d or any two trail number such that, c may not be equal to d you can consider c equal to d, but it will be same. So this is what we have to prove. So let us understand the point number 3 pictorially what it means.

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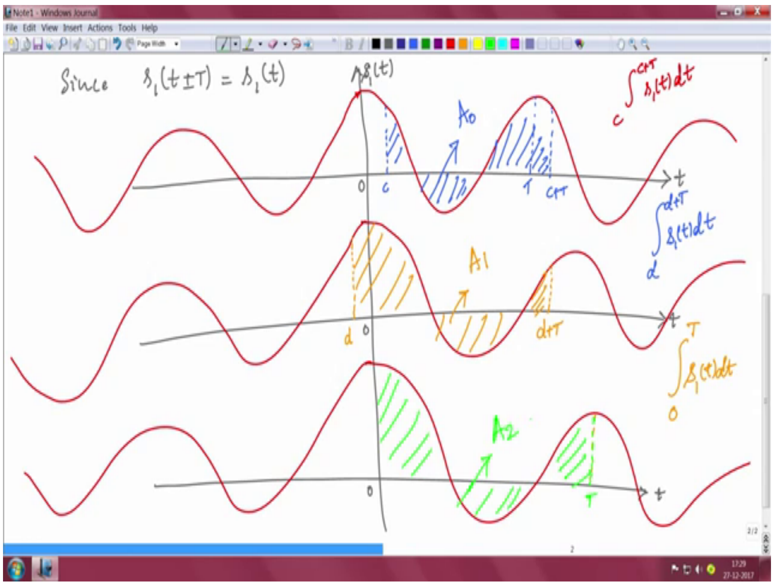


Since s 1 of t is a periodic signal with period capital T, we have this. So for example to understand let us consider a periodic signal for simplicity I am considering it cosine.

I will make 3 copies of it, so this is the 0 point, this is the time axis, this is s 1 of t. So the cosine signal is a periodic signal, which extends from minus infinity to plus infinity like this. Now consider the term integral c to c plus t is 1 of t dt. Let us understand what this integrable integral means, now take any point c and take another point which is c plus t.

So and the signal is periodic with period t that means the capital T is here. Now the integrable integral the meaning is the area under this curve from c to c plus t ok. Let us call the area under this curve as A 0. Similarly consider the integral d to d plus T s 1 of t dt. That means, you consider any point on the real line and call it as t, and the other point d plus T, the meaning of this integral is again area under the curve from d to d plus t let us call this area as A 1.

Similarly the integral 0 to capital T, s 1 of t dt is the area under the curve from 0 to capital T. Let us call this area as A 2.

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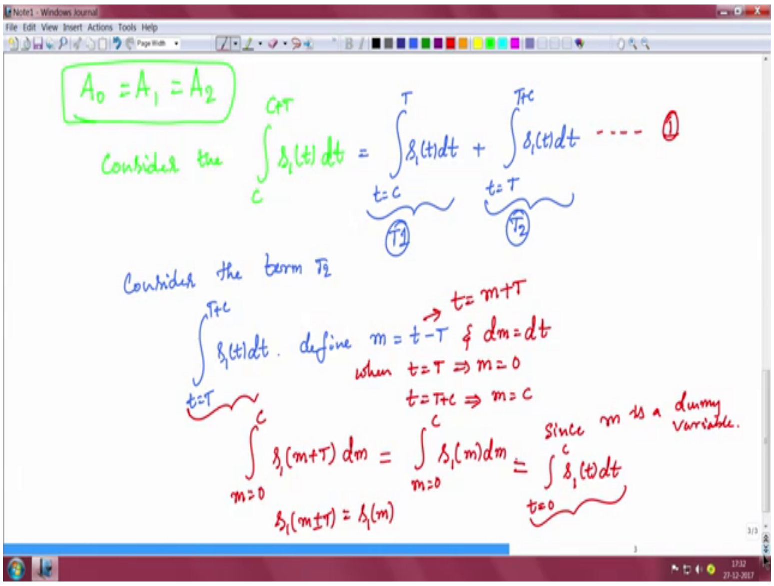

What we have we have to prove is that, you have to prove that A 0 is equal to A 1 is equal to A 2. For this consider the term C to C plus T, s 1 of t dt. Which can be written as the running variable is t. So c to capital T, s 1 of t dt plus t is equal to capital T to t plus c s 1 of t dt. Let us call this term as T 1 and this term as T 2. Consider the term T 2 that is t to capital T, T plus c s 1 of t dt.

Now define another variable m as t minus T. Now when t is equal to small capital t m is 0 and, when small t is capital T plus c, m as c and dm is dt. Now with this new area variable m this integrable integral becomes m 0 to c, s 1 of t. In place of small t we can write n plus capital T.

And in the problem statement it says s 1 of t is a periodic signal with period t right. So the integrable integral becomes; since m is a dummy variable, we can replace m by small t. So therefore this integral is t 0 to c, s 1 of t dt. Now the term t 2 is this; now let us call this equation as 1.

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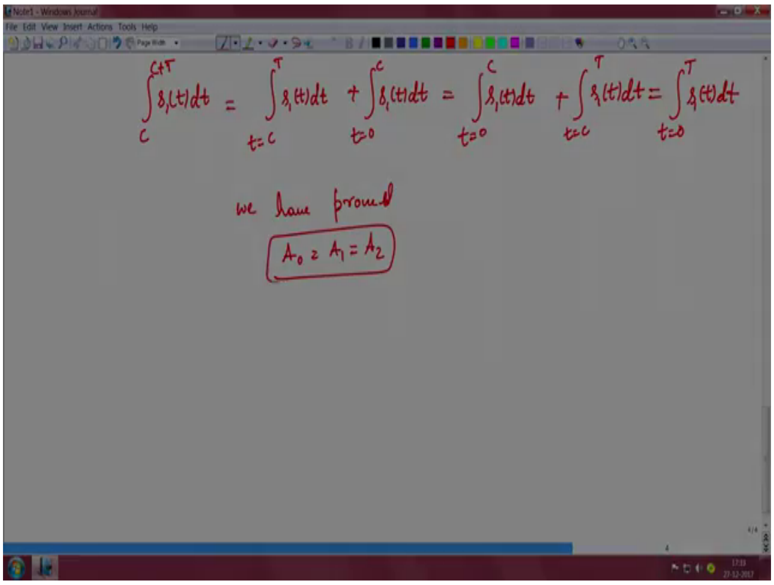


And equation 1 which is nothing but integral 0 to capital T s 1 of t dt. So we have proved A 0 is equal to A 1 is equal to A 2.