**Mathematical Methods and Techniques in Signal Processing - I**
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**Lecture – 53**
**Special filter banks for perfect reconstruction**

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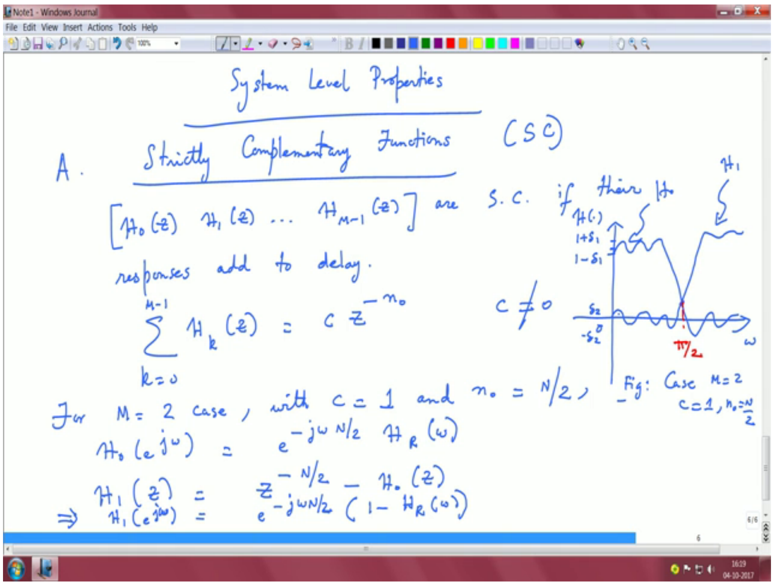


We have studied so far, how half band filters and Nyquist M filters can be designed. So, with this is in mind, we can get into some system level properties. These things are these properties are going to be helpful for us, when we deal with filter banks and the goal is; if you want the overall distortion function.

And we saw when we derived the general form for the M channel filter bank, we said the overall, if you remove all the alias components the overall filter transfer function overall transfer function distortion function would be the first component a naught of Z which is your unaliased component right I mean if you remove all if you set all the aliasing components to 0, then the overall distortion function using your analysis filters and your synthesis filters would be the unaliased part.

Now, our goal is if you want that transfer function to satisfy certain properties, such as the perfect reconstruction property, which is this transfer function has to be just some delay with a gain can we define such; can we realize such filters right. And in the special case, when the delay happens to be 0 no delay essentially right; no delay, but just a gain then you can think about it in this form, where they are like Nyquist M filters, but possibly realizable through some spectral factorization right. So, we will study some of these properties.

So, the first property is strictly complementary functions this is also called SC property. So, H naught of Z H 1 of Z dot dot dot H M minus 1 of Z are strictly complementary, if their responses add to a delay; that means, summa k equals 0 to M minus 1, H k of Z is some C Z power minus n naught. Recall, I just put the subscript k here, unlike in the Nyquist M case where the k was folded in here into the argument with omega power k; Z omega power k right, I just put a subscript k here, because it could be general generally any filter, any band pass filter right.

If this satisfies such that C is not equal to 0 of course, this is trivial condition, then they are called strictly complementary filters. So, for M equals 2 case with C equals 1 and n naught is n upon 2 right, if we start with our base filter H naught of e j omega is some e power j omega n by 2 times, some H R of omega, then I can say H 1 of Z is say Z power minus n upon 2, because a minus H naught of Z is what I have right.

So, now I can say that H 1 of e j omega is basically e power minus j omega n upon 2 times 1 minus H of omega, because H naught plus H 1 should just add up to a delay with a gain of 1 right. We think about how this could be sketched. So, this is case M equals 2 with C equals 1 say n n naught equals n upon 2 right, if you think about equitable filters and this could be your pi by 2 frequency, this is your H naught, therefore 0 H 1 of e g omega right.

So, therefore, the sum would ideally have to be 1, but you could have oscillations around 1 this could be 1 plus delta 1 and this could be 1 minus delta 1, similarly you could have a delta 2 and minus delta 2 symmetrically I mean this is just I have not written the magnitude, if I write the modulus and if I square it of course, you will not see the negative down right; you will need not see the negative ripple down.

So, in general you can think about that for strictly complementary functions the sum of the filter responses should add up to a delay with a scale factor of C, and the special case; when M equals 2 right; and if you choose such that C equals 1 the gain is 1 with some preset delay it boils down to what we are familiar with the quadrature the kind of case right it pi by 2 basically there is a an over an overlap they intersect.

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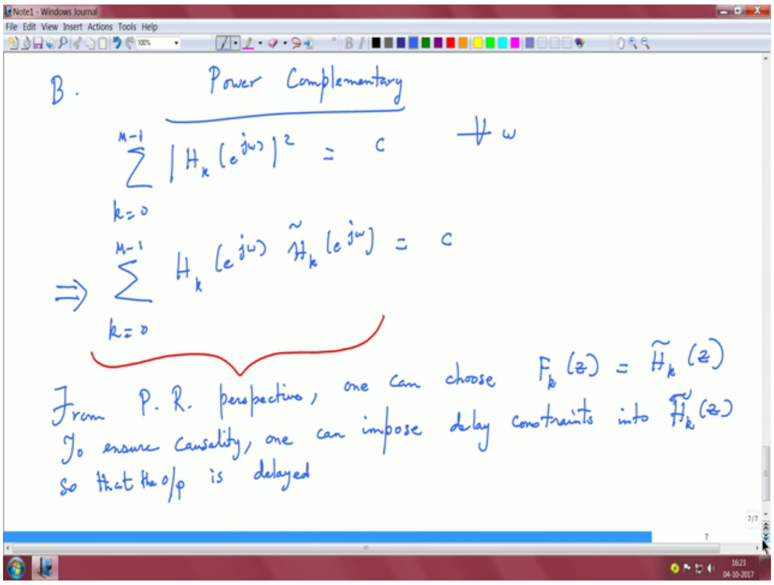


The next special function or special property for filter banks is the power complementary functions. So, if you are given filters H naught H 1, H 2 so on till H n minus 1, they form a power complementary pair, if you look at the magnitude square responses over all the M filters, that should be equal to some constant for all the frequencies. Now this interesting, because the sum of the magnitude square response for all the filters that should be a constant.

Now, let us see what this would imply? Now if you did basically decompose this modulus of H k e j omega square as some H k of e j omega times H k some conjugate of that, then this would be a constant, but if you carefully observe this transfer function, then you can see through inspection this is like my H k with f k, where H k is the analysis filter and F k is the synthesis filter which is a conjugate of the analysis filter right.

So, from perfect reconstruction perspective, one can choose F k of Z to be H k tilde of Z and of course, you may question that; the filters may not be causal it is ok, but we can bring in some delay constraints to H k tilde of Z and then the overall system could be the output could be delayed and therefore, you are still bringing in the perfect reconstruction property here right.

So, the key questions, if I have design tools that can give me filters that satisfy this power complementary property, then I am I am sort of home for good. Now to just write it down carefully to ensure causality, one can impose delay constraints into H k tilde of Z. So, that the output is delayed and the delay would be reflected into a factor that would appear along with C this is sort of straightforward.

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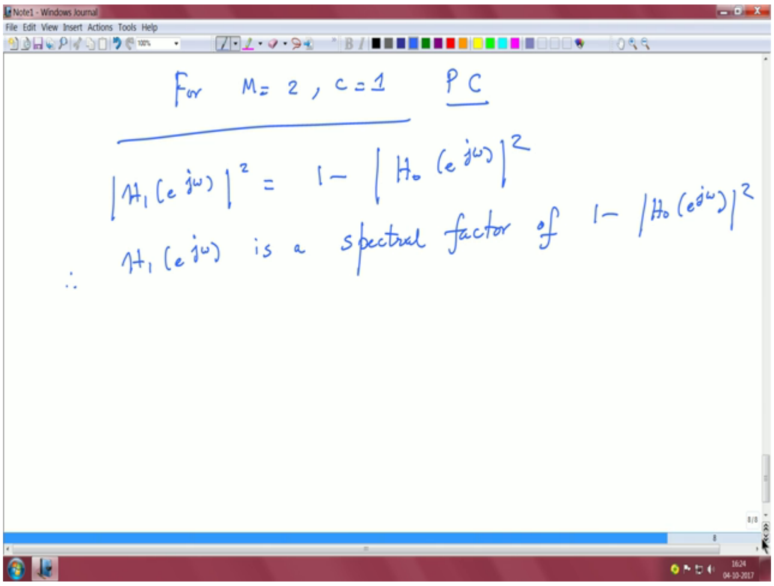


Now, let us look at the case for M equals 2, for M equals 2 and C equals 1, this is power complimentary case, modulus of H 1 of e j omega square is 1 minus modulus H naught of e j omega square. Therefore, H 1 of e j omega is a spectral factor of 1 minus modulus H naught of e j omega square, that is if you took 1 minus this magnitude square and then do spectral factorization, then you can get one of your filters as a spectral factor of this of this term. So, this is sort of straightforward and not be dealing with spectral factorization techniques, because is hopefully would be covered as part of your undergraduate digital signal processing.

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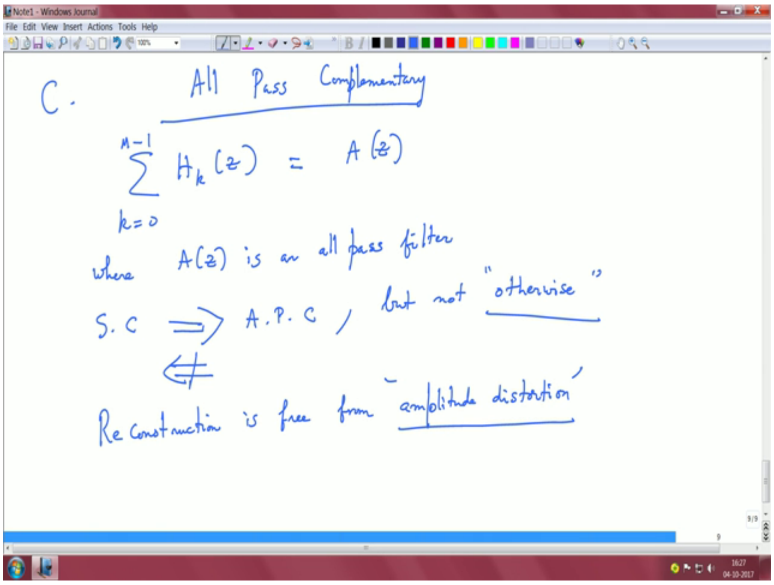


There is another special property, which is all pass complimentary property. So, filter responses are said to be there; they said to be all pass complimentary, if if you take the summation of all the frequency responses over all the M filters, that should result in f transfer function, which is essentially all pass, where A of Z is an all pass filter.

Now, you might wonder is there any link with all pass complimentary and strictly complementary function. Strictly complementary functions are all pass complementary, because some gain times a delay is all pass, but all pass filters are not strict strictly complementary functions ok. Now, SC would imply all pass complementary, but not otherwise; reverse direction perhaps not is not implied.

Now, but if you ensure that, this is satisfied we can at least guarantee that the reconstruction is free from amplitude distortion phase we may not be able to control here, but at least we can say that the magnitude response is unaltered right. So, it is all pass. So, therefore, these are some basic properties.

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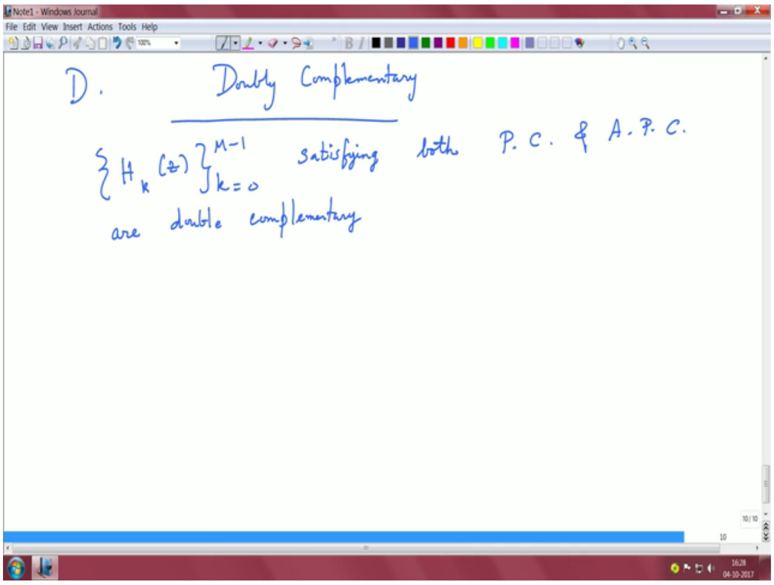


We have the last special property, which is doubly complementary filters; filters satisfying both power complimentary and all pass complimentary or double complementary. So, it is double complimentary, if it satisfies both power complimentary and all pass complimentary.

Now, this gives us a set of properties for constructing filters, that are desirable for example, if I say that well my unaliased distortion; my unaliased component of the overall system transfer function has to be doubly complimentary, then it has to be both power complimentary and all pass complimentary or I want it to be just strictly complimentary, then I can imagine how I could build such filters or I wanted to be just power complementary filters, then I can say that for example, for M equals 2 case that the high pass filter is basically spectral factor of 1 minus modulus of H naught of e j omega square right.

So, therefore, depending upon what constraints you want to bring in, you can essentially let the unaliased part of the distortion transfer function to satisfy one of these four properties. And once you are clear with that then you can think about tools, with further constraints on the filters and properly other properties of the special properties for these filters if they have to be FIR or mirror symmetric etcetera; so that you can realize practical filters ok.

So, I think with this we are done with module 2, which is multi rate systems and filter banks. So, they will give you an idea as to first; how to go about constructing filter banks with the multi rate operations, analysis towards getting the distortion transfer function, seeing through carefully, how you would want to avoid aliasing errors, amplitude distortion, phase distortion etcetera.

And some of these special filters and their properties that you have can come handy, if you want their response to be flat to be just one or some constant C or with satisfying perfect reconstruction property with some delay and then some gain etcetera. So, with this you can get an idea as to how you can go about designing these filters and possibly their translates ok.

So, we come to the end of this and our next say module would be into transforms, where we will discuss with wavelets we will continue our journey from multi rate to wavelets, because wavelets and filter banks are connected it is one topic comprehensive topic, then we will look into k l transforms which are dependent transforms and we will look into special considerations in the in the Fourier series representation and some of the convergence aspects we will stop here.