Mathematical Methods and Techniques in Signal Processing - I
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Lecture – 52
Nyquist and half band filters

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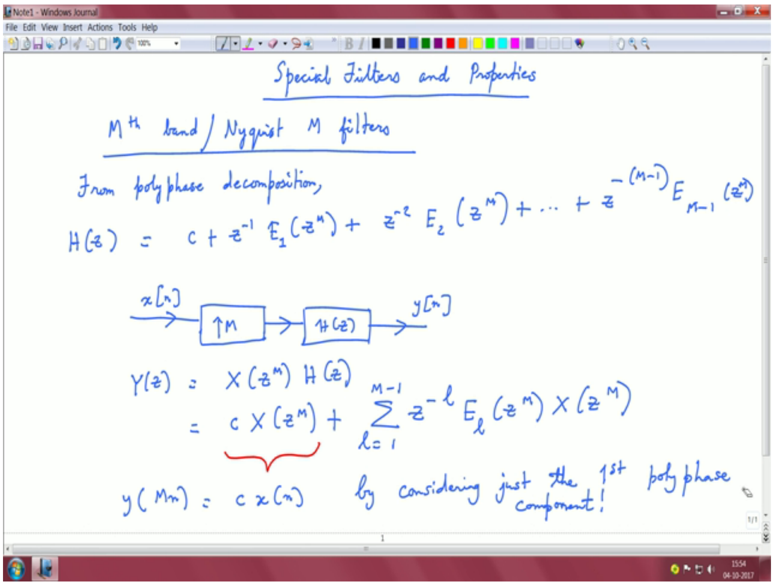

Let us start with Mth band or I would say nyquist M filters. So, these are special filters that you would often see, when you will have to deal with; topics where sampling rate converters or multi rate operations and some of these concepts would be involved. So, the motivation to nyquist M filters stems from polyphase decomposition.

So, from polyphase decomposition, we can write the filter H of z as some constant C plus a delay with the first polyphase component z power minus 2 H 2 of z power M so on; M minus 1 units of delay with; I think this must be E 1 here just ok. So, H of z can be written as z plus z power minus 1, E 1 of z power M z power minus 2, E 2 of z power M and so on and so forth.

Now, when we look at the output right, when we say; if I have some signal x of n, that is possibly up sampled and goes through this filter H of z and I get y of n. So, my y of Z, is basically X of z power M times H of z. And, I could now interpret this as using this filter H of Z, this is C times X of z power M plus summa l equals 1 to M minus 1, z power minus l E l of z power M times X of z power M.

Now, let us consider the first polyphase component, which is what we have here right. So, we can interpret this relation as we have x of n, and when you up sample it you get y of n this is a scalar; this is basically like a gain parameter, if you just ignore this gain; if you take x of n, if you up sample you get this component X of z power M right; if you down sample this signal and you will get x of n back. So, therefore, the relationship between this component to y would be y of M n is x of C times x of n; by considering just the first polyphase component ok.

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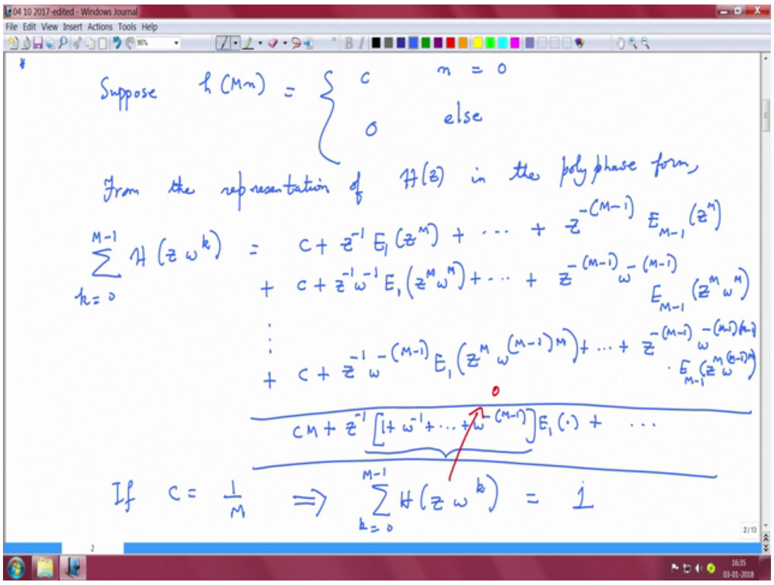

Now, let us go a little further. Suppose, H of M n is C, when n equals 0 and 0 else right; the first component is; the first sample for this filter is is nonzero and everything else is 0. So, from the representation of H of z in the polyphase form, we can consider the following summation; H of z omega power k for k equals 0 to M minus 1; let us say and we could interpret this from our d f t, if you can recall write these for basically translation of the frequencies right take integer multiples of the frequencies and then translate the spectrum this is what it is? Let us add all the translated responses and see what really happens.

So, we start with k equals 0, when k equals 0; it is basically our H of z which is C plus delay E 1 of z power M plus dot dot dot plus z power minus of M minus 1 E M minus 1 of z power M. So, for k equals 1 we have H of z omega, which is going to be C plus you plug z equals the omega here, we have z power minus 1 omega power minus 1 this omega is basically the nth root of unity, you have E 1 of z power M omega power M plus dot dot dot z power minus M minus 1 omega power minus of M minus 1, E is of M minus 1 of z power M omega power M; this is the k equals 1 term you do this till k equals n minus 1.

So, you have C is a dc term plus z power minus 1 omega power minus of M minus 1 E 1 of z power M omega power M minus 1 times M plus dot dot dot z power minus of M minus 1 this times omega power minus of M minus 1 times M minus 1 times E M E E subscript a minus 1 of z power M times omega power M minus 1 times M ok. Now, we add up all these quantities. So, this is going to be C added M times plus so if you look at this term here E 1 of z power M of E 1 of z power M times omega power M omega power M is basically 1, similarly if you if you just take integer multiples of those they will all be 1.

So, you will have E 1 of z power M appearing outside. So, you will have z power M z power minus 1 times you have to add 1 plus omega power minus 1 plus dot dot dot omega power minus of M minus 1 times you have E 1 of something plus these terms. So, if you just notice that these are basically the inverses of the roots of unity and since they form a group you are; you know the, you will circulate back the same elements again. So, this vanishes; so this is going to be 0 and you can conclude with similar argument that the rest of the terms also vanish. So, all you would be with C times M.

Now, if C is chosen to be 1 upon M, this would imply that the summation k equals 0 to M minus 1 of the base filter and all the translates that is M minus 1 copies, this basically is going to evaluate to 1, which is basically a flat response. But, what is the physical meaning of this thing right so; that means, I start with a base filter H of Z, then I look at all it is translations at multiple multiples of the base frequency right you have E power minus j 2 pi by M j 2 pi by M square cube so on right, so all integer multiples of the frequency.

So, then which means you are basically shifting, it start with the base spectrum of H of Z, then you are shifting the spectrum and then you are summing all of them, that is what it means; and that should be flat, that should sum to 1 and why do we need these kind of response, imagine when we started off with this qmf property right; I have a good low pass filter, I good high pass filter and we said basically I take the sum of the responses of the high pass filter and low pass filters at the quadrature frequency there could be certain artifacts, but I really want the responses to sum up to 1. And any deviation that I have would I would what would be basically some sort of distortion, which I would want to minimize perhaps using Johnson's filters and so on and so forth.

So, if I look at all these some frequencies you know across all the translations, they have to add up to 1. And each of these individual filters are basically M band filters or nyquist M filters, because each of them correspond to a certain band of frequencies that they are shown ok.

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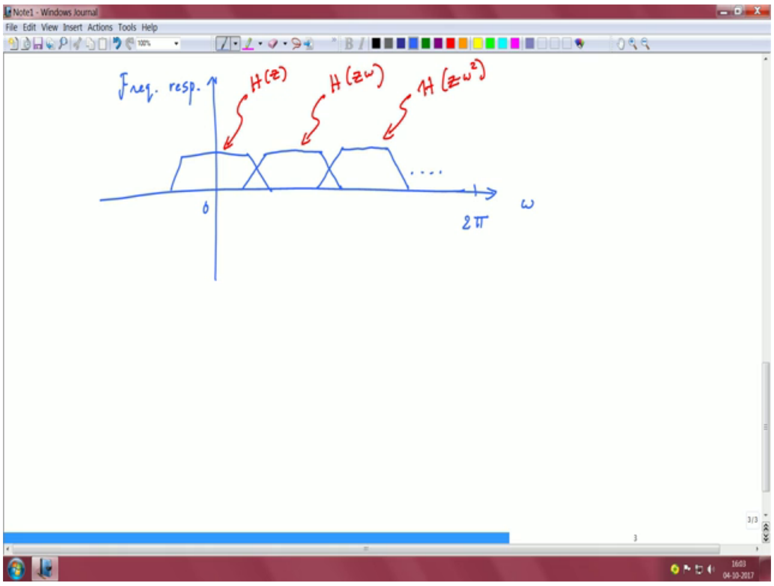

So, if you think about the frequency response you can say my scale is not perfect, but I hope you can appreciate, what is happening here? So, you may have; so let us say this is H of Z, this is possibly H of z omega dot dot dot this H z omega square and so on; somewhere we have to have 2 pi, this is 0, this is frequency response.

So, now, we know the criterion, that we would want some filter H of z to satisfy; if the sum frequency responses have to be 1; that means, one such filter could be the nyquist M filter, which is of the form C plus delay times the polyphase version plus two units of delay followed by the second polyphase dot dot dot and a special consequence of these filters a special k as I would say of these filters are the half band filters.

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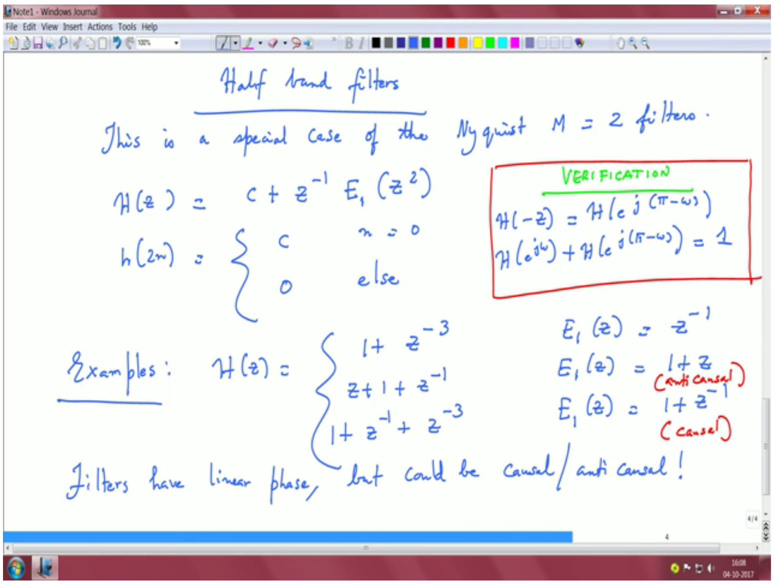

So, as the name suggests this is a special case of the nyquist M equals 2 filters. So, therefore, we can expect H of z to be of the form C plus delay times some E 1 of z square, where E 1 is the poly phase component. So, therefore, H of 2 n is C, when n equals 0 and this is 0; otherwise it is not too difficult for you to see because these are all odd polynomials right. You have z square is z square z power for z power 6 or z power minus 2, z power minus 4, if you want to bring in other ah you know powers negative powers and therefore, this is going to be odd, because you have a one delay component outside. So, basically the even component is basically when n equals 0 it is C is 0 otherwise; it is a special case of the general nyquist M filter.

There are plenty of examples for such filters and let us go through some of them say suppose, H of z is 1 plus z power minus 3 your polyphase component even of z is basically z power minus 1 just a delay; I could have z plus 1 plus z power minus 1, wherein E 1 of z is 1 plus Z, which is anti causal and it could have something like this where E 2 of z is even if it is E 1 here is to be consistent E 1 of z is 1 plus z power minus 1, and in this case this filter is causal, in this case this filter is anti causal. So, the filters themselves could be causal or anti causal, but they all have linear phase.

Now, as it suggests let us verify, if the responses sum to 1 let us sort of verify; so we take H of minus Z, which is H of e j pi minus omega an H of e j omega plus H of e j pi minus omega, this is equal to 1. This means I look at the low pass responsive, if H of z is a good low pass filter, this is a good high pass filter right. If you had a nyquist M filter you will have various band pass versions in the middle. So, this plus this basically sums to 1 and this is symmetric about pi by 2, and you have a case for basically mirror symmetric filters here ok.

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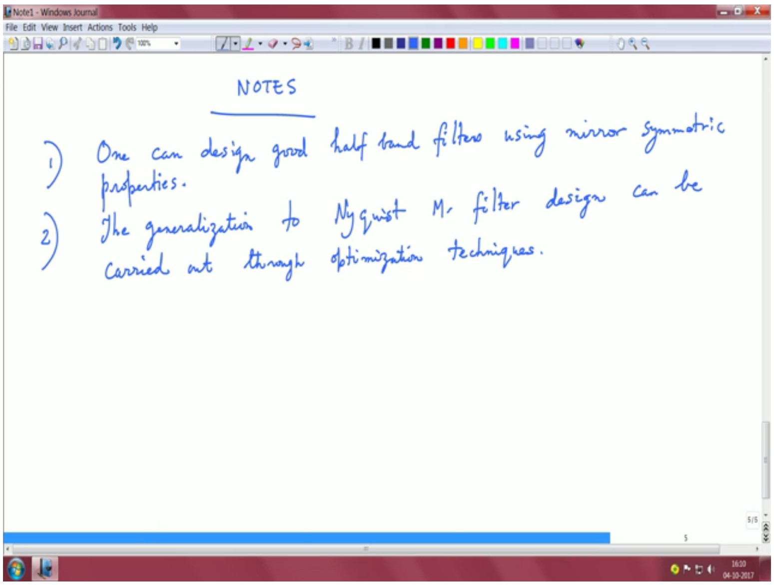

Now, I think with these filters in mind, I think some important notes that we need to kind of but again, one can design good half band filters end out with mirror symmetric properties, and we saw in our homework exercise, how having mirror symmetric properties will help us simplify our computations right. So, basically we can build good half band filters using these and; and second the generalizations to nyquist M filter design and we carried out through optimization techniques.

We saw the case where we could set up our optimization problem for solving for the base filter that led to Johnson filter and the same idea you can extend this towards nyquist M filters. So, these are some basic filters that you will see in multi rate operations.