Mathematical Methods and Techniques in Signal Processing - I
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Lecture – 50
Polyphase representation of M-channel filter bank

So, like the way we did the 2 phase representation for a 2 channel filter bank, we will look at we were looking to the M-channel filter bank and derive a polyphase representation for this. And it will be very useful from our understanding to see how the structure of polyphase fits in to the analysis and synthesis filter banks, ok.

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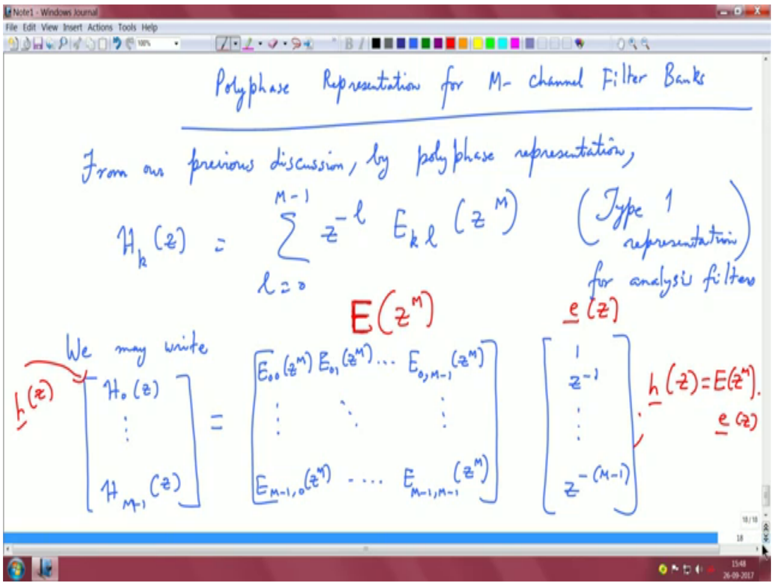

So, from our previous discussions by polyphase representation we can write H k of Z summation as summation l equals 0 to M minus 1 Z power minus l E k l of Z power M and this is the type 1 representation. So, we will look into the type 1 representation polyphase representation for analysis filters right, I choose some k some kth analysis filter, right. So, some k k equals 0 1 2 so on till M minus 1 I will choose one such k in the analysis bank and for that analysis filter indexed by k I have an M m phase representation right l equals 0 to M minus 1 that is the whole idea about this notation.

Now, we can write the representation polyphase representation in matrix form for a stack of all analysis filters right we stack all the analysis filters H naught of Z so on till H M minus 1 of Z, and we link this with the polyphase components and then the delays right.

So, we can write the delay also that becomes easier for us to see through 1, there is no delay unit delay dot dot dot till M minus 1 delays right.

So, H naught will have E naught naught of Z power M we multiply E naught naught Z power M with 1 E naught 1 Z power M with this delay so on and so forth, so on and so forth. It is just a different way of writing things when you stack all the analysis filters as a column vector you can write this in matrix form. So, you have dot dot dot here E 0 M minus 1 of Z power M so on, this is E M minus 1 0 Z power M dot dot dot E M minus 1 M minus 1 of Z power M. And I call this matrix as some bold E of Z power M having all the polyphase components and this is basically a vector of delay elements and this is basically a stack of all the analysis filters right and you can see that in matrix form h of Z is E of Z power M times e of Z ok. So, this is writing it in matrix form.

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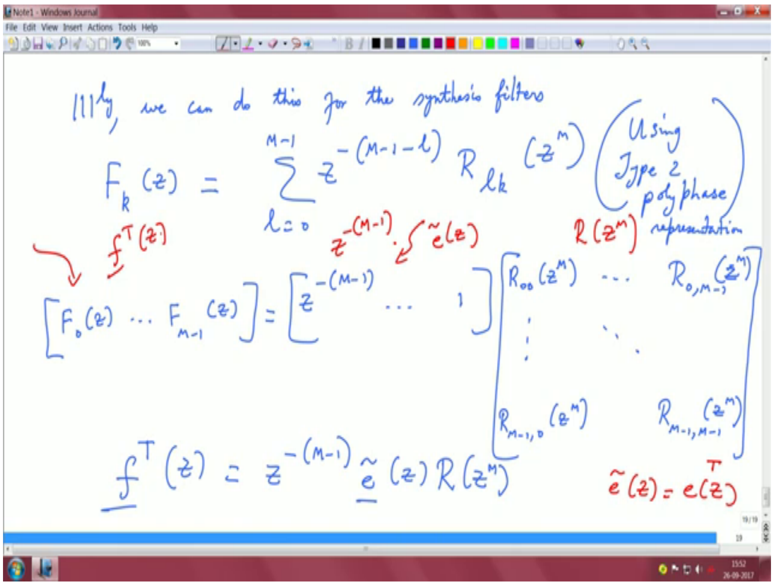

Similarly, we can do this for the synthesis filters, and for this purpose we will look into the type 2 polyphase representation, ok. So, we have this form. So, now, our stack of filters F naught of Z, so on till f minus 1 M minus 1, F M minus 1 of Z can be written in terms of the delay vector and the polyphase components type 2 polyphase components.

At this point you may be even wondering why I need type 1 here and type 2, type 1 in the analysis and type 2 polyphase representation in the synthesis, but things will become very clear in a short while why we need this form of representation, ok. So, I think if we are set with this matrix equation we can write this in this form in a compact way f transpose of Z is basically Z power minus of M minus 1 times some e tilde of Z vector times R of Z power M right, and this e tilde is this transpose version of your E of Z right and I will just add them here this is e tilde of Z, I pull the Z power minus M minus 1 outside times e tilde of Z. So, it is Z power minus of M minus 1 times e tilde Z, where e tilde of Z is e of Z transpose and this is R of power of Z power M and this is f transpose of Z.

So, before we end with a slide a small set of corrections the matrices are typically involved. So, this R matrix has to be in bold following our notation similarly something here, and this transpose is actually transposed a little bit off. So, we can have this transpose ok.

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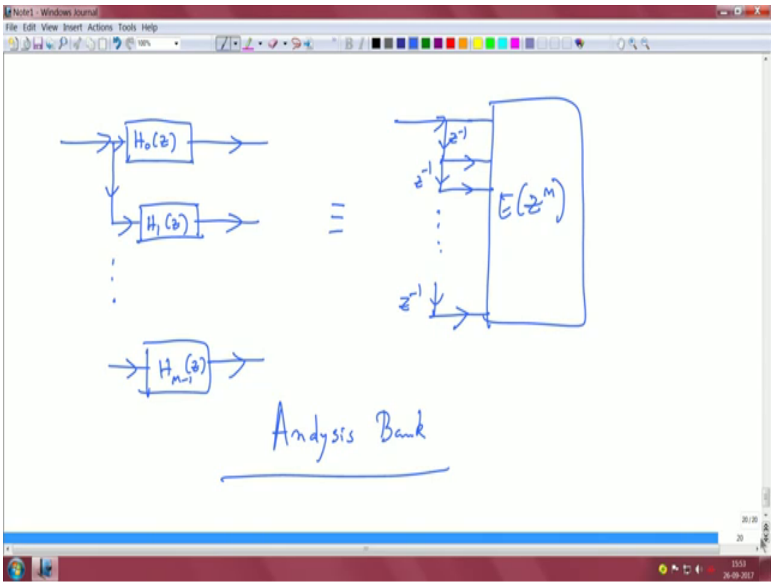

So, with this what we do is we take the analysis bank and write this in polyphase form. So, this E of Z power M is basically a matrix which has all these polyphase components and then we provide delayed inputs to this bank of polyphase components right this is one and the same. So, this representation is basically the architecture schematic sort of schematic for this matrix equation. So, this is the analysis bank. Similarly we can do this for the synthesis bank.

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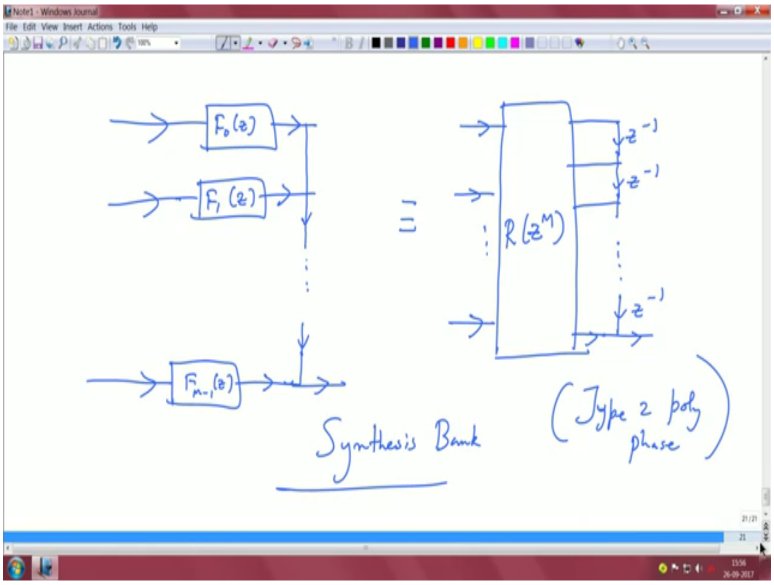

So, let us just focus on the synthesis filters F naught of Z. So, you have all this a band signals coming in and they are going through these synthesis filters M minus 1 of these and this is equivalent to having all these a band signals that go through a type 2 polyphase decomposition followed by delay elements that combine the outputs.

Now, we are not yet complete because we have to introduce our down samplers and up samplers, right. So, let us go one step further.

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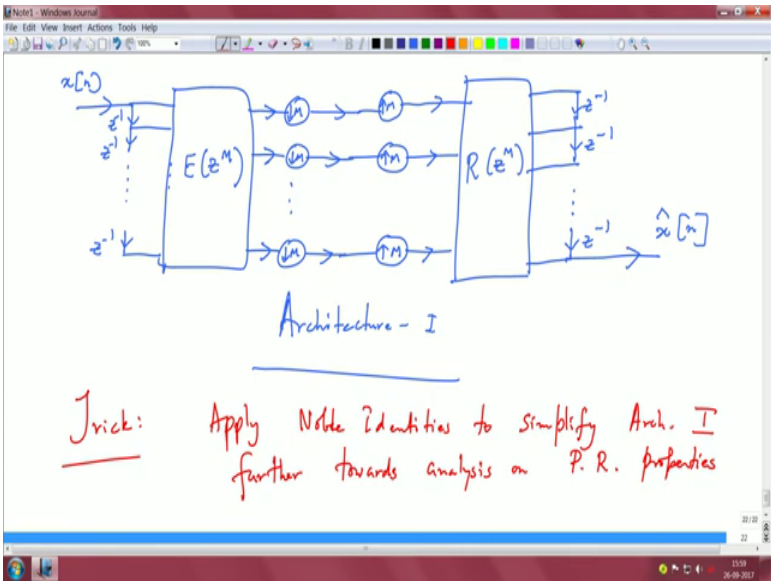

Now, we have this input going through these that goes through these delay lines we have E of Z power M which is our analysis filters in polyphase form. We have down samplers in each of the branches, then we have up samplers followed by synthesis filters in type 2 representation and our reconstructed signal.

Now, this is not enough for us because it is difficult to analyze how the polyphase filters E of Z power M and R of Z power how the polyphase filters behave right. I mean it is impossible to look at the spectrum and then analyze if we if we can get perfect reconstruction property or not, through the analysis of the schematic shown in this architecture here I will call this as architecture architecture 1. With just architecture 1 it is impossible for us to study any properties on perfect reconstruction. But we will apply a familiar trick that we have learnt which is through noble identities we will push these down samplers before this bank and we will push these up samplers after this synthesis bank.

So, the trick would be apply noble identities to simplify architecture 1 further towards analysis on perfectly reconstruction properties. I think once we realized that what that we have to apply noble identities its probably straightforward.

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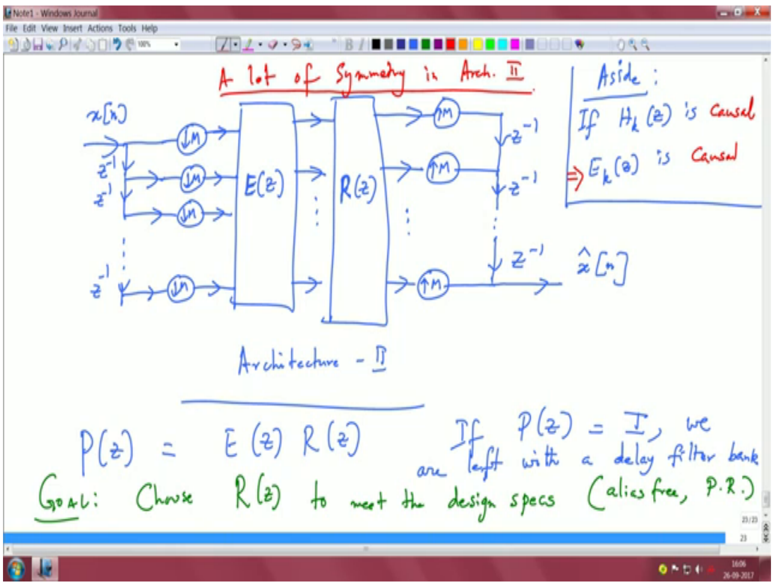

So, x of n we bring in these down samplers here there is a delay followed by down sampler within this branch and this would now, be E of Z naught E of Z power M. And we have our R of Z this bank. We have up samplers followed by delay elements and then we combined the signal to get x hat of n and this is architecture 2. I want to make this distinction very carefully here that this is bold E and bold R indicating that they are matrices.

Now, you have to ponder a little bit why architecture 2 is useful for us. Now, if we say that our overall transfer function P of Z not overall, but I combined E of Z and R of Z to some transfer function P of Z right, and if P of Z is an identity then this is like our delay filter bank. You see this if P of Z is identity we are left with a delay filter bank and we know that the delay filter bank is perfect reconstructing and your intuition should tell you that out after M minus 1 delays you should get your reconstructed output because we saw the example for a 2 channel bank and we got it after unit delay. So, after M minus 1 delays you should get your reconstructed signal right.

So, therefore, this P of Z plays a very important role here, but P of Z need not be identity right we may not land up with an identity. How should we design our analysis and synthesis filters such that P of Z is some delay. So, these are questions that one can one can ask. And aside one has to note that if H k of Z is causal, causality it is also very important here then E k of Z is also causal. And then we may have to choose R of Z I will mention this as a goal, choose R of Z to meet the design specs and what is our design spec it could be it should be alias free it should satisfy perfect construction property etcetera right and maybe some specifications of the distortion as well.

So, this is a very useful representation and that is the reason why we introduce type 1 and type 2, because if you did not do type 2 representation the synthesis side you would land up a difficulties because you could not get the up samplers. I mean the delay elements would not be symmetric here I mean you would have you would have to introduce delays in the middle here. So, you imagine that you have you forward exactly type 1 for synthesis you will have to have delay elements here immediately, and that would be hard to even think about how that would have an impact because it have delayed versions of the sub band signals followed by filtering and up samplers and all that it would be very messy to analyze such systems.

So, this is very elegant it is symmetric because you have two delay elements, two chain of delay elements on either side followed by up samplers and up samplers and down samplers on this side and then the bank of filters. So, there is a lot of symmetry, a lot of symmetry in architecture 2. So, therefore, this is useful for our analysis as well, ok.

So, we will stop here and then we will delve a little bit deeper into the properties for perfect reconstruction for M-channel filter banks and we will see how polyphase design polyphase representation can help us you know overcome some of the issues with stability etcetera if we were slightly clever in the way we design these filters, ok.

So, with this we complete this module we can stop here.