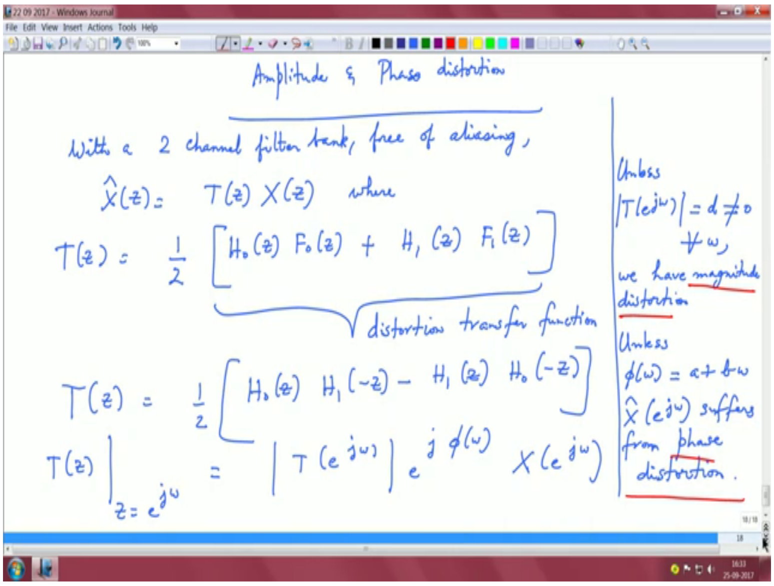
Mathematical Methods and Techniques in Signal Processing - I
Prof. Shayan Srinivasa Garani
Department of Electronic Systems Engineering
Indian Institute of Science, Bangalore

Lecture - 47
Amplitude and phase distortion in signals

(Refer Slide Time: 00:26)



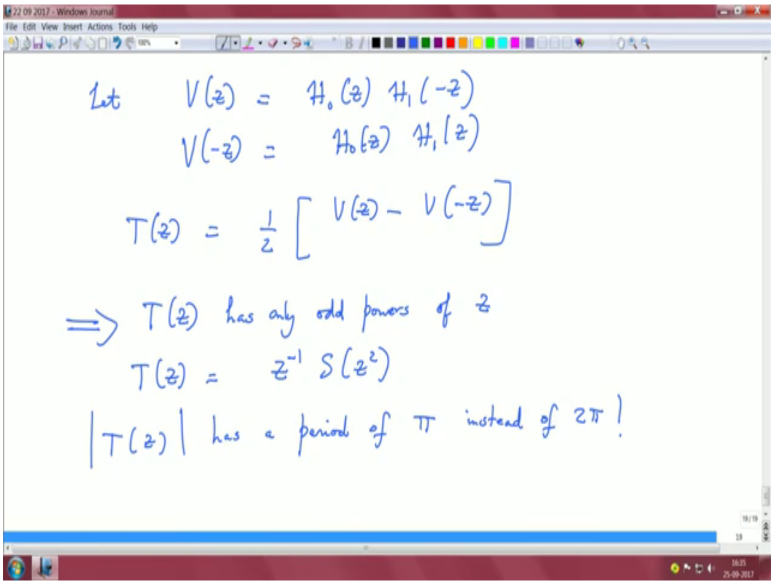
Now, with a 2 channel filter bank, free of aliasing right. We can write X hat of z to be some T of z times X of z where, T of z is this transfer function given by 1 half H naught of z F naught of z plus H 1 of z F 1 of z. And this is called the distortion transfer function. Why? Obviously, as the name suggests the signal X of z is distorted by this transfer function T of z. Now we apply the conditions for F naught and F 1 through the alias cancellation equations and we simplify this as T of z is 1 half H naught of z H 1 of minus z minus H 1 of z H naught of minus z ok.

Now then I evaluate T of z at z equals e j omega, I get some magnitude of this transfer function and then I have some phase phi of omega X e j omega ok. So, I just rewrote this transfer function as having some magnitude response having some phase response. Now, unless modulus of T of e j omega is a constant some d which is not equal to 0 for all omega we have magnitude distortion right. Unless this is some constant; if I know it is a constant gain then I can just attenuate that gain. I can just use some amplifier to just you know either adjust my gain accordingly, if it is a constant gain. If not if the gain varies across frequencies then I have amplitude distortion.

Now, unless phi of omega equals a plus b omega, I mean is of the form unless phi of omega is of the form, a plus b omega. X hat of e j omega suffers from phase distortion. This is very very important this phase distortion because the signal can have different phases for different frequencies. And you have to really compensate them very carefully; that means, if you have a signal this will have different frequency components and imagining imagine that you are sending the signal through a bank of filters and each filter or an each frequency is suffering some distortion in its amplitude and in its phase right.

That means, the delay at the output when you would get these signals would be different for different frequencies that is a physical meaning of phase distortion, right? So, different frequency components will arrive at different times at your output, if you have a phase distortion and it is important eliminate this. So, these are two major points ok.

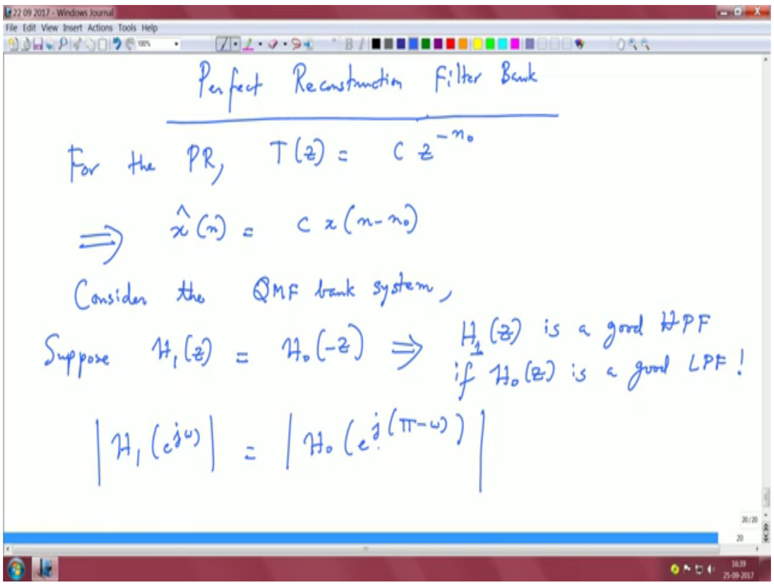
(Refer Slide Time: 05:35)



Now, we will go 1 step further. Let V of z is be equal to H naught of z times H 1 of minus z. And V of minus z is basically H naught of z, H naught of minus z minus here H naught of minus z times H 1 of z right this is just minus z here ok. So, now T of z is 1 half of V of z minus V of minus z. So, this is what we have. Now with this we can we can see that if you have a z and a minus z of z square and a minus z square is the same, z power 4 and minus z power 4 is the same.

So, therefore, you can think that all the even powers get cancelled here and you will be left with odd powers right. So, T of z has only odd powers of z. Therefore, T of z can take this form z power minus 1 that is, you factor out a unit delay times some polynomial in z square right. Because you will have a z cube z power minus see all the odd components you will have. So, just factor the delay out the rest of it would be even right. So, there will be some polynomial in z square. And I think an inspection would tell you that modulus of T of z has a period of pi instead of 2 pi just observe this power here. If it has to be periodic z power 2 j omega right; j 2 omega plus. So, what would you do if you want to translate this by a period T and that has to be basically pi right and just because of the power 2 here?

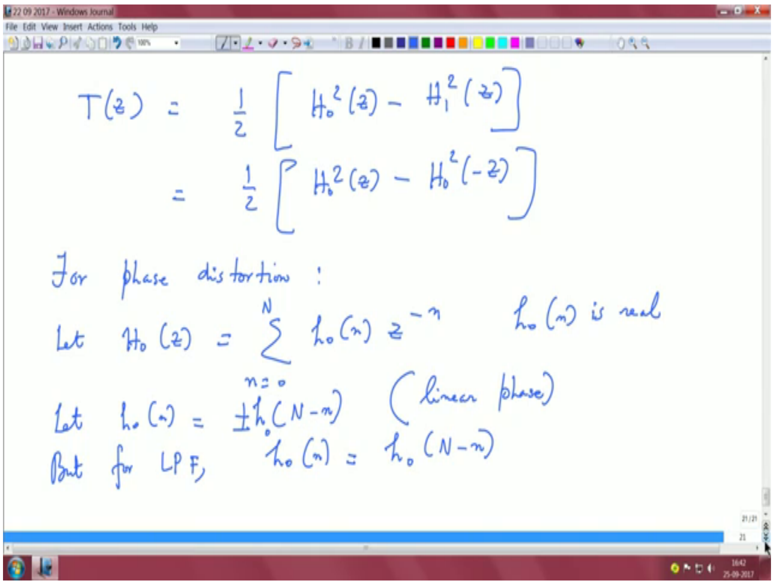
(Refer Slide Time: 08:34)



So, we have a notion for perfect reconstruction. So, for the perfect reconstruction property T of z has to be of the form c z power minus n naught. That is it just suffers from some gain c and some delay. Perfect and the ideal cases where n naught equals 0 and c equals 1 that is an ideal situation right. A less ideal situation would be to have some gain and some delay because you can compensate. So, this implies that x hat of n is basically some delayed version of the input. Now let us consider the Quadrature Mirror Filter bank system. Suppose H 1 of z, that is if it satisfies the quadrature mirror filter property then H 1 of z is H naught of minus z which implies, H 1 of z is a good High Pass Filter if H naught of z is a good Low Pass Filter. If H naught of z your base filter is a very good low pass filter it good properties in H 1 of z is a very good high pass filter.

And it is obvious to see that the magnitude of H 1 of e j omega is basically magnitude of H naught of e j pi minus omega, right? And that minus 1 accounts for this factor. Now, all the filters are completely specified as we discussed earlier that if you have a filter H naught then you get H 1 F naught and F 1.

(Refer Slide Time: 11:44)

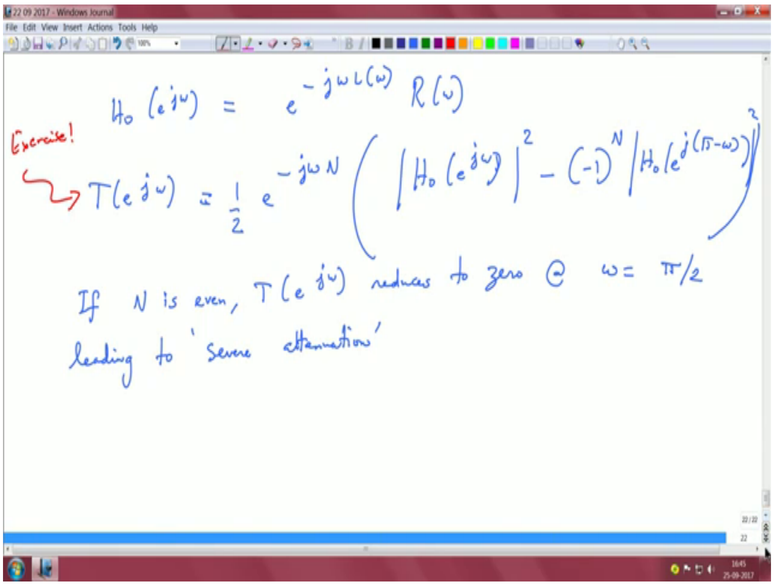


Now, the distortion function in this case is easy. T of z is basically 1 half of H naught square z minus H 1 square z. And then if we can simplify this as 1 half of H naught square z minus H naught square of minus z . So, in terms of 1 base filter H naught of z we can get the distortion function for a quadrature mirror filter bank. So, I think the sequence of steps were as follows. We get the distortion function after we force the alias cancellation conditions right. Once we force a alias cancellation we will have an equation in terms of H naught and H 1 in our distortion function and then we invoke the QMF property and then we eliminate H 1 as well and we get in terms of purely H naught.

So, I think this gives us an idea that we can start with this distortion function and we can basically figure out how we have to design these filters H naught as we have designed this filter H naught of z that can eliminate amplitude distortion ok. Now the phase a little more interesting, so let us just get towards the phase distortion. For phase distortion so let us consider the filter H naught of z to be of this general form H naught of n z power minus n, n equals 0 to n.

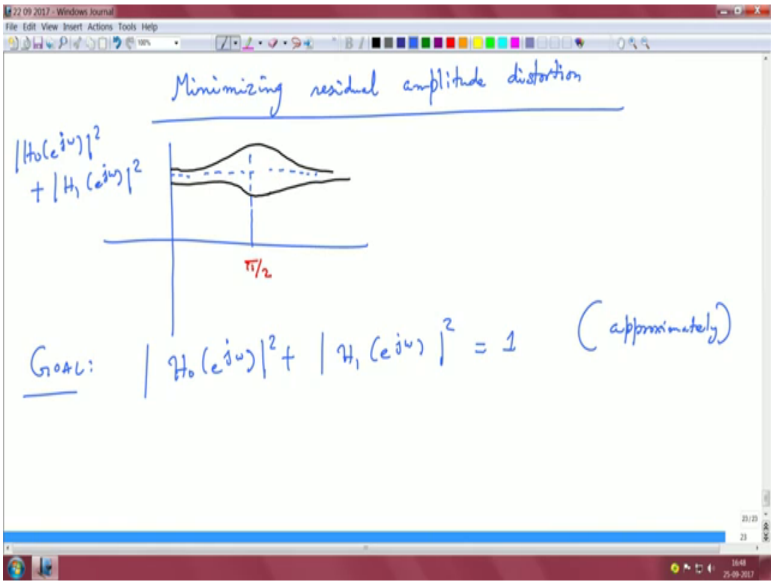
Let us assume that H naught of n is real. So, we also let H naught of n equals plus minus n minus n. So, this will ensure it is linear phase, but for low pass property you have to remove the negative. Because if you have a negative sign it is it is implying it could be a band pass or a high pass right. So, all the coefficients should have the same sign. That is one of the conditions; so H naught of n is basically H naught of capital N minus small n ok; so, with this we can simplify things a little further.

(Refer Slide Time: 14:56)



So, we can say H naught of e j omega is of this form, e power minus j omega times some linear function of omega. It could just be a constant times R omega and you can simplify this to this form T of e j omega is e power minus j omega n 1 half of that times mod H naught of e j omega square minus minus 1 power N magnitude of H naught e j pi minus omega square. This is your distortion function. Now, if N is even, this becomes one. So, this would just vanish. T of e j omega reduces to 0 at the frequency omega equals pi by 2 leading to severe attenuation. So, I will leave the simplification as a homework exercise. I mean this simplification of how you have to arrive how you would get this T of e j omega. In this form, I will leave this has a homework exercise ok.

(Refer Slide Time: 17:27)

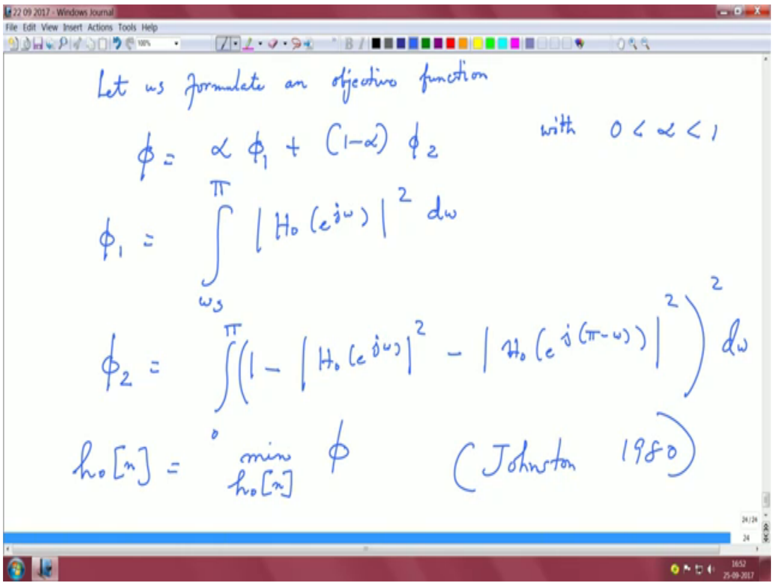


So, with this there are some other details that we have to understand regarding Minimizing the residual amplitude distortion right. We eliminated the aliasing error. Then we looked at the phase distortion right? We said we want to have linear phase. We started with an FIR filter. We ensured that it is the symmetric. It is mirror symmetric right? So now with the mirror symmetric property in mind, then we want to simply simplified it further to get the overall distortion function and that distortion function could have possibly in amplitude distortion. Because it is it is distorted across frequencies the amplitude, is distorted across various frequencies right. So, let us see in a little bit of careful detail what this is about.

So, if you think about how this H naught of e j omega square magnitude plus H 1 of e j omega square how this looks. Ideally you might want it to be at 1 , but at the quadrature frequency which is at pi by 2. You can have a blip up or you can have a blip down. And our goal would be to have the magnitude of H naught of e j omega square plus magnitude of H 1 of e j omega square to be 1 approximately, is equal to 1 is ideal; close to 1 is approximate. And what do we need to do? If this is our constraint we have to choose the coefficients of our filter H naught of z such that this relationship holds. Right? So, H of course, if you use QMF property H 1 can be expressed in terms of H naught we know that.

So, everything can be recast in H naught itself. And then if we recast it in that framework then we have to design the filter that is we have to choose the coefficients of this filter H naught of z such that this is equal to 1 approximately, but that is hard to get. Already we invoked some symmetric property for the filter we ensured it is linear phase right. We took care of all these other constraints. Now to eliminate this distortion completely would be very hard.

(Refer Slide Time: 20:23)



So, let us formulate an objective function that possibly helps us to drive choosing our filter coefficients appropriately. Ok let us assume phi is of the form alpha phi 1 plus 1 minus alpha times phi 2, with 0 less than alpha less than 1.

And let us define the functions phi 1 and phi 2 as follows. Phi one basically goes from your stop band to your Nyquist integrating this function that is this is the magnitude square distortion and you are integrating it across all the frequencies. So, you are basically getting the energy right. You are getting the energy in detail from the stop band to pi. And then suppose if you want if you choose phi 2 as summing up the energy over the residual; that is 1 minus magnitude of H naught of e j omega square minus. So, I will not invoke the QMF property. H naught of e j pi minus omega square. And this if I were to integrate across all frequencies, then I would want to choose H naught of n is minimum overall phi.

Minimum overall H naught of n. Minimum overall H naught of n this objective function phi. That means, if I choose alpha to be 1, I only minimize the tail energy right. If I choose alpha to be 0, I look at just this distortion that is if the H naught and H 1 magnitude squared should deviate from 1 that residual square energy I want to minimize overall frequencies. So, this is like your error that you have. Error square is what you set up here and I want to minimize this error square ok. So, these are some objective functions and I mean one such function is this. So, this gives you a gamete of choosing your alpha just from the tail that is from the stop band to pi versus looking at the entire band from 0 to pi. And choosing this sort of objective functions leads us to an optimization procedure and this procedure was done by Johnston in the 1980, in 1980 and these are called Johnston filters.

So, if you designed your base filter with this objective function in mind, then it gives rise to your optimal low pass filter under this constraint for this objective function and they are called Johnston filters ok. So, this completes our work on analysis of the 2 channel filter bank for alias cancellation, for phase four minimizing the phase distortion and then for minimizing the amplitude distortion. So, you have to keep in mind first you start with the alias distortion.

You figure out how your synthesis filters have to be chosen to eliminate aliasing distortion. Then to simplify your design, you assume quadrature mirror property. So, that given a base low pass filter you can construct the high pass filter as well. Then you analyze the magnitude distortion from you analyze the distortion transfer function and from that you simplify further to look into the phase distortion and amplitude distortion and then construct filters that can minimize these distortions ok.

So, with this we are done with the analysis of a 2 channel filter bank and we will extend this further towards m channel filter banks and then we will study the properties of those filter banks, we will stop here.