Mathematical Methods and Techniques in Signal Processing - I
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Lecture - 46
Two-channel filter banks

Welcome all of you to this lecture. In the last lecture, we covered multistage designs. And we saw the benefit of having multistage implementations over a single stage implementation. And two things to note from the last lecture, one is how multistage designs can drastically reduce the computational burden than what you would have done using a single stage, and second how you can cleverly overcome some of the aliasing effects in lieu of trying to get better computational efficiencies. We saw these concepts clearly through an example. And you can extend this to x you know like interpolation filters etcetera in a very natural way it is not difficult.

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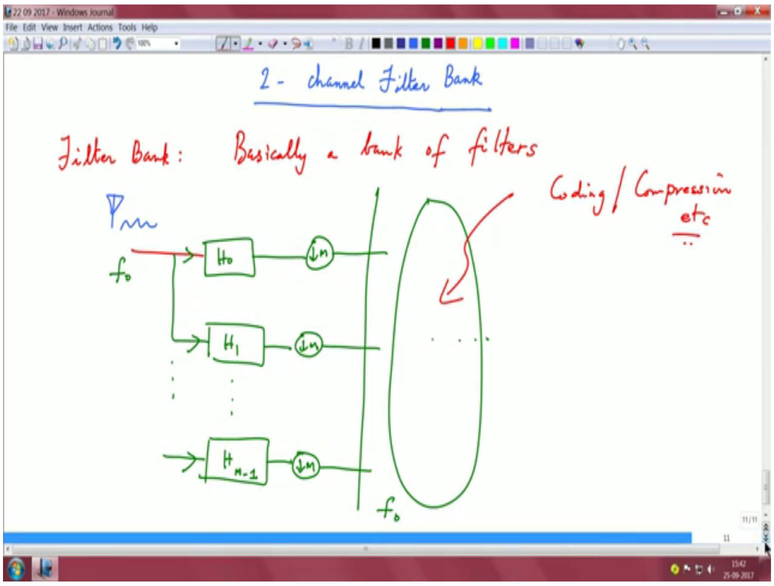

So, now, we have the idea of what down sampling is what up sampling is and how we can use these down samplers and up samplers along with filters. We are ready to embark on a concept leading to filter banks right. So, let us understand what a filter bank is. So, filter bank is basically is basically a bank of filters. And what do these filters do, they have to filter out the spectrum in those appropriate bands right. You take a full band signal you send them through a bank of filters, and these filters produce some spectral shaping properties right. And then to maintain the rate at the input, you basically have to down sample to some extent right.

So, the idea is as follows. Let us say you have something here at the transmission side right. This is your signal coming in. And you have a bank of filters we will call this as H suffix M dot dot dot that is H suffix M minus 1. So, you have n such filters right and the input rate is f s. And here I might want to down sample them by M such that the sum rate here is still f naught. And then I want to do something with each of these signals, and this is one frequency band is another frequency band and so on and so forth these are all called sub bands right.

And I want to do something in these bands maybe I compress the signal in each of the sub bands, maybe I do some kind of computation at this a band level, it could be quantization any operation. And then I want to basically send them through an expansion phase to restore the sampling rate and then I want to synthesize right. A lot of compression coding compression etcetera can happen at this stage when you deal with these sub bands ok.

So, our goal is to design such bank of filters such that you can get perfect first is the first goal is to get perfect reconstruction. I will send something, I take them to a bank of filters, I down sample them. And you may even ask a philosophical question at this point what why should I send them through different filters right, signals sometimes do not have energy which is distributed uniformly across all the bands. For example, I may have maximum energy in the low pass sub band, I may have very little energy in the high pass band right, then I may have to decide how much of compression I want to give, how I want to code, where the dominant energy is and so on and so forth right.

So, this is with respect to this perspective that you want to do some efficient transmission at the channel level that you want to build sub these filters at the analysis stage ok. So, with this objective in mind let us start off with simple version of a two channel filter bank.

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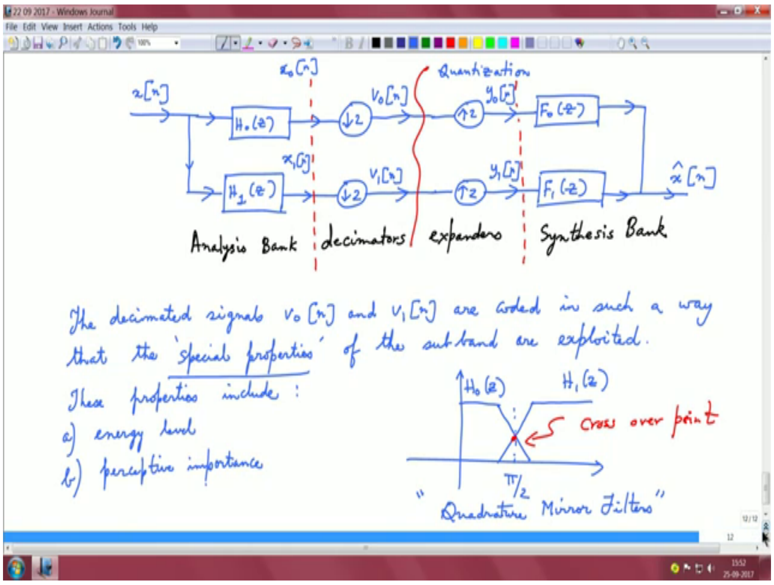

So, we start with the discrete time signal x of n. This goes through two filters H naught of z and then H 1 of z. Then I down sample this at this stage right. I take H naught, I take H 1, I down sample this. And as I told you earlier it is at this interface where you do quantization and some of these things. And you would restore the sampling rate through up sampling pass them through synthesis filters F naught of z and then F 1 of z and you combine the signals from the output of the synthesis filters to get some x reconstructed x hat of n.

So, this is your analysis bank; this is your synthesis bank. And here are your decimeters; and here are your expanders. And let us designate the interim signals also. At this point I have x naught of n; at this point I have x 1 of n; at this stage I have v naught n; at this stage I have v 1 n; at this stage I have y naught n; and I have y 1 n in the second branch of the filter bank post expansion.

The decimated signals v naught of n, v 1 of n are coded in such a way that the special properties of the sub band are exploited. And what are these properties, these properties include a energy level, there could be more energy in the low pass versus what is available in the high pass right. For example, if you look at naturals images, speech, etcetera the energy is more in the low pass than in the high pass right. So, therefore, you might want to look at the energy level. Second is the perceptive importance. For example, if you look through the hearing I mean if you if you if you analyze the hearing process right, there is a sort of a logarithmic response of our ear to various frequencies; that means we do not place the same resolution for all the frequencies that we hear. So, there is this notion of logarithmic kind of rate at which we hear we perceive sounds.

Similarly, when we see images I think the first thing that we observe is the low pass right. We just average the information out in image and that is what we look at. A good example is suppose you are observing a pitcher, probably we will focus on the object the background the object in the in the in the foreground and so on and so forth. The last detail would be what is the color of the hair or one strand of hair on the portrait that would be the last level of detail right the this is a natural course through observations right. So, this is what we perceive through the human visual system.

Similarly, in the auditory system if I give you a lecture for about an hour probably a great student will absorb every little bit of detail in the lecture, but more students would absorb the salient features, the main concepts in the lecture. Some detail of certain variable and what its functionality is perhaps would be retained in the immediately in the class, but when you go out perhaps you will know the concept, but not the variable that was discussed in the class right this is. So, what are you doing here you are averaging information out, you are averaging the signal out. And this is no different then what we want to exploit in when we say we look at perceptual importance of the signal when we want to design these filters ok.

Now, two important filters come into our mind these are basically Nyquist m bright filters and a simplest of that would be this quadrature mirror filter. So, quadrature mirror filters as a name suggests, they have a crossover frequency at 2 pi by 4 which is pi by 2. So, this is the crossover frequency which is pi by 2 and they are basically mirrors of one another. So, if H naught is a very good low pass filter, the mirror of this is H 1 which is a very good high pass filter, and their quadrature mirror filters because their crossover point is at pi by 2. These are quadrature mirror filters. And our goal would be how can we you know build these analysis filters synthesis filters such that the reconstruction is perfect right.

There is a notion for perfect reconstruction; I will get back to that in a while. But intuitively you might just wonder you know we saw the frequency domain effects at the output of the down samplers at the output of the expanders followed by filtering etcetera right we saw all those effects. Now, this is just, so there are distortions that the signal x hat of n can suffer during this process of passing it through a filter bank with multirate operations right.

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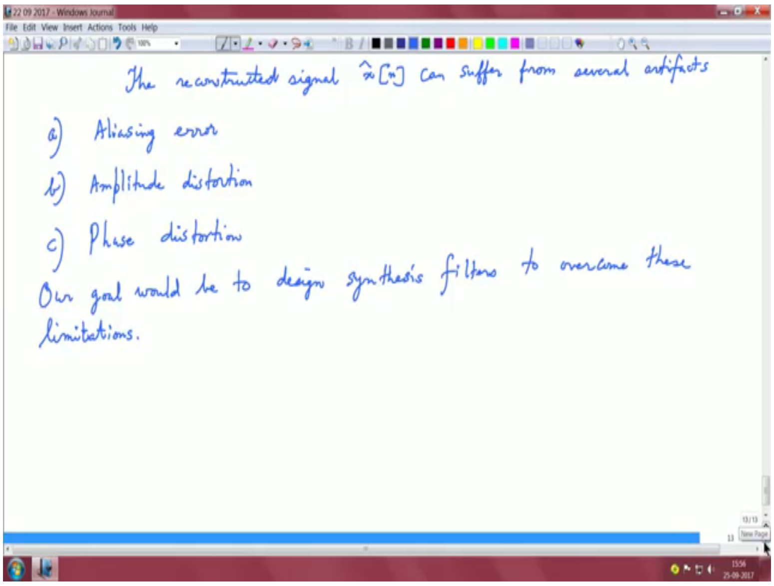

So, what are those artifacts? So, the reconstructed signal x hat of n n suffer from several artifacts. The first artifact is as you can imagine the aliasing error your down sampling because of down sampling process there is a stretch in the frequency. And if something is happening that your filters are not accommodative to this stretch because of the Nyquist rate sampling then there would be aliasing effects. And this is very important that the reconstructed signal should be free from aliasing errors.

Second you can have amplitude distortion. And what you can also imagine would be following amplitude distortion is phase distortion. And our goal would be to design synthesis filters to overcome these limitations. Now, we will go into the details as we analyze the two channel quadrature mirror filter bank, because it is a simple two channel bank. And since I considered a two channel quadrature mirror filter bank, we want to bring in this quadrature mirror filters, and we will sort of analyze from that perspective.

A note here is regarding the amplitude and the phase distortion phase is very important. Like if you imagine a lightning striking right you first see the lightening and then you see the you hear the thunder right, you hear the thunder after you see the lightning. So, why because velocities are different for light and sound right, and therefore, you see this effect.

But now if the frequencies themselves are the same, but the phase is different. A good example is let us say I have images which is part of the video. And I have speech and other things which are coming through the audio. And when I have to merge the sequences I have to be very careful that I do not introduce additional delays or I do delay matching in such a way that the image the lip movement and what comes out through the sound have to be aligned. It will be very awkward if you saw a news announcer whose lip movement is something. And he or she is talking something else which happens you often you might have noticed this is a very good example for phase distortion effects. Or even good example is the speech is coming in some direction, but it is a different frame of a video and that is also phase distortion all right, and we do not want these things to happen. So, therefore, we have to ensure that we take care of these phase effects.

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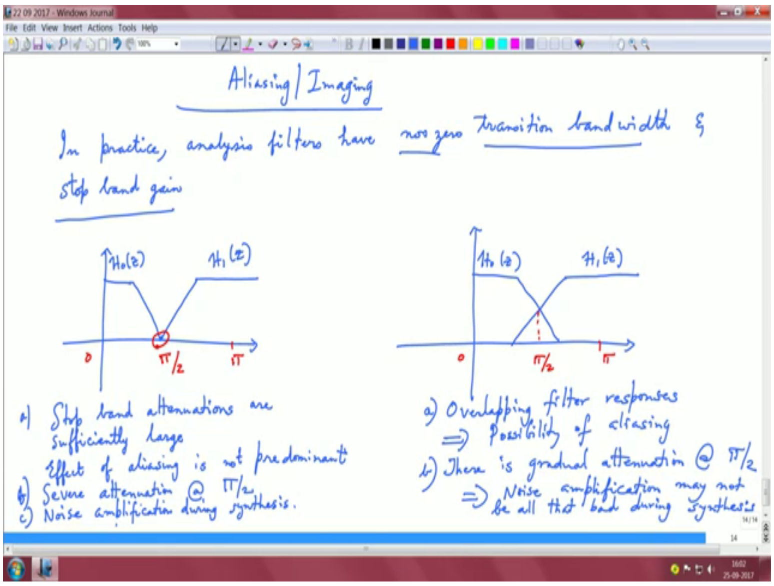

So, let us first start in it aliasing and imaging. In practice, analysis filters have nonzero transition bandwidth, and stop band gain, nonzero transition bandwidth and stop band gain. Now, let us think about few scenarios here. In one case, this is the same frequency at pi upon 2, let us say this is pi, this is 0, and this is my H naught of z, and this is my H 1 of z. I have another case like the quadrature mirror filters, where there is a crossover at pi by 2 ok. Now, you might imagine what is happening here in this case the stop band attenuations are sufficiently large and effect of aliasing is not predominant right.

So, I will I will just note it down here stop band attenuations are sufficiently large. So, which means the effect of aliasing is not predominant, but there is severe attenuation at this frequency pi upon 2. There is a severe attenuation at pi upon 2. So, therefore, if you have to think about the synthesis filters they have to boost the gain at those frequencies. So, if I have to boost the gain at those frequencies, noise can also get amplified because of this process and filters can become very expensive. So, noise amplification during synthesis, so this can be one of the caveats that you can that you can face when you design these type of filters.

Now, if you look at this you have aliasing possibly because you have overlapping filter responses possibility of aliasing. And I mean and then the other point is, but hopefully if you can avoid aliasing through the use of synthesis filter. So, if you use synthesis filters appropriately, you can overcome this aliasing artifact, and we will see how we could do this. And then some of the aspects that we saw here right, if there is a gradual attenuation at pi upon 2. Therefore as a consequence noise amplification may not be all that bad during synthesis ok.

But unless we derive the equation for the reconstructed signal, we will not be in a position to see where aliasing is coming, how can you overcome aliasing right, this is very important. And then what would you do for your other artifacts such as what is the residual amplitude distortion, phase distortion so on and so forth. So, we are now getting a sort of an idea how to go about our analysis of the filter bank. So, let us start with the derivation for an expression derivation of an expression for the reconstructed signal ok.

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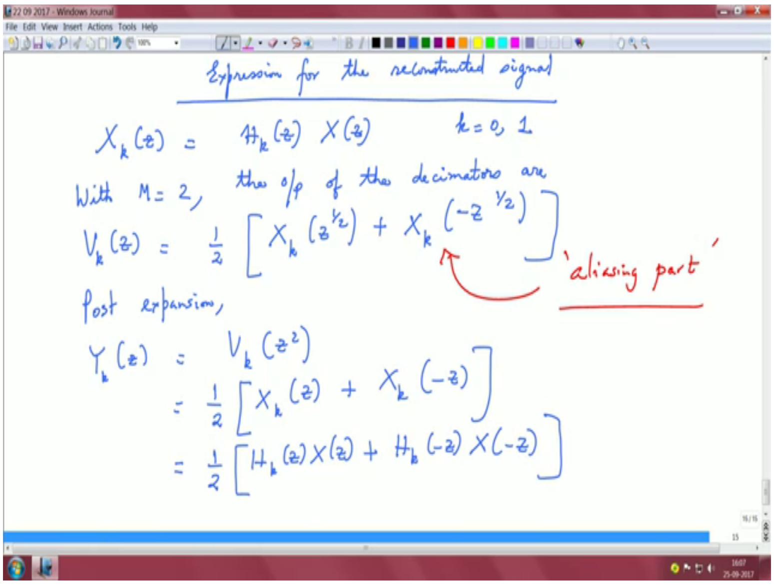

So, we will derive an expression for the reconstructed signal. For this you will have to basically look into your diagram block diagram that you had for the two channel QMF bank. So, right you had x of n going through H naught of H naught of z and H 1 of z, you had these down samplers, then you had this up samplers and then you had the synthesis filters and we have all the interim variables that we defined here the intermediate variables. So, now we will just write down equations for all of these intermediate variables, and then we will get towards the expression for the reconstructed signal X k of z is H k of z times X of z this is for k equals 0 and 1 right, because just you are filtering it through H naught and H 1, so therefore, x naught and x one are basically H naught times x and H 1 times x respectively straight forward.

Now, with M equals 2 down sampling, the output of the decimeters are V k of z is one half X k of z power 1 upon 2 plus X k of minus z power 1 upon 2. And very clearly you see this is the aliasing part. All I did here was write down the equation after the output of the down sampler and the minus 1 you see is because omega 2 power k right, k equals 0 and 1, but k equals 1 you have that minus 1 factor.

Now, Y k of z is V k of z square which is post expansion I get this. Now, I simplify this to be one half of X k of z plus X k of minus z this straightforward plug z equals z square into the equation above. Now, since we know what the equation is for X k in terms of H k and X, we plug that in. So, we have one half H k of z X of z plus H k of minus z X of minus z ok.

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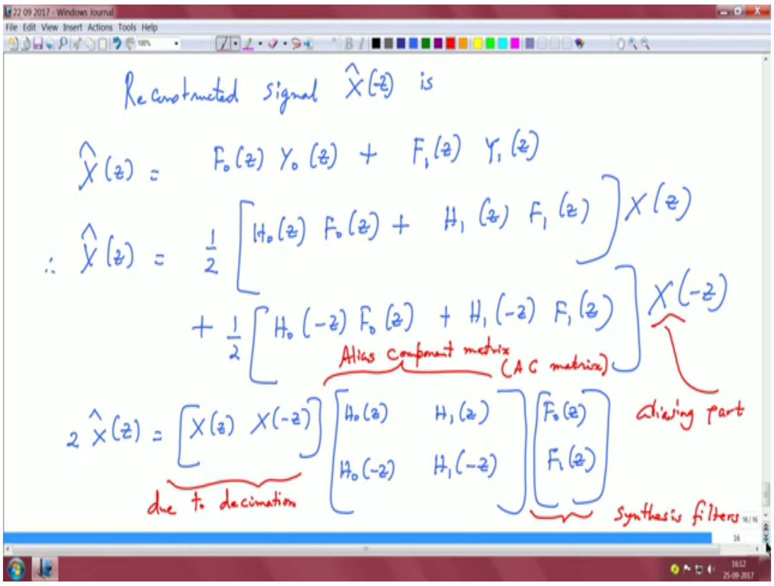

Now, post this is easy for us to figure out what the reconstructed signal is because they have to go through the synthesis filters. So, the reconstructed signal X hat of z is given by X hat of z equals F naught of z Y naught of z because you have Y naught and Y 1 coming from the up samplers during the synthesis stage right. So, therefore, F naught of z Y naught of z plus F 1 of z Y 1 of z this is straight forward. So, now I simplify all this I have one half H naught of z F naught of z plus H 1 of z F 1 of z times X of z, what I am doing here is just plugging in Y naught and Y 1 from my earlier step plus H naught of minus z F naught z plus H 1 of minus z F 1 of z times x of minus z and this is the aliasing part ok.

Now, this will be much easier if get did this in a matrix form. So, I can write it as follows 2 X hat of z is X of z X of minus z times H naught of z H 1 of z H naught minus z H 1 of minus z times F naught of z F 1 of z. And why this appears this is because of due to decimation right, decimation causes aliasing. And these are our synthesis filters and this matrix is called the alias component matrix or it is also called an A C matrix.

Now, with this in place I think it is pretty easy for us to see what we need to do to eliminate alias aliasing distortion right to remove aliasing errors. So, what we have to do is x of minus z is given to us we cannot change this. So, the best that we could do is force this term here to 0. If I force this term here that is the preceding term to X of minus z, if I force that to 0, this aliasing distortion is eliminated right, and that is what we will we will do.

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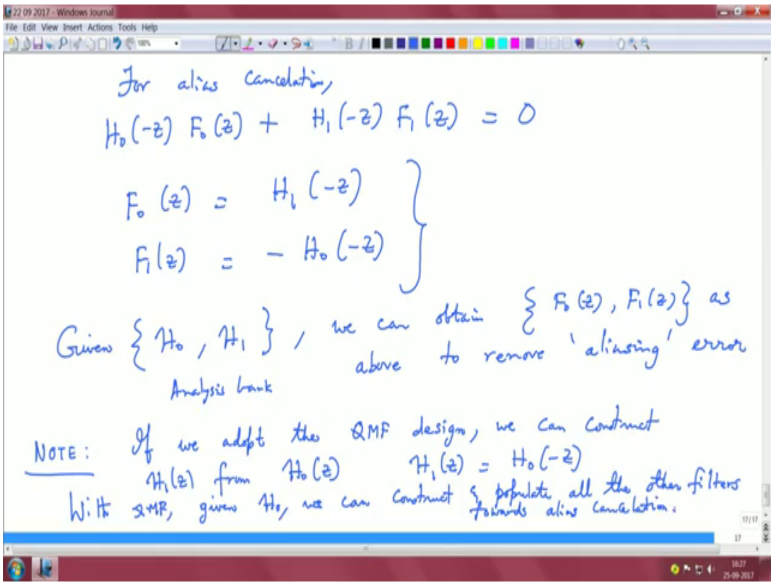

For alias cancellation, H naught of minus z F naught of z plus H 1 of minus z F 1 of z this has to be forced to 0. One choice is to have F naught of z to be H 1 of minus z and F 1 of z equals minus H naught of minus z. Now, you plug in F naught and F 1 to be these values that are given here and you can see that it is indeed vanish right.

So, given H naught and H 1, this is analysis bank. We can obtain F naught and F 1 as above to remove aliasing error ok. This really cancels out aliasing. If you want you can design analysis filters H naught and H 1 separately, a good low pass filter is H naught a good high pass filter is H one or else you can use quadrature mirror filter property, and you can you can design H 1 from H naught that means, if it is enough, if you can build one mother filter which is your low pass filter from which you can derive all the rest of the filters for aliasing cancellation.

You will have to observe carefully that if we adopt the QMF design that is we incorporate the quadrature mirror property into the analysis bank, then we can construct H 1 of z from H naught of z, because H 1 of z is H naught of minus z, and then you use the property here for alias cancellation the equations here which is F naught of z is H 1 of minus z and F 1 of z is minus H naught of minus z.

So, therefore, given one filter one base filter which is H naught of z you can construct all the rest of the filters, all the other filters which is H 1 F naught and F 1 towards the two channel filter back. So, with QMF given H naught, we can construct and populate all the other filters towards alias cancellation. We can stop here.