Mathematical Methods and Techniques in Signal Processing - I

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Lecture – 42
Problems on simplifying multirate systems using noble identities

So, let us have some interactive problem-solving sessions by my students who have taken this course. So, you will see some illustrations and examples into problem solving which is useful to understand and digest the concepts learnt during the lectures.

Ok hello everyone, I am Ankur we are going to solve this problem which appeared in the MMTSP 2013 IASC course in exam 2.

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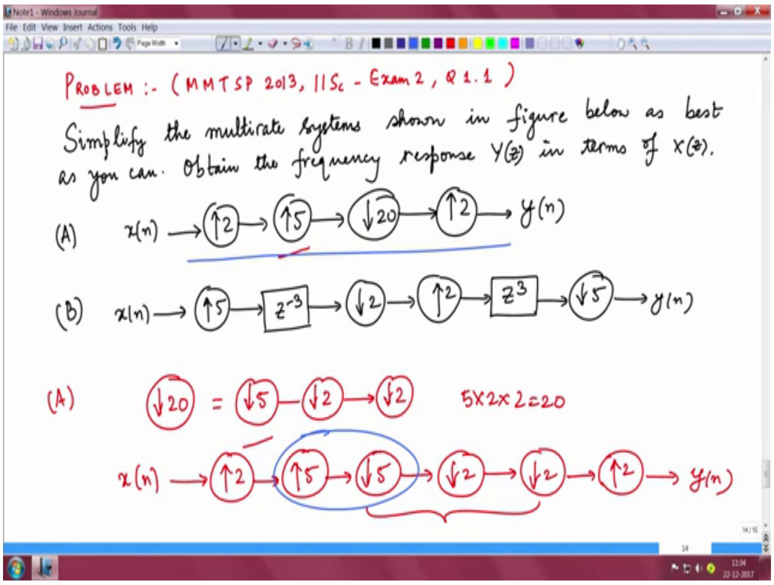

The question is; simplify the multirate systems shown in figure below as best as you can, obtain the frequency response Y of Z in terms of X of Z. So, there are 2 parts A part and B part, we have this signal x of n and it is going through some up samplers and down samplers and we have to get the frequency response Y of Z in terms of X of Z.

Now, if you carefully see we have some up samplers and down samplers and we wish to have a very simplified form. Now to go about the solution what we will do is we will split this down sampler ah, which is down sampling by a factor of 20 as down sample by 5 followed by down sample by 2 followed by down sample by 2.

So, because 5 into 2 into 2 is 20 that is why, we will we will do this and hope to see what happens next, because we have this up sampler 5 and we would like to undo what the up sampler does by having a down sampler 5.

So, we if we proceed in the in this direction we have what we will get is, this up sampler 2 followed by this up sampler 5 which is given now when we substitute this down sampler in this following way. So, this is just a substitution of the down sampler 20 and then we have an up sampler and then we have y of n.

So now we see here what is happening is this particular block is basically identity. So, this is going whatever the up sampler does the down sampler is going to undo it. So, next what we get is simplifying further.

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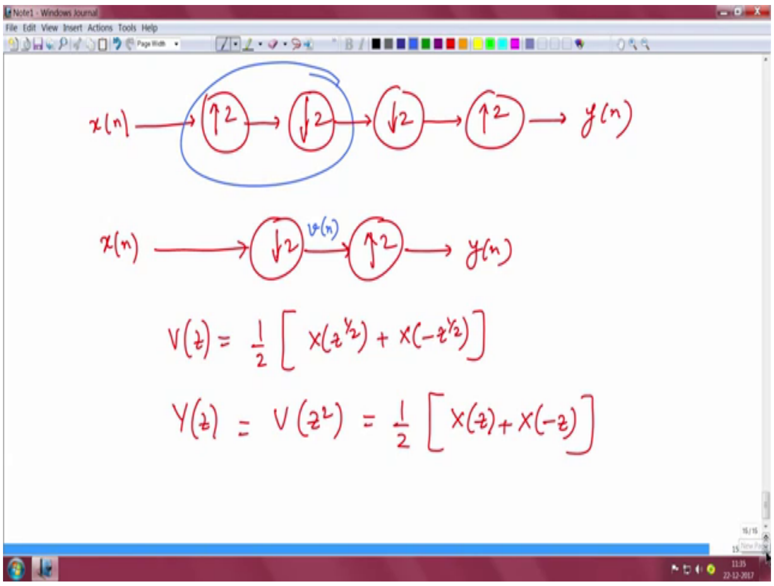

We have the up sampler a factor of 2 and then we have 2 down samplers, then we have the up sampler 2 then we have y of n. Now if you carefully see this factor is also identity the up sampling by 2 and followed by a down sampling by 2 is again identity. So, this again simplifies our work and we get down sample by 2 followed by up sample by 2 followed by y of n.

So, the key thing to note here is up sampling by a factor of m followed by down sampling by a factor of n is an identity; however, the reverse is not true that is the down sampling by a factor of 2 followed by up sampling by a factor of 2 does not lead to identity. So, if we do the simplification of this, if we write this signal as some v of n what we get is V of Z is going to be half of the X of Z power half plus X of minus Z power half and Y of Z is going to be V of Z square which is equal to half of X of Z plus X of minus Z. So, in this way we have got this simplified version of Y of Z in terms of the Z transforms of the signal x of n.

So, the initial block which looked very big this block has been reduced to a very simple form and thus we are able to get a simplified form of Y of Z.

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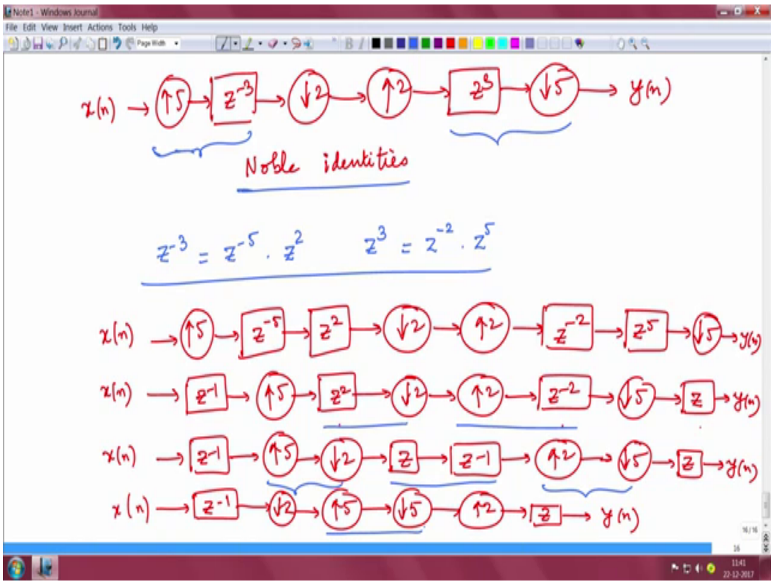

Now continuing with the part B of the question we have this block we will write it again for convenience x of n followed by an up sampler and then we have inter Z power minus 3, then we have a down sampler followed by an up sampler, followed by a filter Z power 3, followed by a down sampler of factor 5 and y of n.

Now, we know we have to use the noble identities here, because we have filters here and followed by up sampler and followed by up sample down samplers all these combinations are there. So, this block this, this block we are going to simplify as follows. So, Z power minus 3 we will write it as Z power minus 5 times we write it again. So, Z power minus 3 we will write it as Z power minus 5 times Z square.

So, in this way we can take the Z power minus 5 to the left-hand side of the up sampler and do some manipulations as we did in the previous example with the up sampler followed by down samplers. And similarly, this block we want the down sampler to come this side we push the filter to the right side using the noble identities. Now we will use Z cube equal to Z power minus 2 times Z power 5

So, using these 2 relations we will push the up sampler and down sampler inside and take the filters to the other side. So, what we get here is up sample by a factor of 5 and this we write it as Z power minus 5, followed by Z square, followed by down sample by a factor of 2, followed by up sampler by a factor of 2 and then we write here Z power minus 2 Z power 5 and then we have a down sample by a factor 5 and we have y of n.

So, this simplifies to using the noble identities, we get Z power minus 1 and we have a factor of 5 here and we have Z square you have a factor of 2, you have a up sample by a factor of 2 and we have is Z power minus 2 and we push this down sampler inside and we have Z.

We see some form of symmetry here we have Z power minus 1 we have Z here and we have Z power 2 here and Z power minus 2. So, something seems to be symmetry and we go forward in this direction. So, we get Z power minus 1 followed by an up sampler by a factor of 5. Now if we see this block we can use the noble identities and further simplify it to down sample by 2 and this becomes Z, and this becomes Z minus 1, followed by an up sample by a factor of 2, followed by a down sample factor of 5 and we have Z then we have y of n.

So, if we see here these 2 are going to be identity Z followed by Z power minus 1 is basically identity and then we have, if simplify further we get Z power minus 1 followed by a factor of 5. So, if we see here we are going to get a down sampler by a factor of 2 here and we have a down sampler by a factor of 2 here up sampler by a factor of 2 and we have a down sampler by a factor of 5.

So, we want the interchange we want to interchange these 2 and these 2. So, that the up sampler by 5 gets nulled by this down sampler by 5. So, if we proceeded in this direction we interchange these 2 because their gcd is 1 their co prime so we can interchange them. So, we put a down sample by a factor of 2 and up samples by a factor of 5, and this becomes one and then we have the interchanging of these 2 down sample by a factor of 5 followed by up sample by a factor of 2 and then we have the regular filter and y of n.

So, these 2 are going to be identity together and what we have next is x of n followed by Z power minus 1 followed by a factor of down sample by a factor of 2 and up sample by a factor of 2 followed by Z and then y of n.

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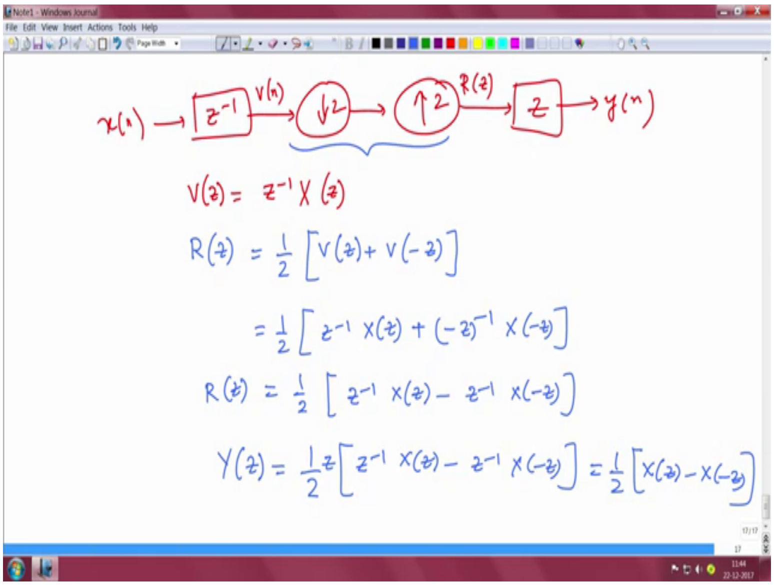

Now, this can be simplified I think this is the most simplified form and if we write if we go further and write this as some V of n, what we have is V of Z is basically Z power minus 1 V X of Z X of Z. And if we write this as some R of Z what we what we have from previous example we had this kind of a up and down sample followed by a up sampler. So, we have R of Z from previous result we have it equal to half of V of Z plus V of minus Z, simplifies to half of Z power minus 1 X of Z plus now we have wherever there is Z we have to put minus z. So, we get minus Z power minus 1 X of minus Z.

So, this is equal to half of Z power minus 1 f of X of Z minus Z power minus 1 X of minus Z this is R of Z, but Y of Z is basically. So, this is equal to half of we multiplied with Z which is Z power minus 1 X of Z minus Z power minus 1 X of minus Z and the Z cancels the factors which are multiplying X of Z and X of minus Z. So, what we have is X of Z minus X of minus Z. So, just the 2 examples that we saw one we had a factor of X of Z plus X of minus Z and here we have X of Z minus X of minus Z.