Mathematical Methods and Techniques in Signal Processing - I
Prof. Shayan Srinivasa Garani
Department of Electronic Systems Engineering
Indian Institute of Science, Bangalore

Lecture - 37
Digital filter banks

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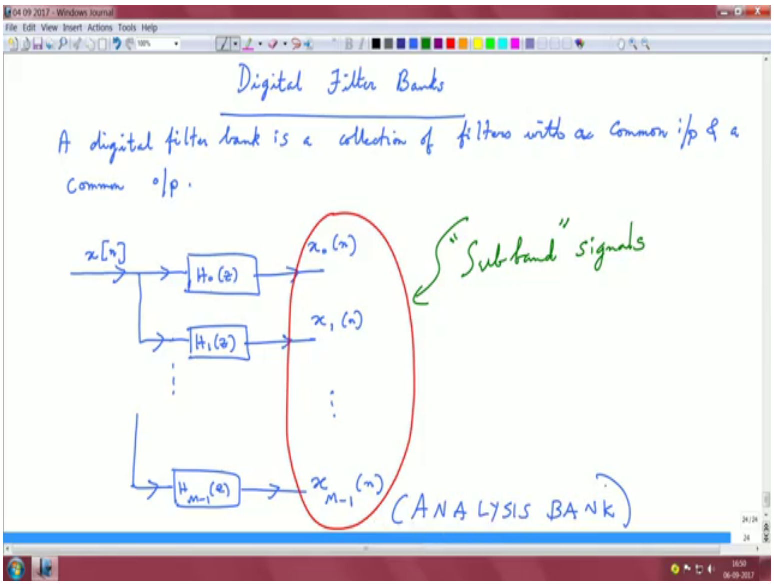

So, let us start with a new topic here in multi rate signal processing itself which is digital filter banks. A digital filter bank is a collection of filters with a common input and a common output. So, the idea is very simple here, we have a discrete time signal and we are filtering it through a bank of filters and that is why it is called a filter bank because it is a bank of filters. So, this is H naught of Z which is H 1 of Z dot dot dot till H m minus 1 of Z.

And let us designate the signal is x naught of n this is x 1 of n so on till x m minus 1 of n. To these bank or filters they are called the analysis bank, where they called analysis bank of filters because you are taking the signal x of n and you analysing this signal x of n through these filters right this could be perhaps h H naught H naught could be basically your low pass filter H m minus 1 could be your high pass filter and all of these other filters could be possibly be band pass versions right.

So, this is basically analysis bank; and these signal x naught of n x 1 of n. So, ion till x m minus 1 of n they are called sub band signals. So, very important they are called sub band signals and the name is pretty intuitive and obvious there called band because you are filtering them in different frequency bands and this is why they are called sub band filters.

So, like the way we have this analysis bank one can think of synthesis bank, so which is again pretty straightforward.

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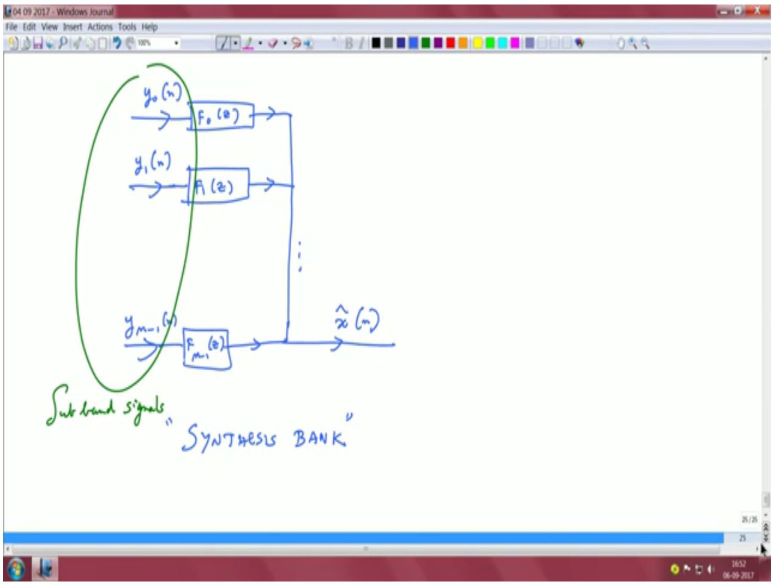

So, we have signals y y naught of n, y 1 of n so on and the past through some sort of filters these are basically synthesis filters, and our hope is we can reconstruct x back right. I mean you can think about is this as basically you are sub band signals and these sub band filters or filtered through a bank of synthesis filters, which is termed as synthesis bank and then you basically sum the output of all the signals coming out of the synthesis filters and hopefully you get your reconstructed signal ok. This is a very crude, crude form there are a lot of details which we will explore during our journey through this model.

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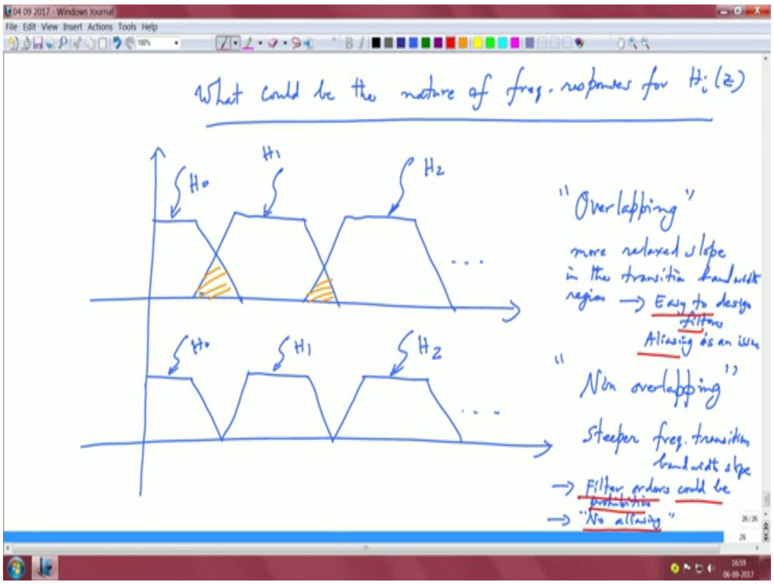

So, a question that naturally comes to us is, what could be the nature of frequency responses for H i of Z and this is a question that you would probably asked to yourself. So, H naught could basically be the low pass version, H 1 could be some band pass version H 2 can be probably next version of this filter basically with some frequency translation. As you can see this is basically overlapping, there is a spectral overlap. An alternative way to think about these filters could be to treat them as non-overlapping.

We may wonder what is the connection to having overlapping frequency responses versus non-overlapping frequency responses right. So, if you are to think about having filters that have non-overlapping responses it is beneficial for us, because there is you know aliasing that you are introducing. So, therefore, when you are trying to reconstruct these things by passing them through these filters there overlapping the spectral domain. So, therefore, you will get a clean signal.

But you think about the kv hat for of this of this structure, you will have to really design filters that have very steep responses, because you really have to notch and null them off at certain points in the frequency spectrum right and this is a critical parameter because if you have to have such sharp frequency responses of the transitions then your filter order just goes up. And if you just look at the formula from bellanger I mean this is the empirical formula is inversely filter order is inversely related to the normalised transition bandwidth right in the if the response is to be very steep, then you know you are you are order really jacks up.

So, this is 1 thing. So, it is easy to visualize conceptually having filters that are non-overlapping in the frequency responses, it helps us overcome aliasing and all these things, but we have a problem because orders can be quite high. But if I have aliasing I mean if I if I bring in this overlapping nature of frequency responses, then my filter order is less etcetera I mean I am able to tolerate because I can I can have a more steeper curt in this in this transition band, in the in the transition bandwidth I can basically have a more gradual slope right it is basically less stringent in terms of my steepness right.

So; however, that would cause aliasing and I have to overcome this alias better, through some means in the process of doing this filtering through a bank of analysis filters, followed by synthesis filters and when I call this is a filter bank and have to do something to this filter bank to overcome aliasing errors; introduce because of overlapping frequency responses. So, I think the message is very clear.

So, non-overlapping means, steeper frequency transitions, frequency transition bandwidth slope is a transition that is a transition bandwidth and if you look at the slope of the transition bandwidth it is steep, and here is more relaxed slope in the transition bandwidth region, which means it is easy to design filters, but aliasing is an issue and in this case filter orders could be prohibited and no aliasing ok. Here filter orders could be prohibited, but no aliasing here because aliasing, but easy to design the filters. So, there is some compromise that we will have to do.

But fortunately we have solutions when we have designed digital filter banks, we can overcome these aliasing errors and have overlapping frequency response, but the design our synthesis filters somehow to cancel aliasing using our multi rate operations. So, I think that is the whole idea in the design of this filter banks ok. And you may wonder why even should be take a signal and decompose this into analysis and synthesis filter banks.

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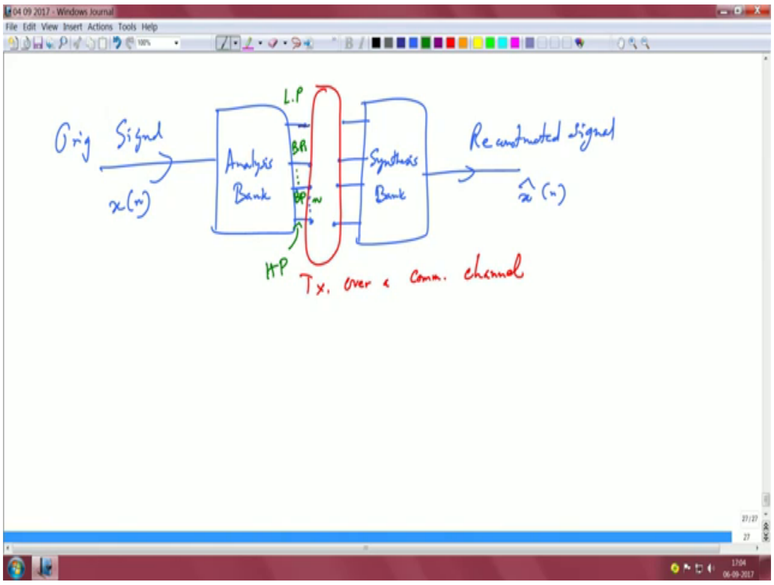

That means, we have a signal, we have an analysis bank let us say we have the sub band signals, and we take the sub band signals passed them through a synthesis bank and we get back our reconstructed signal. So, sum x hat of n this is sum x of n right at this step, which is very important there is a transmission that is happening over a communication channel.

When we say we are transmitting over a communication channel, this means that we are claimed with a bandwidth of the channel in some sense right and if we have stringent constraints on the bandwidth of the channel, then we may have to possibly compress the signal right.

Now, we have different frequency components that get filtered as we go through this analysis bank, I mean this could be essentially a low pass version and all of these could be band pass BP 1 dot dot some band pass say n and this could be our high pass right. And depending upon the information content in each of the bands I may have 2 choose my bit rate in such a way that I can throw away certain samples in each of the banks; that means, I can adjust my sampling rate in the sub band signals and basically quantized and do anything that I want to do to compress and again I restore this sampling rate in the process of web sampling then I just get a set of signals here at the input of the synthesis bank and then I try to reconstruct the signal.

So, the journey here is under what conditions can I do perfect reconstruction. So, if I do not do quantisation and I do down sampling here at the output of the analysis bank followed by up sampling, can I reconstruct perfectly, what are the conditions for filtering or choosing you know, how do I choose my analysis filters and synthesis filters to overcome errors due to aliasing due to errors in you know phase distortion possibly magnitude distortion, in the frequency response etcetera, etcetera, right.

So, these are all various questions 1 would get, but I think you have to appreciate here that this is this practical because one 1 when one thinks about sending the sub band signals over a communication channel then we will have to basically work with the limitations of the channel, and that is the real channel challenge here how to design this this filter banks ok.

So, with this we will stop here, we will look into the discrete fourier transform or the DFT as a filter bank, we will consider d DFT as a case study and then we will generalise this towards other filter banks we stop here.