Mathematical Methods and Techniques in Signal Processing - I
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Lecture - 36
Fractional sampling rate alterations

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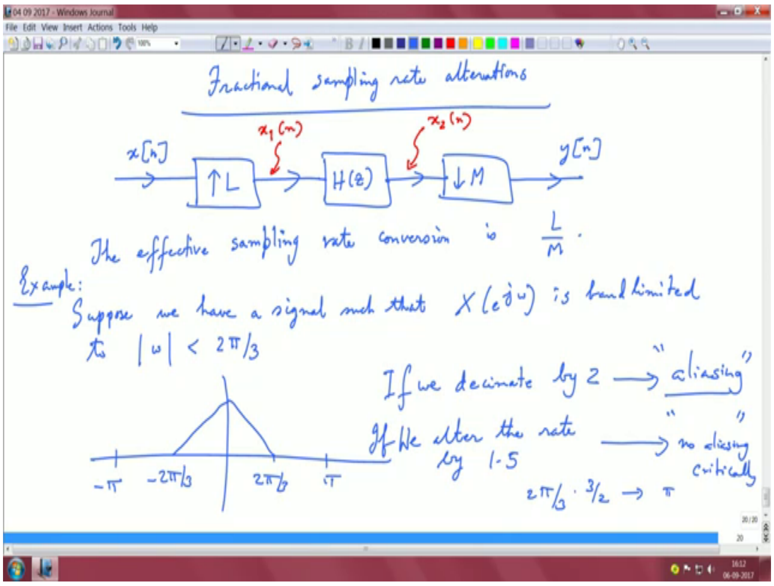

Now, we did integer rate decimation and integer rate expansion. And we saw in the very beginning of the module that is module 2 on multi rate basics for multi rate signalling signals multi rate signal processing, but we are interested in sampling rate conversions right we had a studio frequency at 32 kilohertz and then we want it to be converted to maybe a that frequency at say 44.1 or broadcast frequency 48 you know different sampling rate conversions should be made possible.

So, the question is how can we basically do the sampling rate conversions digitally this is of course, the path where we are basically heading towards. So, a natural question suppose I want a fractional sampling rate conversion right I think this is m down sampling by m. So, basically I am reducing my rate 1 upon m and I am expanding it by a factor L it is. So, basically L upon M is my rate right.

So, let us imagine the situation that we have x of n at the input, this is up sampled first by a rate L right and then we have a filter H of Z in the middle, this is followed by a down sampler at a rate M and then we have the signal y of n. So, let us say this interim point is x 1 n and this interim point is some x 2 n

. So, the effective sampling rate conversion is L upon M right where we are up sampling by L reducing by M. So, it is basically a ratio L upon M. So, let us think about a simple example here suppose we have a signal such that X of ej omega, this is the spectrum is band limited to frequencies less than say 2 pi by 3, 2 pi upon 3.

Suppose I decimate by 2 right if I decimate if we decimate by 2 this results in aliasing right because its 2 pi by 3 right it will stretch itself to 4 pi upon 3 and if we alter the rate by 1.5 right if you alter the rate by 1.5. So, this 2 pi by 3 point is basically 2 pi by 3 into 3 upon 2 1.5 is 3 upon 2 and this is basically this goes from minus 2 pi you know it basically stretches.

So, this is this is this is possible. So, this is 2 this is pi here sorry. So, just pi here 2, 2 gets cancelled and 3, 3 gets cancelled this is pi. So, this is pi and of course, minus 2 pi by 3 is mapped to minus pi. So, if you alter the rate by 1.5 this is no aliasing critically, I am saying it is critical because you are touching these points plus minus pi.

So, this gives you a sort of an idea, how much we can alter the sampling rates such that we do not land up with aliasing errors, ok.

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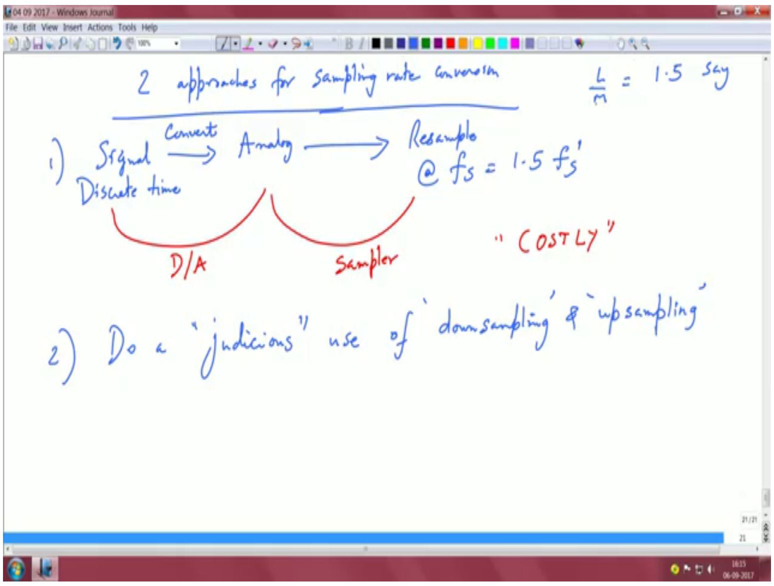

So, you might again question here what are the approaches for sampling rate conversion? You have a signal you convert let us say this is a discrete time signal, you convert it into analog and then you resample at some rate.

So, let us assume that you know let us say we want rate L upon M to be 1.5 say just following the previous example right. So, we want there a 1.5 increase in the rate right. So, a naive approach is take the discrete time signal convert it into analog signal right and then you do resampling at some rate f s is 1.5 times f s dash. So, here you have a digital to analog converter, and then you need a sampler, its costly because you need to have hardware for these operations and it is costly.

I think a better choice would be do a judicious use of down sampling and up sampling with possibly some filtering in the middle that is you up sample you have a filter in the middle and then you down sample figure out what the parameters of this filter have to be what your l and m ratios have to be to get you to what you want, to what ratio you want to alter your sampling rate ok. So, I think this gives us a basic idea now right we get an idea what we really need to accomplish our operations.

So, now let us go a little deeper into the filter design aspects and how we can you know go about designing these filters.

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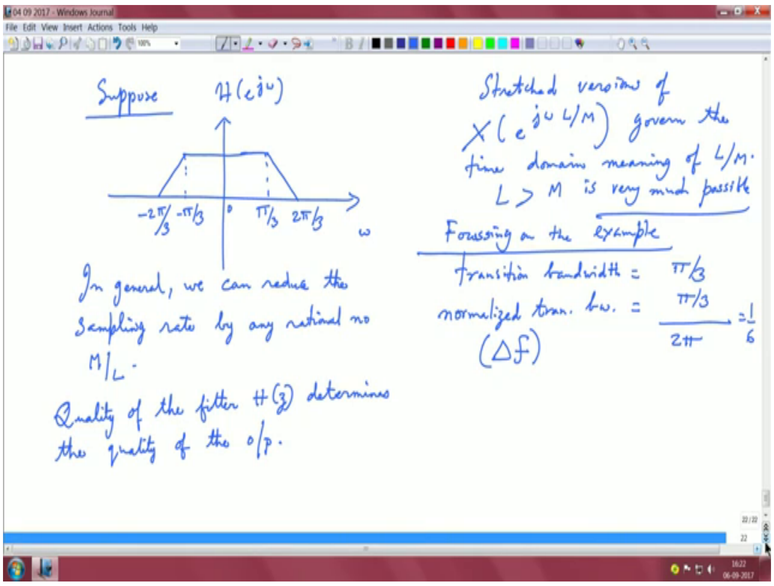

Suppose H of ej omega is given by this response, this corner point is to upon it is the stop band is at minus 2 pi by 3 the pass band is limited here to minus pi by 3 to plus pi by 3 this is the other corner point. And in general we can reduce the sampling rate by any rational number M upon L I said reduce and I put M upon L if I said I increase I would say L upon M, I think that is sort of it should not confuse a confused you here ok.

What is really important is the quality of the filter H of Z determines the quality of the output; and most of you would have learned from your undergraduate digital signal processing some basics of filters right you have you should have studied Butterworth filters, Chebyshev Equiripple filters and so on and so forth and we can we can see in this case, how we can design 1 such filter, which can give us possibly what we want right. So, and you can you can sort of imagine that the stretched versions of the spectrum govern the time domain meaning of L upon M.

So, if we want to alter the sampling rate by a factor L upon M you can imagine that the stretched versions of this spectrum, which is X of e j omega L upon M they govern the time domain meaning of this alteration right the L over M factor is to account for the decimation and the factor L appears consistent with up sampling and you can you can say that L greater than M is very much possible.

Now, let us look at this case here and we focus on this example, the transition bandwidth is basically 2 pi by 3 minus pi by 3, if you look at the positive side of frequencies basically this is the same error here right and between the pass band pass band cutoff is a pi by 3, the stop band frequency is a 2 pi by 3, the transition band width is basically pi by 3 and if you do the normalize normalized transition bandwidth, this is pi upon 3 by 2 pi, which is a sixth and let us call this parameter normalized transition bandwidth is as delta f. So, delta f is basically 1 upon 6.

Belanger Maurice Belanger he came up with an interesting formula that connected the filter order with the tolerances on the ripples in the pass band and the stop band for an equiripple filter and the transition bandwidth. So, he was able to connect all of these to suggest the filter order that we need to accomplish this sampling rate conversion process.

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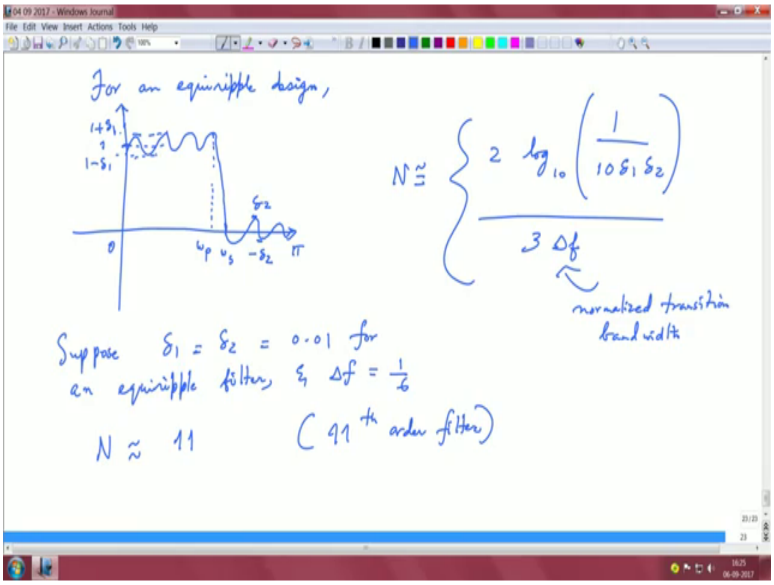

So, for an equiripple design; so if imagine the response would be something like this right. I did not do mod H e j omega I just did h ej omega. So, therefore, I have I have this magnitude then you will have to really. So, basically these ripples could swing between minus delta 2 to plus delta 2, this is your pass band this is your stop band and in this let us say the nominal gain was 1 and I could have 1 plus delta 1 here and a 1 minus delta 1 at this point ok.

Now, somewhere it is say it is pi. So, n he came up with this formula which is basically an empirical result, this is 2 log to base 10 of 1 upon 10 delta 1 delta 2 divided by 3 times delta f. So, delta f this is the normalized transition bandwidth, and delta 1 and delta 2 are the tolerances for this for this equiripple filter. If it is butterworth it is pretty much flat right if it is equiripple you are basically having these ripples in the pass band and this stop band.

So, now suppose delta 1 equals delta 2 is a 0.01 for an equiripple filter right and delta f per our calculations in the previous slide it is basically 1 over 6 1 upon 6 right this is what we said this is what we ideally want ideally we want it like this, but it is not possible, but we say we will tolerate some ripples in the pass band and the stop band we have a normalized transition bandwidth of one-sixth.

So, you plug into this formula you land up with N to be approximately 11, that is we need eleventh order filter and the details of as to how we can we can design, I mean we have equations to get the filter coefficients if we can pull up a manual for the design of such filters right and then we can basically go through the exercise of computing, the coefficients of the filter. And we get an order and once we know the order depending upon the governing polynomial and the equations to get the filter coefficients, we can just extract their coefficients out and that will give us the design right.

So, this sort of gives you an idea as to what is desired right. So, basically I want a sampling rate conversion to happen and the process of getting the sampling rate conversion, I have to ensure that I also have this filter that I need that that has to work with because ideally brickwall filters are not possible, I mean the order is really infinite if you have to realize ideal low pass filters you are not going to get that.

So, basically we have to approximate the brickwall filter by an approximation right. We need some kind of low pass filter with a a practical order that we can implement. To get that we should know what the pass band is, what the stop band is and these other details and once we get those details we can basically figure out where we need to build. This and we will do this as part of some homework problems and projects in this course.

So, we will exactly build some filters towards audio or some applications and you will appreciate, what is happening in theory, ok. So, we can stop here.