Mathematical Methods and Techniques in Signal Processing

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Lecture – 33
Basics of Multirate Systems

So, let us get started with the foundations of multi rate systems multi rate signal processing.

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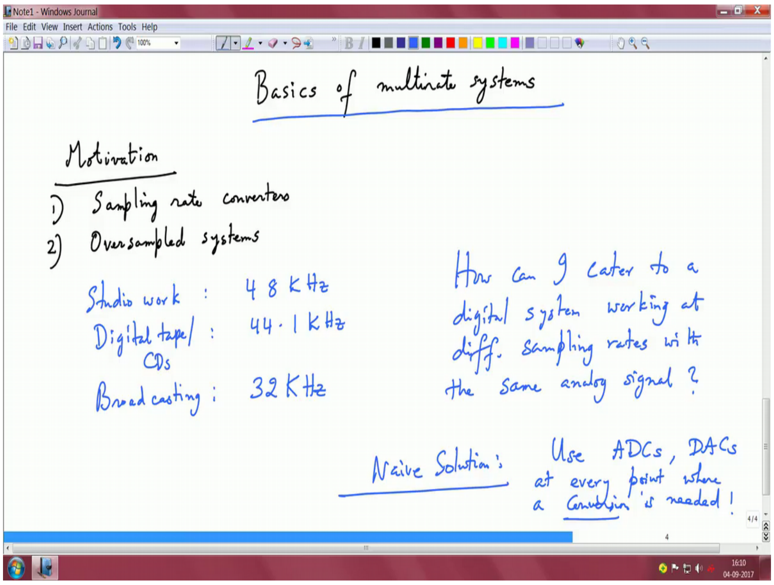

So, the motivation towards this area is in 2 main problems coming from engineering practice. One is sampling rate converters, and the other is in over sample systems.

So, let me brief you with a motivation towards multi rate signal processing. Now the foundations to multi rate signal processing is basically you are sampling the standard Nyquist sampling theory. So, the question, way back in the 1960's -1970's, when people in their labs were developing systems were as follows.

For example, in those days, the digital audio required various sampling rate options. For example, if you think about this studio work that required a sampling of 48 kilohertz for a audio, if you want to think of the digital tape slash CD's, then they required a sampling rate of 44.1 kilohertz. And if you thought about broadcasting applications that required, say, 32 kilohertz; the question is how can I go from an analog signal, and cater to 3 different systems that are working at different sampling rates right.

So, how can I cater to a digital system working at different sampling rates with the same analog signal; It is the same continuous time signal, but it is working at different sampling rates. Now a naive solution one can think about is, well I have digital, I will convert again digital to analog, I do the sampling again, I have these d to a, a to d converters at every point in the configuration and therefore, work with these systems.

Naive a solution is use ADC's and DAC's at every point where a conversion is needed. But there is also a cost, you cannot buy these ADC's very cheaply, it is not possible to buy them.

So, that is prohibitive, if you have various systems which require different sampling rates I cannot use ADC's and DAC's at every point in the system. So, therefore, we need to do something clever. Basically we manipulate digitally, somehow that we can go from one sampling rate to the other sampling rate, that is, I can convert my sampling rate seamlessly from one rate to the other, still preserving all the properties of the underlying original signal that I have and then work with the rest of the system.

So, that is the solution. But how to go about doing that so that led to the birth of this theory into multi rate signal processing and a lot of work has been done from the pioneering works of folks at Bell labs; Ron Crozier, Belanger and many of these people who were there at the time, and then, this has led into a lot of applications from compression to high decimation filter design to trans multiplexers and so on and so forth, the applications are tremendous; and today I think this is something basic. So, it is almost like a 45 years, but I think it is basic now.

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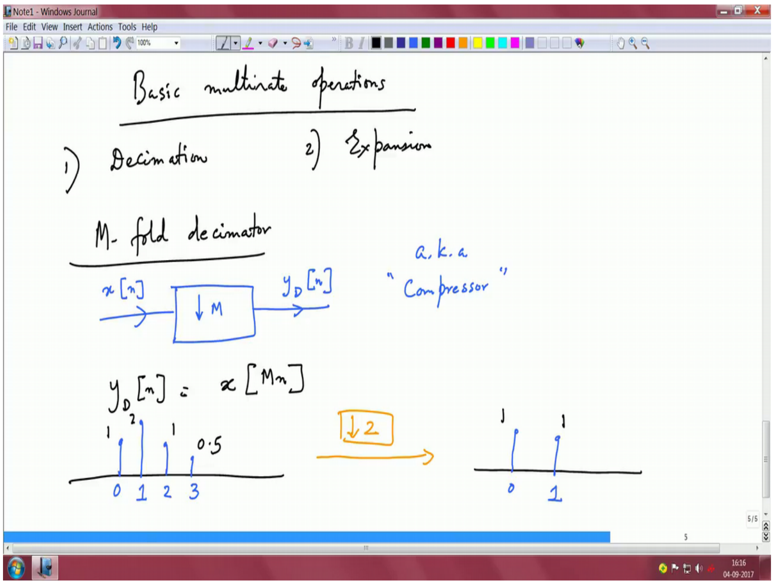

With this motivation in mind let us see; what are the basic operations in multi rate signal processing.

So, when we say multi rate, we mean that we are changing the sampling rate. So, that idea is clear. Now if you think about what possible operations are there, the first operation is decimation, which means I am down sampling or I am reducing the sampling rate. We can assume, it is a integer level of decimation.

Because anything which is a fraction we can always realize as some combinations of increase and decrease of the sampling rate. If you think about decimation pure decimation, let us think about integer decimation; that means, I am reducing the sampling rate, by say, rate half or one-third, one-fourth some number, some integer decimation.

Then, we have this expansion, which is we are up sampling or we are increasing the sampling rate by again an integer amount. Now if I want to get a rate of say 1.5 conversion, 1.5 is 3 upon 2; That means, I can reduce the rate by half and increase the rate by 3; may decimate by 2 and expand by 3. And that might give me the rate that I need.

So, again let us go back to our motivation, to the studio work at 48 kilo hertz, and then which required for the CD’s at 44.1 kilo hertz. The rate conversion is 44.1 upon 48 or if you want to go from other side, you can say 48 upon 44.1, whichever is the way you want to go about.

So the goal is, I do not want to use ADC's in the middle. Directly using the samples that I have, in whichever sampling rate, I started off with; I want to proceed to convert seamlessly from one domain to the other domain.

Now, let us go a little deeper with this idea in mind, and we will say; what is this M fold decimator. So, an M fold decimator takes x of n, which is my discrete set of samples as its input, down samples by a rate M and I get my decimated output which is y D of n and this is also called a compressor. y d of n is basically x of M times n. So, this is the relationship. So, let us see how this this works. So, let us say I have samples 1 say 2. This is say 1 say this is 0.5. Let us assume that this is time step 0, time step 1, time step 2, time step 3 and, If I want to get this through a decimator with m equals 2; that means, I am slashing my rate by half. What I get is, just look at this equation; that means, take the first sample I discard the second one, I take the retain the third, I slash the fourth.

So, every alternate sample, I just chuck it off. So, basically I get this. This is the output of a down sampler by 2. If I decimate by 3 then I retain 1 in 3 samples. So, I think this idea of decimation has some historical perspective. I do not know how true this is, right or wrong.

But imagine if you had a crazy military general, who decides to retain 1 soldier among 10 soldiers in the enemy camp. I mean that is 1 in 10 decimation. So, I think that is probably the idea of decimate, 1 in 10 heads is kept.

So, it is basically you are slashing the rate. Number of people you are just getting them out. And probably here is the same thing with sampling; that means, you retain 1 in 10 samples, but of course, in we want to be very general for down sample by m; that means, we retain one out of m samples.

So, similar to the decimator we have another important block, which is an L fold expander.

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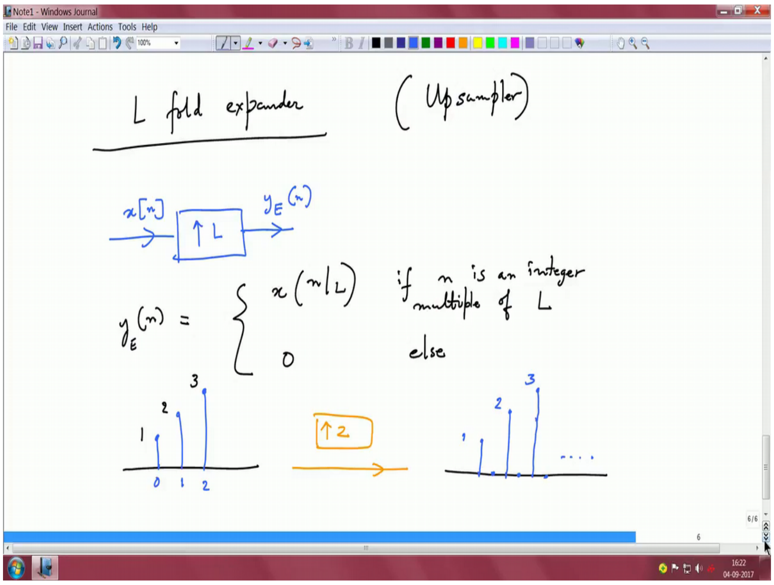

Let us see what this expander does. It is also called up sampler. So, we have x of n as our input. We will not up sample this by a factor of L and let us say we get this signal y E of n, E standing for the expander. And like the decimator, we can we can go about with relationship between y E of n and x of n and y suffix E of n is x of n upon L.

If n is an integer, multiple of L and this is 0 otherwise. So, let us see what is happening with the with the expansion process. Suppose I give you samples. So, time index 0, 1, 2; and let us say the values are 1, 2 and 3. And it goes through an expansion by 2. What we do is the following.

So, we retain 1 at the next time step I have a 0, because it is 0 otherwise. Then I retain 2 0 otherwise, retain 3, 0 otherwise, and this happens which means I am inserting the 0's at alternating points.

I mean this is because it is up sampling by 2 and my rate has now sort of doubled. Now you may ask these questions as I am lecturing, there is no information that you are really conveying, you just have the same information here, except that you have doubled and you have placed the 0's, but usually these operations of expansion or down sampling or up sampling, they have some filtering operations as well.

So, once you up sample, you can imagine I need to have some interpolation filters to fill in these samples, filling these 0's.What sort of interpolating filters would I use. And can I get reconstruction, which is perfect, and these type of questions you know these are things which we will discuss through the succeeding modules; In this succeeding lectures module on multi rate signal processing.

Because this is basically the idea, how can we construct filter banks etcetera. Basically I have a signal digital signal, I just pass them through some analysis filters, I reduce the sampling rate, then at that point I am doing some transmission with quantization or quant quantization in transmission, and then at the receiving side I basically up sample to restore my sampling rate back, and then I have to have;

Because of sampling itself, this process does not let you get the samples that you want. And then you should have some interpolation filter. So, this is sort of the sketch of the idea what we need, but we studied 2 basic operations on a decimeters and expanders and we want to basically go delve a little more deeper into these ideas, but here, I think I would like to sort of give you a simple exercise for thought not so difficult.

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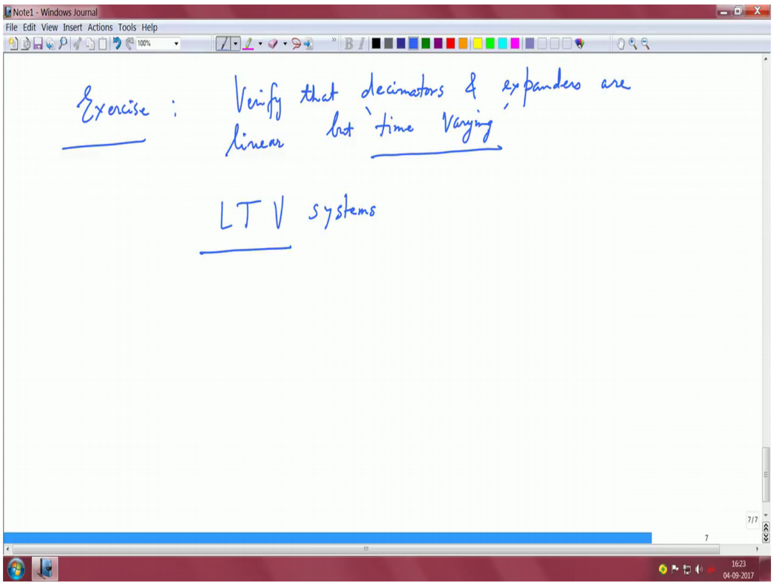

Verify that decimeters and expanders or linear but time varying, is very important. They are not L T I, but this is an L T V operation. They are L T V systems. It is not too difficult to verify this and you could do that.

Thank you.