Mathematical Methods and Techniques in Signal Processing - I
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Lecture – 03
Linear time-invariant systems

In this lecture we will be studying about the basics of LTI systems which is a concept of linear and time invariant systems.

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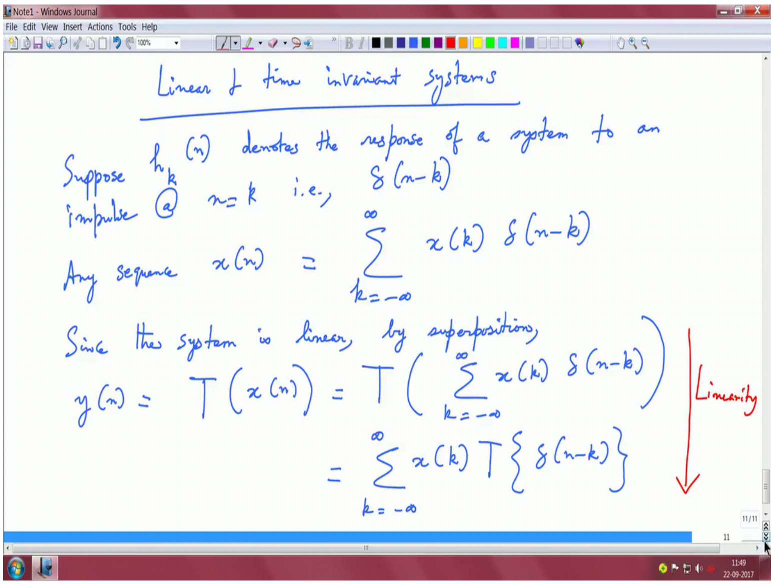

So, we saw what linearity was that means, I should satisfy the property of superposition and we saw what time invariance was that means, if you delay the input by some n naught time steps the output is also delayed by the same time steps. Now, if a system has both linearity and shift invariance or time invariance, it is called linear time invariant system, and we extensively study liner invariant, time invariant systems in signal processing. So, let us see if it has any unique property.

To study this let us consider the following suppose h suffix k of n denotes the response of a system to an impulse at n equals k, that is we are giving some impulse delta n minus k and let us assume h k of n denotes the in response of the system to the impulse given at n equals k. So, we can write any sequence x of n in terms of the delta sequence right. So, this is basically k equals minus infinity to plus infinity x of k delta of n minus k. Why?

Recall that delta of n minus k exist only when n equals k, so that means, when n equals k this is one this thing happens to be one and therefore, when this is just becomes x of n.

So, any x of n can be written as basically moderated through this delta c equals right. Now, since the system is linear by super position. So, y of n is basically the response to this x of n right, y of n is the response of the system to the input x of n and this is response to x of n which is expanded out using the delayed delta sequence and since T is linear we can say this is summa k equals minus infinity to plus infinity x of k you bring in the map T here and T of delta n minus k.

So, the validation to this step is linearity. You may call linearity from a systems perspective, if you are a mathematician you may say this is a linear map. Now we can simplify this further.

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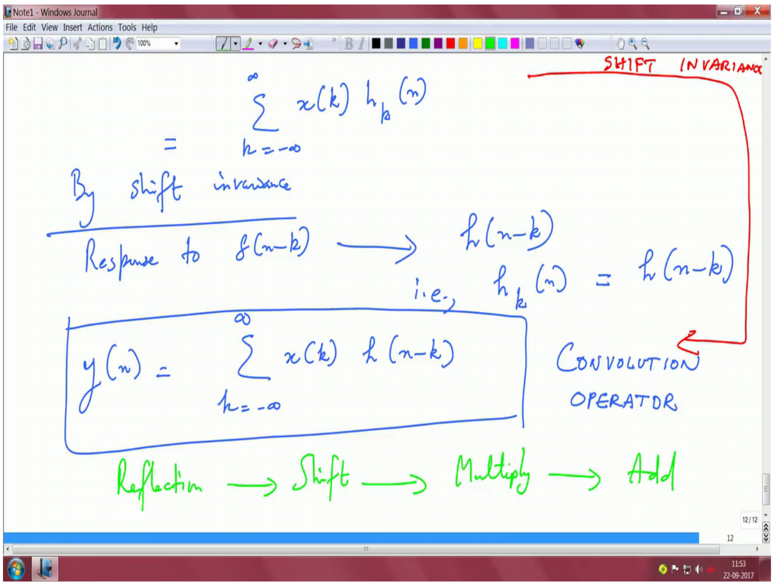

We can say this is summation k equals minus infinity to plus infinity, x of k and we said the response of the system to an impulse a time n equals k is h k of n right and we just replaced it in this form. Now, let us apply shift in variance because we said that the system is end out with shift invariance property, now response to delta n minus k T is h n minus k. So, your h k of n can be replaced by h of n minus k right because it is shift invariance, therefore, response to the input delta n minus; delta n minus k will result in h of n to be delayed by k times steps which is h of n minus k.

So, now we are sort of ready y of n is summa k equals minus infinity to plus infinity x of k h of n minus k. And this is an important relationship because this operation is the convolution operation between sequences x of n and h of n right. So, from this step we can come to this the last step because of shift invariance.

So, now, we have all the tools to study the response of a linear time invariant system to an arbitrary input so that means, we have to apply convolution and conversation does not hold for non-linear systems because you cannot satisfy the superposition principle and therefore, you will be violating at this step itself. A good example would be take x square n or x cube n. I mean the input is some x of n let us say the output just squares to signal so obviously, you have cross terms that are appearing and that is a clear indication that it is a non-linear system because it does not satisfy this property of super position.

Now, if you have to pointed about this a little bit think what is happening in y of n. First there is a shifting because you are delaying h of n by k steps there is a shift after reflection. So, this minus k indicates this reflection there is a shift in reflection, then there is an addition after multiplication. So, you just multiply x k with h of n minus k and then you add. So, there are 4 important operations that are happening which is reflection. So, I give you h of k, I give, I get h of minus k, reflection then shift because you have to get n minus k then you multiply and then you add and these operations, these 4 operations basically defined are part of the convolution. So, these 4 operations are part of convolution and convolution is an important property for LTI systems or linear time invariant systems. So, with this we are ending this.