Mathematical Methods and Techniques in Signal Processing - I
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Lecture - 29
Problem on mean and variance

So let us have some interactive problem solving sessions by students have taken this course ah. So, you will see some illustrations and examples into problem solving which is useful to understand and digest the concepts learnt during the lectures ah.

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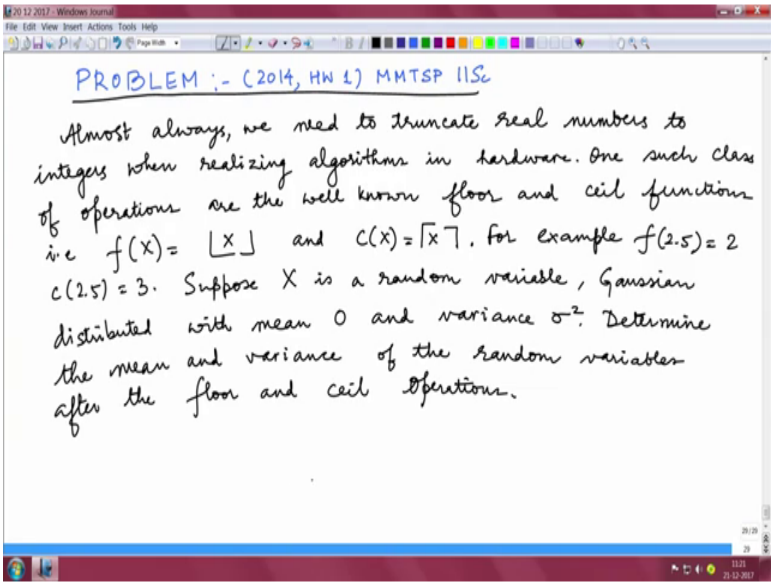

Student: ah. So, hello this problem occurred in the year 2014 in the homework 1 and in the MMTSP course in IISC and we are going to solve it. I am Ankur. I am a student here in IISC. So, almost always we need to truncate real numbers to integers when realizing algorithms in hardware. For example, we have FPGA or we have DSP kits, where the incoming signal will be continuous and we will have to truncate to the decimal points and convert into digital values, so that we can perform certain algorithms.

So, one such class of our operations are the well known floor and ceiling functions namely f of x is shown like this and c of x. So, for example, f of f of 2.5 that is floor of 2.5 will flow rate to the nearest integer which is 2, which is below the 2.5 value and ceiling is the nearest integer which will take it to the next integer which is above it , which is that is ceiling of 2.5 is 3.

So, now consider x is a random variable which is Gaussian distributed with mean 0 and variance sigma square so; obviously, x is a continuous random variable and what we are doing is performing operations that it that is this ceil function and the floor functions are functions of these random variables and we are going to determine the mean and variance of these random variables after the floor and ceil operations. So, hope the question is clear.

Now, we realize that since xs are continuous random variable and c of x and f of x, that is the ceiling and the floor functions are integer random variables ah. So, they will take only discrete values which are integer values.

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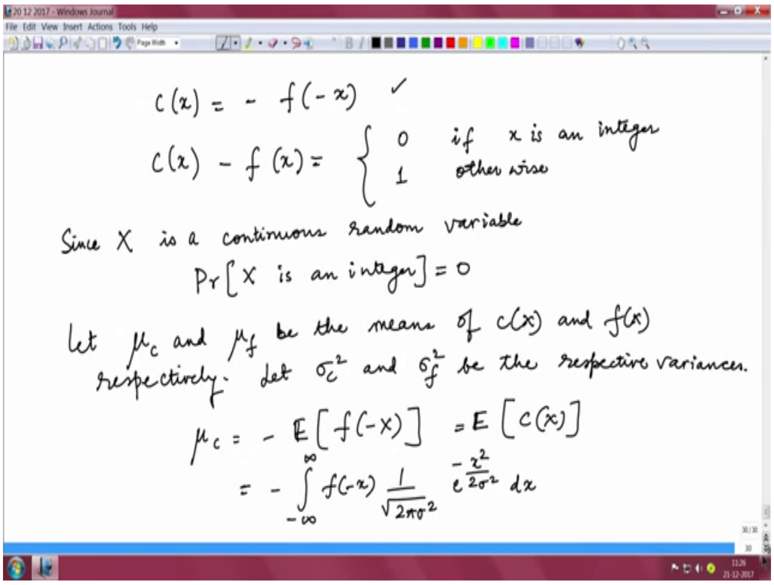

So, we know that intuitive by observation that c of x is going to be minus f of minus x. So, and the second observation is c of x minus f of x is going to be 0, if x is an integer and 1 otherwise.

Now, since x is a random variable, is a continuous random variable. We have probability of x is an integer is 0. So, most of the weight or the measure of x is over the non integer values. So, let mu c and mu f be the means of c of x and f of x respectively and let sigma c squared and sigma f squared be the respective variances.

Now, we realize that of from this equation that mu c is going to be minus of expectation of f of minus x right, which is expect which is equal to ah, for example, the expectation of the c of x. So, this is equal to minus of minus infinity to plus infinity and f of x, f of minus x. So, it takes the value f of minus x and what is the PDF of x it is the Gaussian. So, it is 1 by square root of 2 pi sigma squared sigma and we assume the, I mean we know that the variance of the original random variable Gaussian is sigma squared. So, we use that assumption here and the PDF of that is this.

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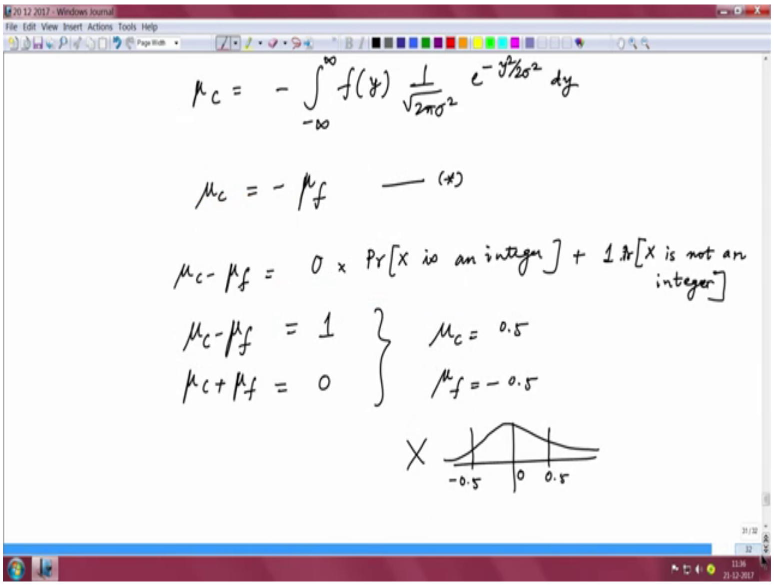

So, this becomes equal to minus of minus infinity to plus infinity by making use of x is equal to minus y , we have 1 by square root of 2 pi sigma squared e power minus i square by 2 sigma squared dy. So, we realize that this is actually the mean of the floor function come operating on the random variable x. So, mu c is equal to minus f, this is one of the equations let us call it star.

Secondly we realize that since c of x is a c of x minus f of x is going to be 0 if x is an integer and 1 otherwise. So, we realize that mu c minus mu f is equal to 0 into probability that x is an integer plus. So, we realize that this c of x minus f of x is 0, if x is an integer and 1 otherwise.

So, what is happening is this is like an indicator function, which indicates the difference is this difference c of x minus f of x what type of values it takes. So, 0 or 1. So, it just becomes 1 whenever x is not an integer and it becomes 0, when whenever x is an integer. So, when we take expectation on both sides expectation of c of x minus f of x it becomes expectation on the right hand side also. So, what happens is it takes the value 0 with the probability x not x is an integer and it takes a value 1 when x is not an integer [noise.

So, when we take the expectation on both sides, we get mu c minus mu f is equal to. So, what is the expectation on the right side? So, 0 the value it takes a value 0 with the probability that x is an integer and it takes a value 1 when x is not an integer. So, now, we realize that from this equation probability that x is an integer is 0 and probability that x is not an integer is 1. So, this equates to 1. So, we have 2 equations which is mu c minus mu f equal to 1 and mu c plus mu f equal to 0. By solving these 2 equations we get mu c is equal to plus 0.5 and mu f is equal to minus 0.5.

Now, originally just to analyze this point just to pointer up and what is what we what results we have got here. x was a continuous random variable and it was, it was a continuous random variable. It was taking both positive and negative values and it had a mean 0. So, what is happening essentially is the ceil function has a mean around 0.5 and the floor function is having a mean around minus 0.5. So, a both of them are equally sort of opposing each other and. So, that the mean of the continuous random variable is x. So, this ceil function will be centered around here and the floor function will be centered around minus 0.5.

Now, we compute we go on to compute the variances for that we compute the expectation of the ceil function squared which is equal to ah, from this equation from this equation which is equation 1 which is equal to expectation of 1 plus f of x, the whole squared, which is equal to expectation of 1 plus 2 f of x plus f of x squared, which is 1 plus 2 times expectation of f of x plus expectation of f of x squared.

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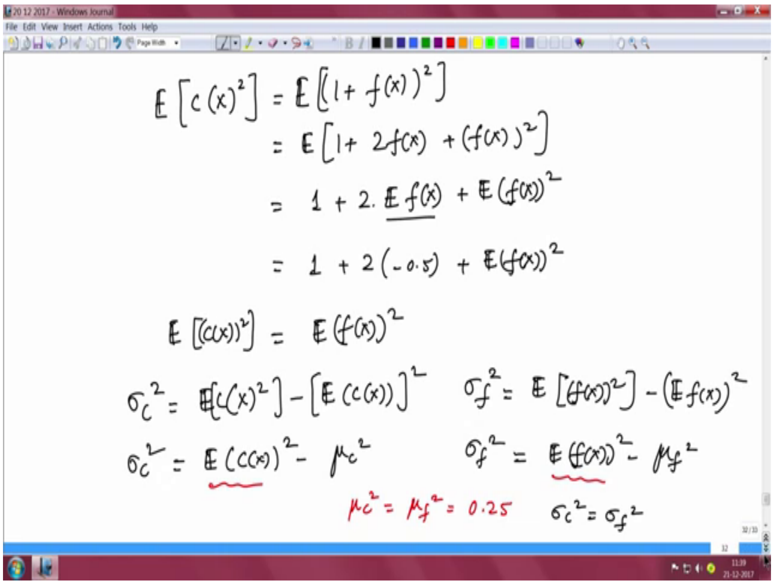

So, now we previously we know that from the result that mu, this is basically mu f it is minus 0.5 and this is expectation of f of x squared. So, this is actually expectation of f of x squared. So, now, we have the. So, variance of sigma c squared will be x essentially expectation of c of x squared minus expectation of c of x , the squared and mu f squared will be expectation of f of x squared minus expectation of f of x the whole squared.

Now, to compute these is little challenging and we from these equations we realize that this is actually expectation of c of x squared minus mu c squared. So, this is sigma squared and sigma f squared is this is expectation of f of x squared minus mu f square and since these 2 parts are equal and mu c squared is equal to mu f squared which is equal to the square of 0.5 which is 0.25. What we have is sigma c squared is equal to sigma f squared. So, both their variances are same. We saw that sigma c squared is equal to sigma f squared ah; however, there is no closed form expression for the variance the variance can be computed using the PMF of the ceil and the floor ok.

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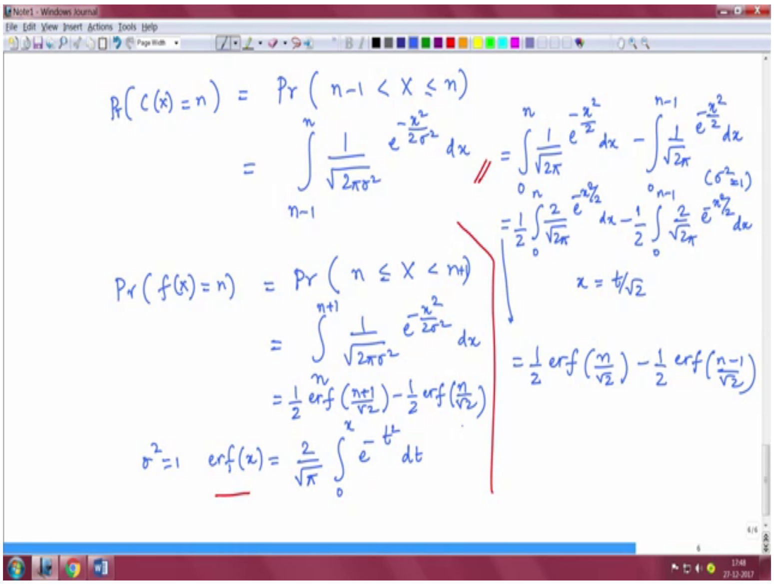

So, the PMF is given by, first we see probability of the ceil function taking a value n is equal to the probability that n minus 1 lies between x and n right. So, this is equal to the integral from n minus 1 to n and dx and the probability that the floor function takes a value n is equal to the probability that n is less than or equal to x less than n which is equal to the integral from n. So, this is n plus 1, n plus 1, 1 by square root of 2 pi sigma squared e power minus x square by 2 sigma squared dx.

Now, if we have to do this on a computer what we will do is, we will assume sigma squared equal to 1 and we make use of this function called e r f of x it is given by 2 by root pi 0 to x, e power minus t squared d t. So, what we have here is this expression and we need to make use of the erf function in mat lab. So, that it becomes computable.

So, this is actually equal to 0 to n, one by square root of 2 pi, if we substitute sigma squared equal to 1. What we have is e power minus x squared by 2 dx minus 0 to n minus 1, 1 by square root of 2 pi, e power minus x square by 2 d x, this is with sigma squared equal to 1.

So, we see that we have a form here and we want it in the form of erf function. So, this evaluates to half of 0 to n , 2 by square root of 2 pi e power minus x square by 2 dx minus half of 0 n minus 1, 2 by root 2 pi , e power minus x square by 2 dx [noise.] Now, if you make the substitution x is equal to t over root 2, what we get here is if you just marked at it what we get here is this evaluates to half of the erf function of n over root 2 minus half of erf function n minus 1 over root 2.

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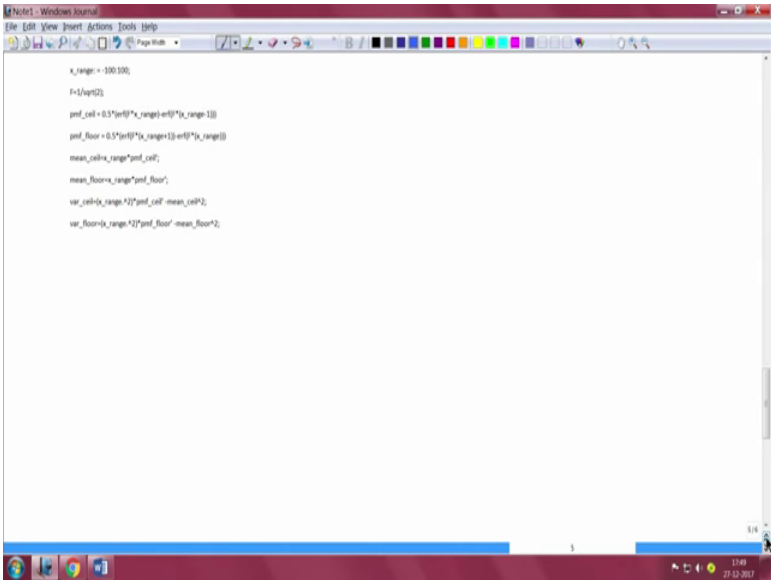

Similarly, this expression evaluates to half of erf function of n plus 1 over root 2 minus half of erf function of n over root 2. So, we have a mat lab code here which does the job and we can select a range from minus 100 to 100 and f is a factor of 1 by root 2 and this code can be used to compute the variance and this can simplify the expressions.

Thank you.

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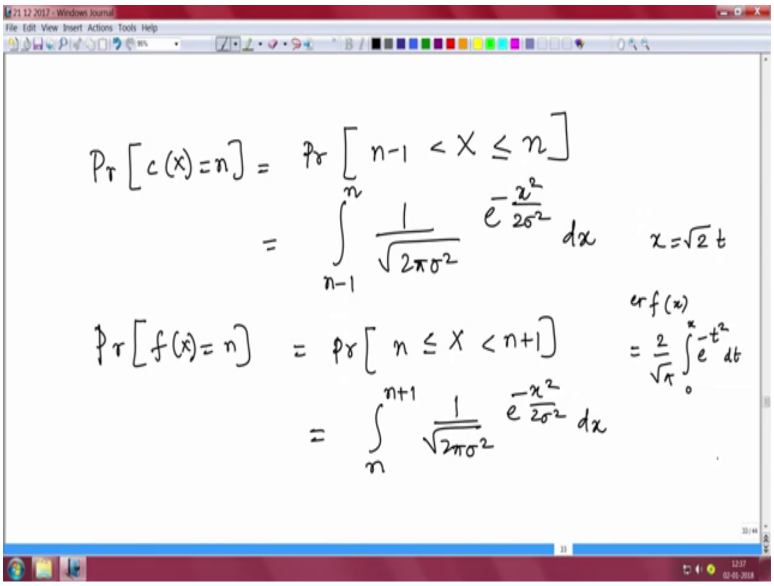

Here a small correction the right substitution is x is equal to root 2 t, this will make sure that the probability that we compute the PMF we get is in terms of the erf function and the mat lab code will execute the code and we get the PMF in the right way. That is it.

Thank you.