Mathematical Methods and Techniques in Signal Processing – I
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Lecture – 27
Statistical specification of random processes

So, we will first define the statistical specification of a random sequence and with this hopefully we should be able to define what stationarity is of a process, we will learn what stationarity is and what it is strict sense stationarity and there is another concept called the wide sense stationarity and we need to know the definitions for both of these results and I think with this sort of you should be set for the rest of the course.

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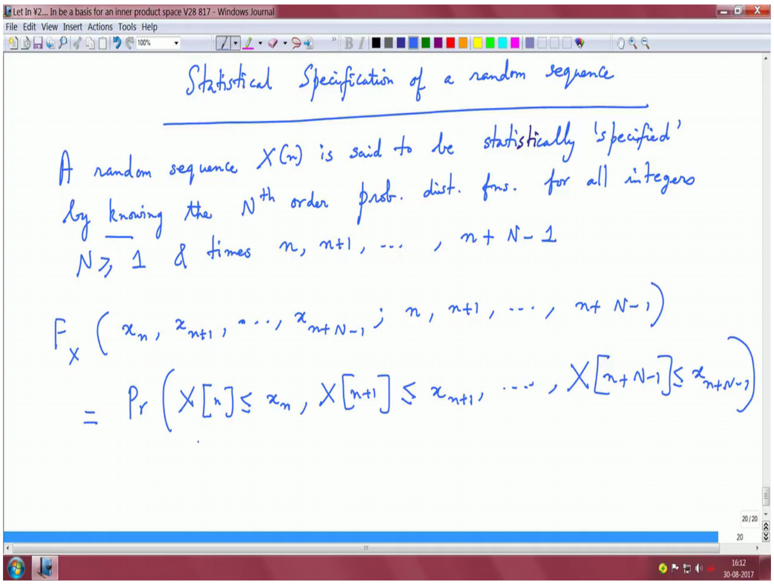

Statistical specification of a random sequence a random sequence X of n is said to be statistically specified by knowing the nth order probability distribution functions for all integer times n greater than or equal to one for all integers n greater than or equal to 1 and times which is small n small n plus 1 so on till small n plus capital N minus 1. So, a random sequence X of n is said to be statistically specified by annoying the nth order probability distribution functions for all integers capital N greater than or equal to one and times small n; n plus 1 so on till n plus n minus 1 so; that means, f x X n, X n plus 1. So, on till x n plus capital N minus 1 and times n n plus 1 dot dot dot n plus capital N minus 1. This is basically the probability that this variable random variable is less than or equal to x of x n X of n plus 1 is less than or equal to x small x n plus 1 dot dot dot x n plus n minus 1 is less than or equal to x n plus n minus n.

Now, how you how do you define the probability right if basically probability that this continuous random variable you know if I have to compute the probability. Basically, I integrate from minus infinity to some point a, right of the density function, right. So, if it is a probability for a discrete I can say probability that x can take this particular value, I can exactly specify right for example, if it is a Bernoulli random variable right 0s and 1s and with probability P and 1 minus p, I can say what is the probability that this x is a 0 or x equals 1, but for continuous random variables, I cannot say exactly what is the probability of a particular that that random variable takes this particular value because it is going to be 0, right.

So, basically you can say the probability that the event x is less than or equal to some number right some numbers specified what is the probability this is that then you can basically integrate if we look at the integral of this density function from minus infinity to that point right that this is exactly what we have here I give you some values x; small x suffix small n x suffix n plus 1 that is at different times I give you these values specified at times n; n plus 1 and so on.

Now we have the random variable X of n right, I told you that if you have a random process if you basically fix a time value it becomes a random variable, right. Now if I want a distribution of this, if I give you a random sequence X of n and I say that this is statistically specified; that means, I am interested in the probability that x x of this random variable X of n is less than or equal to some small x n because its random variable at this time step small n right, I can basically specify that what is probability jointly that at this time the random variable is this is less than or equal to this at this time n plus 1, this random variable is less or it will X of n plus 1 dot dot dot till the times that are specified ok.

So, this picture should be very clear in your in your mind that we have a process we are sampling this process at different times and at different times we have these random variables and over these random variables we want to define the probability distribution function as opposed just to a probability distribution function or a probability density function that you can define for a know a uni-variate case or a multivariable multivariate case that sampling is very important sampling this random process at different times specification of this in terms of this distribution now.

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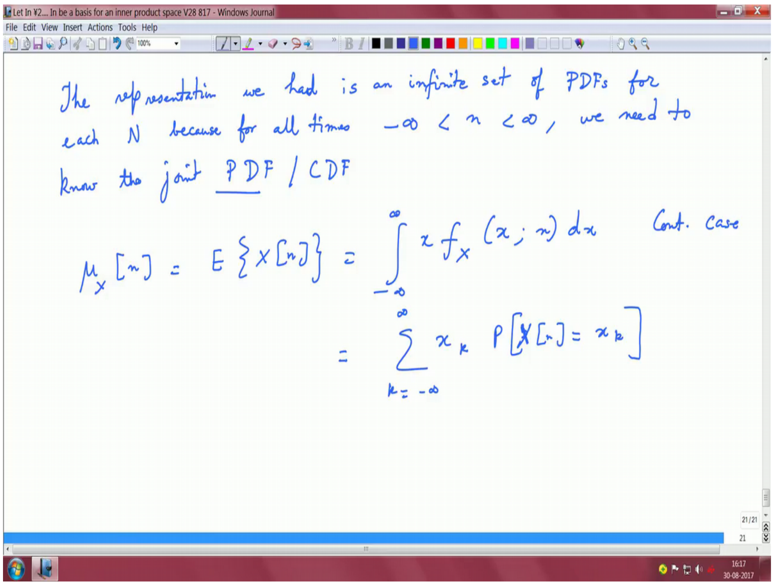

So, the representation we had by specifying the probability distribution function is an infinite set of probability distribution functions for each n each capital N because for all times minus infinity less than n less than infinity, we need to know the joint probability distribution function probability distribution function is also called CDF cumulative distribution function right you have to distinguish probability density versus probability distribution function.

So, we can once you know this distribution function I, this is very important because you for every value of capital N for every value of capital N you have to know all these this distributions for all these times right. So, I give you some integer capital N. So, N equals 1, N equals 2, N equals 3, for each of those integers, I need to compute the joint distribution for all these times right. So, unlike one probability distribution function that you see the concept in random processes is you find that this statistical specification has basically an infinite set of probability distribution functions for each n for each integer n.

Now I think this is sort of consistent, we can define the mean which is expectation of X of n which is basically for the continuous case is this is f; this is x just taking the statistical expectation this is for the continuous case and for the discrete case is basically you have a probability mass function and you can compute that is random variable at this time takes this value average over all possible values it takes clear.

So; that means, you can imagine that this mean is a function of the index potentially could be a function of this index. So, basically we also have some extensions or classifications of random processes.

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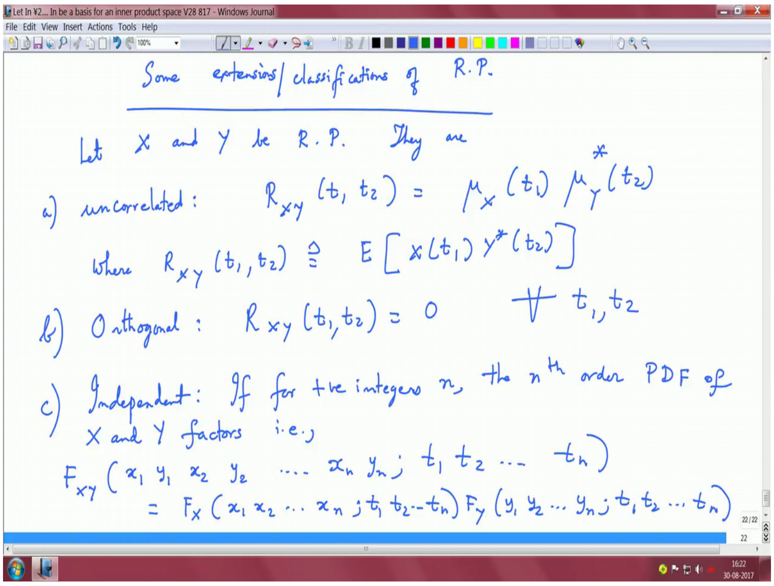

So, let x and y be random processes they are a uncorrelated if r x y of t 1 t 2 is mu x of t one times mu y of t 2 conjugated where r x y of t 1 t 2 is defined by our familiar cross correlation specified a times t one and t 2 you can define orthogonality for random processes. So, if r x y of t 1, t 2 is 0 for every t 1 and t 2 for all t 1 and t 2, then these 2 prophesies are orthogonal, right, imagine this is like the inner product, but we are taking expectation on a product right under that measure this is 0.

So, therefore, the natural dot product I mean thinking about vectors or signals that are deterministic taking the inner product whether it is in whichever space that we will not look at the inner product and or the you know in the in the integral norm or in whichever since we took the inner product right we could take the inner product and if that happened to be 0, we said it is orthogonal similar to that sense we can define orthogonality for random processes that r x y of t 1 t 2 is equal to 0 for all times t one and t 2 then these 2 processes are orthogonal.

It is a very important idea just imagine for every such 2 times you take these this product right x of t one and y conjugate t 2 and then take the expectation averaged it out and it has to be 0, it is a very very powerful; powerful concept, then there is something which is more basic which is independence if for positive integers n the nth order probability distribution function of x and y factors that is f x y of x 1 y 1 x 2 y 2 dot, dot, dot x n y n given a times t 1 t 2 so on till t n. If this factorizes as f x of x one x 2 dot, dot, x n specified a times t one t 2 dot, dot, dot t n and f y of y 1 y 2 dot, dot, dot y n and t one t 2 dot, dot, dot t n then we say that these 2 random processes are statistically independent ok.

So, I specify all the times, I know what values; I need to take when I say f x of x 1 x 2, then I say f x of the random variable capital X at time t 1 is less than or equal to small x one comma the random variable x capital X at time t 2 is less than or equal to t 2 so on and so forth using this definition that we have here right and if it if the if the probability distribution function if it factors in this form, then these 2 processes are statistically independent of course, then you can prove some other results from this if they are independent are they uncorrelated so on and so forth, correct you can sort of exchange some of these results. So, we will pause here we will take a break.