

Mathematical Methods and Techniques in Signal Processing-1
Prof. Shayan Srinivasa Garani
Department of Electronic Systems Engineering
Indian Institute of Science, Bangalore

Lecture - 22
Problem on Orthogonal Complementing

So, let us have some interactive problem solving sessions by students have taken this course. So, you will see some illustrations and examples into problem solving which is useful to understand and digest the concepts learnt during the lectures. So, this problem appeared in MMTSP course in IISC in fall 2016 in the midterm exam 1, so the problem is as follows.

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MMTSP, IISC, Fall 2016, Mid Term Exam 1

Problem :- Consider the space V spanned by the vectors
 $\underline{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\underline{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\underline{v}_3 = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$
 Obtain the basis and dimension of V and V^\perp

Solution :- \underline{v}_2 and \underline{v}_3 are linearly independent
Reason :- \underline{v}_2 and \underline{v}_3 are orthogonal
 $\underline{v}_1 = \underline{v}_2 - \underline{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$
 $\therefore \underline{v}_1$ is a linear combination of \underline{v}_2 and \underline{v}_3
 Basis of $V = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix} \right\}$
 $\underline{v}_2, \underline{v}_3$ are linearly independent and span V

So, consider the space v spanned by the vectors v_1 is equal to $1 \ 2 \ 1$, v_2 is a column vector $1 \ 0 \ 1$ and v_3 is the column vector $0 \ -2 \ 0$. So, we have to obtain a basis for the space v and also a basis for the vector space v^\perp . So, the solution is as follows.

So, we can clearly see that v_2 and v_3 are linearly independent; the reason is that v_2 and v_3 are orthogonal and any 2 orthogonal vectors are linearly independent and is also clear that v_1 is equal to v_2 minus v_3 . So, therefore v_1 is a linear combination of v_2 and v_3 . So, a basis for the space v will contain the vector v_2 and v_3 . So, the reason is that v_2, v_3 are linearly independent and they span the vector space V .

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As the basis contains 2 vectors, $\dim(V) = 2$
Obtaining a basis for V^\perp
Any vector V^\perp is orthogonal to any vector in V
Let v_p be any vector in V^\perp
$$v_p = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\therefore \langle v_p, v_2 \rangle = 0 \quad \text{and} \quad \langle v_p, v_3 \rangle = 0$$

$$\langle [a \ b \ c]^T, [1 \ 0 \ 1]^T \rangle = 0 \quad \text{and} \quad \langle [a \ b \ c]^T, [0 \ -2 \ 0]^T \rangle = 0$$

$$\therefore a + c = 0 \quad \text{and} \quad -2b = 0$$

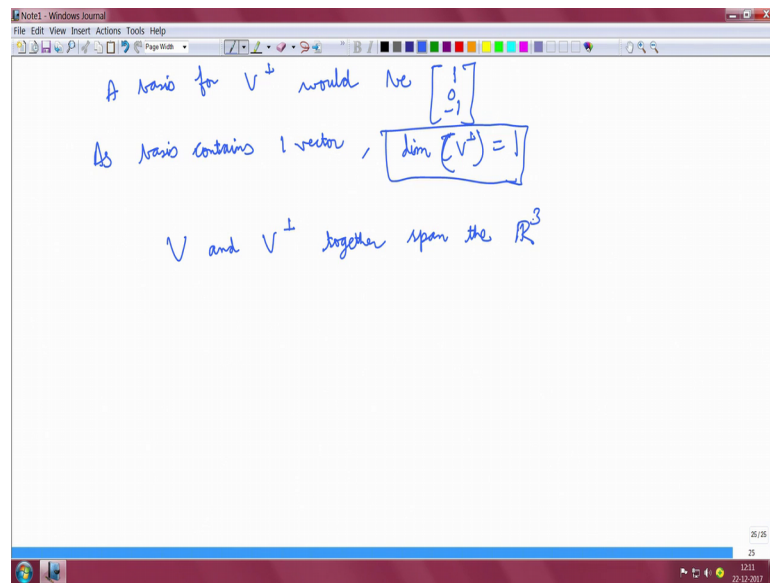
$$a = -c$$

$$\therefore \text{Any vector in } V^\perp \text{ will be of the form } \begin{bmatrix} \alpha \\ 0 \\ -\alpha \end{bmatrix} \text{ where } \alpha \in \mathbb{R}$$

So, as the basis contains 2 vectors dimension of v is equal to 2. Now, the next step is to obtain a basis for v^\perp . So, this v^\perp any vector in v^\perp is orthogonal to any other vector in the original space V . So, let us say let v_p be any vector in v^\perp , so let v_p be $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ and a, b, c are some real numbers. So, therefore as this v_p is orthogonal to any other vector in v , so it should be orthogonal to the basis vectors of v . So, we can write it as so inner product of v_p and v_2 will be 0 and the inner product of v_p and v_3 will be 0.

So, if you find any of the products from this 2 equations, we obtain that $a + c = 0$ and $-2b = 0$. So, a is basically $-c$ therefore, any vector in v^\perp will be of the form let us say $\alpha \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$. So, a basis for v^\perp would be taking α to be 1, you take any value of α it will be a basis of v^\perp .

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So, as basis contains 1 vector dimension of v pub is equal to 1 and you can see that this v and v pub together span the so this ends.