Mathematical Methods and Techniques in Signal Processing – I
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Lecture-21
Gram Schmidt orthogonalization of signals

Let us get started with the Gram Schmidt orthogonalization procedure for signals and that concept should basically lead us to basically treating signals as points in the signal space, right which is derived in the last module that the error in the reconstruction; the squared error in the reconstruction is basically the original energy in the signal minus the norm square of the signal whose components are basically these escays in the signal space.

So, we will basically first get how to get the basis is a first question right because once we get the basis then you can represent them and then you can compute how good your representation is. So, we will start with an example and I think you can now link the signals to vectors and basically invoke the same proof that you did for the Gram Schmidt orthogonalization procedure for vectors.

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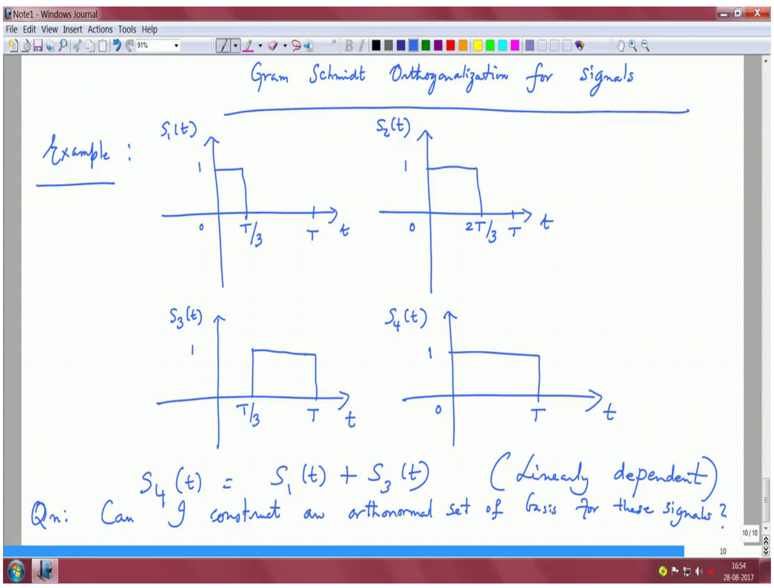

Now, let us start with an example. Suppose, I think of these wave forms like this, S 1 of t is 1, for capital T by 3 seconds some time units, I have S 2 of t which is for two thirds of this time, this is 1, it is 0, otherwise, S 3 of t is 1 between capital T by 3 to capital T. So, many units of time and we have S 2 of t which is one for capital T units of time.

So, if we just observe these signals under the definition of the inner product, in the interial sense, right, you can figure out easily that some of them are orthogonal pairwise, but are they you know or some of these linearly independent is another question that you might ask, but; obviously, for example, if you consider S 4, this signal here right, S 4 is basically S 1 plus S 3; which means that in this signal set that I have S 1, S 2, S 3 and S 4; some of them are linearly dependent, right, the question is can I construct an orthonormal set for these signals.

And this type of you know rectangular pulses sometimes you know in power systems etcetera you find this its type of motors you find this step pulses coming through, right and I do not know if one can think about application of this signal representation problem in the power engineering space, but at least you can kind of relate to these type of wave forms that we see they are not unrealistic wave forms, ok.

So, let us see how we can apply Gram Schmidt orthogonalization procedure for signals exactly the same way that we did you start with the first basis.

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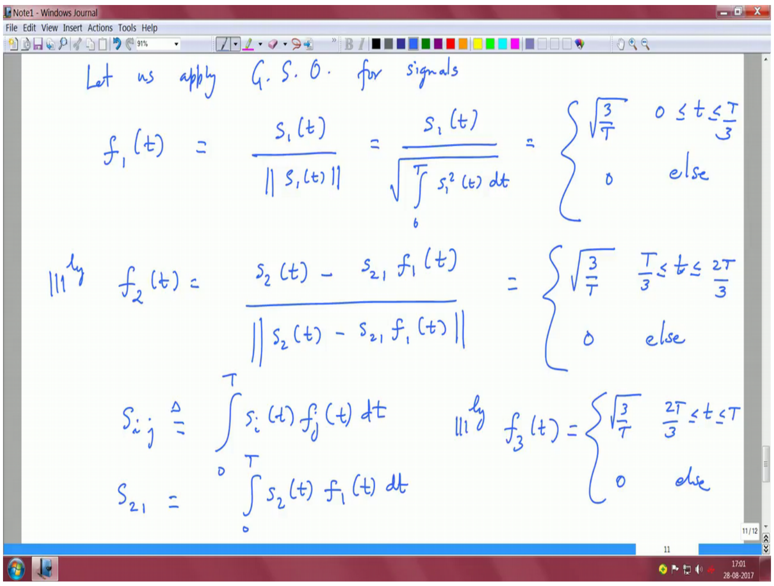

So, I have S 1 of t divided by the norm of S 1 of t, this norm is basically 1, right. So, at least I am setting this to norm 1. So, this is S 1 of t divided by square root of integral of S

1 square t d t, right. So, it is pretty straight forward that this is root of 3 by t for this duration of time and it is 0 else, right, you take the; so, square root of 1 square. So, you just have root of t by 3 in the denominator and you just flip it out, it is root of 3 by t and S 1 is 1 basically and that one root of 3 by t times one for this intervals 0 else.

So, similarly f 2 of t is S 2 of t minus S 2 1, this is basically the inner product of S 2 with f 1 on f 1 of t, right, in you have to remove that component out and if you want to normalize this; this is basically the norm of; now how do you compute. So, now, S 2; so, if you take the integral of S 2 with f 1, you just with S 1 u dot it right S 2 with S of t, S 2 with f 1 in the direction of f 1 is what we need to do, right, you can take the take the inner product, right.

So, I would say S i j here in general is basically. So, basically if it is S 2 1 is S 2 of t inner product of S 2 of t with f 1 of t under this norm under this; this definition, right, S ij is S i of t S j of t and so, S 2 1 is S 2 with f 1, right. So, if you if you carefully did this math. So, you just observe the signal when I take S 2 with S 1, right, if you just observe this diagram when I take S 2 with f 1, this exist only in 0 to t by 3 because from t by 3 to 2 3 by t, it just does not exist, it the inner product is just going away.

So, what you are left with the pi is basically this signal 0 to 3 by t defined from 0 to capital T by 3 right, 1 from 0 to capital T by 3 that is what you have for that pi of the signal. So, basically when you subtract all these things and when you normalize, you will land up with root of 3 by t. So, basically now you can think about when you subtracted it, S 2 is basically a signal when you take the inner product is what you get when you subtract it, you will land up with basically the pi from t by 3 to 2 t by 3, right, is very straight forward line this example is very illustrative not much of math you have to do.

So, if you just see through the figures you can kind of imagine what they are. So, basically this is from one third of a time to two thirds. So, typically this capital T is a signal in period in terms of signals, I mean from communication signals or something called signal in period for these pulses and that is what it means in the signal processing sense, but you know for math, you do not have to worry about it from t seconds. So, this is 0 else and you do the math for f 3, the same way take S 3 compute the inner product of S 3 with f 2, then S 3 with f 1, remove those components out and just normalize the signal is just routine trick you will have to do.

So, this is basically if you can guess; it is root 3 by t 2 t by 3 0, else.

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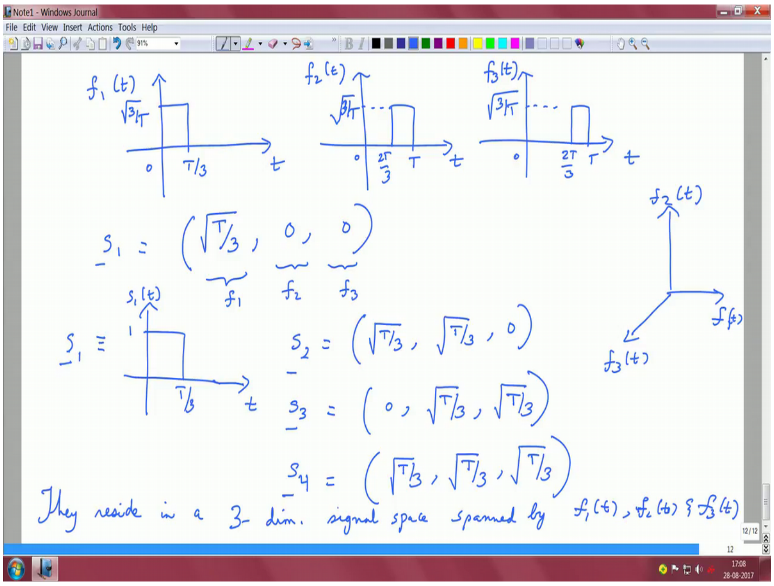

So, now, we have f 1, f 2 and f 3 and let us see how they look. This is my f 1 by f 2 is from 2 t by 3 to t and f 3 of t is from 2 t by 3 into t this is root 3 upon t, right, this is how I have and if I want to represent my S 1; my S 1 is basically 1. So, S 1, if you just recall is this way form which is one from 0 to t by 3, right. So, this is S 1 t. Now this is basically if you want to express it in terms of f 1. This is basically root t by 3 times root of t by 3 root t by 3 times root 3 by t gives you 1.

So, therefore, this is square root of t by 3 in terms of a coordinate on f 1, 0 component on f 2, 0 component on f 3, right. So, you can think about this corresponds to f 1, this corresponds to f 2, this corresponds to f 3, it makes sense because f 2 and f 3 have no components and in f 1. It is root of t upon 3 of f 1, right. This is the coordinate representation for the signal S 1.

So, S 1 which is vector is basically sort of I would say, it is equivalent to this signal is basically this coordinate in this space and I think, it is very important to get this feel just by inspection if you observed, right, the support really there is no overlapping support for these f 1, f 2 and f 3, right; that means, f 1 and f 2 they are orthogonal mutually all these f 1, f 2, f 3 are orthogonal and they are normal as well I mean they are orthonormal and because they are normal you can write them in this in this coordinate form because you have to normalize the length; normalize.

So, therefore, normality is taking care this root t by 3 is this basically normalizing and it is very very important to normalize. So, now, you get a sense here. So, S 2 this vector can be is. So, S 2 is what? So, if we go through we have from 0 to t by 3 and then from t by 3 to 2 t by 3. So, therefore, you have components from f 1 and f 2 as well, right. So, therefore, you have a root t by 3 a root t by 3 is 0. So, you can just do this and convince yourself, S 3 is basically 0, root t by 3, root t by 3 and S 4 has all these components.

So, we are done because this was the aim where we wanted to start within the beginning of module 1 where we wanted to write signals as vectors and indeed we have written the signals of vectors. So, now, these signals S 1, S 2, S 3, S 4; though I have given you 4 signals, they reside in a 3 dimensional signal space spanned by f 1, f 2 and f 3 and now, it is very easy, now for you to figure out, what is a angle between S 4 and S 3; what is the length of S 3 or S 4 everything you could easily figure out, right.

So, the length and the angle that they are used to in the vectors can be equivalently transformed or translated rather to the lengths and the angles for signals as well; so, the geometry of vectors can translate to the geometry of the signals in the signal space and now, since we have vectors; signal is transformed to a vector using this appropriate coordinate system a lot of linear algebra can be applied to signals, basically, it is now a vectors now. So, we are ending this module here.