Mathematical Methods and Techniques in Signal Processing - I

Prof. Shayan Srinivasa Garani
Department of Electronic Systems Engineering
Indian Institute of Science, Bangalore

Lecture – 02
Basics of signals and systems

So, in the beginning lecture in the first you know in the introductory lecture. So, let me briefly touch upon signals and systems, some basic sequences you know some of the details that we that deal with when we, I would say we would deal with signals and systems. So, let us try to get a hold of what basic signals are, what basic systems are and how we apply these ideas towards the rest of the course.

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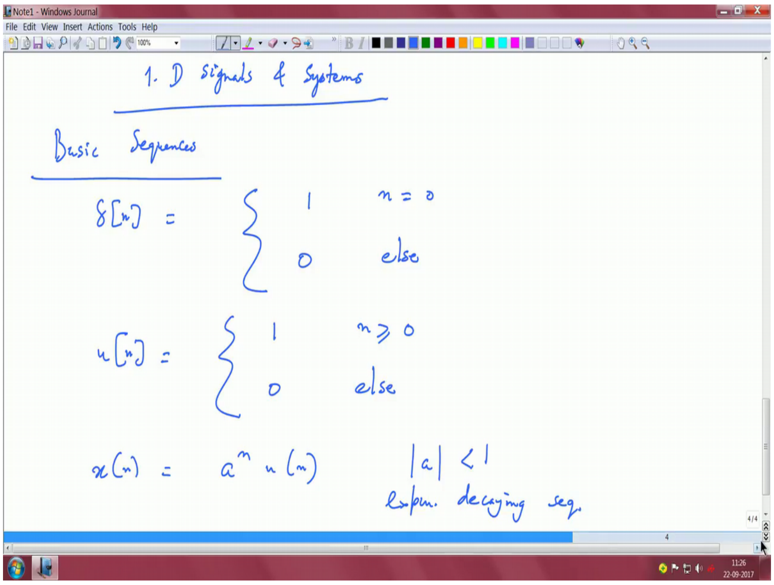

So, I would start with 1-D signals and systems. So, let us start with basic sequences. Now, one of the basic sequence one can think about is a delta subsequence which is one when n equal 0 which is 0 otherwise. Then we have the unit step which is 1 for n greater than or equal to 0 and 0, 0 else. And we can think about other sequences such as exponentially exponential sequences and if say x of n is a power n u of n, where u of n is this unit step sequence and if modulus of a is less than 1 it is an exponentially decaying sequence, even if encounter many sequences.

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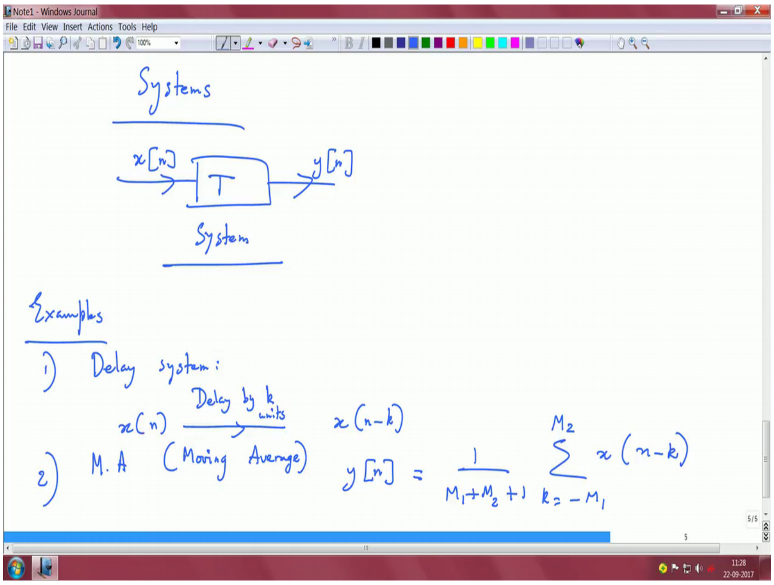

Now, from systems perspective there is a differentiation between what a sequence is and what a system is. A sequence goes through a system, a system is basically you can think about it as having an input sequence and it produces an output sequence right. So, if you can think about it in this form x of n is some input sequence it goes through some system T and it produces this output y of n and the system T is this, is an instance of a system, T is an instance of a system.

There are many systems the simplest system is the delay system. So, if I take an input x of n and I delay by k units right I get x of n minus k and that is a delay sequence. So, how can you realize a delays system by just taking a signal and pass this through a tap delay line right, I mean if you pass this to a through a tap delay line and you can realize a delay of k units.

We can have a moving average system and what is this moving average system this can be described by this equation, the output is basically a moving average of the input over a window of M 1 plus M 2 plus 1 units right. There is something in the past, there is something in the future and I just take an average and I basically get a moving, running average.

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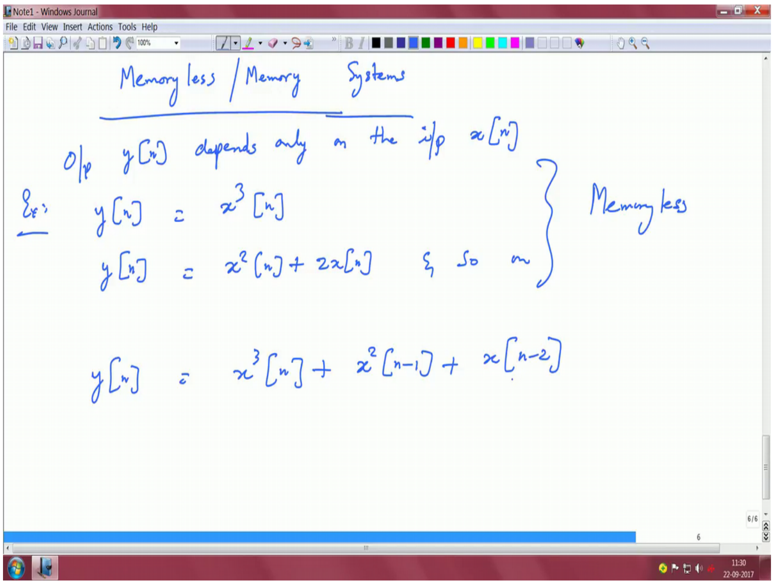

Systems could be memory less or they could have memory. So, what does this mean say if the output y of n depends only on the input x of n then it is a memory less system because it is only depending upon the current sample and nothing in the past. A good example is y of n is x cube n or y of n is some x square n plus 2 x of n and so on right because the output y of n is depending only on some function of x of n.

Now, if say y of n this is memory less and if say y of n is some x cube of n plus some x square of n minus 1 plus some x of n minus 2 say suppose then we can think about this as a system having memory because you are requiring some inputs from the past. That is at a time step n minus 1 at time step n minus 2 and so on and so forth.

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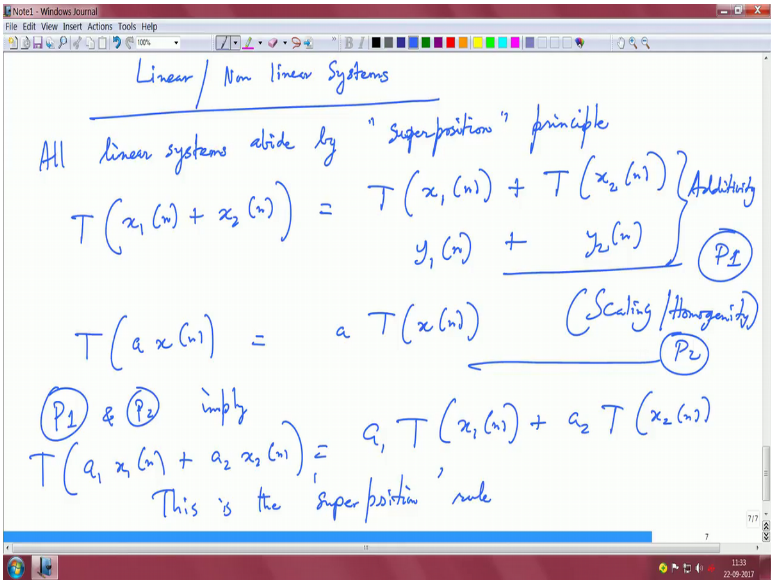

So, there is this notion of linear versus non-linear systems and this is something really important in signal processing. So, all linear systems abide by something called the superposition principle and this superposition principle has two components, one is additivity and the other is scaling or homogeneity. So, we will see what these properties are.

So, let us say the system T is acting upon inputs x 1 of n and x 2 of n and it gives you some y 1 and y 2, where y 1 of n is the response of the system to x 1 of n and y 2 of n is this the response of the system to the input x 1 of n. So, y 1 corresponds to x 1 of n and y 2 corresponds to x 2 of n and this, if this relationship is satisfied T of x 1 plus x 2 is T of x 1 plus T of x 2 then we call this as additivity, but it satisfies the additivity property. And T of some scaled version of the input if it gives us a scale of the response of the system to the input then it is called the scaling property or homogeneity right.

Now, if we invoke properties p 1 and p 2 we can say that T of a 1 x 1 of n plus a 2 x 2 of n is a 1 the response of the system to x 1 of n plus a 2 times the response of the system to the input x 2 of n and this is the superposition rule or the superposition property. And you will see this in electrical circuits when you have to look at elements such as resistors capacitors and inductors and these are essentially linear circuits. And if you bring in diodes transistors MOSFETs and other circuit elements it is basically non-linear system, and you can see a non-linear system does not satisfy.

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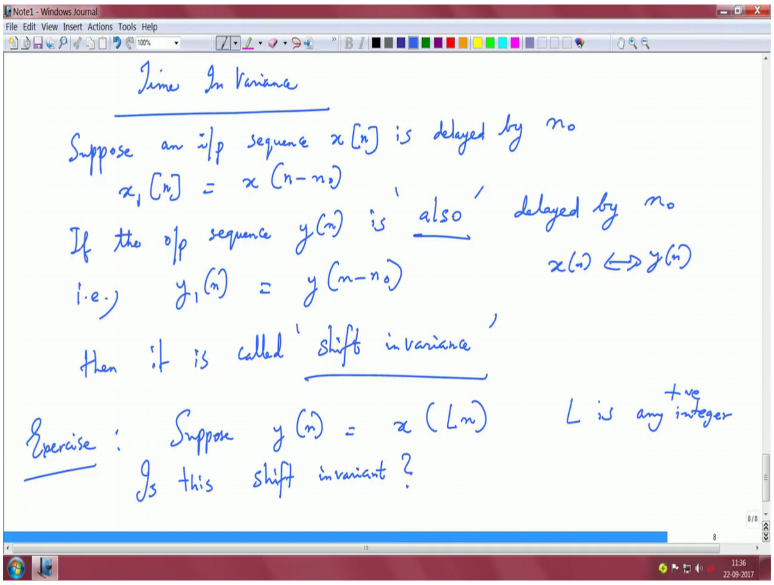

The superposition principle and there is something called time invariance when we when we bring this into systems understanding right.

What is this notion of time invariance? Suppose an input sequence suppose an input sequence x of n is delayed by n naught and let us assume that the sequence is x 1 of n which is x of n minus n naught. If the output sequence y of n is also delayed by n naught that is y 1 of n is y of n minus n n naught and x of n corresponds to y of n and if you delay x of n by x of n minus n naught my output is also delayed to y of n minus n naught. And if this property holds then it is called shift invariance.

Now, you might want to ponder about if I give you sequence y of n which is x of some L times n where L is any integer. So, it is a positive integer. Is this shift invariant? You know whatever I did just now I mean I said y of n is x of L times n and this is basically this process is called decimation and we will deal with this extensively in the multi rate signal processing module. So, I want you to examine if this decimation process is shift invariant.

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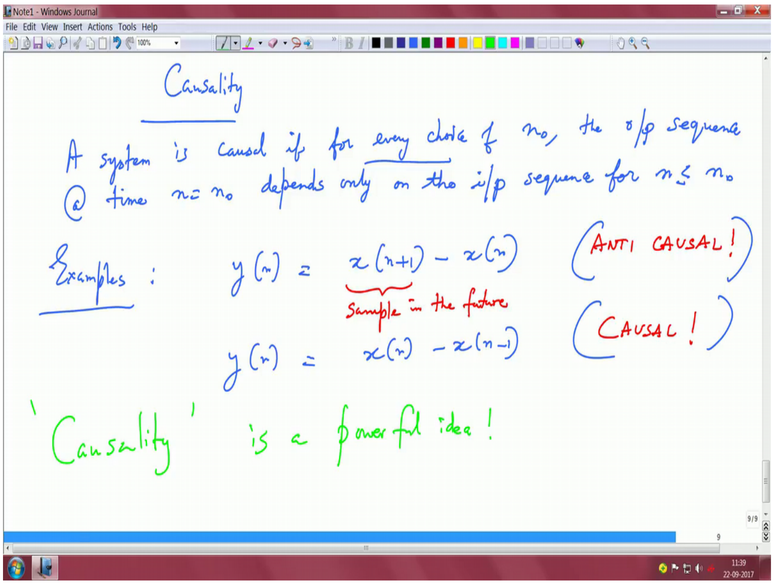

Another concept which is important is causality. A system is causal if for every choice of n naught the output sequence at time n equals n naught depends only on the input sequence for n less than or equal to n naught. That means, a system is causal if for every choice of n naught the output sequence at time n equals n naught depends only, depends only on the input sequence for n less than or equal to n naught. That means, if I have all the past inputs then and the current input then I can specify my output sequence; that means, there is this is the notion of causality.

If I am anticipating something then it is anti causal. So, let us see some examples, y of n is x of n plus 1 minus x of n this is anti causal. Why? Because the output at time n depends upon the input at time n no doubt and also the input at time n plus 1 that is some sample in the future. So, this is a sort of a very powerful idea because something now is depending upon some input in the future right and that is anti causality. And if y of n is say of this form x of n minus x of n minus 1 this is causal.

Causality is a powerful idea and one has to think if the universe is causal because this is depending upon the time reference at the moment you remove the time reference then you lose this notion of causality right. If you lose the notion of time and everything is just in space right is just you know there is no notion of this causality, I mean there is everything seems to be the same there is no notion of phase or no reference to time and this is a very important concept. And sometimes you may encounter anti causal systems and you could make them causal by you know by delays and delaying things.

For example, if I put a unit delay on the system that I described here right x of n plus 1 minus x of n. So, with a unit delay x of n plus 1 would give the x of n and with a unit delay x of n would give me x of n minus 1. So, I can convert an anti causal sequence to a causal sequence by introducing delays and you will see how this could be useful particularly when you are building systems.

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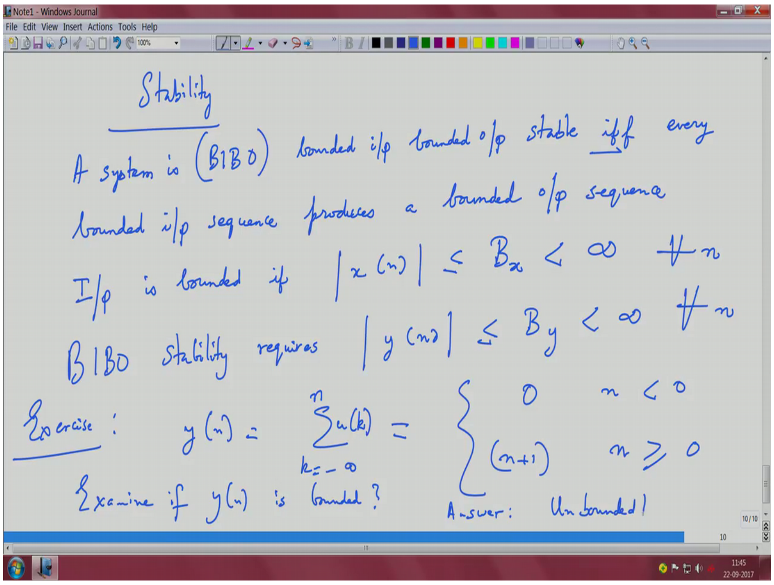

Now, another notion in this study, another notion in the study of systems or a concept, another concept in the study of systems is stability and we will discuss what stability is. A system is bounded input bounded output, bounded input bounded output. A system is bounded input bounded output, stable, if and only if every bounded input sequence produces a bounded output sequence. So, what does this mean?

So, if every bounded input sequence produces a bounded output sequence then it is called BIBO stable. Now, in precise language of mathematics what do what do we mean by bounded input? Input is bounded if mod x of n is less than or equal to some quantity B suffix x which is strictly less than infinity for every n. And BIBO stability requires that the output of the system to the input x of n which is y of n that is bounded; that means, there exist some Bs of x y which is strictly less than infinity such that the magnitude of y of n is strictly less than or equal to this Bs of y for every n.

Now, some things that you want to ponder about is suppose I give you y of n to be the summation of the unit step from the past that is from minus infinity to, minus infinity to the time step n right and we can say that this it is a 0 when n less than 0 and it is n plus 1 because you have to count u of 0 and there are n such time units. So, this is going to be n plus 1 for time n greater than or equal to 0. Now is this bounded? Probably no, it is not. Why? Because as n goes to infinity because this holds for all n right and n could be also infinity, so when n goes to infinity y of n heads to infinity, therefore, it is not bounded. So, this is an example of a unbounded system.