Mathematical Methods and Techniques in Signal Processing - I
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Lecture – 16
Problem on sum of subspaces

So, let us have some interactive problem solving sessions by my students have taken this course. So, you will see some illustrations and examples into problem solving which is useful to understand and digest the concepts learnt during the lectures.

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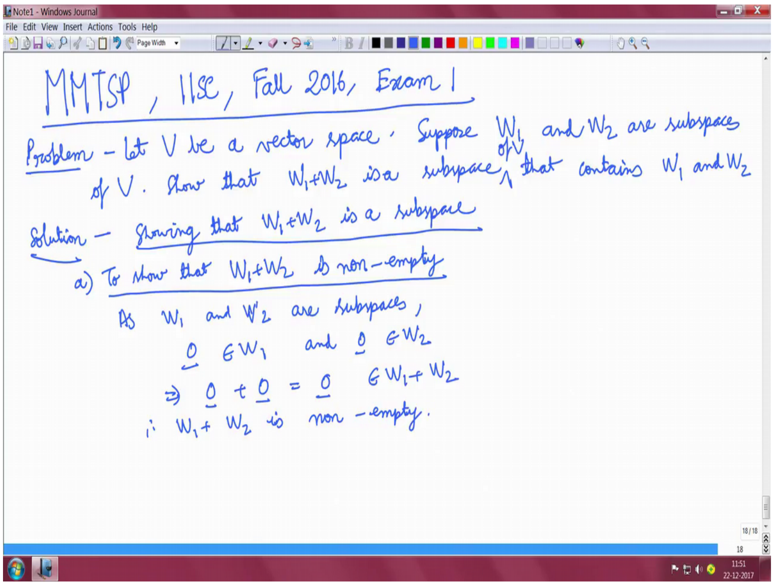

So, this problem appeared in MMTSP course in IISC in fall 2016 in the first midterm exam. So, the problem is as follows. Let V be a vector space and suppose W 1 and W 2 are subspaces of V you have to show that W 1 plus W 2 is the subspace of V of V that contains W 1 and W 2. So, let us proceed as follows.

First of all you have to show that W 1 plus W 2 is a subspace. So, for this we have to prove 3 properties which are as follows. First of all you have to show that W 1 plus W 2 is non empty. So, as W 1 and W 2 are subspaces, so 0 vector belongs to W 1 and 0 vectors belongs to W 2 as well. So, therefore, 0 plus 0 vector which is 0 vector that belongs to W 1 plus W 2. So, we have proved that W 1 plus W 2 is non empty.

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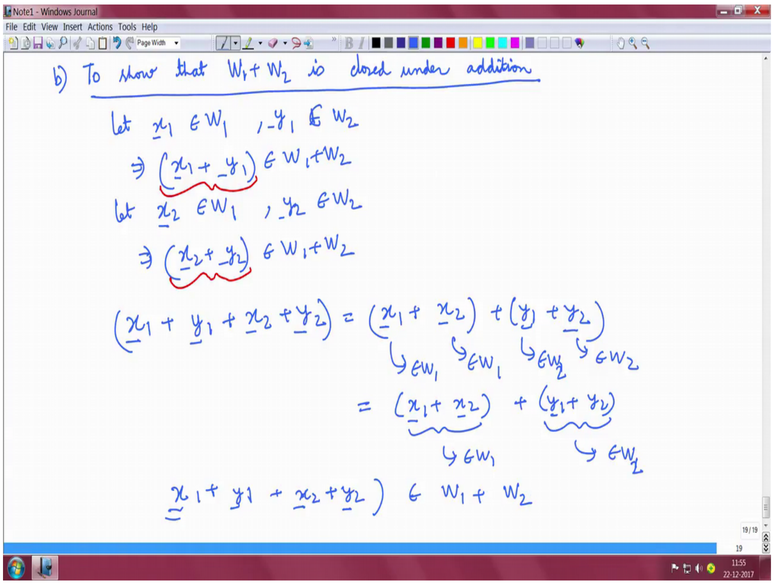

So, next we have to show that W 1 plus W 2 is closed under addition. So, for this we proceed as follows. So, let x 1 be a vector in W 1 and y 1 be a vector in W 2. So, therefore, x 1 plus y 1 belongs to W 1 plus W 2, similarly let x 2 be a vector in W 1 and y 2 be a vector in W 2. So, it implies that x 2 plus y 2 belongs to W 1 plus W 2.

Now, let us consider these 2 vectors x 1 plus y 1 and x 2 plus y 2. If you saw that the sum of these vectors also belongs to W 1 plus W 2 then we have proved that W 1 plus W 2 is closed under addition. So, let us do that. So, the sum of the vectors can be written as it can be rearranged into x 1 plus x 2 plus y 1 plus y 2. Now, we know that this x 1 and x 2 this vectors belongs to W 1 and y 1 and y 2 belong to W 2. So, the sum of those vectors will also belong to W 1. So, if we, therefore, the sum of the complete thing belongs to W 1 plus W 2; therefore. So, I have shown that W 1 plus W 2 is closed under addition.

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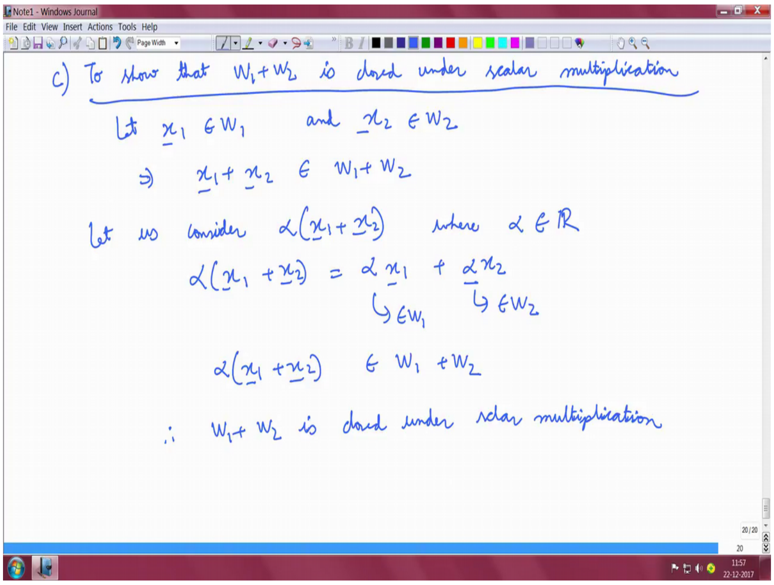

Now, the next thing you have to prove is that W 1 plus W 2 is closed under scalar multiplication. So, for that let us consider a vector x 1 which belongs to W 1 and x 2 which belongs to W 2. We know that x 1 plus x 2 belongs to W 1 plus W 2. Now, if we show that a scalar multiplication of this to this vector x 1 plus x 2 this also belongs to W 1 plus W 2 then we have proved that it is closed under scalar multiplication.

So, let us consider alpha x 1 plus x 2 where alpha belongs to the real field, now this alpha x 1 plus x 2 can be written as alpha x 1 plus alpha x 2. Now, this alpha x 1 belongs to W 1 because W 1 is a subspace and it should be closed under scalar multiplication similarly alpha x 2 belongs to W 2. So, the vector alpha x 1 plus x 2 belongs to W 1 plus W 2. So, we have shown that W 1 plus W 2 is closed under scalar multiplication. So, we have successfully prove that W 1 plus W 2 is indeed a subspace.

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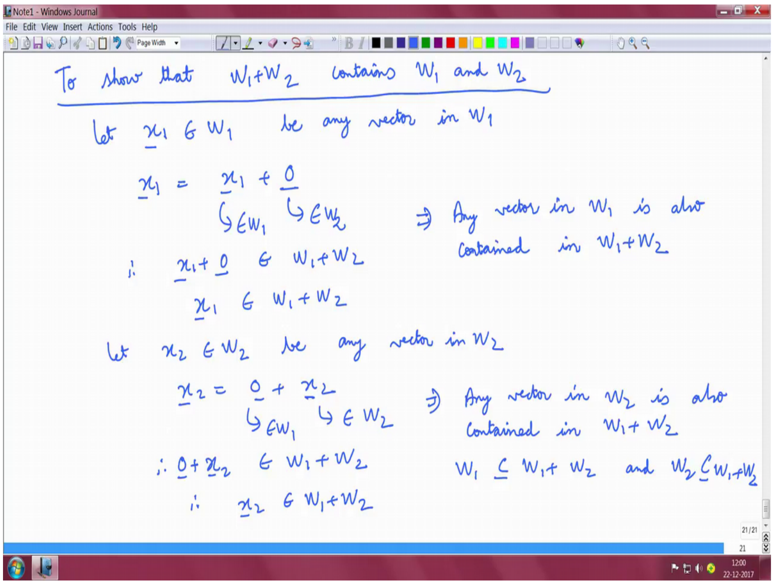

Now, the next thing you have to prove is that W 1 plus W 2 contains W 1 and W 2. So, for that let x 1 be any vector in W 1, so this x 1 can be written as x 1 plus 0. Now this x 1 belongs to W 1 and as 0 vector belongs to any subspace. So, this 0 belongs to W 2. So, therefore, this x 1 plus 0 belongs to W 1 plus W 2 which is nothing but the x 1 vector belongs to W 1 plus W 2. So, this implies that any vector in W 1 is also contained in W 2 at W 1 plus W 2. Similarly let x 2 belongs to W 2 any vector in W 2, this x 2 can be written as 0 plus x 2 this 0 vector belongs to W 1 and x 2 belongs to W 2.

So, therefore, 0 plus x 2 belongs to W 1 plus W 2 therefore, x 2 also long subspace W 1 plus W 2. So, any vector in W 2 is also contained in W 1 plus W 2. So, I have shown that W 1 is contained in W 1 plus W 2 and W 2 is contained in W 1 plus W 2. So, we have proved that W 1 plus W 2 contains W 1 and W 2.

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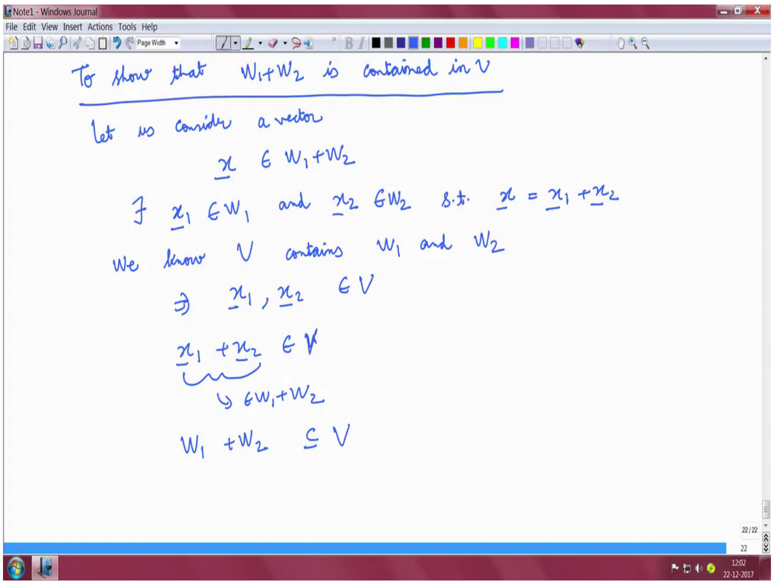

The next thing we have to prove is that, we have proved that W 1 plus W 2 is a sub space we have to shown, we have to show that W 1 plus W 2 is contained in V, show this. Let us consider a vector x which belongs to W 1 plus W 2. So, according to the definition x can be broken down in 2 vectors one in subspace W 1, one in W 2. So, there exists x 1 belongs to W 1 and x 2 belongs to W 2 such that x equal to x 1 plus x 2.

Now, we know V contains W 1 and W 2 therefore, x 1 x 2 this are present in the vector space v. So, as V is a vector space, x 1 plus x 2 should also be in V. So, an x 1 plus x 2 this belongs to W 1 plus W 2 which is x itself right. So, therefore, we have proved that W 1 plus W 2 is contained in the vector space V. So, we have proved, first of all we prove that W 1 plus W 2 is a subspace then we showed that W 1 plus W 2 contains W 1 and W 2 and then we finally, showed that W 1 plus W 2 is contained in V. So, in the sense the problem.