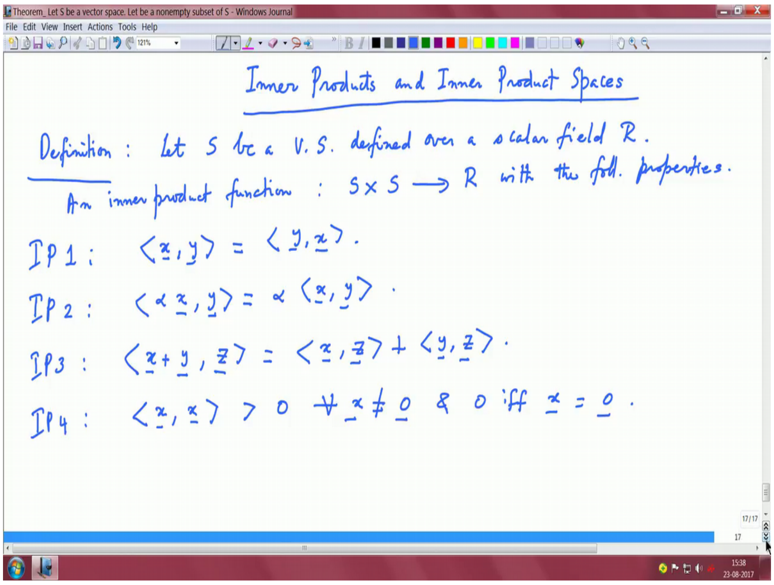
Mathematical Methods and Techniques in Signal Processing - I

Prof. Shayan Srinivasa Garani
Department of Electronic Systems Engineering
Indian Institute of Science, Bangalore

Lecture – 13
Inner products and induced norm

So, in this lecture we will basically define what inner products are and delve into the inner product spaces. So, we will start with a definition.

(Refer Slide Time: 00:32)



Let S be a vector space defined over a scalar field given by this R. An inner product function is basically a function from the Cartesian product of the vector space to R with the following properties. So, IP 1 stands for inner product property 1, the inner product of x with y assume that x and y are vectors and each of the coordinates are basically real entries. So, inner product of x y is inner product with y x of course, these definitions will change if I define the inner product function as a Cartesian product of the vector space 2 perhaps a complex number so that would basically be a different definition. But here we will stick to real, reals. So, therefore, this property holds the inner product of x y is a same as that with y x.

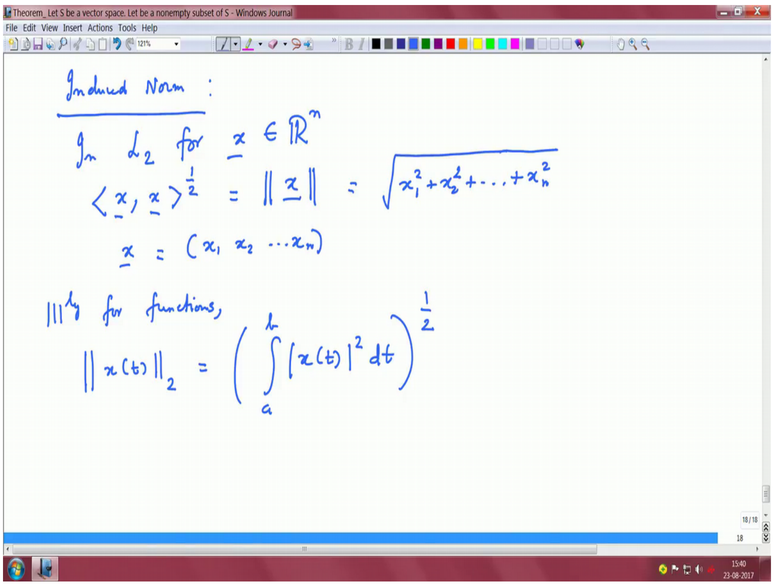
So, we have the second property. Suppose I scale the input you know one of the vectors by alpha, let us say we want the inner product of alpha x with y this is basically alpha times the inner product with x and y and of course, we say that alpha is basically a real number, here belonging to the real number.

We have the third property which is basically distributive property. We take the sum of two vectors and we take the inner product with another vector. So, the inner product of alpha y alpha plus y with z is basically the inner product of alpha with z and inner product of y with c. And we have the last property the inner product of a vector with itself is greater than 0 for all x which is not the null vector and it is 0 if and only if the vector itself is a null vector right. These are several properties for an inner product function.

Now, this inner product is a very important tool for signal representation because we would like to figure out how much of distance we have to go along a certain basis right, along a certain basis function how much of distance should we go and that basically sets up for the coordinates in the signal space. So, this is a very very important parameter.

So, once we have the inner product defined we have something known as the induced, induced norm right.

(Refer Slide Time: 04:59)



So, in L 2 spaces if you recall we did L 1, L 2, so on L infinity right we defined the basically the geometry of these L p spaces right. So, in L 2 for a vector x belonging to R n its n dimensional vector, if we look at the inner product of x with itself take the square root of this quantity, this basically is the norm of x. It is trivial to figure out and this is basically x 1 square plus x 2 square plus so on till x n square and if you think about x is basically x 1, x 2, dot dot dot x; that means, you have n coordinates.

Now, similarly for functions we can define this induce norm, that is we look at the inner product of the function with itself right and that quantity is basically x t in the L 2 sense and this is basically integral a to b mod x t square dt, the square root of this, this quantity right. So, this is basically induced norm. This is basically giving you a metric of the inner product of a vector with itself or a function with itself. Now, once we are comfortable with the induce norm we can this is idea of having one vector, now we can think about the difference vector and we can think about the norms over these difference vectors.