Basic Electrical Technology
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Lecture - 9
Sinusoidal Steady-State

Hello everybody, till now we have been discussing about modelling of circuits and in the last class we also did some analysis using the MATLAB software. Let me make a brief review. In modelling a circuit we use the state-space approach. That is we define the state variables of the circuit or the system which is basically the energy the energy storing variable associated with the energy storing element in the circuit like the C and the L and after that we obtain the state equation the state equation is of standard form which is X dot is equal to Ax plus Bu and the output equation Y which is equal to Cx plus Du.

Now after having obtained the state equation the state equations fully describes the dynamic behaviour of the circuit. After obtaining the model we input this model into MATLAB and then we perform the analysis by viewing in three different domains. One is the time domain that is where the circuit and the signals all these exist and it versus the time axis, the other domain was the frequency domain where you view the magnitude plot and the phase plot with respect to the frequency. That is, at various frequencies what will be the gain of the circuit that is the input to output and what is the phase relationship of the output with respect to the input. Now these at various frequencies are noted down and plotted and we use the bode plot in MATLAB to check this out.

And then, the third domain is the pole-zero domain. This is an important domain in the sense you get the idea of the excitation modes because the poles indicate the exponential growth or decay of any disturbance or noise which can occur in this system. So a pole would mean e to the power of minus something T.

Now, in the case of the particular example last class we saw that for different parameters for the same circuit for different parameters the pole locations can change and we saw how a fully damped system gradually became kind of a oscillatory damped oscillatory system because the pole locations shifted started shifting towards the imaginary axis and it also became complex poles. So, various analyses can be performed on these circuits.

Now in this session today we shall discuss one more important analysis that is the steady-state analysis and specifically steady-state analysis with respect to sinusoidal signals. Let us say the inputs are sinusoidal in nature which is one of the most common inputs that you will be giving in electrical in the electrical circuits and the electrical domain. So, for a sinusoidal input signal because the components are most of the components that you will be using in the circuits are linear you will see that all the branch currents, node voltages they all will be sinusoidal however there will be phase shifts with respect to the input voltage that you will be applying.

Now how do we characterise the system under steady-state condition, under the constraint that sinusoidal inputs have been given. Some important features of this circuit can be extracted in this mode. So today that will be the focus of the discussion. Given a sinusoidal excitation under steady-state condition how will the system behave?

Of course it is a very specific nature; the problem is a special case. The state equation is a more general case; you can apply the special case to the state equation and obtain the steady-state equations. Of course we will see how we go about doing that in this session. Before that we have to understand the concepts of the J the rotation the rotation factor which will be used commonly in the steady-state analysis. So let us try to understand the concept of the cos and the sin rotation.

Now let us consider the time domain access. I am having y axis here and likewise I will also have the x axis and the x axis is time t. Now here on this x axis let us say that we have a sinusoidal signal like this (Refer Slide Time: 7:12). So let me call this one as V a this is the sinusoidal signal. Now the sinusoidal signal can also be imagined as a phasor which is rotating in the complex frame. What I mean to say is let us have alongside here a complex frame aligned along this axis. This also has an orthogonal axis which is the y axis.

Now this being……… that is you have the real and the imaginary so I will call this one as sigma and the imaginary axis. Now imagine that there is a vector along this axis. Now this vector we are going to make it rotate in a circular fashion like this. So this vector is going to be made to rotate in a circular fashion like that all along. Now we are going to just note down the projection of this vector on the vertical axis and that projection value will be projected on to this time axis as shown here so let us see what happens.

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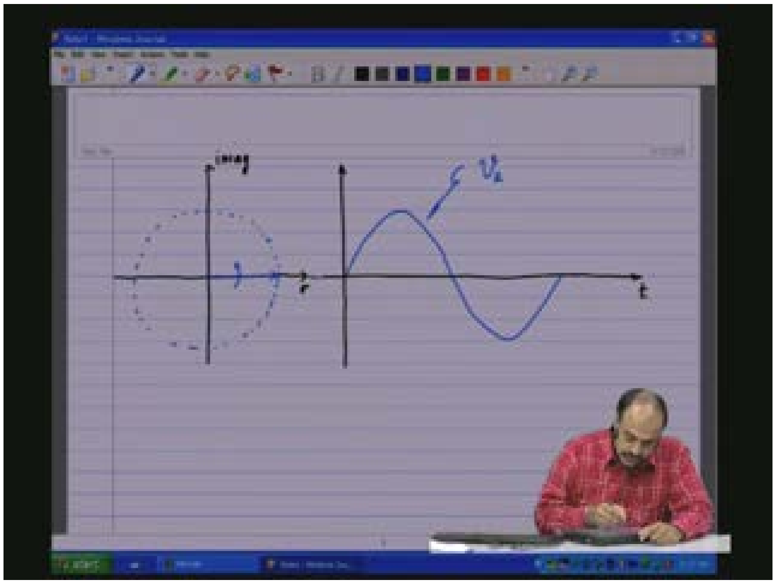

So what you say now at this point the projection of this vector at this point is going to be zero so let us say this is a point here. Now let us say this vector has taken a position here, it has moved to a position here, now this is the projection along the vertical axis and we project it along to this and we have a point here and there this vector is now moved and taken this position (Refer Slide Time: 9:48) gives projection on the vertical axis and then taken on to this because this is occurring at a time which is much later than this, this is occurring at a time much later than this and therefore there is the time progression of these points and so on.

Now you see that we have the peak occurring at this point that is when it is when this vector is 90 degrees in the complex frame. Therefore, you see that there is a phase shift of…….. there is a phase shift of 90 degrees 90 degrees here also in the time domain the 90 degree point with the respect to the zero point. So, so on if we keep writing all the vectors we get the various points which on projection projecting to the time frame will give you will give you the various points; here this, this, this and so on (Refer Slide Time: 11:21).

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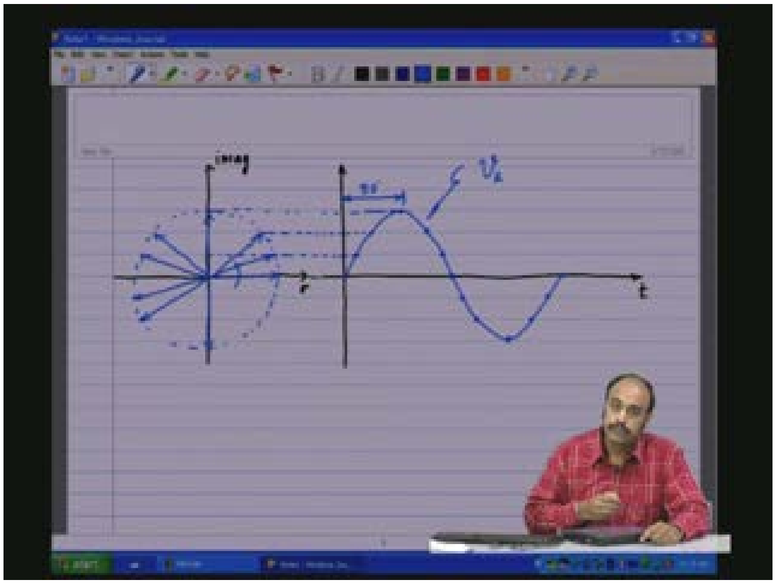

So we can say the sine wave can be considered equivalently as a phasor or a space-phasor will call it as a space-phasor because it is in the complex domain and it is rotating continuously in a circle; the amplitude is fixed and that amplitude is equivalent to whatever our maximum amplitude here let us say this is V m and that amplitude is also V m. So, with amplitude of V m this is rotating; and if we just keep taking the projection on the vertical axis and then project it on to the time axis we get the sinusoidal signal evolving out of this one.

Now let me consider another phase vector simultaneously. When the blue is at this point I will say that there is a red space vector at this point. The blue was called V a and let us say the red is called V b. This also has the same amplitude V m and with the amplitude V m………….. and this is also starting to rotate and this also starting to rotate and once the whole cycle is completed one period is completed here which means the red space vector on projecting will start from here at time zero; when the blue space vector is starting here the red space vector is starting here at time zero and then you would of course have after sometime this would have come to this point

(Refer Slide Time: 13:34) and then its projection will let us say be along this curve so we have something at this and then the vector will progress in this fashion. You have a point here and then the vector progresses here like that you have a point here and so on. So we see that we will be obtaining a wave something like that and keeps going on like this. So you will have a point here and corresponding points here of course this is…….. this is how it will look like.

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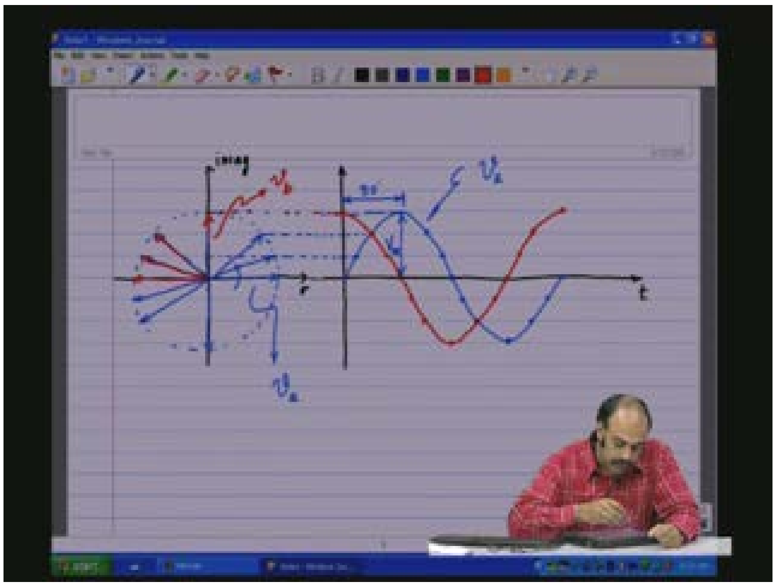

So if I am having a space vector starting 90 degrees away from the V a space vector let us say I have a vector V b which starting 90 degrees of sector 90 degrees phase shifter with respect to this V a space vector then you see that the resulting time projection of the space vector tips is a cosine wave so what you get here is V b which is a cos wave. So V a is a sine wave and V b is a cos wave.

Now let me go to the next page but before that let me select the wave pattern and then sorry I did not copy it; let me select the whole thing, well, copy go to the next page, let us paste.

Now if you say that V a equals V m sin omega t into V m sin omega t amplitude V m and sin omega t; V b is V b is V m cos omega t or we could also say that at every instant of time the V a space vector and V b space vector are shifted by 90 that is phase shifted by 90 degrees. So you could also write V b is V a into a phase shift of 90 degrees e to the power of j pi by 2 90 degrees which is equal to V a cos pi by 2 plus j sin pi by 2. This is zero of course, this has a value equal to 0 (Refer Slide Time: 17:56) of course not there, this has a value equal to 1 and therefore you have which is equal to j V a.

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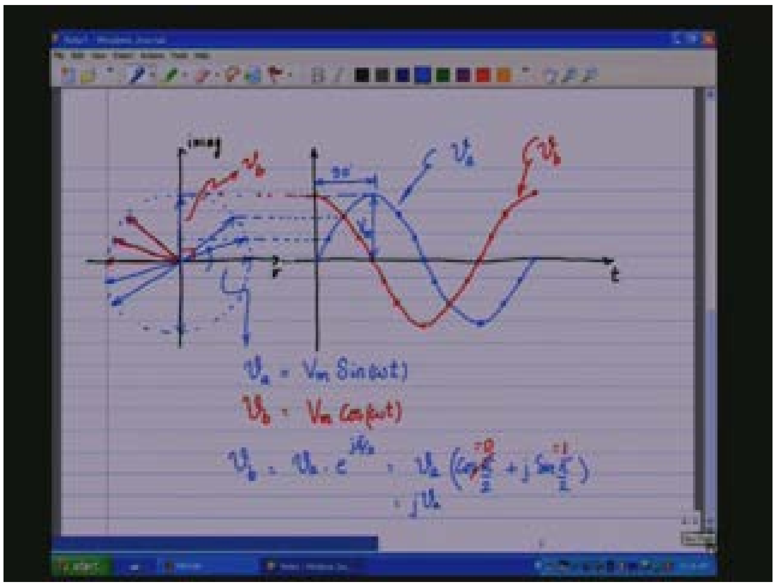

Therefore, if we have V a is equal to sin pi by 2 V m sin omega t and V b equal to V m cos omega t we could also say that V b is equal to j V a or which is equal to j V m sin omega t. This is a useful relationship which you will be using a lot in the sinusoidal steady-state analysis.

Where will we come across such a thing?

Now let us say for example; we have d by dt of V a which is equal to d by dt V m sin omega t this is equal to V m omega cos omega t which is equal to omega into V m cos omega t which is equal to omega into j V m sin omega t or which is equal to j omega V a. So d by dt of V a is going to result in j omega V a; so what does it mean that wherever you have d by dt you are going to get this stuff. So we can have a mechanism where we replace d by dts' with j omega to obtain the sinusoidal steady-state equations.

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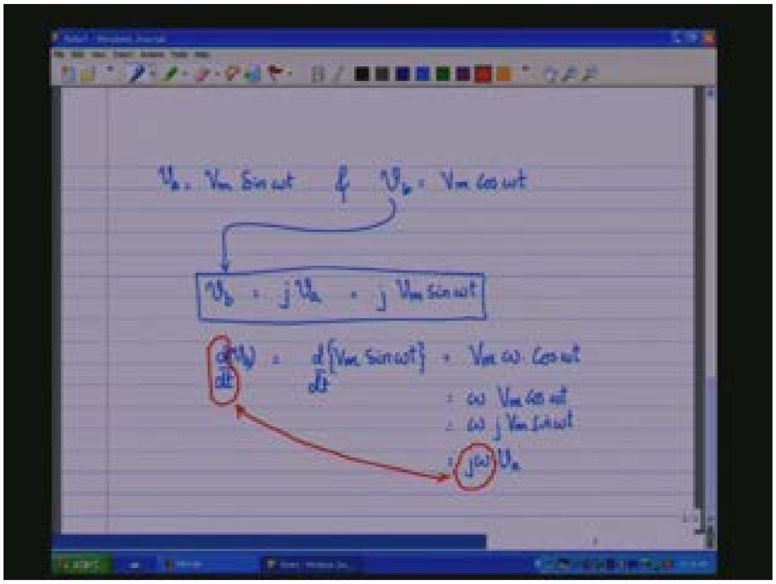

So in the state equation wherever there is d by dt replace it with j omega; that will give you the sinusoidal steady-state equations. We will see that shortly. Now we have two major dynamic components which is the capacitance and the inductance in the electrical circuits; the capacitance which is storing the potential energy and the inductor which is storing the kinetic energy by virtue of the flow in it. So let us consider first the capacitance.

We have the capacitance here C, there is a flow of current through the capacitance let us say i c (Refer Slide Time: 21:26) which is going to result in a voltage across C and that is V c with this positive and this is negative. So for this dynamic element C what is the equation we have? C dV c by dt is equal to i c. This is what we started with.

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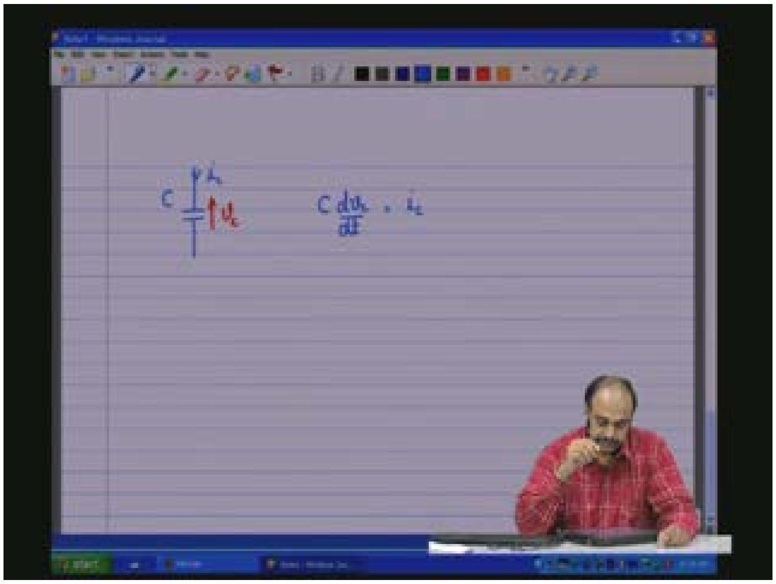

Now suppose all the quantities were sinusoidal; so the quantities are sinusoidal then V c will be a sinusoidal function something like V m sin omega t minus pi or something like that. So it could be something like d by dt V m sin omega t minus some phase shift arbitrary phase shift let us say this is equal i c.

Now here we are operating by d by dt. So when you differentiate V m sin omega t it is going to give the V m cos omega t minus phi with an omega term coming into the picture there. So it will be omega C V m cos omega t minus phi or this can be written as omega C j into V m sin omega t minus phi or we call it as j omega C into V c and this is equal to your i c.

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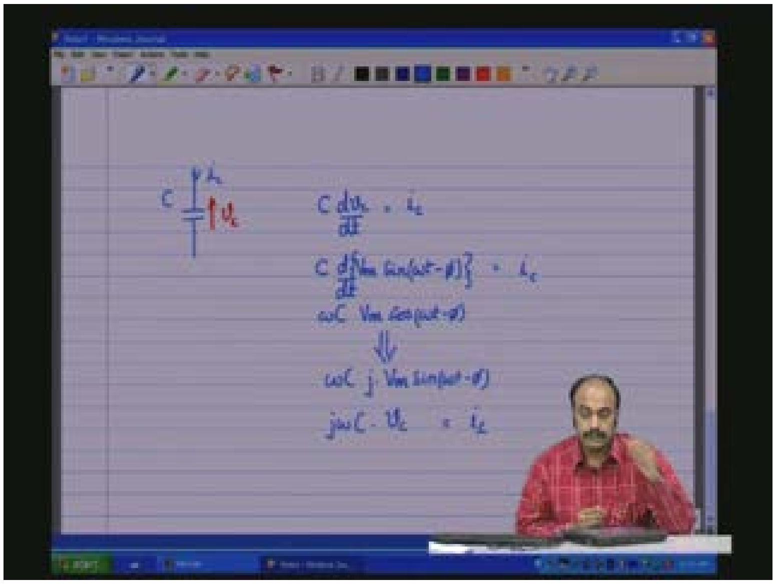

Now notice this equation j omega C into V c is i c. This has a unit of amps, this has a unit of volts (Refer Slide Time: 23:51) is it not this is volts, this is amps so what should be the unit of this? It should be something connected with the ohms. We know that we have a volts divided by ohms which will give you the units of amps. So we have V c into j omega C which is i c or V c divided by 1 by j omega C is equal to i c. Therefore, comparing these two V c is volts, i c is amps, and this should have the units of ohms and this we call it as reactants or capacitive reactance. Of course there is a j term, so 1 by j omega C can be written as minus j by minus j square omega C; minus j square is 1 therefore we have minus j into 1 by omega C. So this we call it as capacitive reactance X c X c and there is a phase shift which is corresponding to this j here and it is a minus j which is minus 90 degrees minus 90 degrees of rotation in this space vector which will mean it leads that is it occurs before the normal phase vector. Therefore, this has a leading effect and here we have capacitive reactance the capacitive reactance and that is j X c which is this and this has a unit of ohms (Refer Slide Time: 27:10). X c is 1 by omega C, j is a rotation parameter in the complex domain and minus is to indicate that it is a leading phase with a line.

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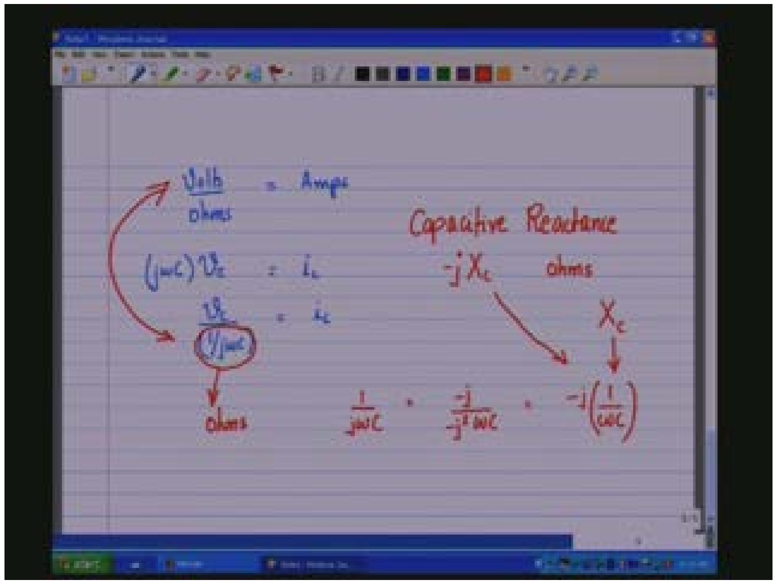

Now consider the other energy storing element which is the inductor. An inductor L which has the state variable i L flowing through that and there is a voltage across the inductor which we will call it as V L. the dynamic equation for the inductor is L di L by dt which is equal to the voltage across the inductor which is V L; L di L by dt is equal to V L is the voltage across the inductor.

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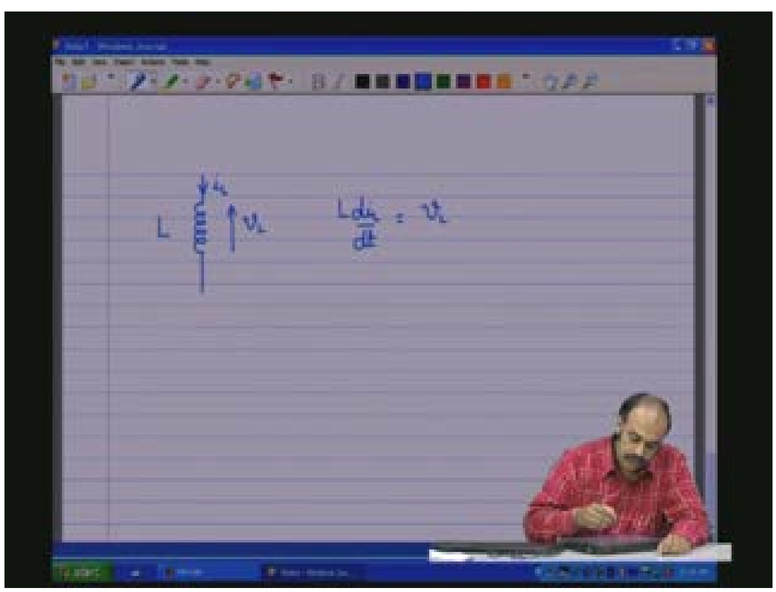

So, if the current the state variable through that is a sinusoidal quantity then you have L d by dt Im let us say sin omega t minus phi which is equal to V L; even V L will be a sinusoidal quantity with a phase shift of course. Now we see here again along the same way we have a d by dt I can replace it by j omega; so you have j omega L Im sin omega t which is Im which is equal V L. Here you have the voltage V L which has the units of volts, the current here having the units of amps so volts will be amps into ohms and therefore we can say that (Refer Slide Time: 29:53) this should have a unit of ohms and that factor j omega L into i L is equal to V L this factor is written as j X L and it has a unit of ohms. So X L is equal to omega L this is called the inductive reactance reactance.

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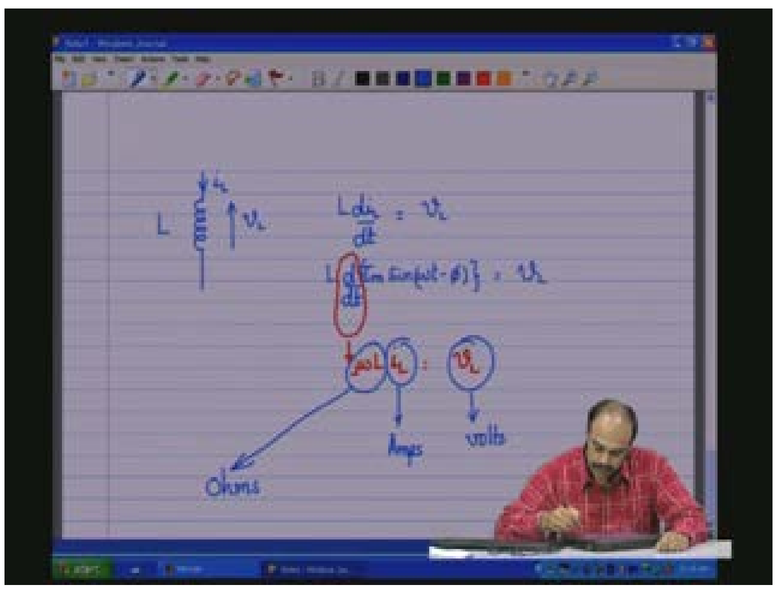

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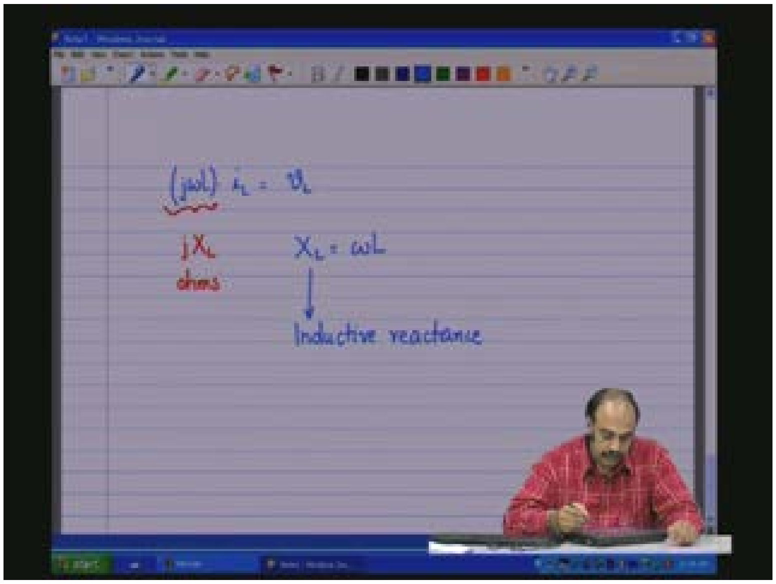

See the rotation of the positive j there which means that it is rotated in the positive direction by an angle of 90 degrees and with respect to the capacitive reactance it is 180 degrees because capacitive reactance gives minus 90 and this gives a plus 90 and it is 180 degrees. In fact, it is very evident in the case of the pole-zero plane; you will always have complex pole mirroring along the real axis. So if you have a pole on the left of top you will have a corresponding mirror pole on the left of bottom and there is a 180 degrees phase difference between these two to indicate that there is a resonance action and the resonance can actually occur and an oscillation can now only occur if there are two different types of energy storing elements where one which is kinetic based and the other which is potential based.

So anyway coming back to the equation here we have the inductive reactance which is written here. So, summarizing we have for the steady-state sinusoidal condition………. we should remember that only for the sinusoidal condition and for these two components C and the L this is going to provide a resistance or an impedance which is given by minus j X c and this is going to provide j X L. X c is 1 by omega C or I could write as minus j 1 by omega C and this is j into omega L where C is the capacitance value and L here is the inductance value, omega here is nothing but 2 into pi into f; it is radiance per second, this frequency f here is in hertz written as H z.

So at any particular frequency radiant frequency the capacitance provides the reactance which is 1 by omega C and for the inductance at any particular frequency omega the reactance provided by the inductor is omega L.

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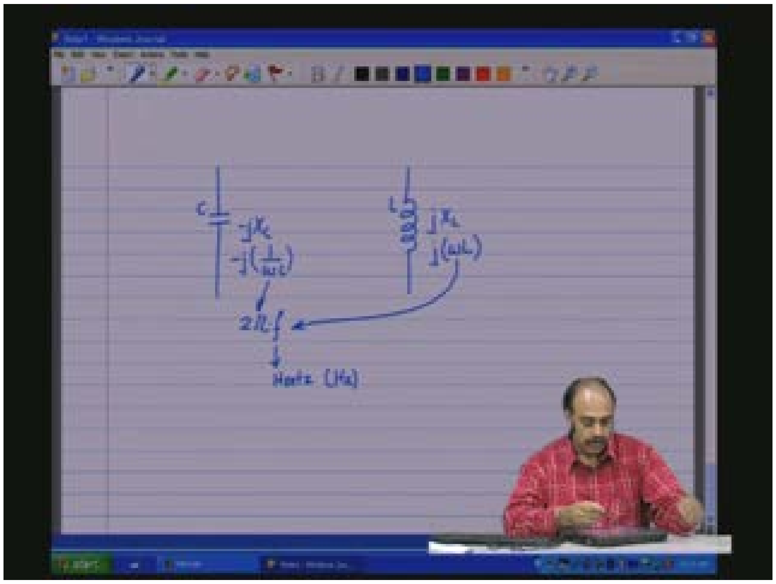

So if you look at this here at omega is equal to 0 what does it mean?
When omega is equal to 0 this means the signals are busy. So at omega is equal to 0 observe that this tends to infinity because omega is at the denominator which means capacitance provides very high reactance infinite reactions which is basically the reason why the capacitance blocks DC. And in the case of the inductance at omega is equal to 0 there is no inductive reactance, it is appearing as a short which means that the inductance has saturated and no longer provides any…….. at DC it does not provide any impedance, this will be 0.

Now, at omega tending to infinity very high frequency very high frequency omega becomes very high, so during that time the impedance the reactance or the impedance provide the capacitance is very negligible so this will tend towards 0 at high frequencies which means it provides a very low impedance path at high frequencies and at frequencies the inductor gives you a very high reactance or impedance and this tends towards infinite which means it tries to block high frequencies which means it will drop across it and on the high frequency this will pass all the high frequency components whereas on the lower frequency this is going to pass the low frequencies whereas this is not going sorry this is going to provide high impendence at low frequency which means it is not going to pass low frequencies and this will pass the low frequencies. This is one of the important features that one can know about by studying the steady-state character.

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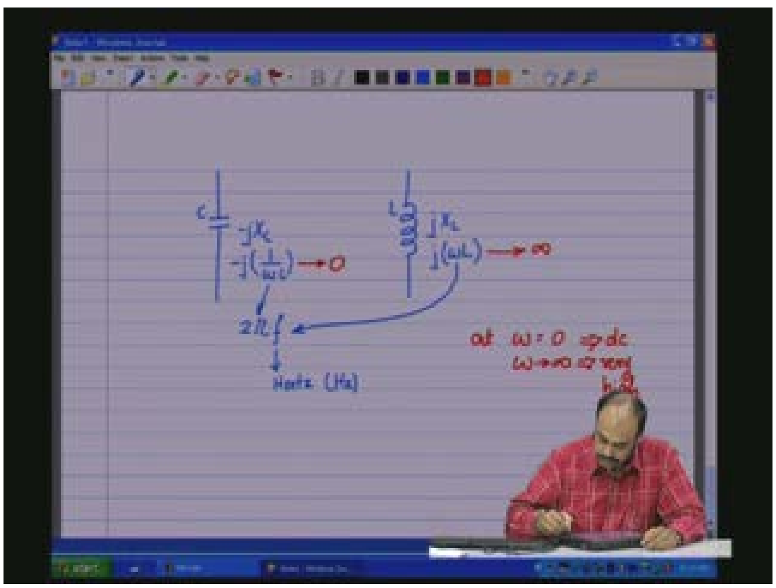

Now let us look at this simple equation which is obtained from this circuit. You have an R and let us say we have a C. So we have an R and we have a C and let us say we have a V i which provides voltage in this fashion; let us say we have a voltage here and of course a voltage here (Refer Slide Time: 37:13). Now R is going to provide a resistance, C is going to provide a reactance and that value is minus j X c which is equal to minus j by omega C that is the reactance this capacitance is going to provide.

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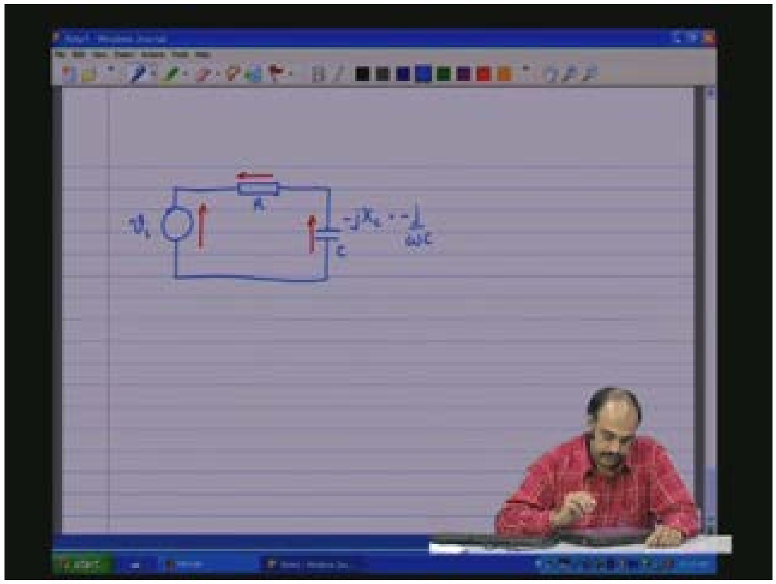

Now what will be the current which flows in this loop? What will be the current that flows in this loop?

It is nothing but the voltage divided by whatever comes in series. Now R…………. what comes in series………….. all the impeding parts would be R minus j X c. Now this (Refer Slide Time: 38:15) is called the impedance Z so the current i will be nothing but V i by Z the impedance, now this is also having the unit of ohms which is basically nothing but V i divided by R minus j X c R is equal to V i by R minus j by omega C to include the omega and the capacitance values in the equation.

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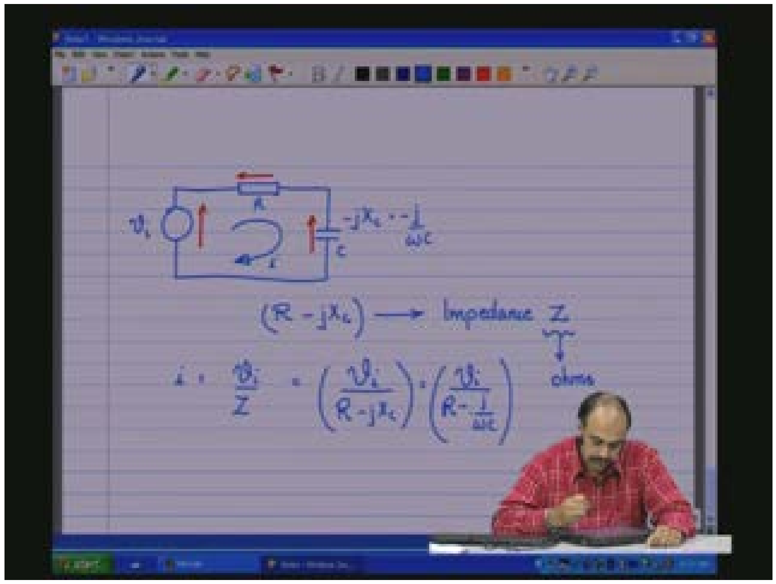

So this is the steady-state value. One could also obtain………….. you must note that the state equations are the most general and this is a specification of the special case. Let us look at the state equation of this particular circuit and then see if you substitute the j omega for the d by dts' you get back these steady-state equations where you will see that for this circuit we have already obtained the state equation in a previous session in a previous session; we have V I, this is R, this is C (Refer Slide Time: 39:59) we have voltage and the voltage across C V c which is………… so the state equations d V c by dt is minus 1 by RC into V c plus V i by RC.

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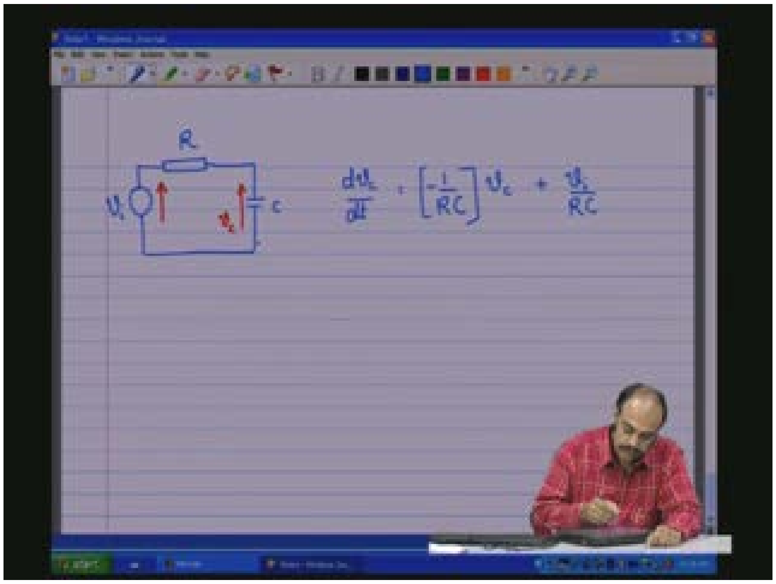

Now the step is that for all the d by dts' you just replace by j omega. In fact, for whatever be the system which is represented as x dot which is equal to Ax plus Bu the dynamic equation part of the state equation you can replace this by j omega x that is what we are trying to do here. So we have j omega V c which is equal to minus 1 by RC V c plus 1 by RC V i and let us take C on this side it becomes j omega C V c which is equal to minus 1 by R V c plus 1 by R v i and we know that this is nothing but the capacitive reactance part which can be written as V c by minus j X c which is equal to minus 1 by R V c plus V i by R.

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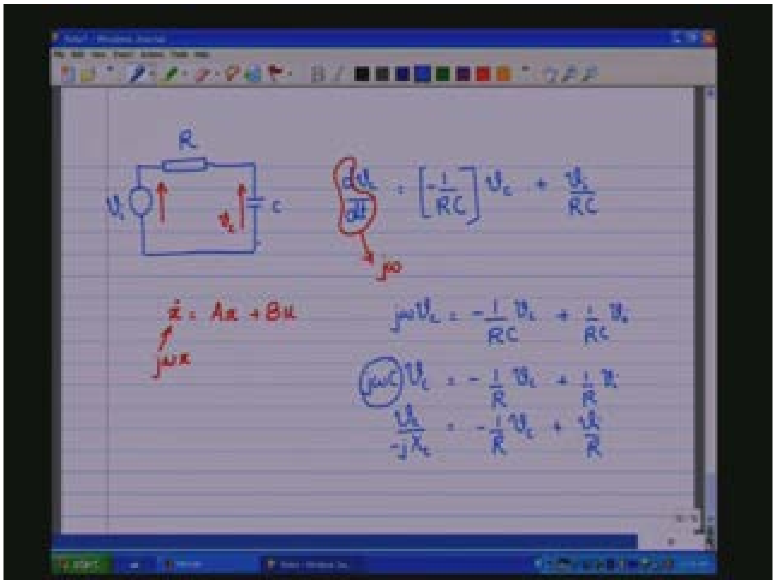

So this when you rearrange it, that is when you take V c on this side and then on rearranging you obtain………. let us have this circuit with us, copy, let us paste it here we will obtain by rearranging that V c will be V i by R minus j X c into minus j X c; you see this has a reactance minus j X c.

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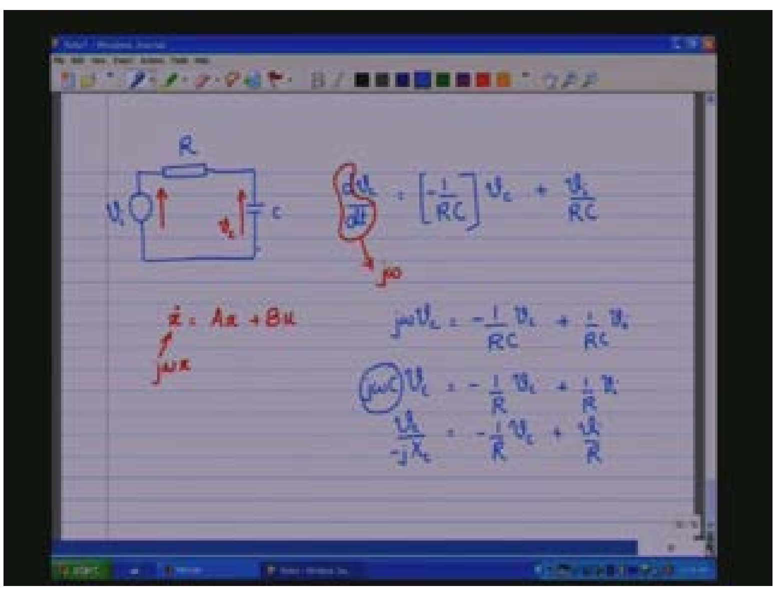

Now the current through this is nothing but V i by R minus j X c this is the current; this is the current through this loop i into the reactance occur into the reactance current into the ohms which will be the voltage across that one to be a straightforward analysis and this is being obtained directly by the original state equation. So state equations can be brought into the steady-state former steady-state equation analysis just by replacing d by dt with j omega.

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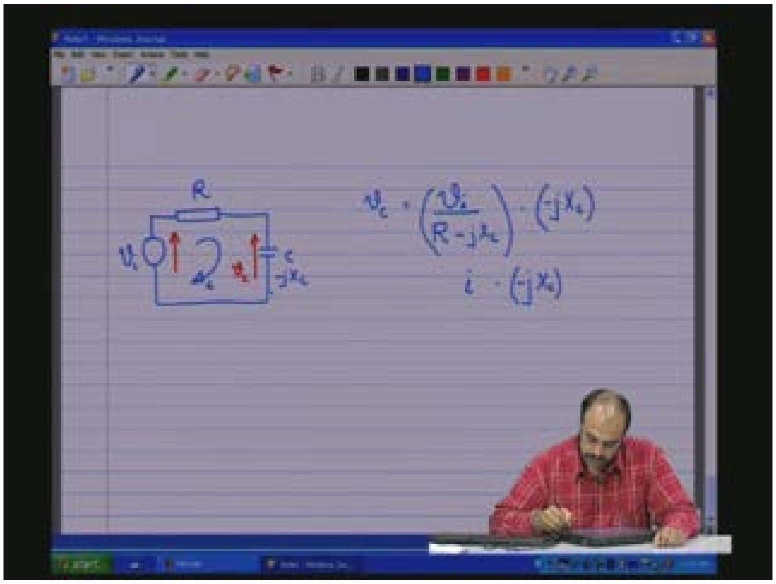

Likewise if you see the inductive circuit also we have R and a L V i, this is the R and the L and of course here the state equation is i L. We have L di L by dt equals V i minus i L into R. This is basically the state equation which was formulated. This actually is not the full complete state equation but the state equation is always the dynamic equation plus the output equation but I mean that we need to consider only the dynamic equation here to obtain the steady-state relationship. So this portion this is the d by dt we replace by j omega; so what do we get?

We get j omega L into i L which is equal to V i minus i L into R. So what is j omega L? It is nothing but j into X L you see that, omega L is the inductive reactance into i L which is equal to V i minus i L into R. So take i L into R into this side (Refer Slide Time: 46:01) you see that V i equals i L into R plus j X L into i L. You see, this is the voltage; i L into R is the voltage drop across the resistance; j X L into i L which is the voltage drop across the inductance which is V L and we have V i which is here so observe that the KVL is obeyed here and in the summation of the instantaneous voltages V i i L into R and j X L into i L they all sum up to zero every instance of the time therefore Kirchoff's voltage law is obeyed even under steady-state conditions so that the energy conservation principle is still a way held.

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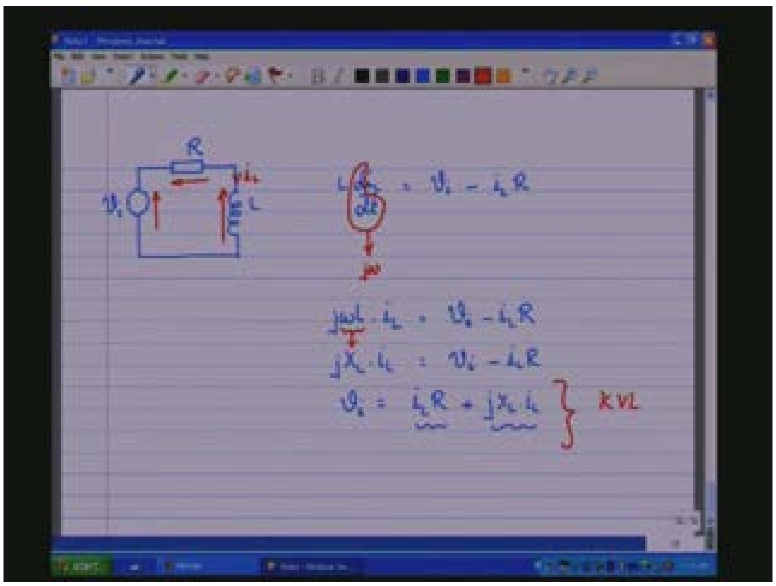

So this portion is the inductive reactance. So, if I take out i L it becomes R plus j X L. This is the inductive reactance and the resistance portion together you have a complex impedance Z. So v i by i L which is equal to Z the complex impedance which is R plus j X L in this case. So actually analysis can be done from the state equations. But we know that we are going to do only sinusoidal steady-state analysis, you can directly do the simplifications on the circuit.

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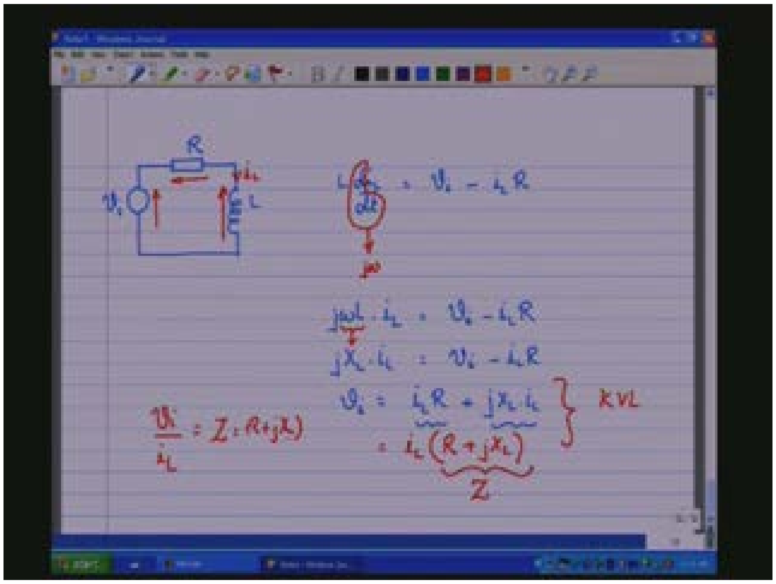

For example; let us say we take a circuit like this; you have R, you have L, you have C; rather let us say another R which is shown like that. So this is R 1 L C R 2 and then you have a v i.

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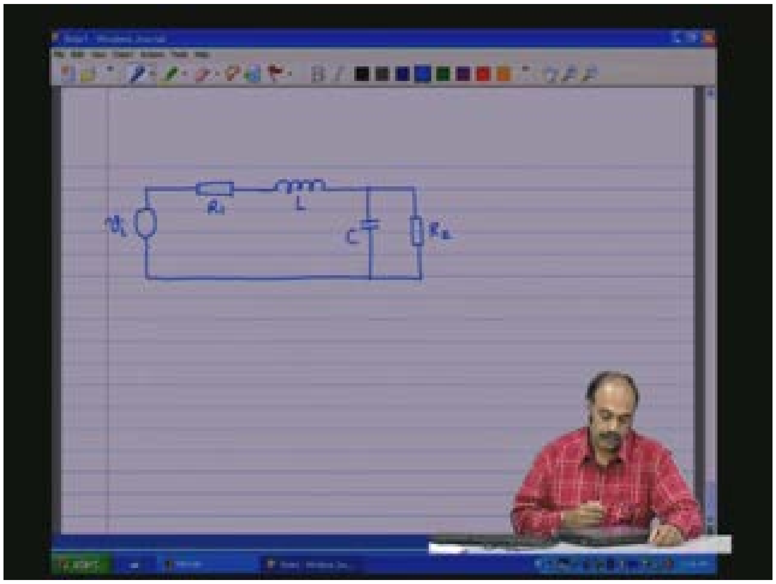

Now this L is providing an impedance which is j X L or J omega L at that frequency and for the case of the capacitance it is providing minus j X c which is nothing but minus j by omega C this is the impendence that it is providing. Now you can just make all the analysis with just these variables on your circuit. So you have V i the voltage source, R 1 is the resistance, j XL is the inductive reactance, minus j X L is the capacitive reactance, R 2 is the mean resistance which is connected across the output.

So, for example; if one needs to know what is the voltage across the output which is basically this value, so we have in simple terms like what do you do for a resistance R 2 parallel minus j X c divided by R 1 plus j X L plus R 2 parallel minus j X c this is the attenuation factor; this impedance divided by all the impedances taken in series is going to be the attenuation factor into V i will be your V 0. So this is the input output relationship for sinusoidal excitation V i at the frequency omega at a single frequency omega given as such under the steady-state conditions.

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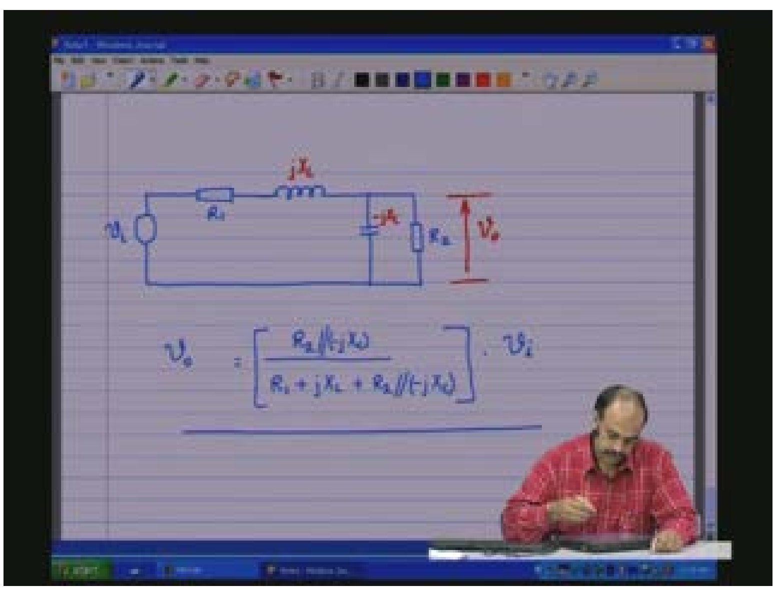

One could also use to find what is the impedance as seen from here. It is also an interesting thing to view or study. Let us say we have this, we take that circuit, copy, go to the next page, let us paste, push this a bit above. So let us say we have the circuit here we need to find what is the impedance as seen from here that is what is the impedance as seen from here which basically means that (Refer Slide Time: 52:22) that if this was not there we just have this portion of the circuit and we would like to see what we are seeing as an impedance from here to the output portion. So it is nothing but this (Refer Slide Time: 52:42) in series with this in series with this in series with this together so which basically would mean R 1 plus j x L plus R 2 parallel minus j X c these three in series is going to be the………….. now what does this imply? This basically means that this is R 2 minus j X c by R 2 minus j X c plus j X L plus R 1 this is equal to Z.

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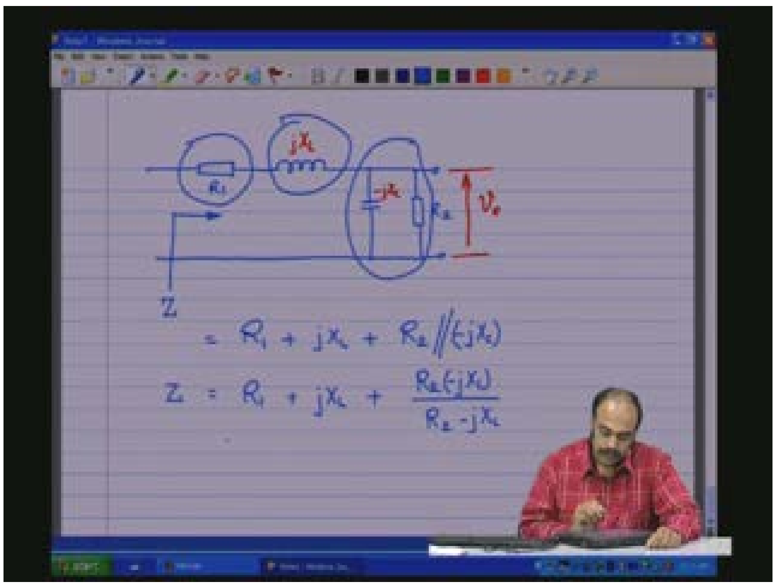

Now this can be further simplified into real and imaginary parts to get the complex impedance. Now let us take this equation (Refer Slide Time: 54:01) let me copy that, let us go to the next page, let us paste that, push that above. Now this can be written as: R 1 plus j X L plus minus j R 2 X c divided by R 2 minus j X c R 2 plus j X c R 2 plus j X c this is an algebraic manipulation so that we try to eliminate this.

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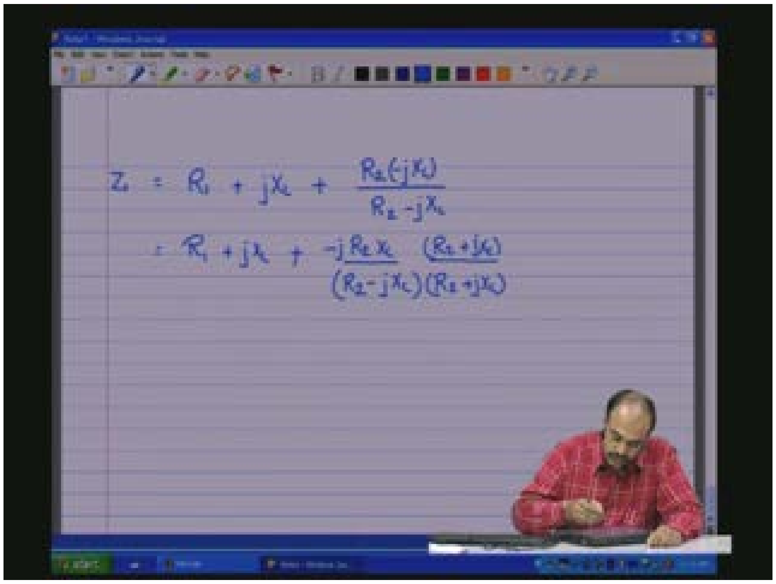

So if you see this, this will be R 2 square minus j square X c square all of the terms cancelled and therefore we have here R 2 square plus X c square. So there are no j terms here, we want to put all the j terms to the numerator and here you have R 2 square X c minus j plus minus j square R 2 X c square, there is a j X L here and an R 1 here. So this portion here is 1 so that becomes a real component R 1 plus R 2 X c square divided by R 2 square plus X c square that is the real portion plus j X L minus R 2 square X c divided by R 2 square plus X c square. So we have basically split the whole impedance into a real part and an imaginary part. This is equal to our Z the complex impedance.

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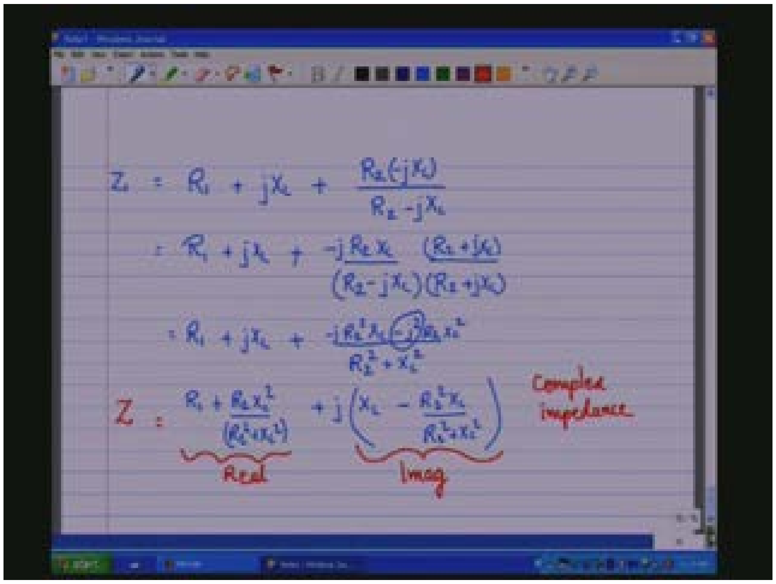

This is a complex impedance of this circuit as viewed from the input. So this is nothing but the input impedance of that circuit. So this is some of the types of the analysis that one can keep doing with the sinusoidal steady-state models. With that let us conclude this session. Thank you.