Basic Electrical Technology
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Lecture - 33
3 phase System – 4

Hello everybody. So, in the last class we have been discussing about the star and the delta circuits. We continue our discussion with these 3 phase circuits now with special emphasis on unbalanced loads till. Now we have been seeing all the loads were of the equal value equal magnitude but we should also know what happens if the 3 phase loads that is the loads in each phase are different which is what would happen in most of practical cases.

In the case of the star we said that if if we have a source 3 phase source like this (Refer Slide Time: 02:06) this is the 3 phase source it has a central point neutral that is star connected source and let us say we also have a load which is in star let us say this is a b and c and this is also a b and c and there is a neutral point N. We connect the a to a of the source, b to b of source and load and c to c source and load, this is E aN E bN E cN.

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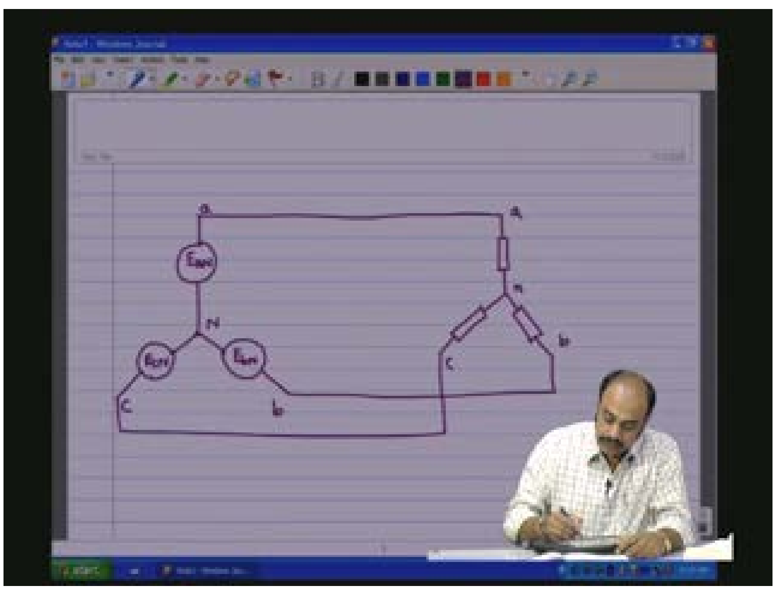

Now if the loads were not equal but unbalanced that is R 1 R 2 R 3 that is R 1 is not equal to R 2 is not equal to R 3, under such con under such conditions we saw that the currents through each of the phase loads are going to have different effective values and as a result what was supposed to be I a plus I b plus I c is equal to 0 instantaneously in in a wire that goes out from here is not zero. So if we make a connection if we make a connection between the neutral of the source and the neutral of the load then I a plus I b plus I c instantaneous values will flow through this and that will have an rms effective value and this value will not be equal to zero. When such a connection is made then the E aN and its single phase load becomes an independent circuit, E bN along with its sorry the E bN circuit is E bN along with its circuit is one another independent circuit and E cN, E c N along with its circuit which along with its circuit along this becomes another circuit.

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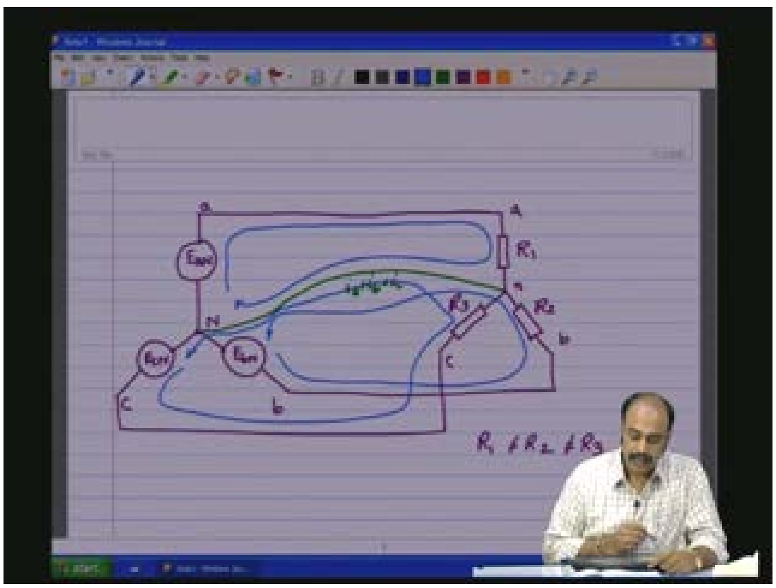

So we have these three independent circuits and the voltages across each phase is E aN, E bN, E cN which is as decided by the source so there will not be any anomalous behavior or any any voltages due to the unbalanced load the voltages across all the 3 phases will still be the same as decided by the load whereas if this green line the neutrals are not connected then the unbalanced currents will flow through will get mixed up and flow through the other loads and then you can have voltage distortions. So, in a star to take care of the unbalanced load you would normally connect the neutral points.

Now in the case with delta what would happen because there is no concept of a neutral? So let us take an example and try to work out what could happen in the case of a delta circuit.

Now let us say we have the following delta circuit; there is....... I have a capacitance, I have an inductance and I have the resistance R L c three each phase is having totally different types of impedances. Let us say this is the a phase a point b point and the c point and we have the three the three source terminals a b c 120 degrees apart from each other, they are connected as indicated here and we have the b which is connected to the b terminal of the load and we have the c which is connected to the c terminal of the load. So these are the three delta point terminals. You have X C, X L and R; each phase is having a totally different type of impedance and the impedance values are not same and therefore the load is a unbalanced load. So under these conditions what is expected to happen? Let me shift a bit okay.

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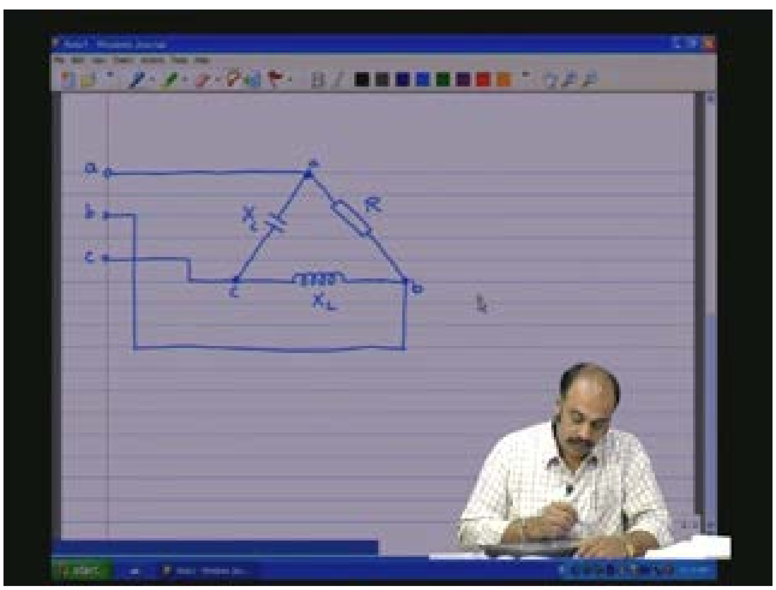

Now we have the voltage across R which is the line voltage or the phase voltage because in the delta circuit line and phase voltage are same it is E ab. The voltage across L E bc and the voltage across the capacitance is E ca see that is the sequence. Now, based on this sequence we have the vector diagram let us say; we have E ab, E ca we have E ca and we have E bc all 120 degrees apart. So a b b c is lagging E ab by 120 degrees and E ca is lagging Eb c by 120 degrees and a b by 240 degrees okay this we know. And there are three line currents this is I a, I b and I c and of course 3 phase currents I ab, this is I bc b to c and then I ca c to a. So this is cyclic in nature I a to b b to c c to a.

Now if we take I ab and E ab the load here is a resistor and therefore we expect I ab to be in phase with E ab what is applied across the resistor. therefore we shall we shall put I ab in phase, I will take the effective value I ab phasor is in phase with E ab phasor these I's is what I am showing the instantaneous values but what I am going to plot here are the effective values because it is a phasor diagram.

Now if you take the if you take bc the voltage bc here and the load there in the bc arm is an inductor gives you an inductive reactance X L so what should be the nature of the current in terms of phase angle; we should see that the current I bc should lag the voltage across the inductor by 90 degrees because it is a pure inductor. So what do we do we take E bc and with respect to E bc the current will lag by 90 degrees. So I bc is going to lag by 90 degrees. So if this is 90 degrees this should be 30 degrees; 90 plus 30 is 120 degrees so I bc is lagging E bc by 90 degrees because of the inductive load.

Now, coming to the capacitive for the nature of the capacitance?
The current through the capacitive load pure capacitive load will be such that it will lead the voltage across the capacitance by 90 degrees. So E ca when it has the current has to lead E ca and therefore we will we shall draw a phasor which is 90 degrees to this E ca and that will be I ca.So if it is 90 degrees to that it will be 30 degrees from here.

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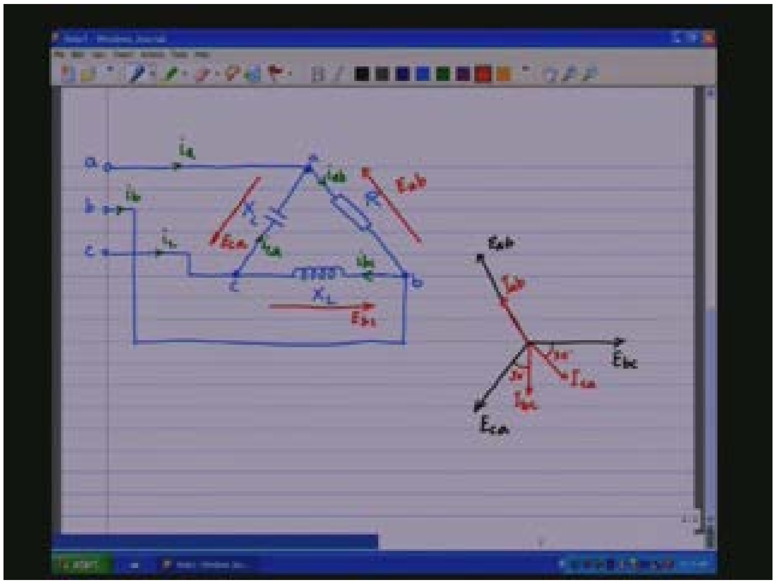

So if I draw extend the E ab line down so you will have a 30 and a 30 here. okay So this is how the phase currents and the line voltages would look like the phasor diagram of such an unbalanced system.

Now let us see what is the what is the equivalent values of the currents which are along the three voltage axis. Or if we project this current I ca on to this axis and all the other two axes then we know the equivalent 3 phase currents which are flowing along the three axis. So let us find out let us find out the currents along the E ab axis, along E bc axis and along E ca axis.

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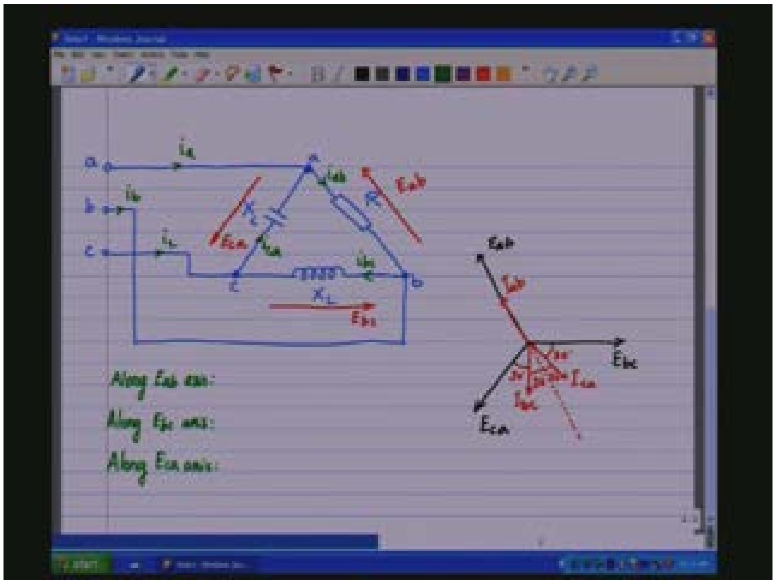

So if we take along the E a b axis you will see that along this axis there is an I ab term then there is a projection there would be a projection of I ca on to this axis, a projection of I bc and on to this axis all these three together are going to form the equivalent current along this axis. So that would be I ab, now that is in the positive direction, now I ca cos 30 and I bc cos 30 are in the negative direction minus I ca cos 30 degrees minus I bc cos 30 degrees which are the projection on this axis. So that would be the value of the current along the E ab axis.

Now let us see along the along the E bc axis. Now if we take the E bc axis I bc this current here I bc is 90 degrees with respect to E bc because of the inductive load so there is no projection of I bc on the E bc axis. Okay So we have only the projection of I ab on the E bc axis that is negative I ab; now this angle here now if this is 120 degrees this is right through in between across so therefore this has to be 60 degrees, so I ab cos 60 there is a projection on this axis and then I ca cos 30 there is a projection on this axis; I bc has no contribution because it is orthogonal to the E bc axis. So therefore if you take the E bc axis along E bc axis the currents are minus I ab cos 60 that is there then we have I ca projected on the positive direction which is plus I ca cos 30 on the E bc axis.

Now I will take the third axis along the E ca axis, what are the projections?

So this if I project it again we have a 60 degree because this is right through and this is 120 degrees between a b and b c. Now there is a projection of I ab cos 60 on the c a axis. Now I ca is exactly 90 degrees from the E ca axis and therefore there is no projection of I ca on E ca axis. Now I bc has a projection on E ca axis which is I bc cos 30. So we have I ab cos 60 which is on the negative part of the axis so we say it is minus I ab cos 60 plus I bc cos 30 I bc cos 30 degrees. So these are the projections of the various currents on all three axes so equivalently we can say that these currents or all currents which are in phase with the E ab, Ecb, E bc and E ca axis like resistive load the equivalent resistive loads put along these lines.

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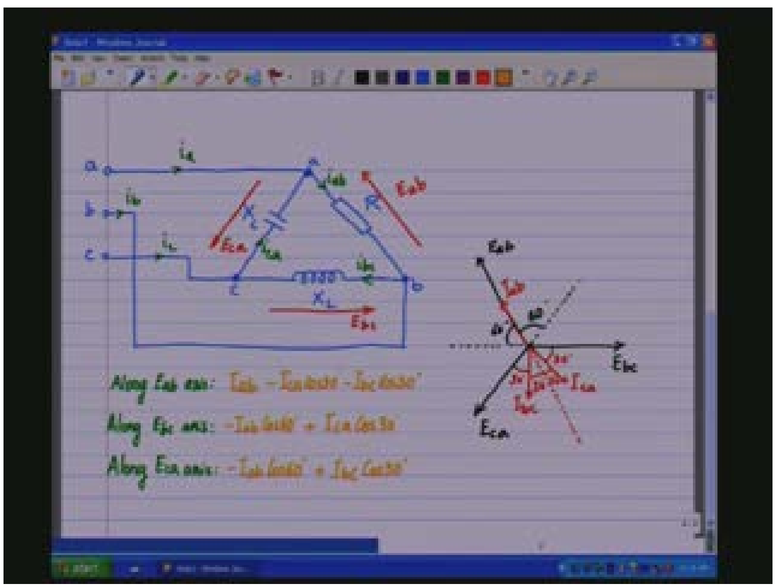

So let us just take this, copy, go to the next page, let me paste this (Refer Slide Time: 20:00). So let us say that if we want to make it balanced we want to make we want to make this load balanced which means we it should it should look like I should have a current along this, current along this, current along this all equal amplitudes then we have equivalent of something like a resistive load and all having the same effective values which means the whole system would be balanced. So for that the equivalent current along the ab axis that is this, equivalent current along the bc axis that is this, equivalent current along the ca axis they all should be of equal values. So let us equate these two (Refer Slide Time: 20:49) and then this with this. So let us see what happens

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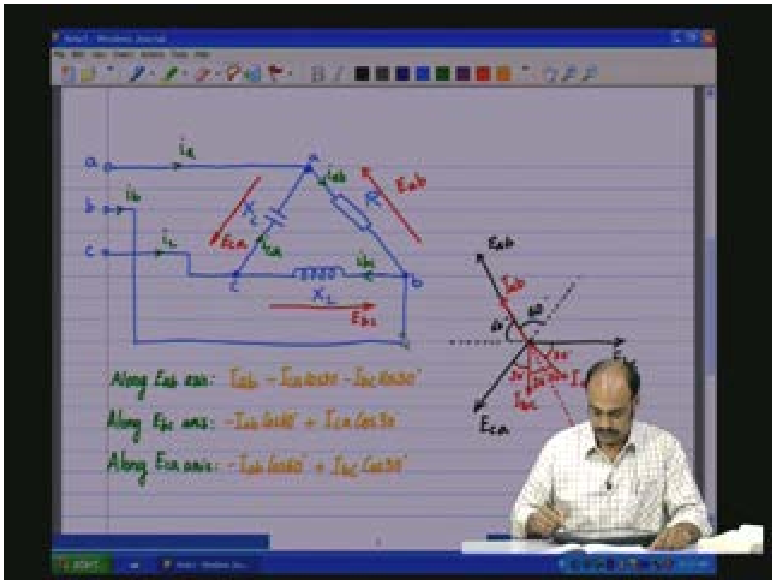

Now (Refer Slide Time: 21:00) let us equate these two. now this of course is equal to I ab minus root 3 by 2 I ca minus root 3 by 2 I bc and this is equal to minus I ab by 2 plus root 3 by 2 I ca and this is equal to minus I ab by 2 plus root 3 by 2 I bc so you have equation 1, 2 and 3 so let us first equate 2 equals 3 so let us equate 2 and 3. So this means we have minus I ab by 2 plus root 3 by 2 I ca should be equal to minus I ab by 2 plus root 3 by 2 I bc. So here we remove this, we remove this so we have a result which is I ca should be equal to I bc and we will say that is equal to some value I, this is number 4.

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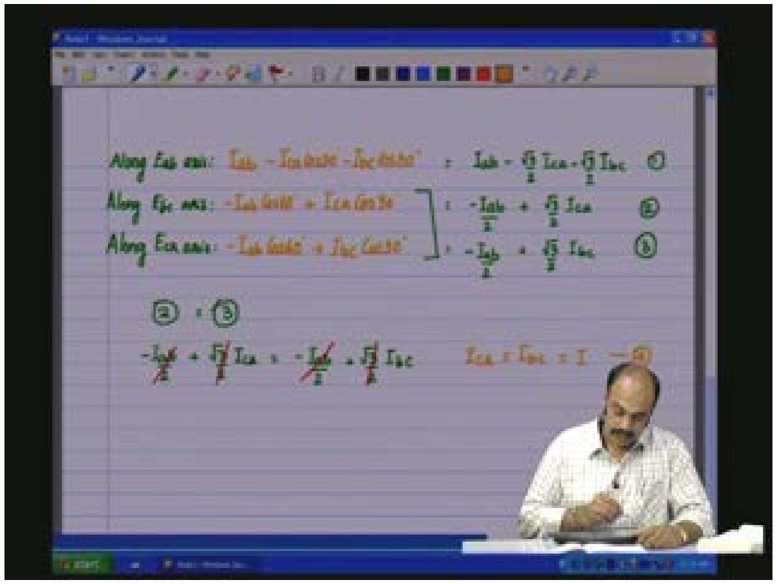

Now let us substitute I for I bc and I ca in one and then solve for I ab. So now we have I ab minus root 3 by 2, I ca minus root 3 by 2 I bc is equal to any one of these two currents because we have already equated these two any one of these so let us say it is equal to the two minus I ab by 2 plus root 3 by 2 I ca so we know that I ca equals I bc equal to I so we make that change I ab I ca and I bc is equal to the same value and that is I so we have substituted I in the equation.

So now 3 by 2 I ab taking I ab on to the left hand side, taking these on to the right hand side you have 3 times root 3 by 2 into I. So this leads to the following nice relation: I ab is equal to root 3 I or I should be equal to I ab by root 3 be equal to I ab by root 3 this is a nice relationship.

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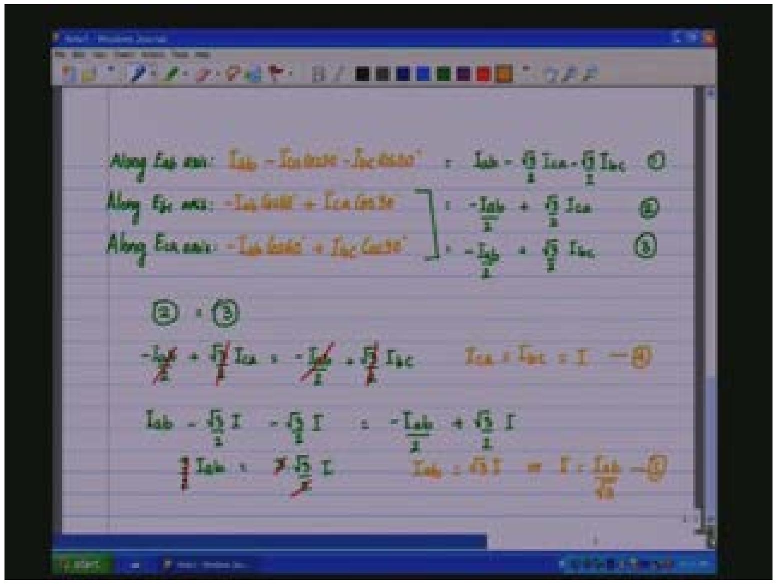

Now using this, what can we gain?

If we go back here (Refer Slide Time: 25:02) let us try to find the values what should be the values of the capacitance reactance and the inductive reactance; copy, paste (Refer Slide Time: 25: 37). So what we have right now said is we have an effective value we have effective values here which after having been solved the values are like this. So here I is flowing, here also we are having I and here we are having I ab which is root 3 times I so we have root 3 times I, this is I ab.

So now we can find out what should be the reactance which will give this kind of a solution. Now the effective values E ab is equal to E bc is equal to E ca is equal to E line for a balanced source which is applied and therefore X L is E line is the voltage across the inductor divided by I the current which is supposed to go through that. Now I is nothing but I is I ab by root 3 so this is E L by I ab by root 3 which is into root 3. so this is Now this is nothing but R the resistive component of the load so this is R root 3. Likewise, X C also the reactance capacitive reactance is E L by I which results in R root 3 value-wise in ohms. So this has a nice important implication.

What it means is that........... let us say we have a load R and you have a 3 phase system; we have a 3 phase system and we have just a single phase load R this means that only currents will be flowing through two of the lines: one will be the forward path and one will be the return path and the third line will not have any current flowing so it is an unbalanced system. In order to make the system balanced meaning you have the same effective value of currents flowing through all the three lines displaced 120 degrees apart, you need to add two reactances: one X L that is the inductive reactance and X C the capacitive reactance as shown such that such that X L value will be equal to X L equal to X C equal to the resistive load into root 3 and if that is done then automatically you will have the currents flowing here as this: root 3 I I and I and they will be balanced when looked at from the point of view of the equivalent phasor diagrams because equivalently if (Refer Slide Time: 29:59) I ab I bc is equal to I ca is equal to I then what flows through this, what flows through this and what flows through this will all be equal that is it is equivalent to saying now that we have values here in this fashion so we have root 3 I, this is I and this is I so this is equivalent to saying that we have in this direction root 3 I by 2 is minus direction that is this plus I ca I cos 30 that is root 3 by 2 into I and in this direction also we have the projection root 3 I by 2 is getting projected here and plus direction so that we will get exactly cancelled out and you have just only the current in this direction.

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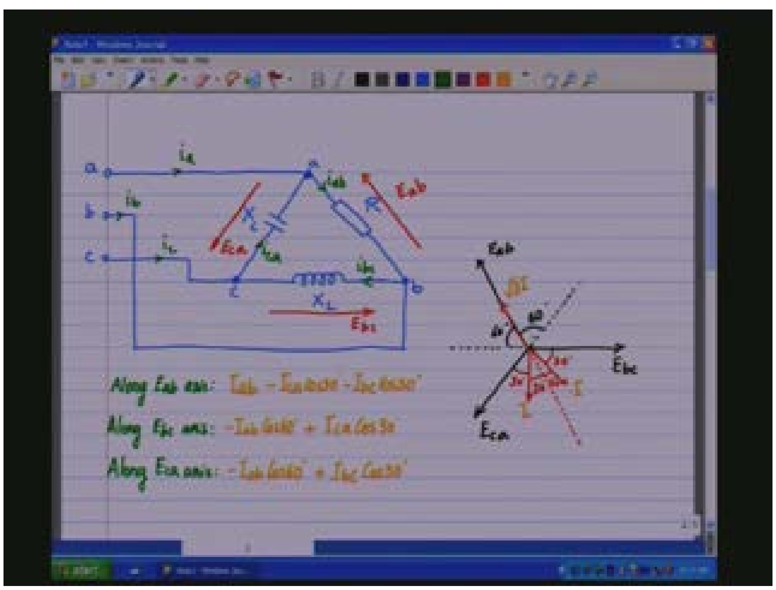

So it will look and it would appear as though that the currents are balanced (Refer Slide Time: 31:42) and equal in all the 3 phases but the but the load is just only the active load is just only one value which is R if you take this condition; is that clear?

Probably we could consolidate it with some numbers, now numerical values. Let me take a simple very quick numerical presentation (Refer Slide Time: 32:26) so that you get the feel for it. So let let it let me take a text book example. So you have E ab E bc E ca all 120 degrees out of phase and there 440 440 volts at a phase angle of zero degrees, 440 volts at a phase angle of minus 120 degrees, 440 volts at a phase angle of minus 240 degrees. these are the E ab E bc E ca that we applied and the values of the resistance the value of the resistance is can be calculated; let us say we have a system which is being supplied by this and the single phase load that gets connected between is a 800 kilowatt load let us say so then R would be 440 square by 800 kilowatts which is 0.242 ohms so this is 0.242 ohms.

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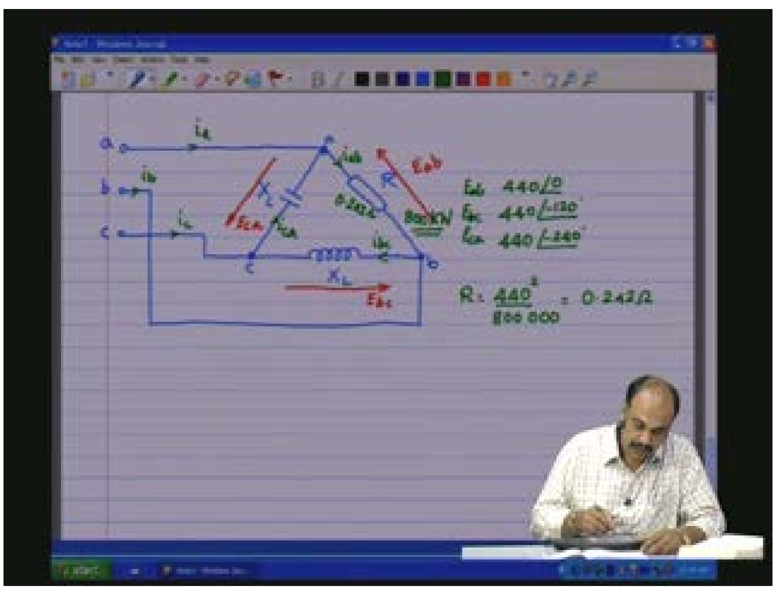

And now what should be the value of X L and X C?
The value of X L and X C should be the value of X L and X C should be X L equals X C should equal to R root 3 and that is 0.419 ohms. So we put here 0.419 ohms 0.419 ohms here also; one

is capacitive, one is inductive. So now if we look at the current through each of these, now the current through let us say the reactant the current through R let us say I ab will be E ab by R and that is equal to that is equal to 1.87 amps is it not?

So this is 440 volts at an angle of 0 divided by R which is 0.242 will give you so much amps that is 1817 angle 0 degrees this is the amps.

Now if I want to find out I bc, so now I bc will be E bc divided by jX L or which is equal to minus j E bc by X L which is equal to now E bc is 440 angle we have we have angle minus 120, so all these angles will get that is a minus 90 provided by this j. So this is going to be divided by 0.419 ohms. So this would be 1047 amps at an angle of 210 amps. Then I ca is equal to E ca by minus j X C which will be j E ca by X C which is 440 angle minus 240 plus 90 divided by 0.419 ohms and that would be 1047 at an angle of minus 150 150.

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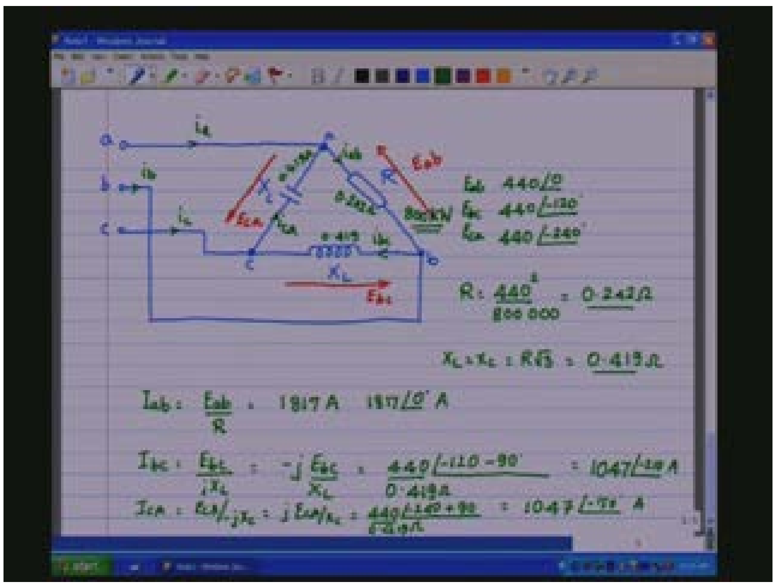

Now if these currents flow then the projections of these currents as we saw these currents are now at something like this (Refer Slide Time: 39:12) these are the 1047 1047 amps and this is 1817 amps so there will be a projection on this axis that is negative that will cancel out and give only the active power, there is a projection on this and there is a projection along this which will all tend to make it into a minus balanced load as we perform the calculation so that this is equal to this is equal to this. So there is one more small topic that we need to discuss and that is and that..... oh just one moment, we go back here, we have calculated this; now what is I a I b I c the line currents; I a...... the effective value of I a I b I c that is also to be calculated, this is I a I b and I c this is also to be calculated.

I a is this current that is I ab current and minus I ca current. So I ab current minus I ca current so this is equal to this comes to around let us say 1047 at a reference angle let us say 0 then one second then I ab. Now let me just calculate the........ what happens at a reference angle of minus 30 degrees and I b....... now I b is I bc minus I ab I bc minus I ab which is also 1047 at a reference angle of at a reference angle of 210 and then I c is (Refer Slide Time: 42:33) if you look at this figure again here I ca minus I bc I ca minus I bc which is 1047 at a reference angle of 90 degrees. So you will see that there is a phase difference of 120 degrees between the 3 phases of the line currents and the line currents are balanced.

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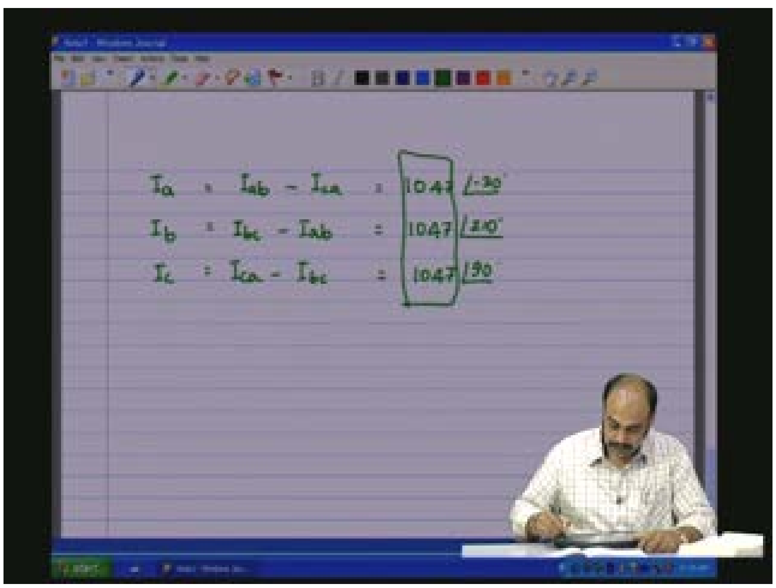

However, you will see that in the case of the phase currents (Refer Slide Time: 43:10) the phase currents we have this artificially included I and I and a root 3 I but effectively they all will cancel out if you put it along the three axis as we show along these using these equations (Refer Slide Time: 43:32) which effectively cancels out on all the other axis and you will have just only the I ab component which will flow through that which is the effective power. But from the line side from the source side it will look as though they are balanced.

Now the new topic here is 3 phase power measurement, there are....... I would say two methods: one is to use oscilloscope with voltage probe and current probe which would be the best method because you would get the actual wave shapes of the voltage and the current waveforms and you can probe the voltage and the current waveforms across each phase or across line to line and once you get the line current and the line voltage once you have line current and line voltage and also the phase difference between the phi which is the phase difference between the line and the current looking at the wave shape of the voltages and the currents we have root 3 E L I L cos phi this is the power measurement. This would be one of the most accurate methods of obtaining the power.

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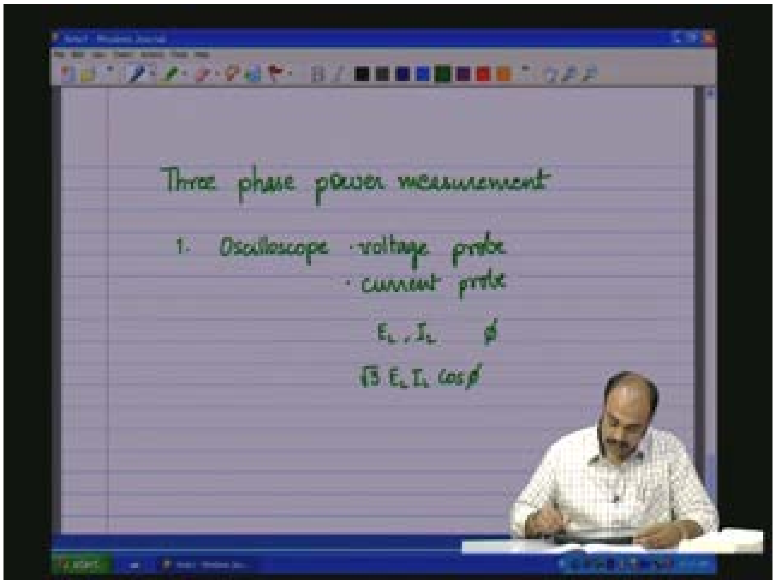

The other method is to use Wattmeters. So, if we use wattmeters the connection has to be done in the following manner. So let us say we have the a, b and the c phase which goes to the load. So let us have the wattmeter like this which and so we will call this wattmeter and the potential coil is between this and likewise the potential coil is this. So this is a wattmeter W 1, this is a wattmeter W 2, this is the potential coil connection and this is current coil connection. So the current of the a phase, current of the c phase is taken, the b phase is taken as the reference point, the potential of a phase with respect to b, c phase with respect to b is measured like that and this of course goes to the 3 phase load goes to the 3 phase load 3 phase load.

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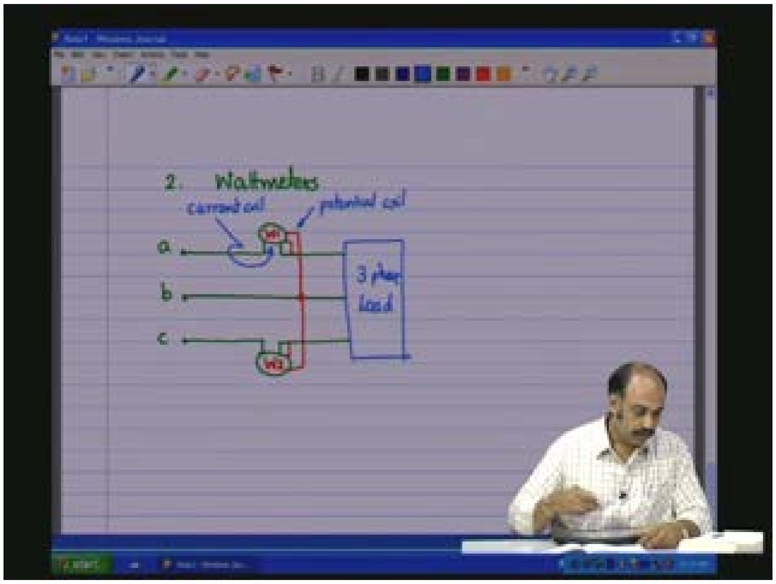

Now, on taking these measurements the power P total is equal to the reading of W 1 plus the reading of W 2. This will give you the.......... this is called the two wattmeter method. It is preferable to use a center zero wattmeter because if the power factor is less than 0.5 then one of the wattmeters can show a reading on the negative side in which case you will have to take the absolute value and then add up for the total power to get the total apparent power which is fed to the load. So if there is a center zero wattmeter then you can take the readings easily otherwise if it is just only a one direction wattmeter you will have to reverse the direction if it goes negative when the power factor is pretty low.

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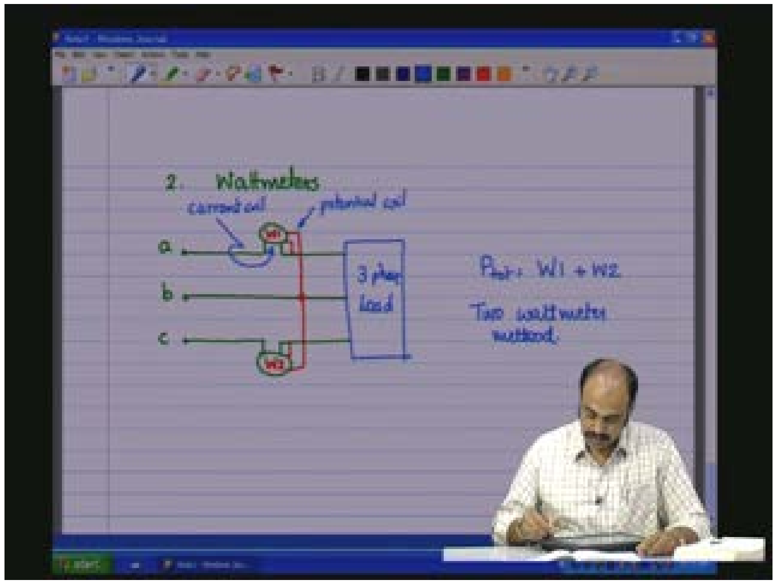

In the case of 3 phase circuits where the neutral is also available meaning 3 phase four wire systems then we just use.......... that is in case of a load which is 3 phase four wire that is a b c and neutral then with respect to the neutral we have wattmeters which are connected with respect to this with respect to this. So you will have W 1 W 2 W 3. So the P total would be W 1 plus W 2 plus W 3 would be the total of the total power. This is especially useful when you are having a 3 phase four wire system. So this is about the basic concepts of the 3 phase circuits 3 phase systems we have been discussing about, they consist of essentially three sources and two important constraints are applied for these three sources and that is the amplitudes the effective amplitudes of these three sources are equal, the peak amplitudes are also equal and there is a phase shift in their waveforms and each waveform is 120 degrees phase shifted with respect to the other, they are equally phase shifted with respect to the other which means when you look at it in the spatial coordinates so you will have 3 phase which are in the form of a star and the form of a star equally spaced along the circle which means 120 degrees with respect to each other.

And then there are two major categories of 3 phase systems: one is called the star connection or star connected systems and the other is the delta connection or the delta connected systems.

In the star connected system one end of the phase circuits of all the phase circuits are connected together and that is called the neutral point, the other end of the phase circuits are brought out as terminals which can be connected to the source or any other load. The neutral points in the star circuits play an important role in the sense that if the loads are balanced the neutral points can be left floating because under balanced load condition I a plus I b plus I c is equal to 0 which we saw and discussed in the previous classes. So therefore, there is no need to connect the central neutral point to the neutral point of the source, there need not be a return path wire. So, as the consequence we save in copper in the sense that you just need three wires to connect from a 3 phase source to a 3 phase load. However, if the load is unbalanced then I a plus I b plus I c is not equal to 0 as we saw and therefore we need to provide a path from a return path from the load side to the source side and that return path is through the neutral wire and under such conditions the neutral point of the load, the neutral point of the source are connected and it becomes a 3 phase four wire system. Under unbalanced conditions if such a neutral wire is not neutral connection is not provided then there will be distortion in the voltage wave shapes of the 3 phase circuits or the 3 phase loads.

In the case of the delta there is no concept of a neutral point the 3 phase circuits are connected sequentially where sequentially one of the other in the form of a ring and the 3 phase the 3 phases are directly connected to the three points, or the three vertices of a triangle are directly connected to the three three points or the three terminals of the source.

In the case of the delta the line voltage and the phase voltage are of the same and in the case of the star the line voltage is equal to root 3 times the phase voltage. The line currents and phase currents are the same. In the case of the delta the line current is root 3 times of the phase currents so these are the two important relationships voltage and the current relationships between the line side parameter and the phase side parameter of the two circuits. However, when it comes to power the power equations with respect to the line side parameters is root 3 E L I L cos phi where phi is the phase angle difference between the line voltage and the current.

If it is phase side parameters then it is three times phase voltage into the phase current into the power factor into the power factor or the cos of the phase angle between the phase voltage and the phase current. So this is the power equations for both the star and the delta circuits. And in the case of the delta circuit we discussed one important unbalanced load condition that is in the extremity if we had just only one single resistor as a load then two of the lines will have currents in them that is one will have a forward current and one will have the return path current of the resistive load the third line will not have any current and it will be just open in order to balance the line currents that is the line currents should see the same amplitude effective values 120 degrees displaced with each other in the spatial coordinate system, the line currents. So therefore, we do a modification in the delta circuit by including two reactive loads in the other two phases. So we have a resistive load which is the active power which is going be drawn from the mains, it is still going to be only the active power which is going to be drawn from the mains but we add a capacitance and an inductance in the other two phases in a way such that if a value of current effective value of current I is flowing through the reactive phases that is the capacitance and the inductance reactive phases root 3 times I will be flowing through the resistive phase that is the actual resistance.

Or in other words, if some load current is flowing through the resistance load resistance that current by root 3 should flow through the reactive components that we are going to put in the other two phases. This resistance, capacitance and the inductance will form a delta and these phase currents will cancel out in such a manner that there will only be the current in the that will only result in the active component of the current and on the line side it will appear as though you have all three balanced currents flowing through it.

Of course one has to be careful in connecting these three components one should not interchange the values should not interchange the positions of the components in which case you will have an unbalanced situation.

And then we discussed about the measurement of power. The best method to measure the power in the 3 phase circuit is by means of oscilloscope. You measure the voltage using the voltage probe and a current probe together, you measure the voltage in the current, line voltage in the line current and then perform root 3 E L I L cos phi and the phase difference can be obtained from the voltage in the current waveforms that can be seen on the oscilloscope that would give you the best method, it is independent of whatever be the power factor.

The other method is by using wattmeters. One may use two wattmeters in the case of 3 phase three wire system that is neutral is not available. So the one wattmeter may be connected to one of the lines and the other wattmeter can be connected to the other line with the potential coils having the same third line as the reference. Then the wattmeter values of the two watt meters are summed up to get the effective or the total power that is drawn from the source of the total VA that is drawn from the source now the total sorry the total watts that is drawn from the source.

In case the power factor is very low in case less than 0.5 then one of the wattmeters can show a negative value in which case we must reverse we must reverse the......... if it is a single direction wattmeter we must reverse the switch such that you could see the negative direction power flow or you you use a center zero wattmeter in which case you take a negative value convert it into absolute value then add up that two powers that would give you the effective that would give you the equivalent total power that is drawn by the 3 phase load. If it is a 3 phase four wire system then just connect the wattmeters, add up the three values you get the power.

We stop here and then continue the next class with 3 phase transformers, thank you.