Basic Electrical Technology
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Lecture - 31
3 Phase System – 2

Hello everybody, in the last session we had begun our discussion on the 3 phase system and 3 phase circuits, we completed a major topic on the motors which are the DC motors and the DC generators. We also discussed about two major categories of 3 phase circuits they are the star and the delta circuits. We take up our discussion from that point and continue to consolidate our ideas in this aspect in this particular area.

So we saw that there are two major types of 3 phase circuits: one is called the star or WYE. This WYE means the shape of the letter Y and because of the circuit is in that shape it is called WYE and it is spelt as W Y E. So in the literature you will see that star circuits are also used, W Y E WYE circuits are also mentioned they are one and the same. And then we have the delta circuit because the shape of the circuit resembles the Greek upper case letter delta, it is in the shape so all your 3 phase circuits will fall into one of these categories.

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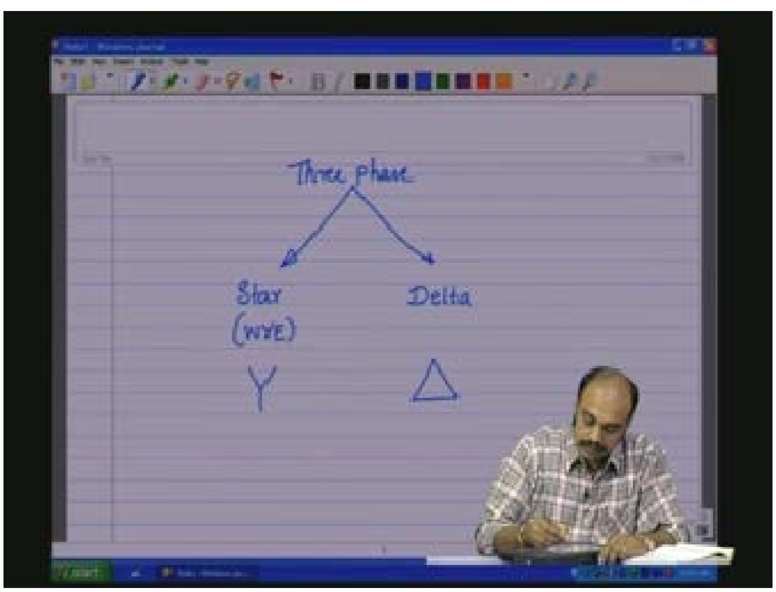

In the in the star circuit we saw that if there is any load one end of the load or the source gets joined connected as shown here and this point is called the neutral point (Refer Slide Time: 3;22) and the other end will of course go to the source. Now these loads we are showing right now as resistive loads but they can also be reactive impedance loads which are the combination of resistance and reactants.

Now let us try to get the relationship between the various voltages and the currents in the star circuit and the delta circuits. And in the case the delta circuit this is a star or the WYE which we have been discussing since the last class. We also have the delta circuits composed of three loads or the resistors. But unlike in the star circuit where one end of the loads are joined together at a point called the neutral here the loads are connected in a ring form. One end is connected to the other the other end the other load and the other end of the other load is connected to the next component and so on which forms a kind of a ring or a delta as shown. And these points (Refer Slide Time: 4:53) are brought out and connected to the sources to the sources. So this is the delta circuit.

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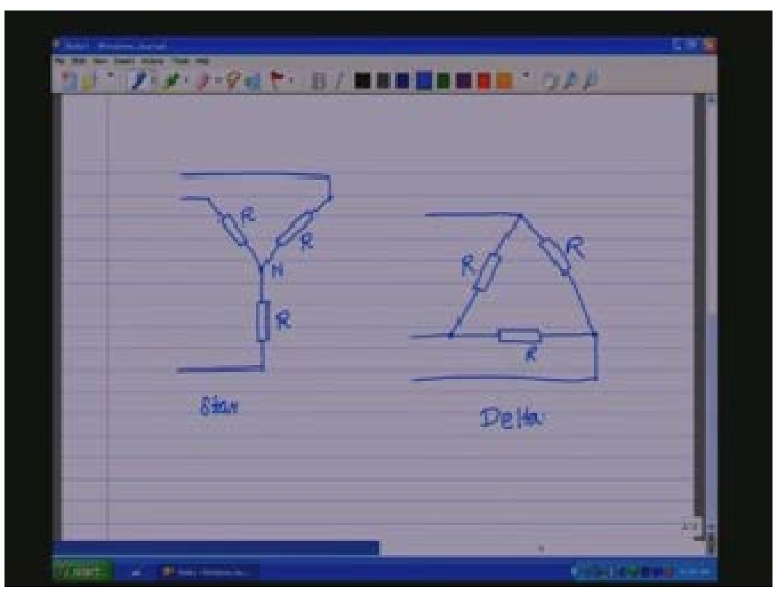

Now these two circuits have different voltage and current relationships which we shall discuss in this forthcoming class in this coming class here. Let us now first consider the star circuit the star circuit.

Now let me write the star formation. We have three loads in this fashion (Refer Slide Time: 5:57). Now it is written in this fashion so that it gives an idea that the phasor diagram is 120 degrees displaced with each other that is the idea is it not? In fact that is the constraint that we applied on general three sources and general three loads to obtain the 3 phase circuit. what was that

What were the constraints?

The first constraint is that the effective values of all the three single voltage sources should be the same of the peak values and the second is that the phasors of each of the three independent sources should be displaced with respect to each other by 120 degrees. So these were the two conditions that we applied and this picture here....... actually the circuit, this is actually the circuit but the circuit is superposed on an underlying phasor diagram to give you clarity on the voltage relationships between the two. So let us call this as terminal a, this as the neutral point n, this as terminal b, this is terminal c and to these terminals you are applying the voltages a, this is a b, b has to be connected to b, b gets connected to b and c gets connected to c; so a b c these are the three terminals of the 3 phase load which are brought out.

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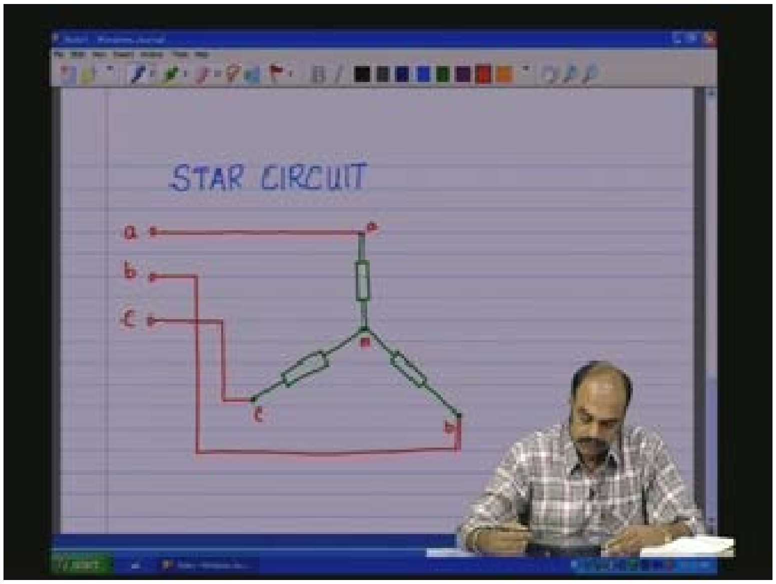

Now we saw that the neutral is not brought out because in a balanced 3 phase load meaning where all the three loads are equal same than the instantaneous current we saw in the last class that the instantaneous current that is the sum of i a plus i b plus i c which flows through the neutral wire to complete the circuit like the fourth wire is always equal to zero and therefore we can effectively remove the fourth wire that is the neutral wire.

So now we shall say that this voltage here is E an let me take the effective values E an, the voltage in the b phase this is E bn and the voltage in the c phase with respect to the neutral E cn. So E an E bn E cn are with respect to the neutral they are across the loads of every phase and they are called the phase voltages. Remember that E an E bn and E cn are called the phase voltages they are called the phase voltages.

Now the phasor diagram is also like this. This is E an E an (Refer Slide Time: 10:49) 120 degrees lagging E bn 240 degrees lagging E cn you see. So this is 120 degrees, this is another 120 degrees between c and b and then another 120 degrees between a and c. So, as the phasor diagram is in the shape of a star superposing the circuit on that you get the circuit in the shape of a star. But in the actual it is the phasors which are 120 degrees displaced by with each other but the circuits can be written in any other shape also with all one ends joined together. But however, for clarity and aesthetically pleasing look this is a much more descriptive way of writing the circuit.

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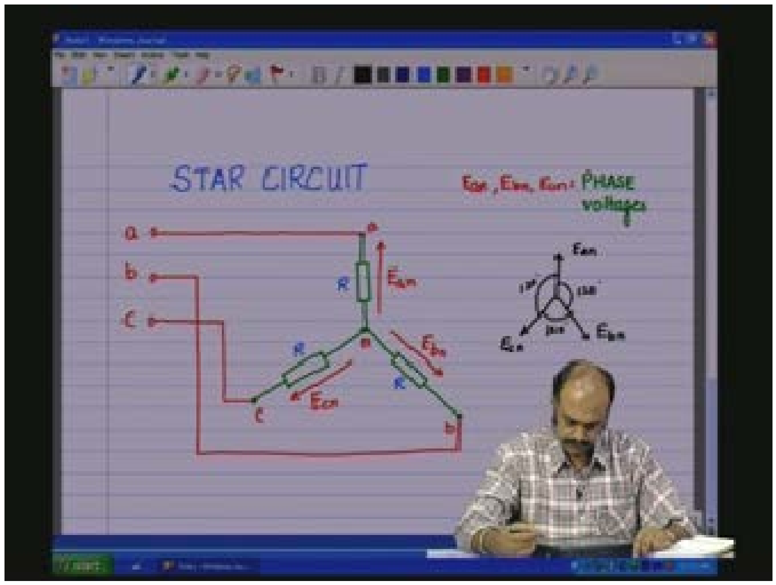

Now let us look at these voltages. Now let me take another color. Now this voltage between these two lines this is called E ab and this voltage is called E bc and (Refer Slide Time: 12:20) this voltage is called E ca cyclically; a to b b to c c to a and here E ab E bc E ca they are called the line voltages they are called the line voltages.

Now one more parameter. let us call this as i a and what flows through this line here as i b and what flows through this line as i c; then i a i b i c are called the line currents. And if we take let us say the current here as i a p and the current here i c p and the current here as i c i b p. Then i ap i cp i bp and i cp they are all the phase currents. So we have the phase voltages phase currents basically the voltages across the each load phase and the currents through the phases they are called the phase currents i ap i cp i bp and the line voltages the voltages across the line this is the line the source so you have E ab, b c and E ca as the line voltages and the currents which are flowing through the line are called the line currents i a i b i c. In the case of a star circuit the phase current and the line current are the same and therefore i a is equal to i ap, i b is equal to i bp, i c is equal to i cp the phase and the line currents are the same.

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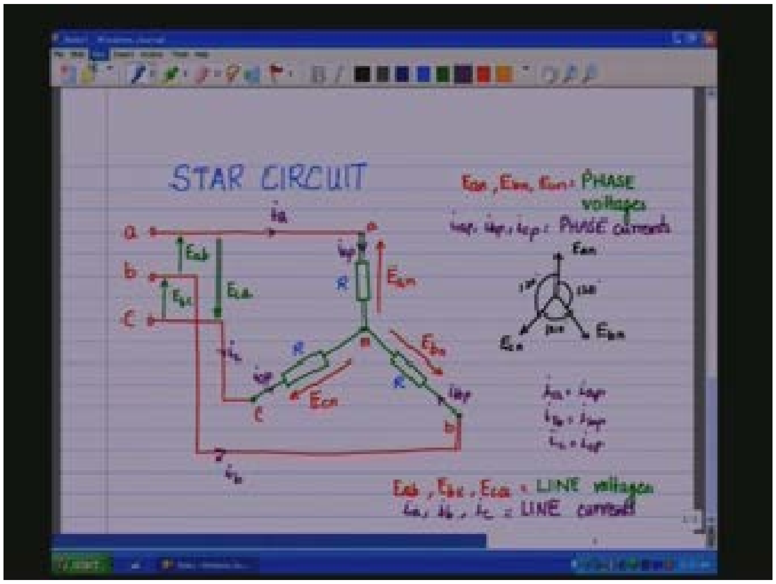

So now let us bring about a relationship between these various voltages and currents. For that let us have a copy of these circuits so that we need not repeat draw, let me copy it, let us go here and then paste it in the next page.

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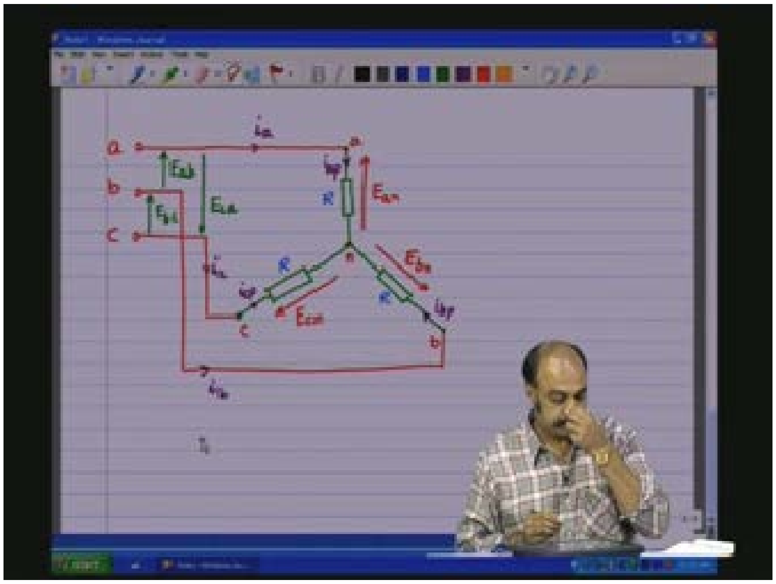

So we have the circuit here. First let us have a picture of the phasor diagram. So let me draw E an this is E a n; 120 degrees away same amplitude we have E bn you see E b E bn and then we have E cn 120 degrees this we know this is the voltage across every phase.

Now what is E a b?

E a b is the line voltage between line a and line b. Now look at this portion of the circuit between a phase and the b phase. So let me draw this in blue. Look at this portion of the circuit. We shall apply the Kirchoff’s' voltage law for this portion the circuit. So, for this loop so for this loop E ab is equal to E an E an minus you see E an is opposing E ab minus E bn according to the directions that we have assumed. So E ab minus E bn E an minus E bn is E ab the line voltage this is the line voltage, these two are the phase voltages phase a phase b. So look at it across in the phasor diagram shown here.

Now E an; what is minus E bn?

The minus E bn will be along this axis but in the minus direction, same amplitude but minus direction. So therefore, if we complete the parallelogram if we complete the parallelogram we have E ab we have E a b. So this is minus E b n. Let us have a look at the angles. So we see that this angle is 120 degrees and if we extend this this angle would be 60. So if this angle is 60 degrees this angle will be half of that which is 30 degrees. So E ab is going to lead E an by 30 degrees or E an lags E ab by 30 degrees. Clear?

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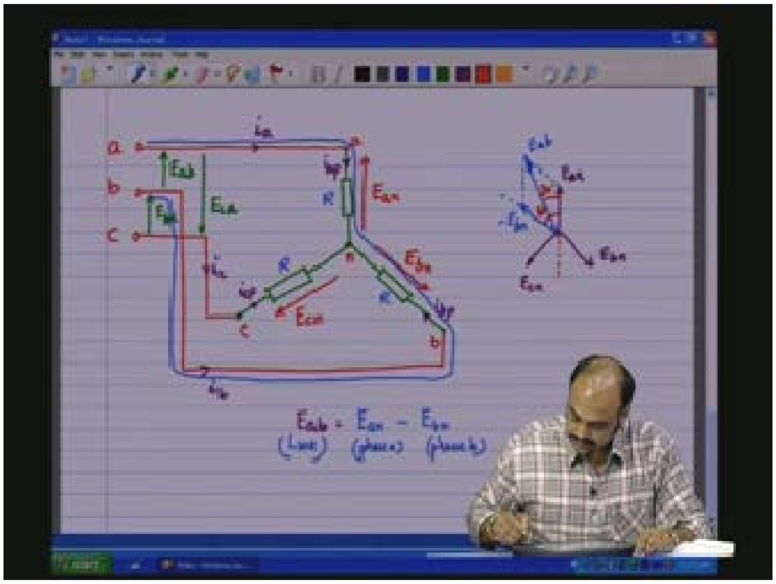

So let us expand that and have a clearer view. So we take that, so we draw let us draw a bigger picture of this. So I have E a n, this is E c n, this is E b n. So I am going to take E bn negative so the E bn negative e will be this is minus E bn and the resultant of that this is E ab and that is leading E an the phase voltage by 30 degrees. So this is the this is the resultant that is the line voltage with respect to the phase voltages E an and E b n.

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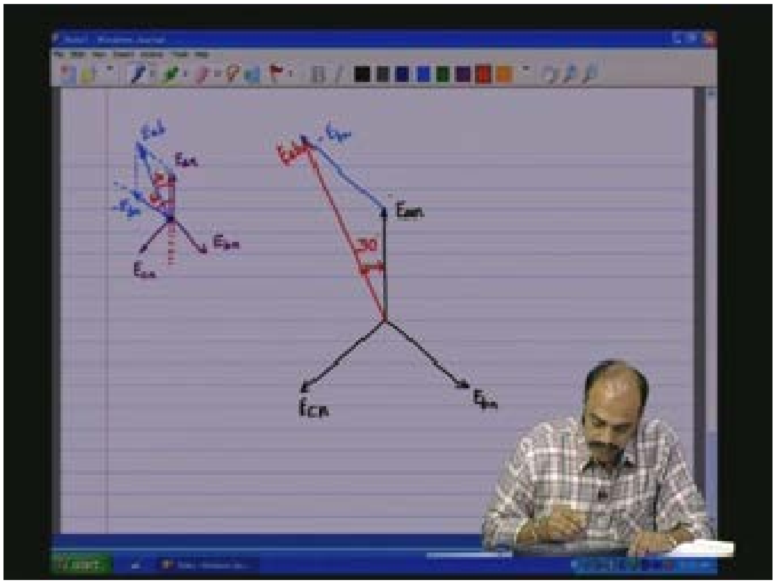

Now what is the value of E ab with respect to E bn and E cn E an?

Now we said that the amplitudes of all the phase voltages effective values are the same. So therefore let us let us say E an amplitude is equal to E bn amplitude is equal to E cn amplitude which we shall call it as E phase. Likewise, E ab amplitude will be equal to E bc amplitude will be equal to E ca amplitude we will call this one as E line sorry the E L is the line voltage. So let me take the projection of the phase voltages on to this.

So E an cos of 30 is going to be the projection of E an along the a b line. So this much this value here is going to be E an cos 30 and this value from here to here is going to be the projection of minus E bn on to the E ab line, this again is also 30 degrees thus this is an isosceles triangle and this amplitude will be E phase let me see E phase cos 30 degrees and this E an can also be now written as in terms of a common variable E phase.

So E phase cos 30 plus E phase cos 30 will be E ab therefore E ab E ab which is E line the line voltage will be equal to 2 into E phase cos 30 degrees what is cos 30 degrees; cos 30 degrees is root 3 by 2 so which will be 2 into E phase root 3 by 2 so we have a cancellation there and therefore we have an important relationship line voltage is root three times phase voltage line voltage is root three times phase voltage. So this is applicable in general to all the line voltages.

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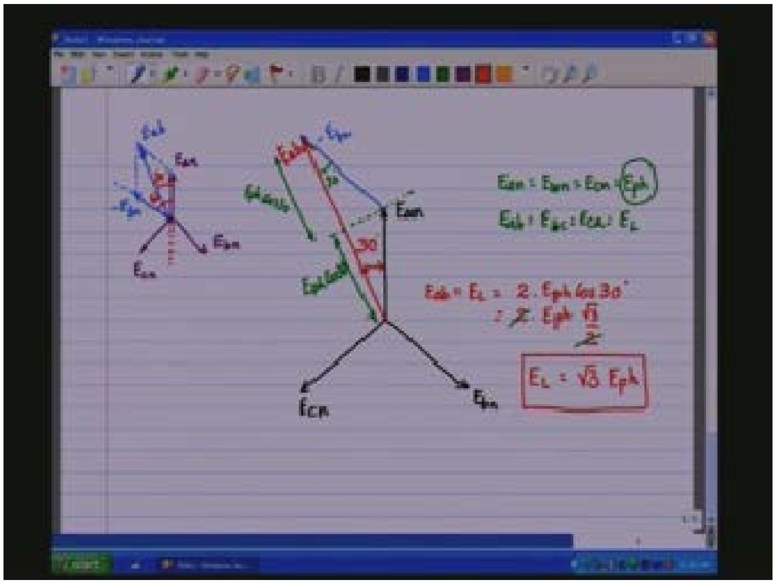

We saw the diagram here for this one. But if we go back to our previous this one (Refer Slide Time: 25:13) previous picture you can also see that E b c......... so for E bc we are talking of this loop, we are we are considering this loop the one which is shown by the cursor (Refer Slide Time: 25:36) that loop. So what do you see; you see E bn minus E cn will be E bc. So it will be E bn minus E cn and likewise E c a; so for E ca we are considering this loop as shown by the cursor. So, if we look at the voltages E ca will be equal to E cn that is this one minus E an and then back to that. So this is equal to E cn minus E an. These are the voltages symmetrically applying.

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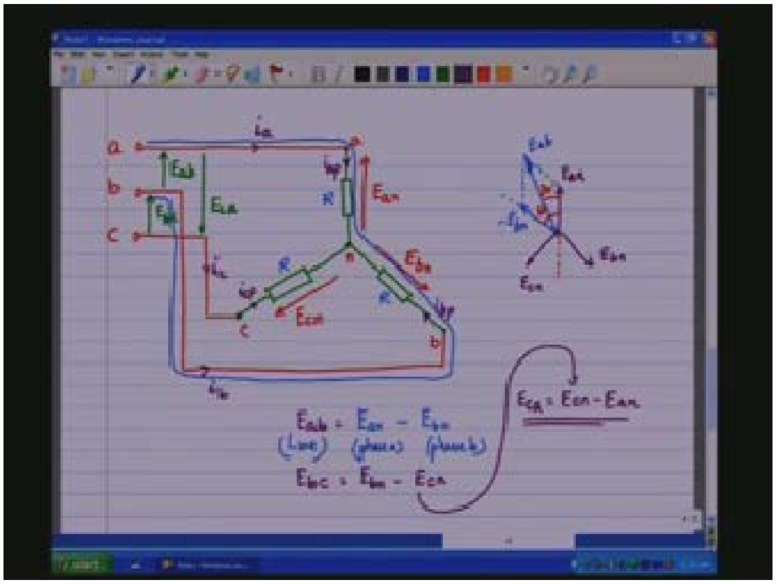

So if you look at this particular picture (Refer Slide Time: 26:57) you will see that there is a symmetry about all the axis. So now let us say we we want E c we want E ca. Now if you take about E ca you have a n which will be shown in the negative. So you have E an minus E an and you have this vector which is E ca.

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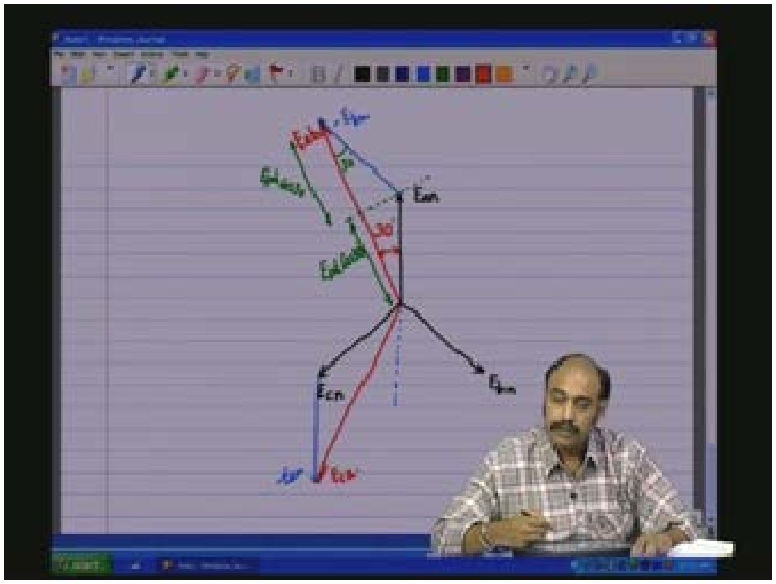

Now let us say E bc. So in the case of E bc you have c c vector will be........ this is E cn minus and we have E bc. You see symmetrically all the phases will have similar type of a vector phasor construct and in all cases the line voltages are nothing but projections of these two phasor voltages and in all cases the line voltage is leading the phase voltages by 30 degrees and therefore the line voltage here will be twice the projections of the phase voltage which is again root 3. So line voltage will be root 3 times phase voltage; one important relationship that you have to keep in mind.

Now in the case of the star circuit we are discussing the star circuit; what is the relationship between the line currents and phase currents? So likewise let us give similar symbol for the line currents; I L is the line current and I phase is the phase current and we have the line current equaling the phase current because in the case of the star circuit (Refer Slide Time: 29:43) we see that i a the line current the same current that flows through the phase circuit, i c the phase line current the same current that flows through the phase circuit and so also i b. So the line current and the phase currents are same in the case of star connected circuits.

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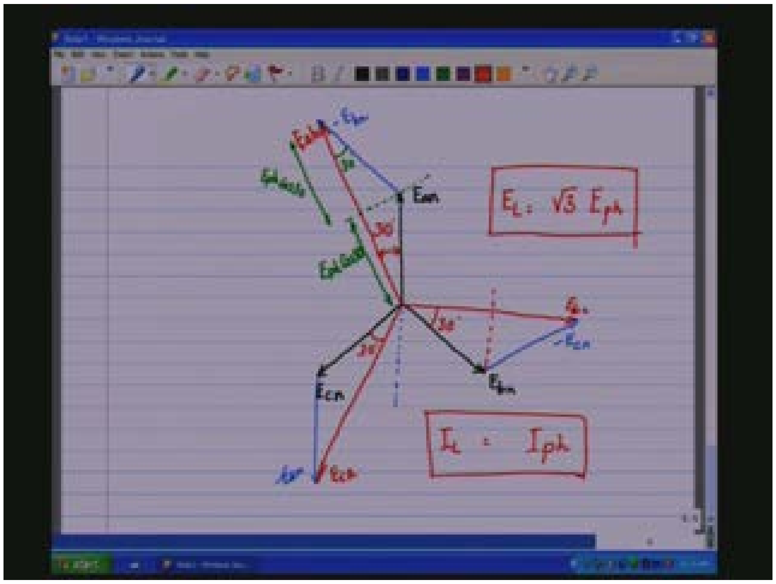

So, in the case of the star circuits these two are the important relationship that you have to remember between the line voltages between the line side parameters and the phase side parameters; between the line voltages and the currents and the phase voltages and currents.

Now, coming to the delta circuit delta circuit delta circuits.......... so, in the delta circuit the picture is as follows. We have the load connected in the pattern of a delta a triangle basically, an equilateral triangle shape as shown here as we are constructing here. It has of course three R's three loads and let us say we have the a terminal, the b terminal and the c terminal and let us connect the a terminal to the a terminal, a terminal is connected to the a line of the source, the b line of the source is connected to the b line of the source then you have the c line of the source connected to the c terminal of the circuit here as shown.

Now here let us now define the various parameters. Now a to b we shall call this as E ab, then we shall call this as E bc and we shall call this as E ca cyclically, cyclically we have E ab E bc E ca. Now we need to define three line currents. So you have i a, this is i b and this is i c three line currents. Note that the phase voltages are the same as the line voltages. In this case all phase voltages you see; across all the phase circuits the i a b which is directly connected across the line a and b E bc the phase circuit......... this R is connected directly across b and c and so also E ca.

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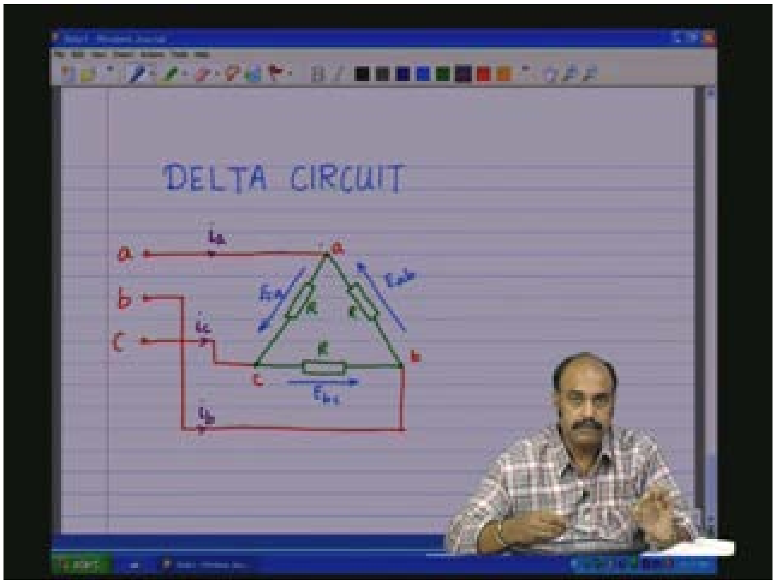

So the phase voltage and the line voltage are the same whereas in the case of star the phase currents and line currents are the same. And in the case of the delta circuit the line currents and the phase currents are not same as it was in the case of the star circuit, so keep that in mind and let us let us say that there is a line current a to b in this direction let us call that one as let us call that one as i a b and then we have a current b to c and we will call that one i b c and then we have a current from c to a and that we should call that as i ca. So these are the directions of the currents that we are indicating here.

Now we need to find the relationship between this line current this line current here i a and the phase currents here, so also the line currents here and the phase currents here and the line current and phase current because we know that between the phase voltage and the line voltage there is no difference so they are all equal so what is remaining is to find the relationship between the line and phase currents.

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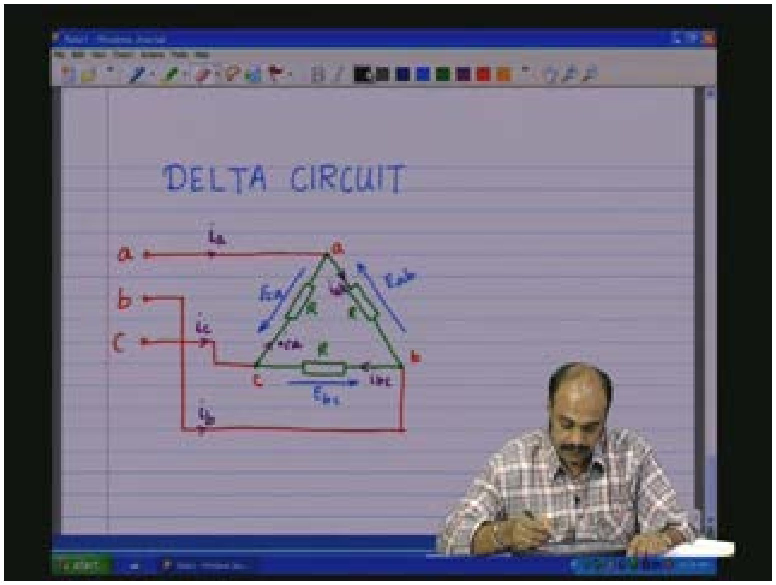

So first let us draw the phasor diagram. So in the phasor diagram we first draw the phasor diagram of E ab E bc E ca. We know that the line voltages are all having the same amplitude same effective amplitudes phasors but they are all displaced 120 degrees out of phase with each other. So let us let us let us draw the E ab component in this fashion E a to b and E a to b as the circuit is drawn here. So this I will let us call it as E ab E bc E bc as shown here, then we see that E ca so we draw from here the E ca E ca E ca. So we have the three line voltages drawn here and they are all as you know 120 degrees phase shifted with respect to each other that is the characteristic of 3 phase circuits 3 phases systems.

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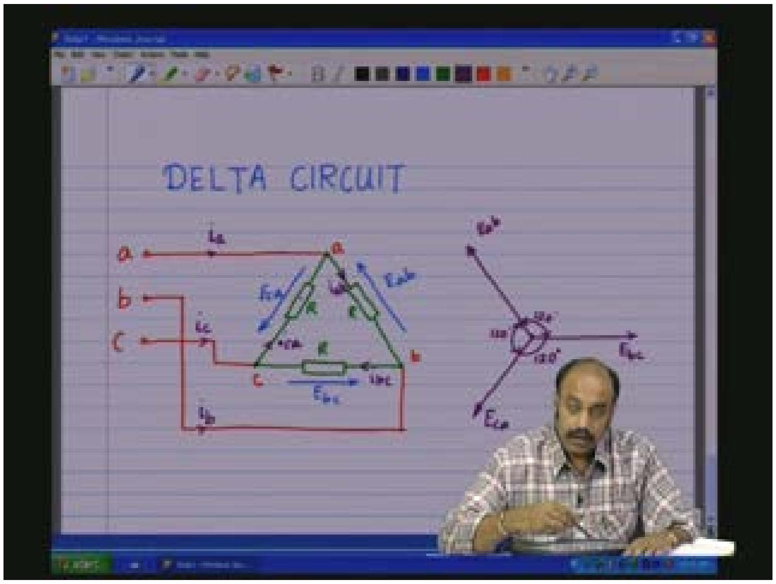

Now all are resistive loads. So as all are resistive loads the currents will be in phase with the voltages the respective phase voltages. Now the phase voltages here in this case are the line voltages themselves and therefore we have the currents which are flowing through here the i ab b c and c a will be in line or in phase with the voltages across those terminals E ab. So therefore the currents phase currents i ab will be along E ab i bc i bc here will be along E bc same amplitude because the load is the same and then i ca which is flowing in this direction here will be along c a. So this will be i ca, this is going to be i bc. So we have the 3 phase currents.

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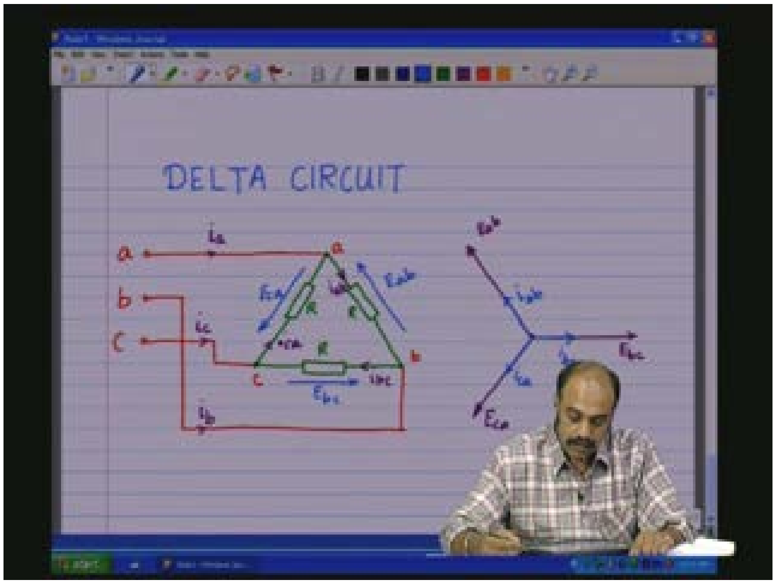

Now how do we relate the 3 phase currents with the line currents i a i b i c?

So if we take the Kirchoff's currents law now we apply the Kirchoff's current law i a is equal to...... so we look at this load here; i a is equal to the current which is going out here which is i a b minus the current which is coming through here because they are now in two different directions so therefore we have i ab minus i ca because i a and i ca are coming to the node, i ab is going out of the node that is the reason that the sign difference is coming in to the picture here.

Now here if you look at i a minus i ca so minus i ca is going to give you the current will give you the current the resultant current. So let me draw this; i ca minus should be like this is it not, so it goes........ this is minus i ca and the resultant is like that, this is i a, you see i a is i ab vectorial sum minus i ca so this would be i a and note that i a and E ab are having a phase difference and that phase difference is again 30 degrees.

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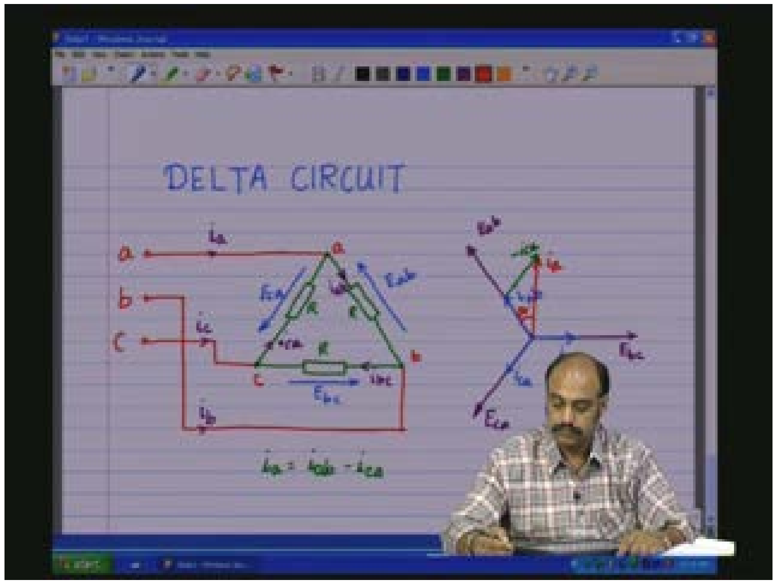

So here too if you say that i a i b i c they are all having the same effective amplitudes and we will say that it is the line current I L; i ab, i bc, i ca they are all having the same effective values I phase. And looking at the diagram and if we draw the projections so you will see that i a b projection on this i a line and i ca amplitude project on this i a line (Refer Slide Time: 43:04) so you have an I phase cos 30 and I phase cos 30 so 2 times I phase cos 30 is added up to get I a amplitude so I a is nothing but the I line amplitude so I line will be 2 times I phase into cos of 30 degrees which is 2 times i phase into root 3 by 2 because cos 30 is root 3 by 2 and out of which these two are going to get cancelled. So we have the relationship I L is equal to root 3 I phase pretty important relationship and here E line will be equal to E phase because they are the same.

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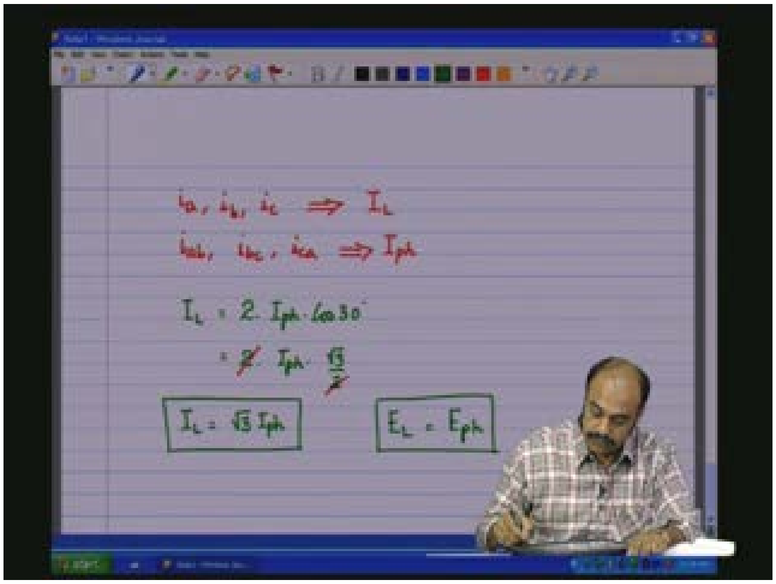

So if we summarize the star and delta circuits so you have the star and you have the delta circuit we can put it like that and this is like this let us say. The star circuit E line equals root 3 E phase I line is equal to I phase and the delta circuit E line equals E phase line voltage is same as the phase voltage and I line will be root 3 times I phase. These are the two important relationships with respect to the star and delta circuits.

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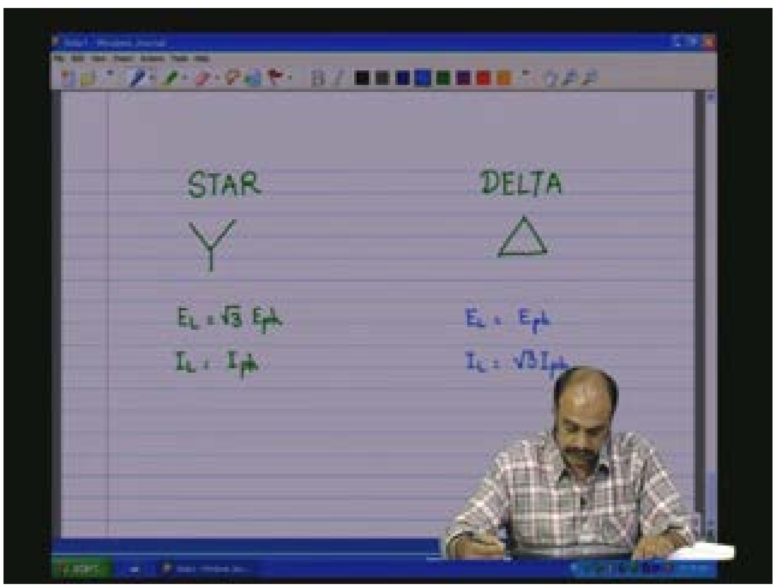

Now if you consider the power we consider the power there are in a 3 phase circuit there are three single phase or 3 phase circuits which are connected either in the star or the delta. So if you take per phase in each phase the let us say we are considering right now the resistive load therefore the whole power is active power so E phase into I phase will be the power per phase however there are 3 phases and therefore the power of all the 3 phases get added up therefore the total power the total will be 33 into the phase powers which is equal to 3 into E phase I phase.

Normally we express the power in terms of the line parameters. So if we have to express it in terms of the line parameters so let us say we first take the star circuit star circuit. We know that P total is equal to 3 times E phase into I phase. So in the case of the star circuit what is the relationship that we use; we use E line equals root 3 E phase I line is equal to I phase. So substituting these here you have P total is equal to 3 into E phase is E L by root 3 E L by root 3 I phase is the same as I L and therefore we put it as I L so which is root 3 E L I L that is the total 3 phase power; root 3 line to line will change into line line current.

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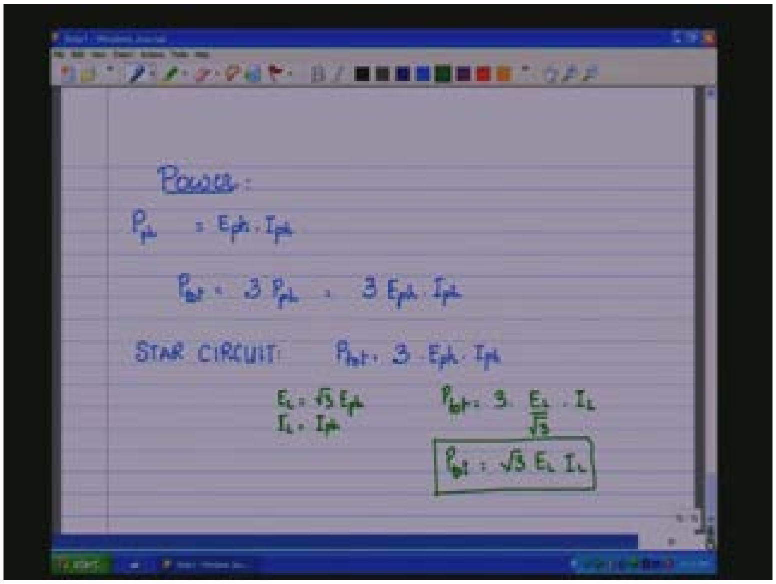

Now the case is delta circuit delta circuit, P total is 3 times phase voltage into phase current and here the relationship that we are trying to use is E L line current will be the same as the phase current in the delta circuit, I L the line current will be root 3 times the phase current this we saw. So substituting this here we have P total 3 times E phase is same as E line so we write it as E line in terms of the line parameters and I phase looking at this equation we have I L by 3 by root 3 I L by root 3. So this gives you the same relationship E L into I L this is P total. So whatever may be the type of connection whether star or delta in a 3 phase circuit if we are using the line parameters line voltage line currents the total power is always given by root 3 E line I line. This will be the total power P total the 3 phase circuit.

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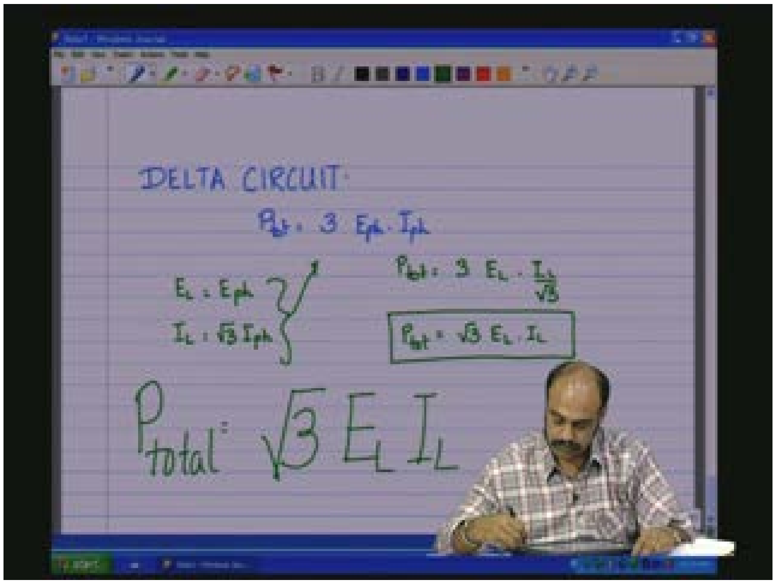

So a 3 phase circuit also you can have non non-resistive loads like it need not be pure resistance it can also be impedances which means combinations of resistances and reactances which means some portion of the power is going to the reactive in nature and in some portion of the power is going to be active in nature. And the concept of the power factor that we developed earlier is still valid. We we discussed the earlier that the power factor in the case of sinusoidal circuits power factor is given by cos of phi cos of phi phi being the phase angle between phase angle between the voltages and the currents in the phase.

So, if we have a resisting load we saw that the current the phase currents and the phase voltages they were in phase. But if they are non-resistive that is you have an impedance load then the currents are going to either lag or lead depending upon the type of the load. If it is an inductive type of impedance then the current is going to lag the voltage and if it is a capacitive type of impedance the current is going to lead the voltage. So cos of that angle that lead lag this is the lead or lag angle in the phase is going to be power factor. So the active power in each phase that is power phase will be E phase RMS value of course I phase R M S of the effective values into cos of phi as we had discussed earlier in the single phase circuit and the total power in all 3 phases because all the 3 phases where all phases are symmetric is going to be 3 times P per phase power which is going to be 3 times E phase into I phase into cos of phi.

Just to give an idea here so if we have let us say the phase voltages let us say a star circuit this is E an E bn E cn; in a resistive network the currents are going to be in phase. But in the case of let us say an inductive circuit R L circuit which is there in the three in the three load arms you will have a current which is lagging each of the phases. So this would be i a i c this is i b; how this comes about you will see that if you have a circuit something like that as shown here (Refer Slide Time: 54:18) this is the a, b and c, this is inductive in nature R L circuit R L R L R and L same Rs and Ls balanced load so you will see the currents lagging the voltage in every phase.

So in such a case the total power all 3 phases put together is given in this fashion and still the relationship.............. let us say in the case of the star in the case of the star we would have E line is equal to root 3 E phase and in the case of the delta we would have I line which is equal to root 3 I phase and then if we substitute here in both the cases you will see that P total will be root 3 E line I line cos of phi so this will be the total active power active power that is drawn from the load in the case of the 3 phase circuit.

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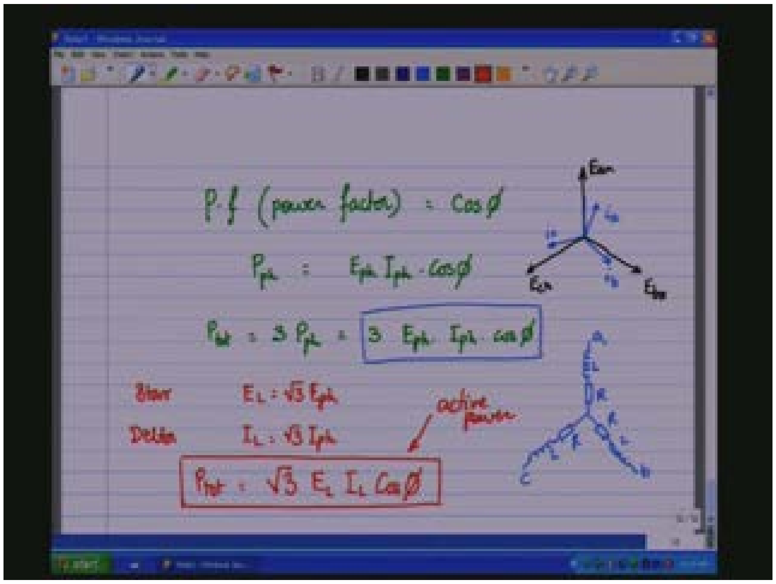

So let us take a simple example from the book. Let me consider line voltages line voltages 865 volts effective value and the load resistance R is 50 ohms load resistance R is 50 ohms. Now the load is connected in star circuit in the star circuit so let us calculate let us calculate first voltage across each resistor first step. Second: the current in each resistor and third the total power which is drawn from the source.

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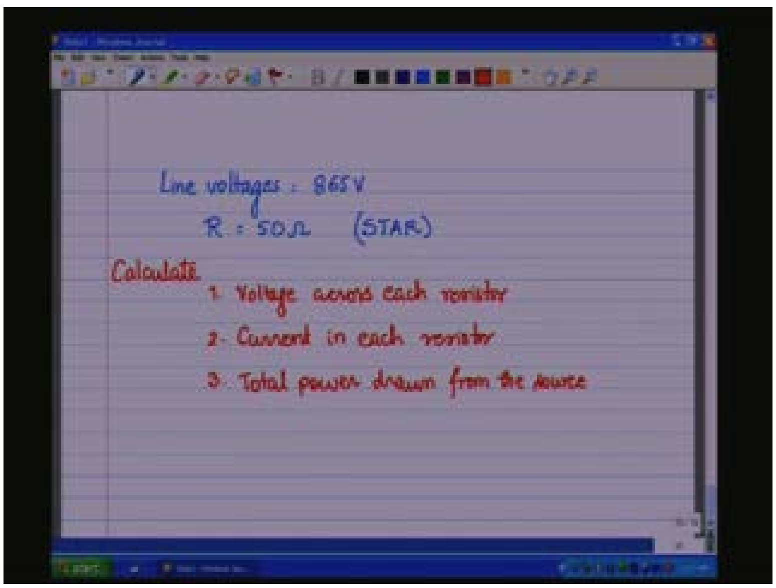

So we have a star circuit which is of this form and let us say this is a b c so we make the connection like that and here the a b c this is 50 ohms 50 ohms 50 ohms and we have the line voltages 865 volts.

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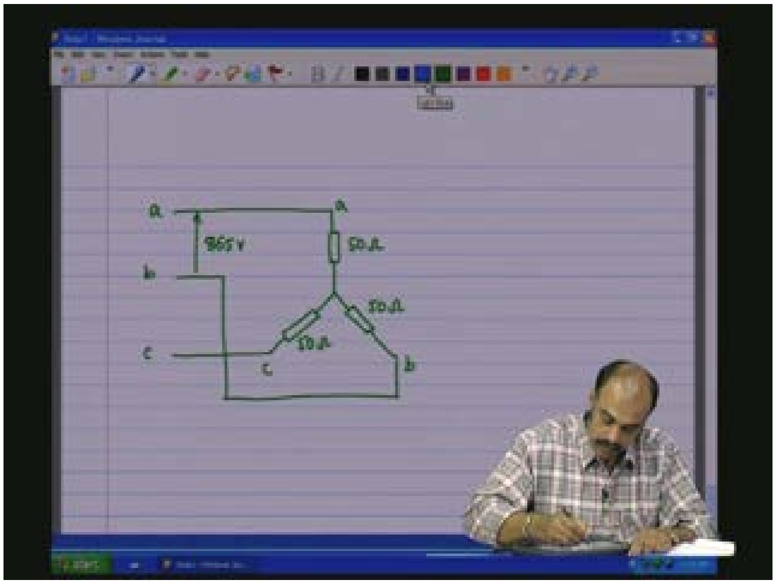

Now what is the phase voltage?

The phase voltage will be line voltage by root 3 which is 865 by root 3 and that is equal to 500 volts. So that is the voltage across every resistor, so every resistor we are seeing that there is a voltage across of 500 volts in the phase voltage and we know that the line current is equal to 10 amps and through each of the resistor it is going to be the same because in the star circuit the phase current is also 10 amps so 10 amps into 50 ohms which is 500 volts. And the third thing is total power P total which is equal to root 3 E L I L as resistive loads so cos phi is equal to 1 so this will be equal to root 3 865 into 10 amps which is equal to which is equal to 15 kilowatts or 3 into E phase I phase is also 15 kilowatt of 15000 watts.

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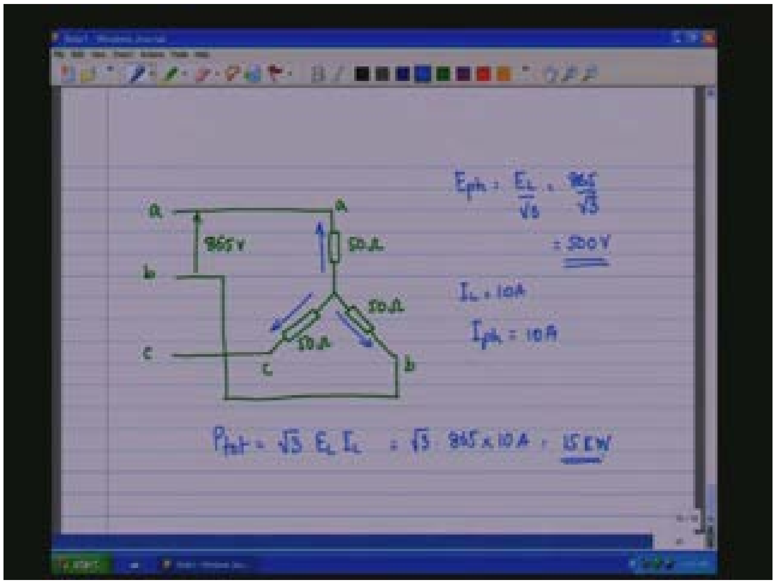

We shall stop here at this point of time and then continue in the next class. Thank you very much.