Basic Electrical Technology
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Lecture - 30
3 Phase System

Hello everybody, in the last class we had gone through the discussion on DC motors and we almost completed the discussion on the DC motors towards the end of the session. We just very briefly discussed on how we go about performing the dynamic model of the DC motor. In this session we shall carry that through that is make a dynamic model of the DC motor which will complete our discussion on the DC motor and then we shall take up a new topic and that is on 3 phase systems.

So first, the DC motor. So we have been studying quite at length about the DC motor. Now you are familiar with symbology; now we have the armature and to the armature is attached the brushes through the commutator and we have two non-idealities in the dynamic case: one is R a and the other is L a; R a is the winding equal and winding resistance of the armature, L a is the equivalent inductive reactants of the armature.

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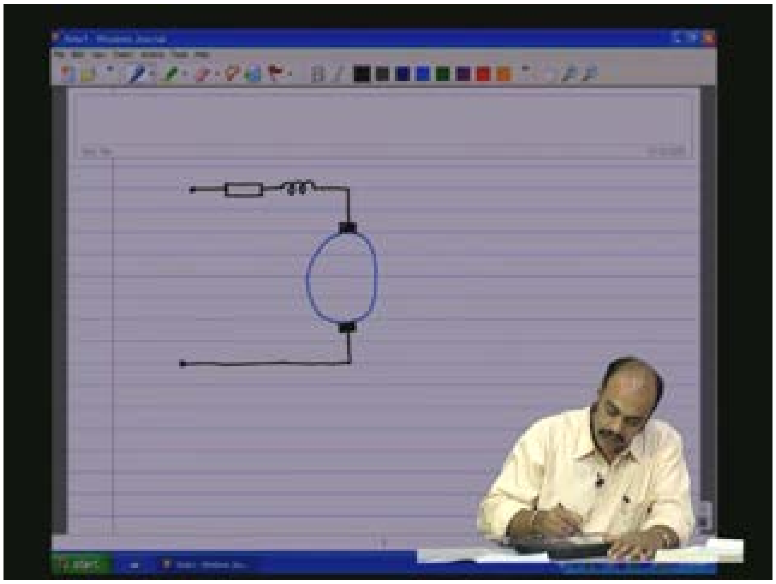

Now this is the DC motor which has a mechanical shaft and we said that in the case of the motor the energy is moving from the electrical domain to the mechanical domain and the mechanical loading gets reflected on the electrical domain as back emf across the brushes.

Now we apply a voltage E a and there is a armature current I a which is flowing through it, this is R a and L a and then in the mechanical domain we have a torque T g which is getting generated and because of the torque T g there is a speed angular speed of rotation omega in radians per second or in rpm which gets generated. Now this is on the mechanical domain. Now this mechanical domain also contains its own load or the reflected inertia J which is like the mass of a vehicle or the mass of the load or the mass of a very big wheel which has an inertia to it and then when it is rotating it gets stored in the kinetic form and that is called the inertial energy which is stored by virtue of an object being in motion which we know that it is stored in the kinetic energy form and therefore J or the inertial parameter is similar to L by inductive parameter and therefore the state variable in the case of the mechanical domain which is involved with the parameter J would be omega this speed. There could also be at the bearing some friction associated and we will call it as friction B.

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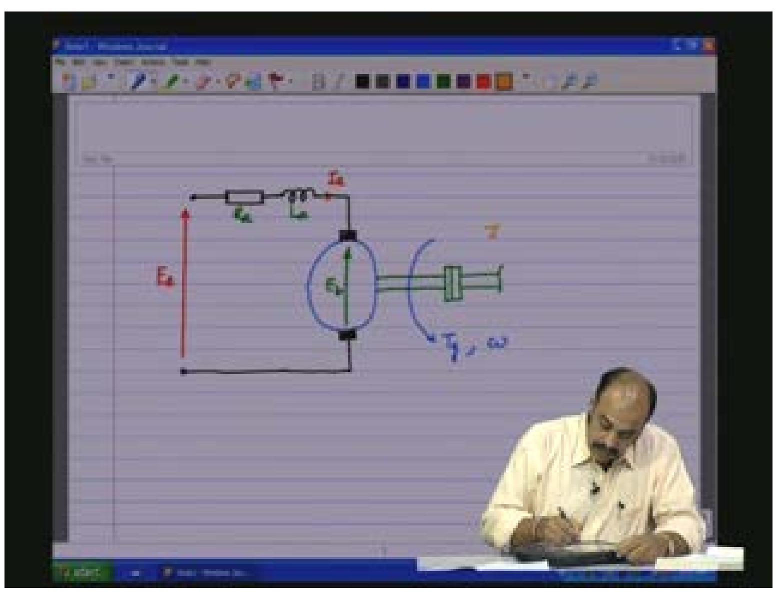

Now there is this is the electrical domain and this on the right side we have the mechanical domain, the link between the electrical and magnetic domain is the motor. There is a back emf we said and this back emf can be expressed in terms of the mechanical domain parameter or mechanical domain state variable and that is k phi omega we saw this in our earlier discussion and that is equal to the back emf. The torque component here the torque that is generated can be related to the state variable on the electrical side by the same k what we have written phi i a.

Now as we are doing a dynamic model we do not want to take the rms values, let us take the instantaneous values. So therefore I will replace these by the instantaneous values. So we have e a we have i a and E b so this is the relationship. Now we can apply the Kirchoff's voltage law to the electrical domain here. You see that the applied voltage is e a, there is a voltage drop r a there will be a drop across the ℓ a inductor and then of course there is a drop that gets developed across the brushes because of the rotation of the armature that is the generated emf or the back emf and that is called e b. Therefore we have we can apply the Kirchoff's voltage law in this area in this electrical area.

Likewise, on the mechanical area the mechanical side also; one can apply equivalent to the Kirchoff's voltage law the potentials or the potential variable being torque. The speed is common; like the current being common in the electrical circuit the speed is common to all the things connected to the shaft of the armature. the sum of the potentials which means in this case the torque should be equal to zero like the Kirchoff's voltage law.

So, generated the generated torque is T g should supply J d omega by dt and also d omega and if there is any other load torque that component also. So we are applying the Kirchoff's voltage law on both the electrical domain and the mechanical domain with the state variables which are the state variables; on the electrical domain we have one dynamic element L a with associated state variable i a and then we have J as another dynamic variable in the mechanical domain which is the reflected inertia and then the associated state variable omega. So we have two state variables at this point.

Now there is one more issue and that is phi. how are we generating phi; because the excitation, the field is being generated differently with respect to different topologies of the motor as we saw; in the case of the shunt motor, the series motor, the compound motor the phi is being generated separately so which means as far as this motor is concerned there is one more input which is phi that we need to take care.

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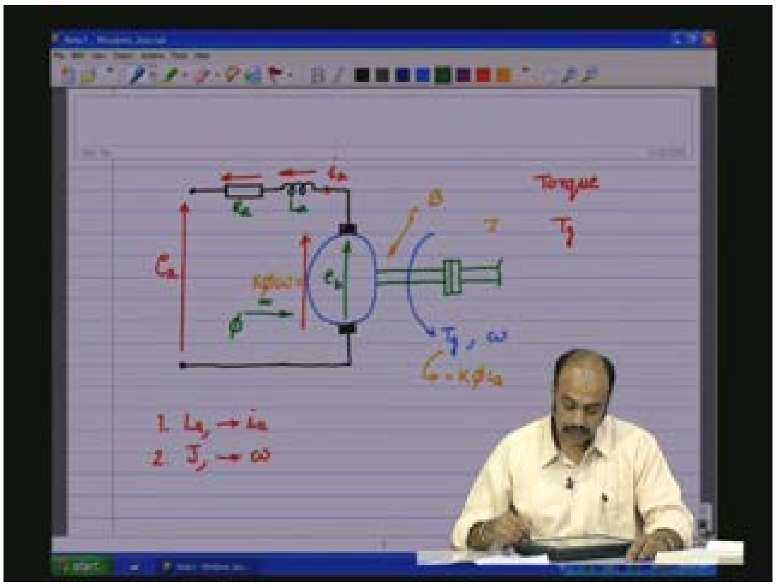

So first let us consider the case of the shunt motor where phi is a constant or a separately excited motor where phi is a constant, we are not touching phi. So under that condition we can now write the equations. Now the electrical domain side first so we have L a dia by dt so the voltage across that is e a minus R a e a minus i a R a minus e b. but e b can be expressed in terms of the state variable which is given by k phi omega so which is e a minus k minus i a R a minus k phi omega. So, for the moment we are considered considering phi as a constant that is this.

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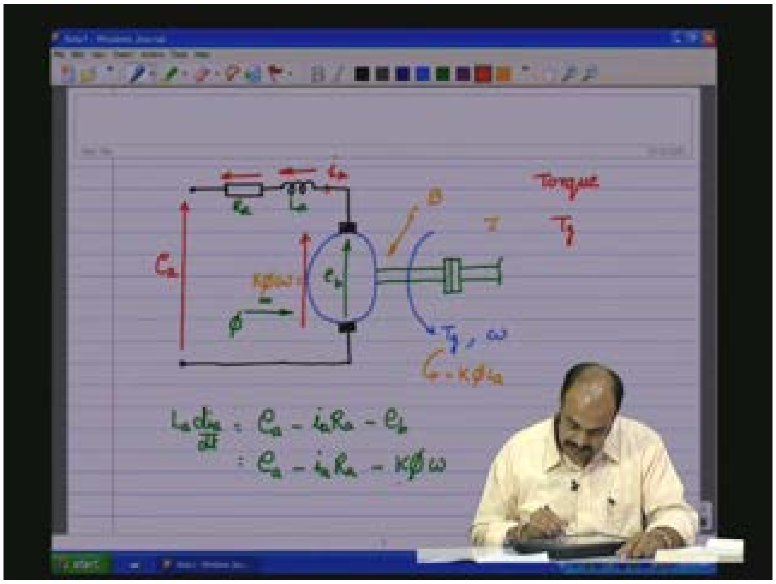

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So now (Refer Slide Time: 11:29) on the mechanical side there is a generated torque T g. Now this is going to supply the inertial component d omega by dt which is similar to L di by dt plus i r a equivalent to the resistive drop B omega in the mechanical domain.

Now plus any other load torque, any other load torque which may be applied to the mechanical shaft, all these should be generated by T g. So J d omega by dt will be equal to T g. Now T g can be expressed as k phi i a in terms of the state variables k phi i a minus B omega minus T load. So, as a mechanical input to the system there is T load and as the electrical input to the system we have e a.

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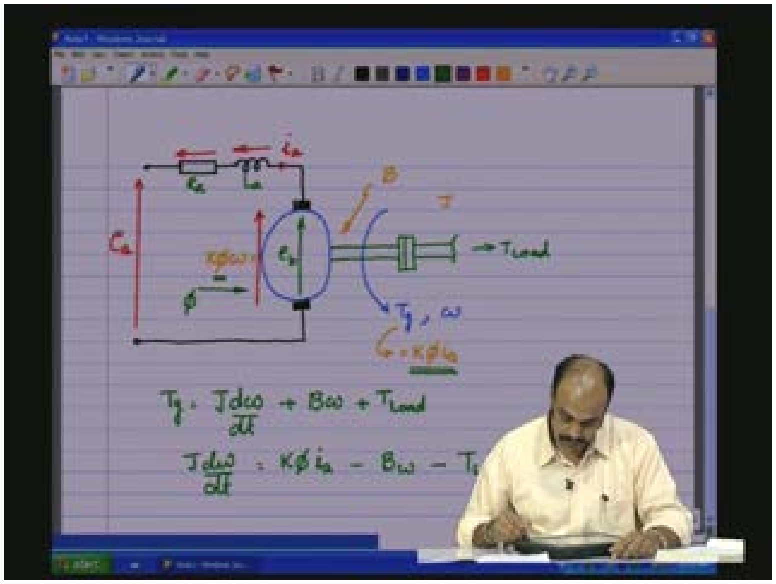

So combining these two we have dia by dt equals............ look at this (Refer Slide Time: 13:23) e a by L a R a by L a into i a and k phi by L a into omega. So I will rewrite it as minus R a by L a into i a minus k phi by L a into omega plus 1 by L a into e a so this is one equation. Then we have d omega by dt which is the mechanical equation so we have for the state variable i a k phi by J k phi by J into i a minus B by J into omega minus 1 by J into T load. This is the second equation.

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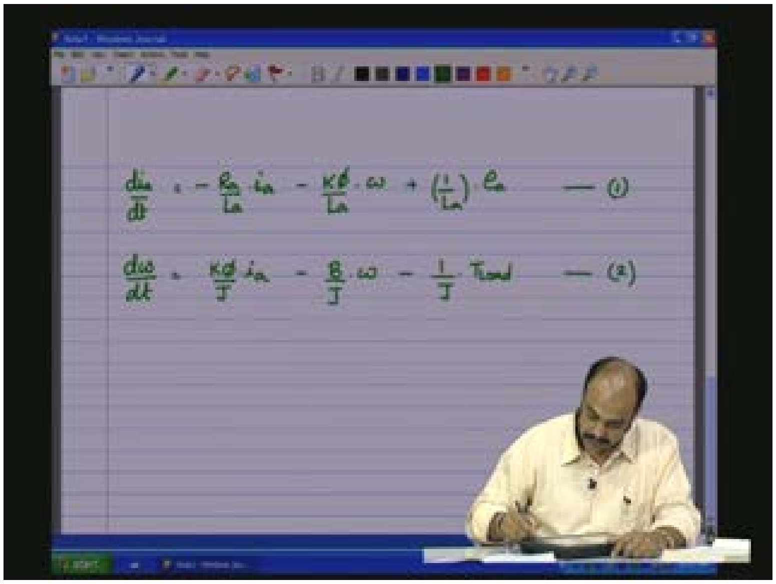

This equation has been rewritten (Refer Slide Time: 14:47) this one. These two equations completely define the dynamics of the motor. Now if it is a separately excited motor or a shunt motor where we can assume phi to be a constant where we can assume phi to be a constant then we can write it in the standard form of i a dot d phi by dt omega dot d omega by dt which is equal to minus R a by L a minus k by L a k by J k phi by J this is and minus B by J; i a and omega being the state variables in the state vector plus 1 by L a minus 1 by J sorry 0 and minus 1 by J and then we have two inputs e a and T load. So this is of the form x dot is equal to A x plus B u the standard form.

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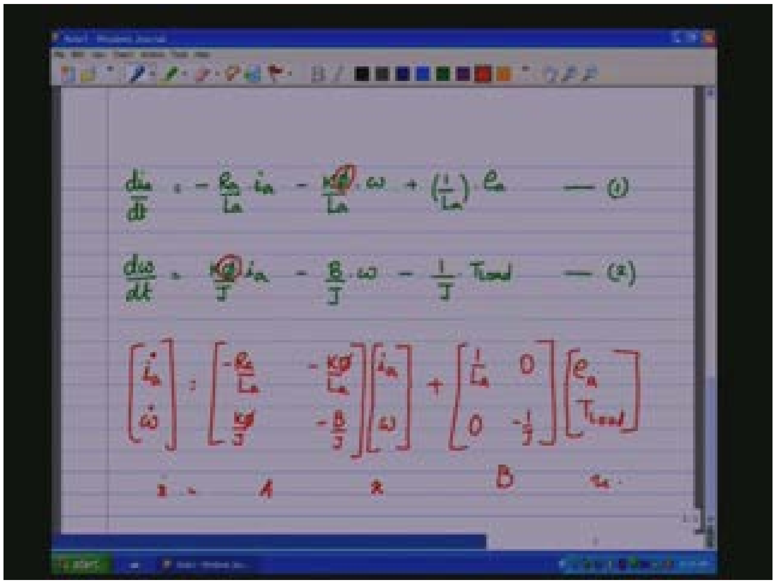

Now if phi is not a shunt but let us say a series motor in that case phi itself is a function of i a so which means here (Refer Slide Time: 16:50) phi becomes proportional to i a which results in......... let us say this becomes now k 1 i a k 1 i a so we do not..... because there is now straight multiplication i a into omega i a into i a this is i a square this is a nonlinear system therefore we do not put it in the standard matrix form we just leave it as differential equation system so that would be become a nonlinear system but still that would give you the entire dynamic model of the DC motor system.

So the phi should automa should appropriately be modified whether it is a constant or whether it is a now a function of i a or so or any other variable that should automatically be put in the........ and that will give you the complete dynamic model of the DC motor in the form or differential equation form.

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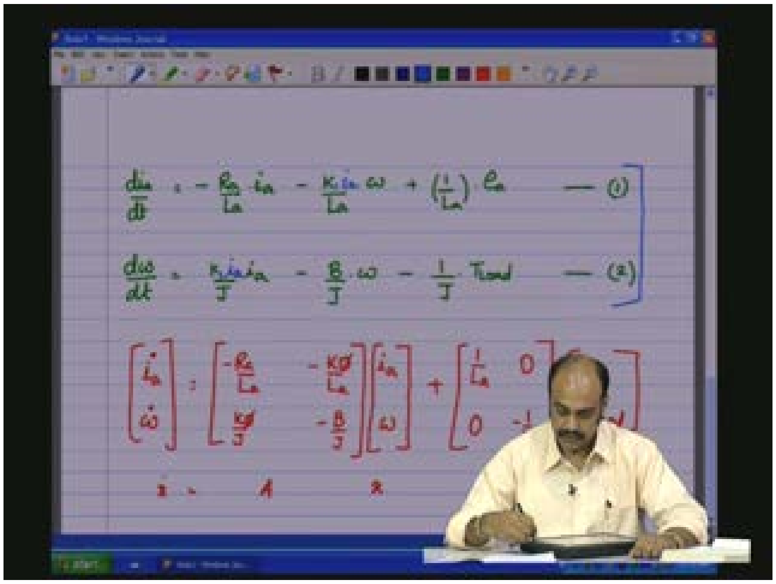

So we thought we conclude our discussion on DC motors, the DC generators and DC motors are a class by themselves, they both are similar machines, similar in structure but the only difference being that in the case of the DC generator the energy is being input on the mechanical domain, you are taking out the energy in the electrical domain and therefore from the electrical domain point of view it is a generator. And in the case of the DC motor you are applying the energy input in the electrical domain and taking out the energy in the mechanical domain.

But constructionally both the machines are very similar and a DC generator can be operated as a DC motor and DC motor as a DC generator and vice versa.

Internally inside the machine inside the armature the currents that are flowing in the coil are always AC but only by means of the device called the commutator which we have discussed and the brush combination external to the motor in the electrical circuit or external to the generator in the electrical circuit the signals are DC otherwise within the motor it is always an AC signal.

Now the mot the idea the concept of energy being passed through many domains is the underlying principle in which most of the machines in most of the applications applications are being used by many of the applications for example the induction motor induction motor or in the case of the alternators for generators or a combination of all these domains. And most of the in the electrical technology the central what we call the central concept or the central theme is some prime mover, propulsion, movement on the mechanical domain and the electrical domain is just the currents and the potentials. The core of the transformation of the energy from one domain to the other is always most of the cases in these set of equipments being done in the magnetic domain. So the central..... what we call them is generally a magnetic domain in most of our electro electrical technology applications magnetic domain. So you could get power or energy input from the electrical domain, take out power in electrical domain, you could take out the power in mechanical domain or you can put energy into the magnetic domain from the mechanical domain and take out power into the electrical domain all these are possible.

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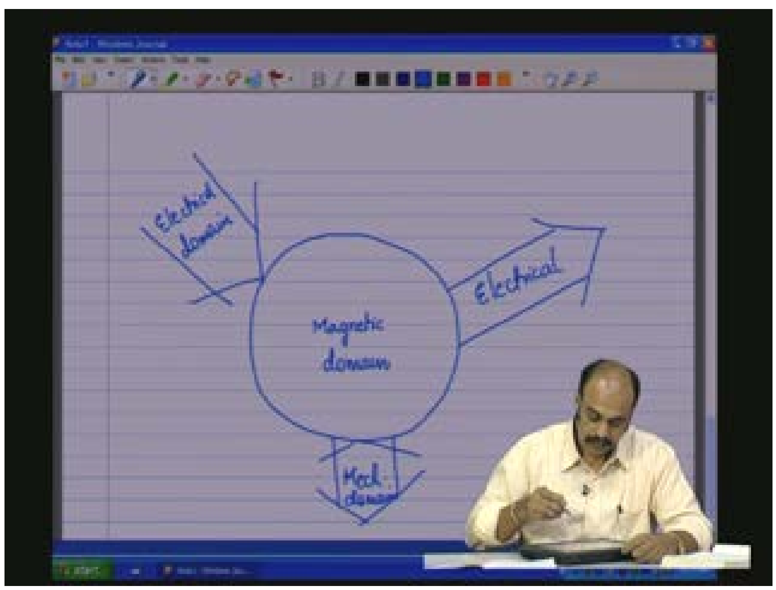

So the central mechanism which makes possible the conversion between many of these domains is the magnetic domain in the case of most of the electrical equipments and all the all these area (Refer Slide Time: 22:35) including the interfaces between the electrical magnetic and within the magnetic they are always AC and in many applications it is not single phase AC it is 3 phase AC. So before we try to understand and learn about the equipments electrical equipments which fall into these categories like the induction motor, the synchronous motor and generators, 3 phase transformers they all form into the fall into the category of this multi-domain principle. In the case of 3 phase transformers of course this domain is not there, there is only that is the mechanical domain is not there, there is only electrical transformation between electrical and the magnetic domains.

In the case of the induction motor with shorted rotors we do not have you have only one electrical domain which is inputting and this other electrical domain is not there and you have the mechanical domain. In case of the wound rotor where you have the rotor windings not shorted but taken out to a resist to resistor to a set of resistors then you have the input electrical domain or output electrical domain, you also have the mechanical domain so on and so forth. So like that you can have many combinations but all falling into this class of this type of concept.

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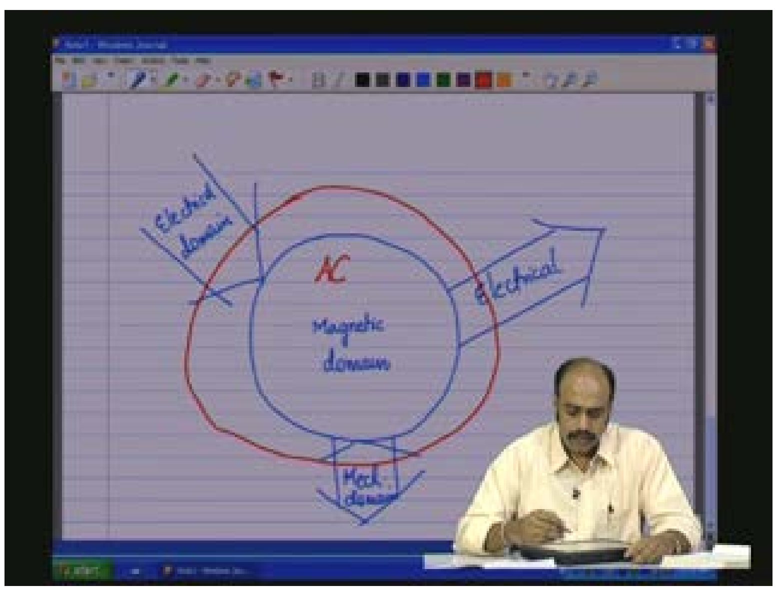

So, before we go further at all into the understanding of these equipments like induction motors, 3 phase transformers, synchronous motors, synchronous generators we should have a good understanding of the 3 phase system and what is this 3 phase systems.

So the major part of this session is going to now deal with 3 phase systems. This is also written as 3 phi for 3 phase system. So in the literature you will see that phi being used for the term phase. So now what is this 3 phase system, how does it look like and what is its character and what is its what are its features.

Now let me take a source a sinusoidal voltage source let me arrange it in this fashion. So let me say that this source is connected to resistive load, let me connect it to a resistive load as shown like this.

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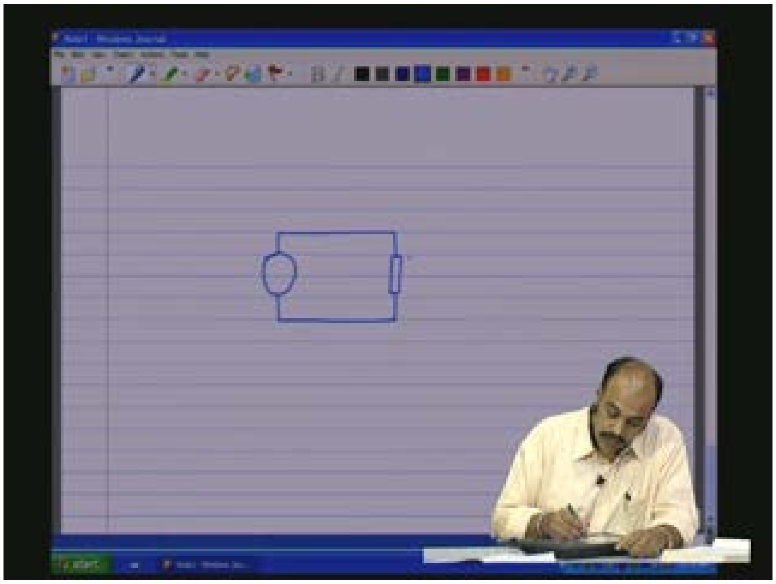

So this resistive load R is connected across this source which has two terminals here and let me name that terminal a and let me name this terminal as n 1 and the voltage across these two e a n 1. So this is the voltage and there is going to be a current through this one and we will call that one as i a.

Now I will put one more source one more source and let me connect it in this fashion (Refer Slide Time: 26:48); electrically they do not have anything in common except that I am putting one within the other and that is also having the same resistor R and let me call those terminals as b and n 2 and this is e bn2 which also has a potential like that and it has a current i b which takes this path along this resistor to end to back again to the other terminal of the source; that is also an AC source.

Now I put one more the third so I have one more source which is going to also be supplying a load resistance R and that is also not going to be electrically connected to the other two sources and it has two terminals which I am going to name it as c and n 3 and the voltage source itself is named between the terminals given the terminals names e c n 3 and there is going to be a voltage and a current i c.

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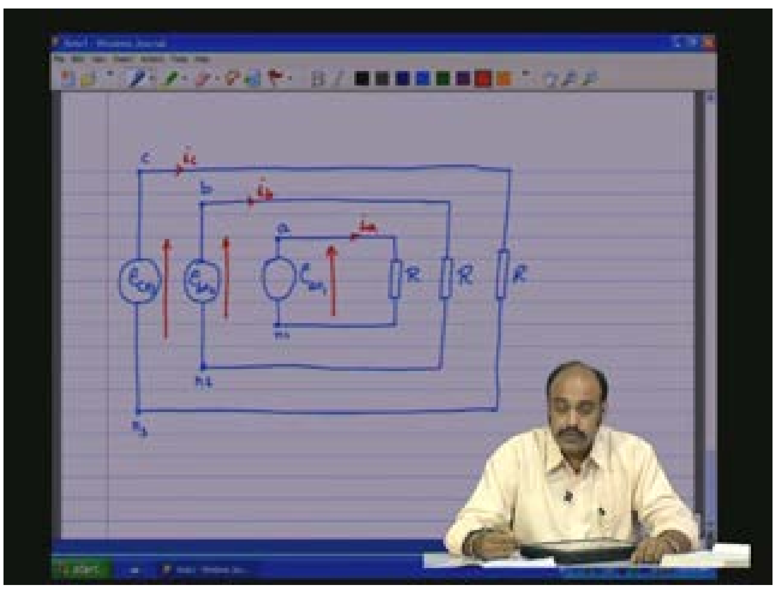

So you see there are three sources they are not at all linked electrically, they are absolutely independent of each other each of them supplying their own loads each of them supplying their own loads.

Now suppose let us apply some constraints. So what are the constrains? Let us make the amplitudes of the voltages same for all the three. So what is the first constraint; we make we make not the instantaneous mind you, we are making the peak amplitudes peak amplitudes or peak values or the sign waves of each of these AC sources sinusoidal sources same for all three that is e an1 this is n 1 e an1 peak will be equal to e bn2 peak will be equal to e cn3 peak the peak values of all the three sinusoids will be the same that is the first constraint. And then, of course we have made R the impedances of all the three resistive and all the three are same, the loads that all the three see are the same.

Now we apply the second constraint. Each of the sources have a phase shift with respect to other of 120 degrees phase shift with respect to each other. Each of these sinusoidal sources has a phase shift of 120 degrees with respect to each other that is the second and the most important constraint that we are going to apply.

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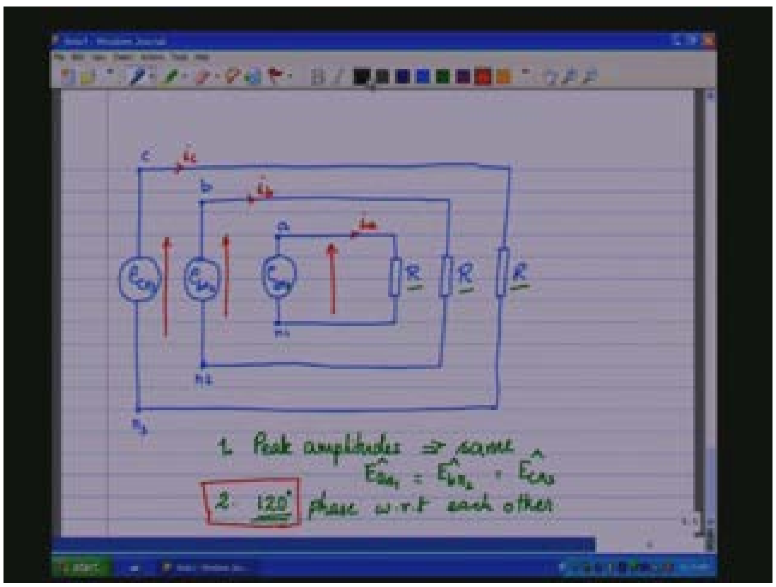

So, if such a thing is applied what happens to the phaser diagram?

You see; now let me have E an1 rms value of the E an1 source is being taken as the reference. Now let another voltage and let me call it let us say E bn2 lag E an1; note that the rms amplitude is the same because the peaks are same the rms is the same is lagging by 120 degrees. The E b source voltage is lagging the e a source voltage by 120 degrees. Now let me have the E c source voltage E cn3 lagging E a source voltage by 240 degrees then the second constraint gets established. what does the second constraint say; (Refer Slide Time: 33:25) it says there should be a 120 degrees phase shift phase shift with respect to each other that is all these three sources.

So we have 120 degrees phase shift between E an and E bn, 240 degrees phase shift between E an and E cn on the negative side but this means that E cn is leading E an by 120 degrees and between and between E b and E c because it is 240 with respect to E an and 120 with respect to E a n for e b this becomes also 120. So you see that E c is 120 degrees displaced from E a and 120 degrees displaced from E b, E b is 120 degrees displaced by E c and 120 degrees displaced with respect to E an this satisfies the second condition.

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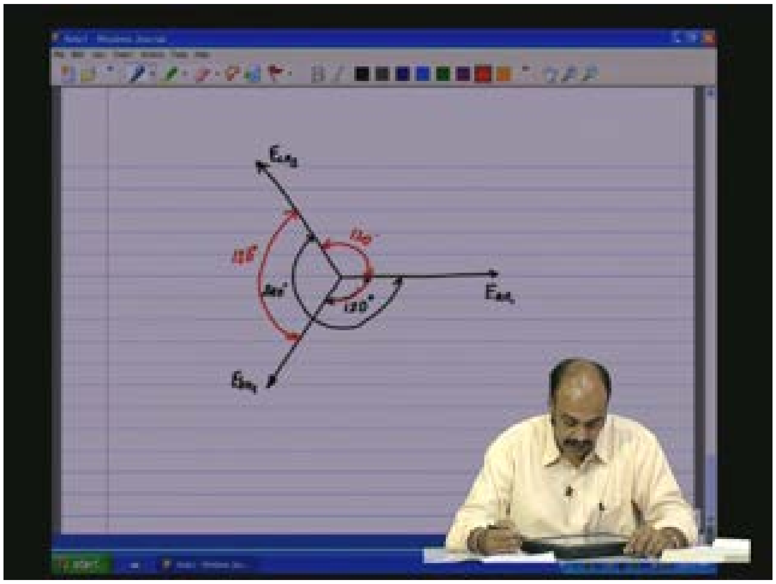

The first condition of course being the amplitudes are all same the phaser amplitudes and the second condition is they are equally displaced with respect to each other in a circle and that is 120 degrees 360 by 3 which is 120 degrees with respect to each other so this is the second constraint.

Now this is the nature of the sinusoids that will be applied; so which means that though electrically they are not connected we have applied a constraint on the three source voltages which is equal in amplitude equivalent in effect effective amplitudes or the peak amplitudes and the second is the phase displacement between each other should be equivalent which is 120 degrees, this also implies that we have the effective values of all three equal the peak and the effective values.

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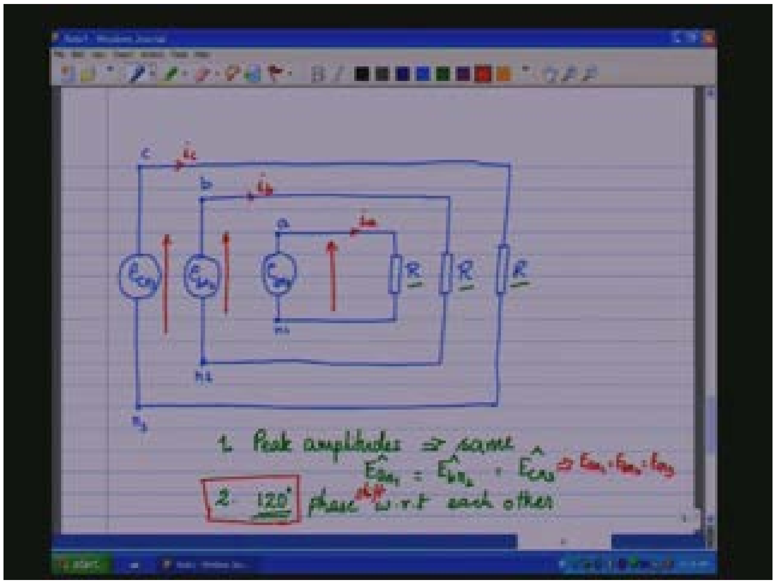

So now we can make some conclusions here.

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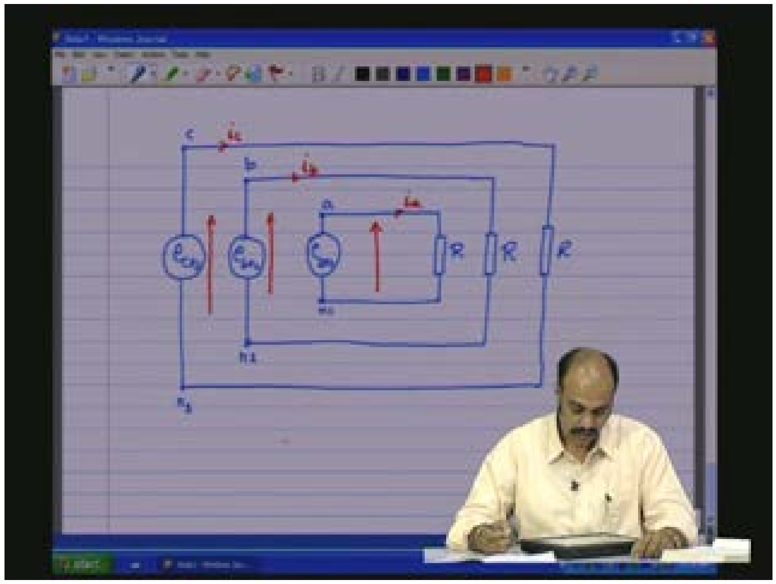

Now all the return paths of all these things can be clubbed together because there is nothing here in this no other components which comes but just plain wires, conductors, so let us say we remove all these conductors (Refer Slide Time: 36:32) and club them together like this and join them. So what have we done? The six wire system has now become 1 2 3 and a 4 a four wire system. From a six wire system it has been reduced and made more compact into a four wire system and of course i a i b i c is going to flow here; i a plus i b plus i c will flow here.

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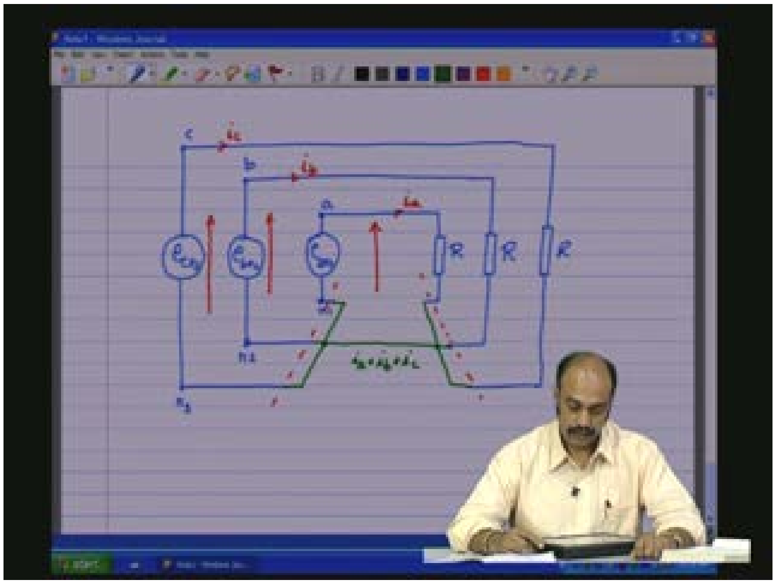

Now there are two things that we need to have look at that is the voltage waveforms and the current waveforms; how do they look like and what is the instantaneous values of these three put together and what is the current resultant current which flows through this line the return path line and this line is called the neutral; this is called the neutral line, the return path line which is common is called the neutral line through which i a plus i b plus i c flows.

If the loads R R R all are balanced and the voltages are all as according to the constraint, equal effective values, 120 degrees phase shifted then you will see that i a plus i b plus i c is equal to 0 and not a very large value so which means the conductor here need not be very thick. In fact if it is equal to 0 one can just eliminate this conductor one can just eliminate this conductor and make it a three wire system.

You see a six wire system became a four wire system and from the four wire system because of the applied constraints and loads being balanced and equal you have i a plus i b plus i c is equal to 0 at every instant of time so it is equivalent to having no wire here and you can just make it into a simple three wire system.

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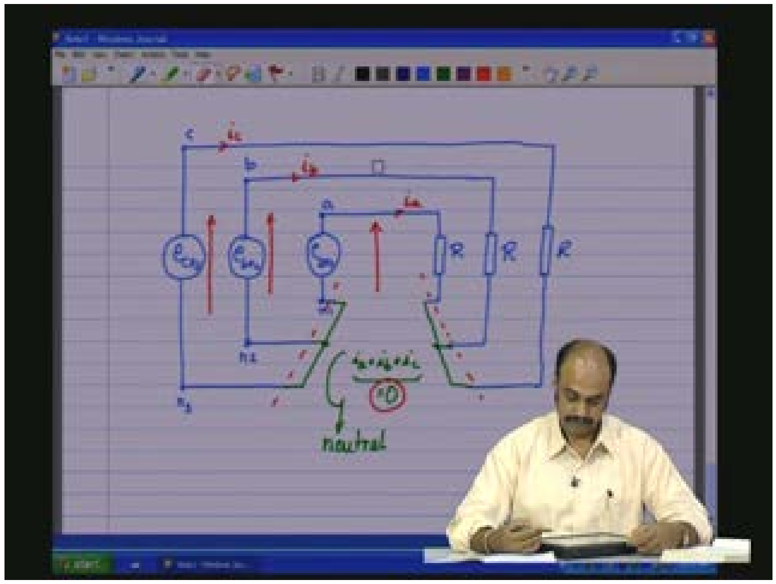

Now let us have a look....... let us as such not erase this line let this continue to be there, we shall erase it after some time after we just make some study on the currents and the voltages. So now let us have a look at the voltages here.

So we draw the 3 phase we draw the 3 phase voltages e a; now we start from zero this is the time t now e a is the reference so let it start from zero like that the sinusoid and then reaches one full cycle (Refer Slide Time: 40:40). Now the e b is displaced 120 degrees it is lagging e a by 120 degrees so this was e an1 so this is 90 so 120 will be somewhere around here. So we have a waveform which goes like that completes the cycle, this is 120 degrees lagging and that is e bn2.

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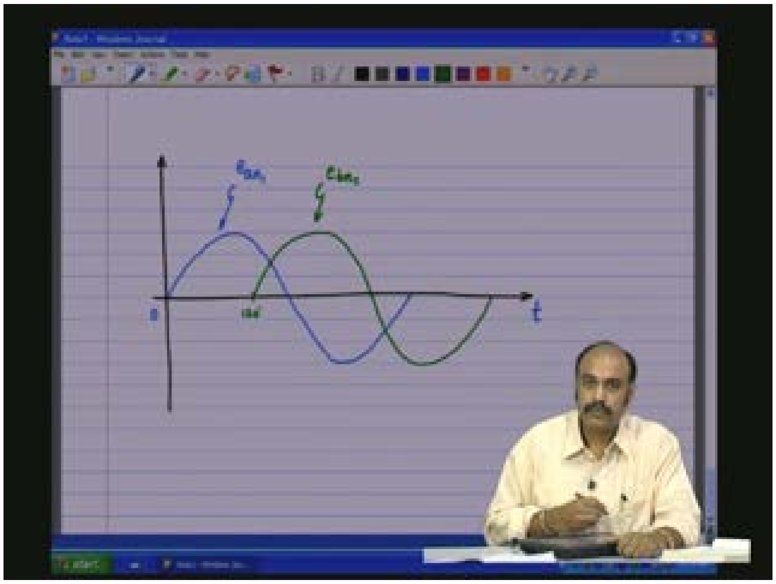

Now if we join all the things n 1, n 2 and n 3 become same and we can say that all these are now same and it is equal to n node n and we can erase this and say that these are c n b n a n.

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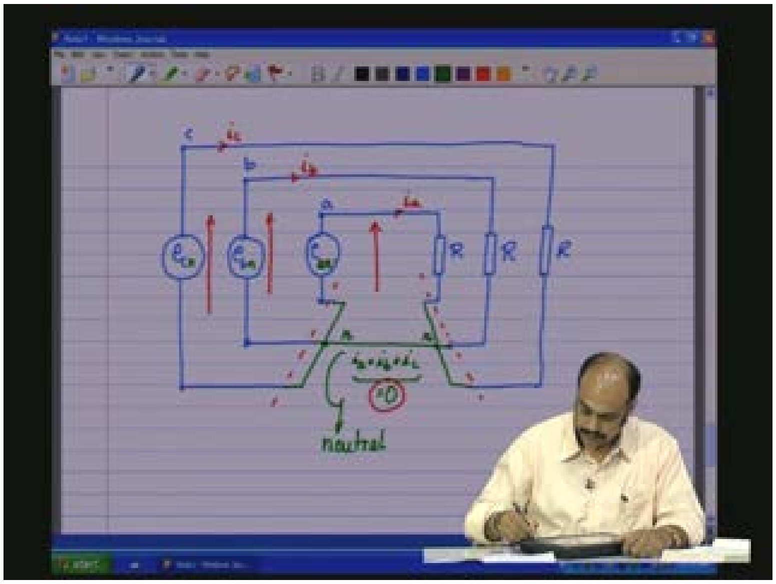

Then we have the third the c wave form which is 240; this is around 270 and therefore 240 will be around somewhere here, these are 240 degrees and so on we have sine wave which continues. So this is e c n 3 and because we have common the return paths it is now e a n e b n e c n it is a four wire system. Now this will get projected like that and this will continue like that; the cycle repeats.

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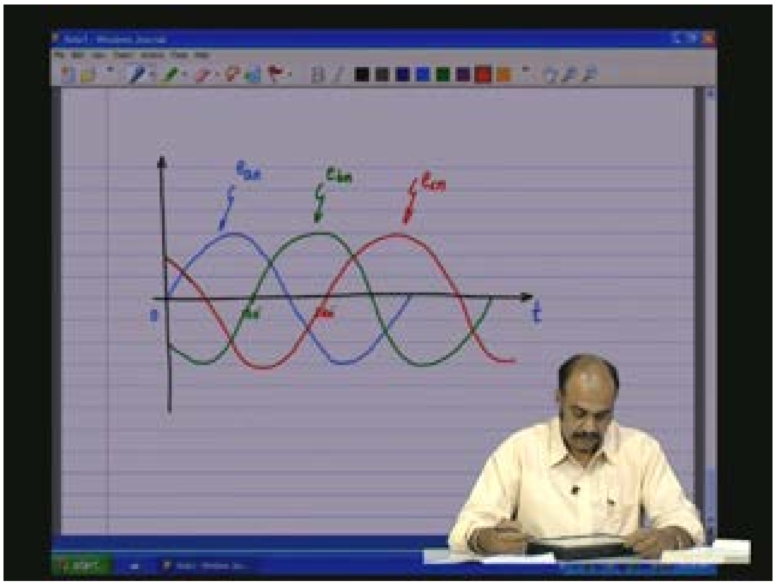

So now if we look at these waveforms one thing that we observe is that at any section at any section you can take any section any section this is positive the other two are negative and here these two are positive and the other one is negative maximum so there will always be a cancelling and you will see that any section the sum will be equal to zero.

Let us say this point this is the this is having a value which is half so the red one which is the e c n is having 1/2 let us say normalized value, green one is minus 1/2 so minus 1/2 plus minus 1/2 is 1 this is 100 percent peak so they both cancel and becomes 0. Likewise, let us say at this point this is plus 1/2 and plus 1/2, blue is plus 1/2, green is plus 1/2, red is minus 1 and gets cancelled it becomes 0.

Likewise, at every point you will see that the addition of all these three become zero and as they are passed through resistive loads what is i a this waveform will be also similar to i a i a i a i b and i c will also have similar only the amplitudes get changed by the factor of R thus all are having Rs. So e an by R i a e bn by R is i b e cn by R is i c so they also will have similar. So you will see that every instant of time i a plus i b plus i c is equal to 0 and this means but this will happen only if Rs are same balanced loads and the voltages are same so this means that this wire is redundant and you can now remove that wire here. So we have a strange connection between these two which is written in this fashion.

So let us say I have three loads three sources and I have three loads which are written in this form R R R. Now all the three sources the return path points I am going to common them called n and in the case of the loads also all these are going to be common and then the other ends of the sources are connected in this fashion to the loads. So this is a 3 phase three wire system.

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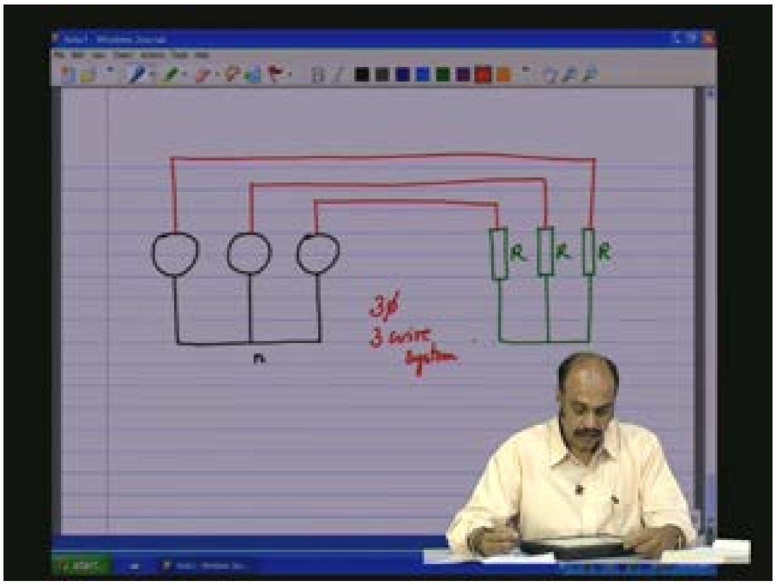

Now look at the way the source and the loads are connected. And we saw that e an e bn e cn had a phasor diagram which is e an e bn e cn 120 degrees, this is 120, 120, 120 degrees with respect to each other all having equal amplitudes, same. Now this can also be written differently in this form because we can take anything as the reference; 120 120 120 so you have e an e bn e cn or you can also write it in this form e an e bn e cn all 120 degrees out of phase. So in the literature you will see that they use these two ways of writing the phaser diagram for the 3 phase circuits.

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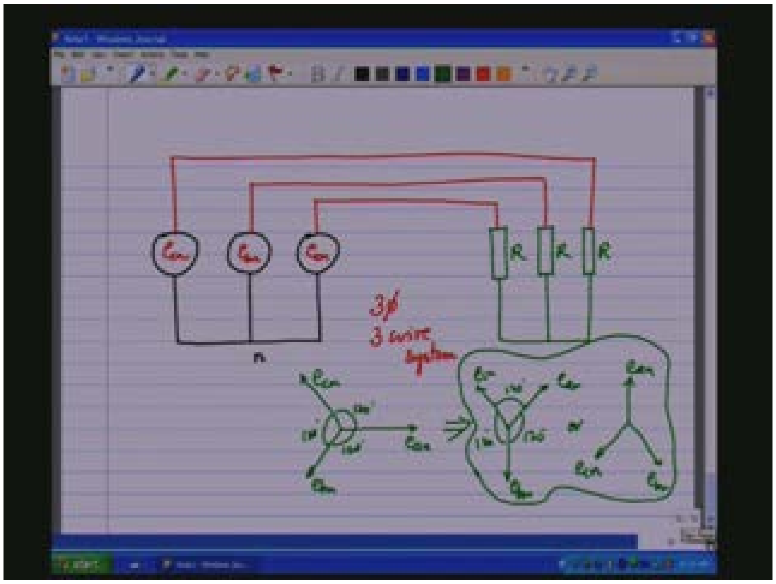

And the circuits are also built in such a way that it tries to give you a better feel that is along the vectors; that is what was supposed to be the vectors let us say we have the three vectors here the circuits are written along these vectors to give a better feel and this common point is called N. So we so we have the a phase circuit let us say connected to......... (Refer Slide Time: 50:42) we have the a phase connect circuit connected to the a phase load or the a phase resistor R so this is e an and then we have the e bn the b phase circuit which is connected to the b phase load. So let me connect this to the b phase load and then we have the e cn the c phase source which is connected to the c phase load R.

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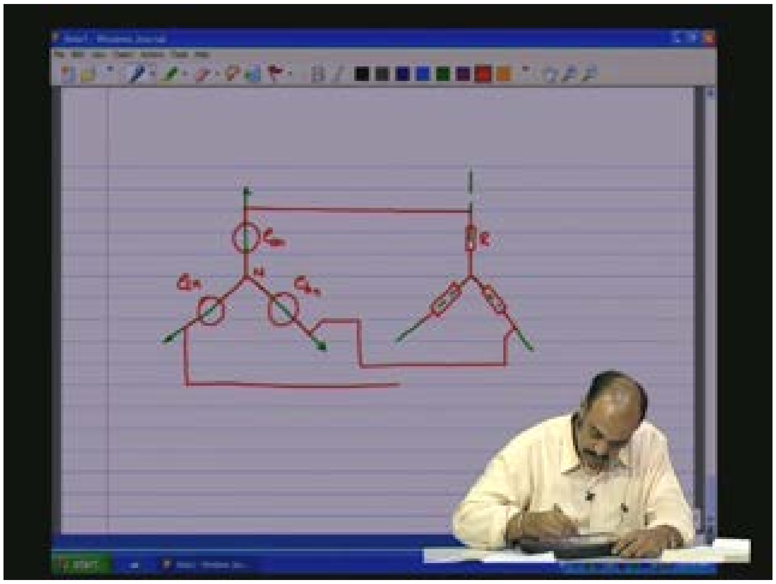

So you see that this is much more aesthetically pleasing and gives an idea that they are all 120 degrees phase shifted with respect to each other in a phasor manner both the load and the source. So, in the literature you will find these as a pretty common way of expressing 3 phase circuits and by the way it looks there are two indications: one is that this looks like a star and sometimes this is called a star circuit. Star load is also is in the star form. And if it is in the inverted way that is if some in some times in some cases the circuits may be written in this form here the a b c so this looks like a WYE and this is called WYE circuit. So, for this type of circuits where you have one end of the sources or the loads connected joined together in common and the other three terminals are taken out and use for connection to the source or the load then such a connection is called star or WYE connection. So this is a star or a WYE connected circuit.

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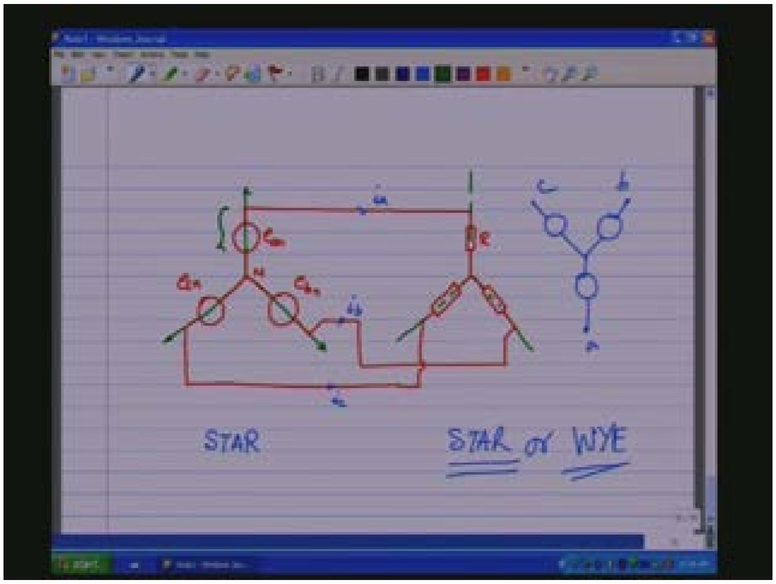

Now some terminals terminologies are in order, this is called the phase. Let us have a...... so let me have let me have a load of this form which I am writing here. So this is a star circuit or a WYE circuit as it may be called conveniently. So let us say this is point a, this is point b, this is point c so (a b c)s are connected to a 3 phase load a b c are connected to a 3 phase source.

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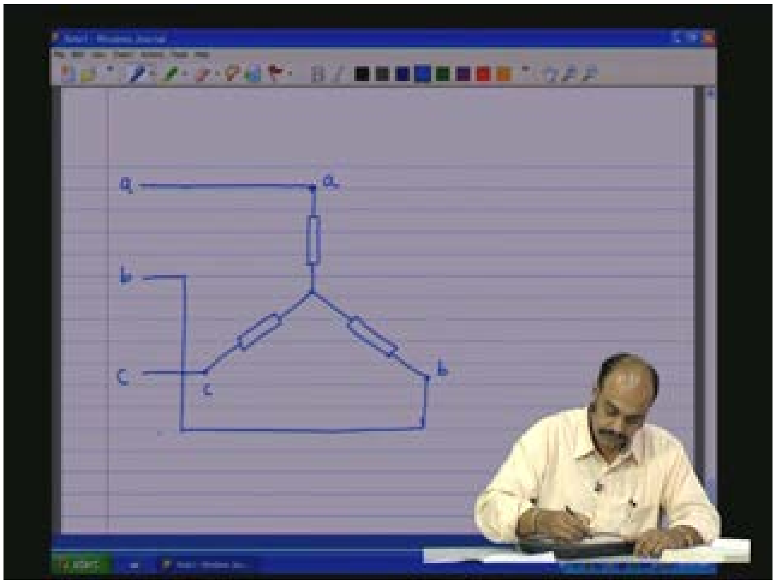

Now the voltage across each of the phase and let me call this one as N or the neutral across each of the phase that is this is called line, this is called line a, this is called line b and this is called line c; line a line b line c; between the line and the neutral is called the phase and this portion is called the phase circuit the phase circuit and the voltage across a and N is called the phase voltage. Likewise, the voltage across b and N is called the phase voltage e b N and the voltage between c and N is called the phase voltage e c N these are all phase voltages.

Now the voltage e ab is called the line voltage, e bc is line voltage, e ca is also line voltage. So e ab between the lines e bc e ca they are all called line voltages. The current going through the a to N phase let us say this is called the phase current and the current flowing in the lines are called the line currents. So you have the line currents and the phase currents. So, current flowing through the phase circuit is the phase current, current flowing through the line is the line current of course in this case the phase current and the line current are the same, the voltage across the lines is called the line voltage, the voltage across the phase circuit is called the phase voltage.

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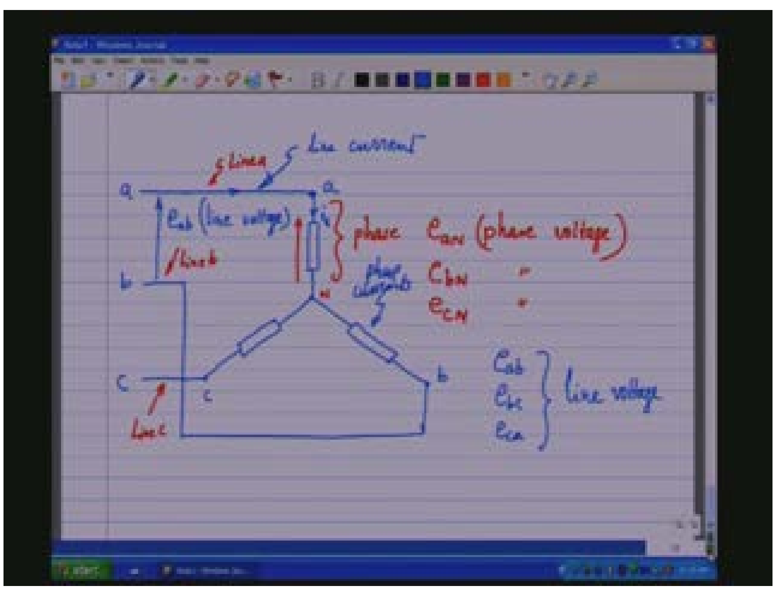

Now there is one more way in which the circuit load or the source can be connected between the lines. So let us say I have line a line b and line c and we saw that we connect it with respect to the phasor diagram in this manner in the manner of a star. But it can also be connected between these terminals as shown here like this (Refer Slide Time: 58:43) R R R. So this type of circuit looks like a delta the Greek letter delta and that lines can be connected as shown like this. So here also the voltage across the phase circuit.......... this is called the phase voltage, in this case phase voltage line voltage is the same, the current you have the line currents which are flowing in the lines phase currents which are flowing in the phases here let us say this is the phase currents, line currents, phase voltage and so on.

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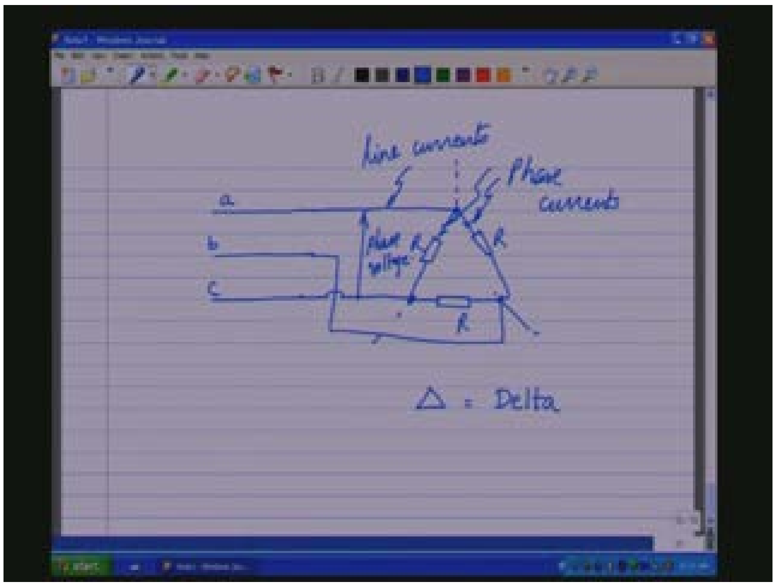

So in general we have two major types of circuits: we have the star circuit and delta circuit. So in the literature star circuit it is also called as WYE circuit. So in all the 3 phase systems you either have the star circuit or the delta circuit.

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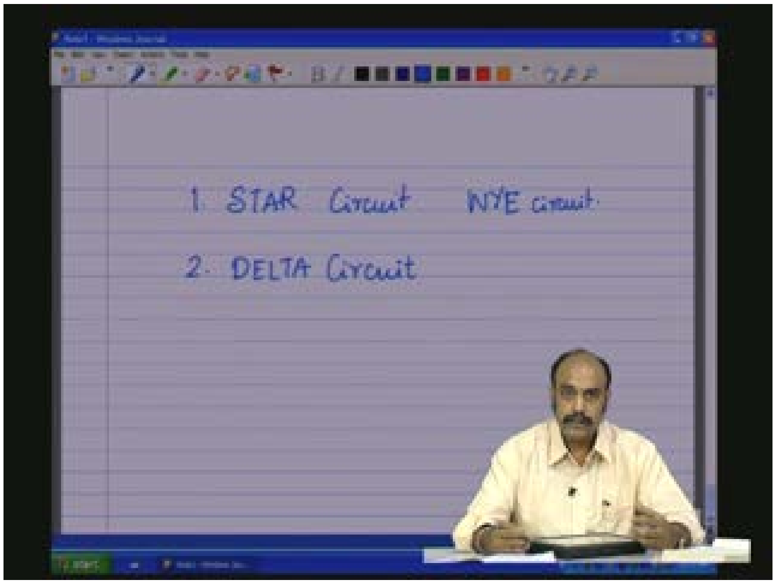

We shall discuss more about this in the next class. Thank you for now.