

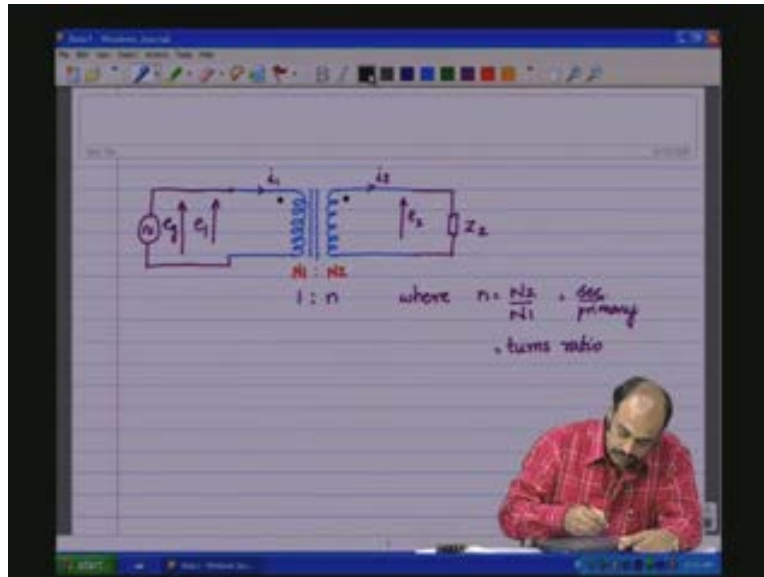
Basic Electrical Technology
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Lecture - 19
Transformer Basics Part – III

Hello everybody, today in this session we continue the basics of transformer where we left off in the last session. The last session we had been discussing about the transformer and its phasor diagram and how it is represented as phasors in the spatial coordinates. We also represented the symbolic form of the transformer and today we will continue the discussion from that point onwards.

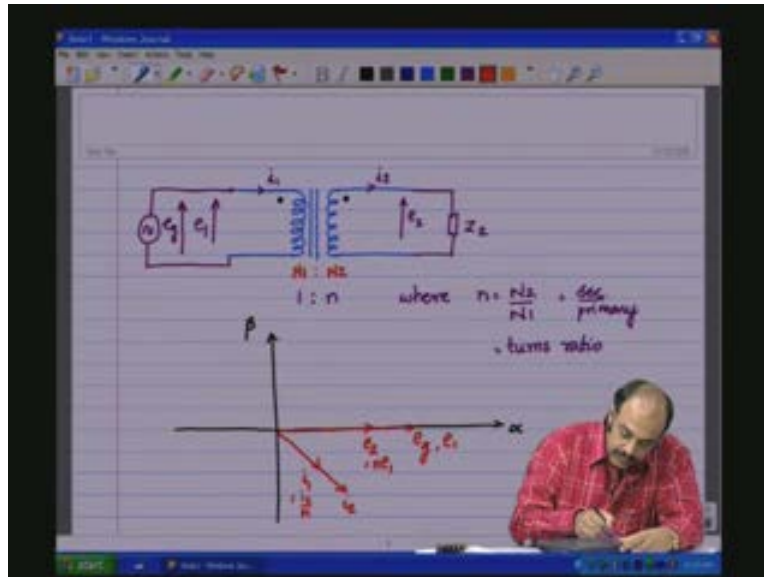
So we have this transformer with a primary, core and the secondary. We should not forget to put the dot polarities, this is to indicate that this part of the windings is in-phase with this part of the winding and there is one more information that we have to provide that is N_1 is to N_2 that is the N_1 number of turns or N_2 number of turns. Alternatively, one may also provide the ratio that is 1 is to N where N is equal to N_2 by N_1 , this is also a frequent representation of the transformer. Instead of the actual number of turns they provide the turns in ratio, N is called the turns ratio, this is called the turns ratio secondary turns number of turns to the primary number of turns. And of course we have the primary voltage even with the primary current i_1 , secondary current i_2 , secondary voltage e_2 with some node connected at the secondary some Z_2 and there is a source here which is connected across the primary which is the generator; it is an AC generator and we call that e.g.

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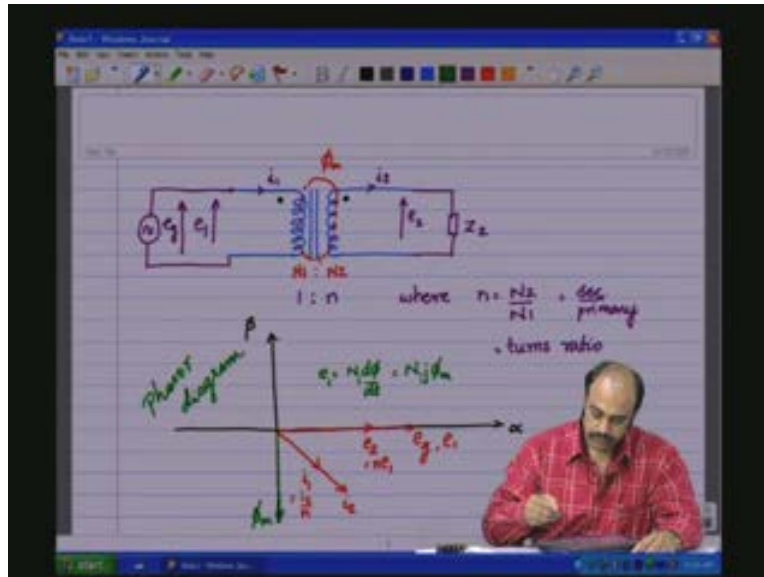
So we saw that the phasor diagram in the spatial coordinates that is the alpha beta coordinates the beta alpha we have e_g ; e_g and e_1 are in-phase because they are one and the same voltage. Now the dot point of e_1 **e 1** and the dot point of e_2 will also be in-phase and therefore let us say we have e_2 and then i_2 is going to lag e_2 by lag or lead e_2 depending upon the impedance. If it is RC or if it is a lead network then i_2 will lead e_2 , if it is a lag kind of a network that is inductive kind of an impedance then i_2 will lag e_2 . Let us say it is a lagging impedance then you will have i_2 just like that. Then i_1 and i_2 will be in-phase; only the difference is by the turns ratio and this is going to be i_1 . So e_2 is equal to $n e_1$, i_1 is equal to i_2 by n . This is the phasor diagram.

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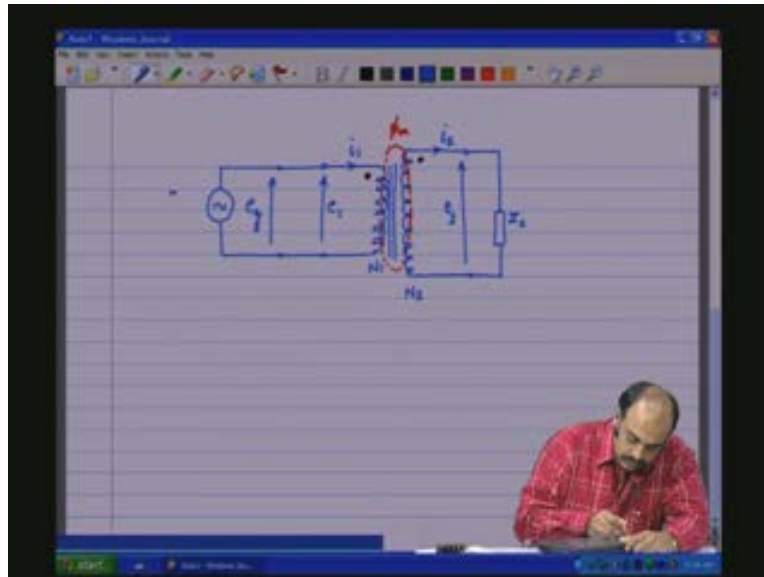
Then we also noted that inside the core there is a flux linking ϕ_m . And, from the Faraday's law we see that the induced emf is equal to $N \frac{d\phi}{dt}$ which is equal to $N j \phi_m$. Now, induced emf e is in this direction this is e_1 direction that if we are taking the induced emf across the primary $e_1 = N_1 \frac{d\phi_m}{dt}$. So either e_1 or e_2 one could take and we know that they are in-phase and they are along the alpha axis. Now ϕ_m lags e_1 by 90 degrees because of the j operator. Or it basically means that if I have ϕ_m now the vector of ϕ_m direction is rotated by j which is rotated anticlockwise plus j rotated anticlockwise by 90 degrees to obtain the direction of the induced emf for the induced voltage which is e . So this would be ϕ_m direction. This is the phasor diagram for this idealised transformer.

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Now let us have a simple example to understand the various points here. we have a source, an AC source, so the AC source is connected to the transformer terminals, there is a core and it is a step up transformer let us say and this is connected to a load as shown like this. This is our e.g. generator voltage, this is e_1 the primary induced voltage across the primary of the coil, there is an e_2 the secondary induced voltage across the secondary of the coil, this is Z_2 the secondary impedance that is applied across the secondary terminals, we have N_1 number of turns here, N_2 number of turns here, there is a primary current i_1 flowing into the dot, there is a secondary current i_2 flowing out of the dot and of course there will be a flux which links the two coils and causes the energy to flow from the primary to the secondary.

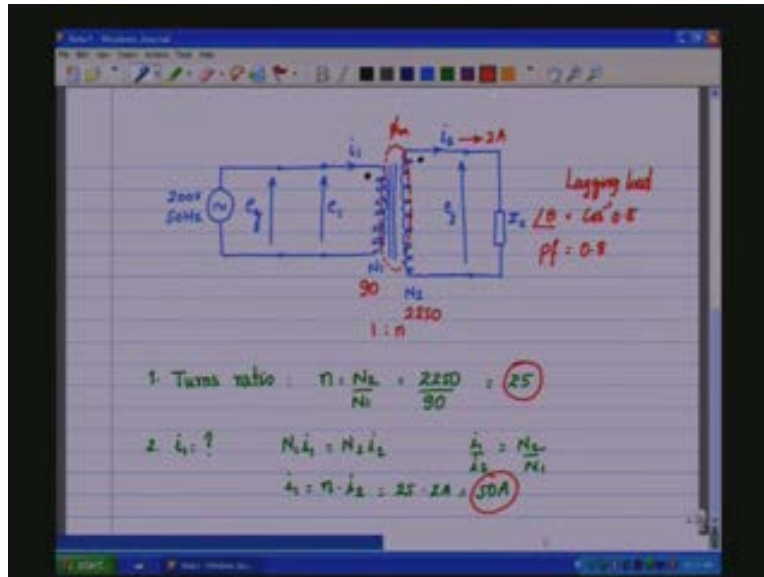
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Therefore, for this circuit let us say we have 200 volts 50 hertz as the source and let us say this is 90 turns and this is 2250 turns N_2 , let us say that there is a load current of 2 amps that is flowing with a lag angle of with a lag angle θ of \cos^{-1} of 0.8 or power factor equals 0.8. Let us say lagging load which means Z is inductive RL kind of a load.

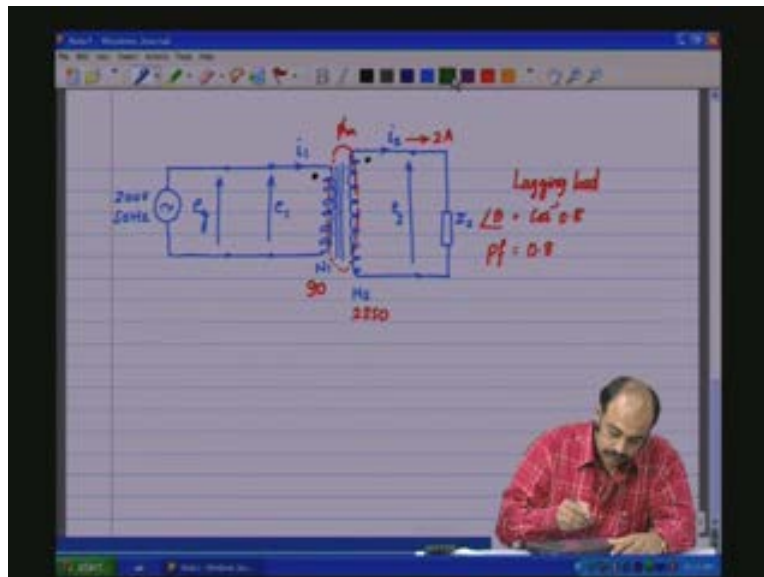
Now this being the description of the circuit let us try to see what information that we can extract and this is a 1 is to n transformer. So let us first see what is the turns ratio. The turns ratio is n which is equal N_2 by N_1 which is equal to 2250 by 90 which is equal to 25. Then let us see what is the primary current i_1 equals what? The secondary current is 2 amps because rms; all the values generally that are mentioned are effective values or rms values unless otherwise specified. So we know that $N_1 i_1$ ampere-turns should be equal to $N_2 i_2$ ampere-turns. and also from the power equation we also have earlier seen that let me use the same notations $N_1 i_1$ is equal to $N_2 i_2$ because we see that i_1 by i_2 is equal to N_2 by N_1 inverse proportionality for the currents. Therefore we have i_1 which is equal to n into i_2 which is 25 into 2 amps is 50 amps. So, for a 2 amps secondary current the primary current is 50 amps rms because it is a step up because it is a step up transformer.

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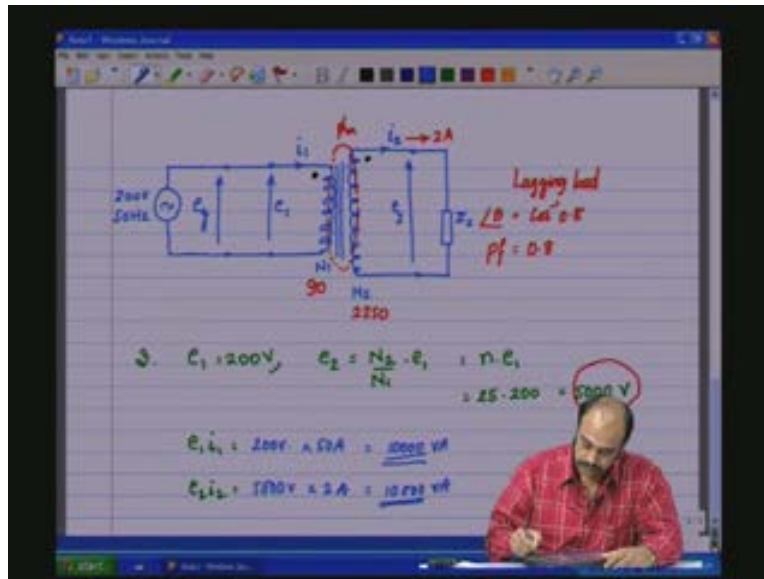
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Third point: e_1 equals 200 volts. So what is e_2 ? e_2 is equal to N_2 by N_1 into e_1 . Or in other words, n times e_1 which is equal to 25 into 200 which is 5000 volts **5000 volts**. Note: e_1 i i is

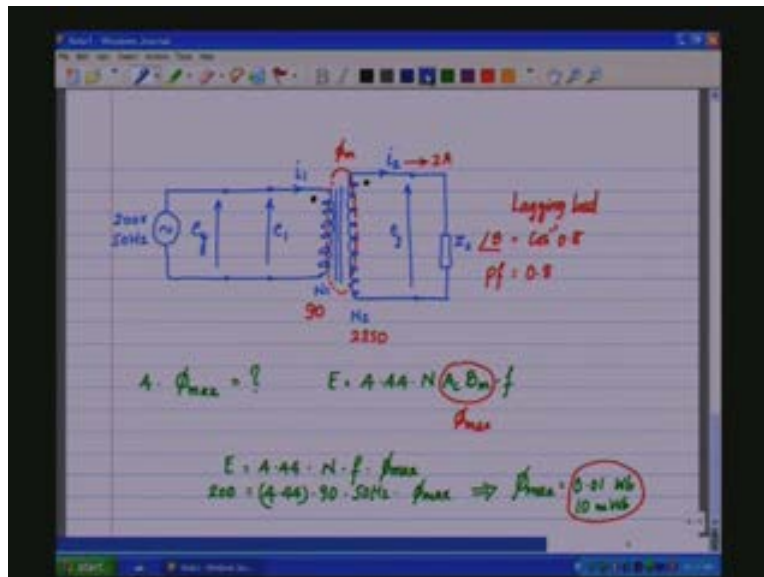
equal to 200 volts into 50 amps which is equal to 10000 volt amps; $e_2 i_2$ which is equal to 5000 volts into 2 amps which is equal to 10000 volt amps VA, this power balance should happen.

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Then what is the information can we extract from this particular diagram; what is **what is** phi max that one can extract from this diagram. We know that e is equal to $4.44 N A c B m$ into f . What is this one? Flux density into $A c$ is nothing but phi max or phi m. So e is equal to 4.44 into N into f into phi max or phi m. One could take either the secondary side or the primary side. taking the primary side we have e equals e is $200 \cdot 4.44$, N is **90 turns on the primary side** 90 turns on the primary side, frequency is given as 50 hertz of the..... and then phi max this is the only unknown and therefore phi max can be calculated which gives about 0.01 Weber or 10 milliwebers.

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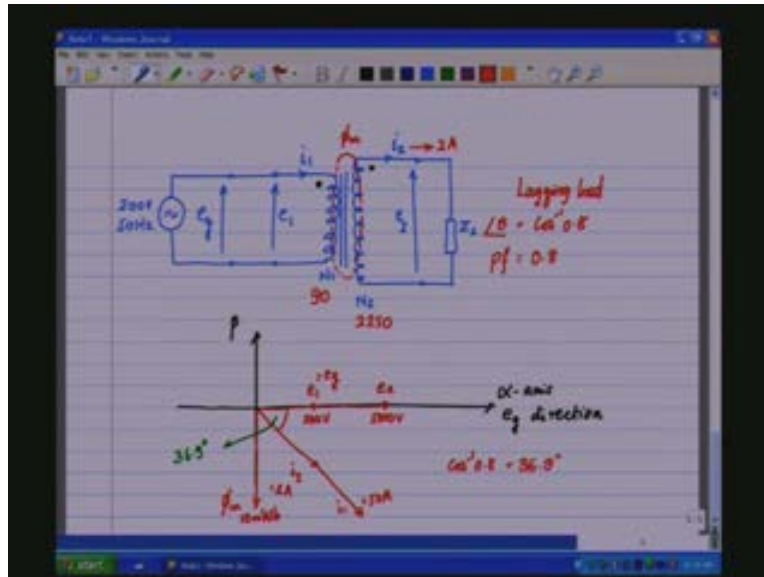
Like this we get the information about the transformer using basically the fundamental laws law which is the Faraday's law of electromagnetism and also one more law which is the Ampere's law. Now let us draw the phasor diagram for this circuit.

To draw the phasor diagram (Refer Slide Time: 18:27) we take the two axis in this spatial coordinates, this is the beta axis and this is the alpha axis. So the alpha axis direction is taken as e_g direction, the generator voltage direction itself. So we have even which is 200 volts and we have e₂ which is 5000 volts; e₁ equals e_g of course and what is given is lagging load, i₂ is lagging e₂; this is i₂ this is lagging e₂ by an angle which is cos inverse of cos inverse of 0.8. So cos inverse of 0.8 is 36.9 degrees. Therefore this angle is 36.9 degrees.

And what about i₁?

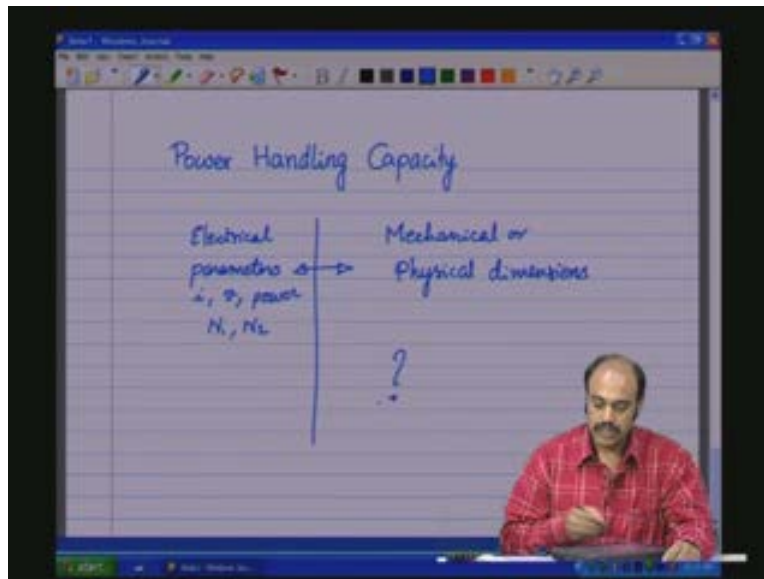
i₁..... now this i₂ is 2 amps, i₁ is 50 amps; of course I am not..... This is i₁ which is equal to 50 amps and then there is phi_m, this is phi_m which is 10 milliwebers, 5 m is also in rms quantity because it was calculated using rms quantities. so this would be the phasor diagram for this transformer circuit.

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Now let us try to understand one more important aspect in the transformer which is the power handling capacity of the transformer. You have the small transformers, the big transformers, the huge transformers, the oil cool transformers so on and so forth. So you have transformers with varying sizes right from small segment to very high power transformers. Now what decides the power handling capability of the transformer? So, to do that we need to understand the mapping between the electrical and the mechanical or the physical dimensions **or the physical dimensions** of the transformer, **physical dimensions**. So you have the electrical parameters in terms of the currents the voltages, the power, number of turns N_1 N_2 so on and so forth and to that we need to have some kind of a relationship with the physical dimensions of the core with respect to the core sizes the sizes of the wire gauge and things like that one. so what is that mapping and how is that done?

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Let us have a look at the transformer **to understand** to get a better understanding of what are the physical dimensions that we are going to talk about.

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Let us have a look at the transformer to understand its physical dimensions. You see the picture of the transformer here. The transformer consists of the terminals there. So, in a practical transformer the terminals are brought out and that is where the coil windings are terminated by soldering, the rectangular solid block that is the core. In this case the core is CRGO that is Cold Rolled Grain Oriented silicon steel and as the steel is conductive electrically we would have lot of eddy currents if we have a solid block of steel **as we discussed in the last class** so to reduce the cross-sectional area for the eddy current path and thereby increase the resistivity to increase the resistance for the eddy current path the laminations have been put and see that all practical transformers will have **the core** the solid bulk of the core which is laminated and then after laminating **it is** all the air pockets are vacuumed out and it is varnished so that black colour or the darkish colour is due to the varnish.

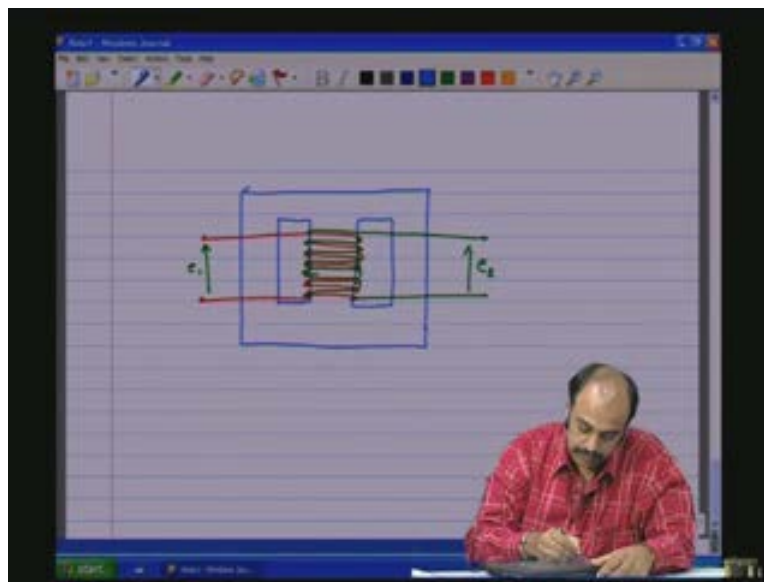
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Now you see at the winding at the centre which goes through a central limb. Now this consists of both the primary and the secondary. So the primary is wound first and then the secondary is wound on that one about the central limb. So the core here looks slightly different. **Let me give you a better idea by drawing the two dimensional view.**

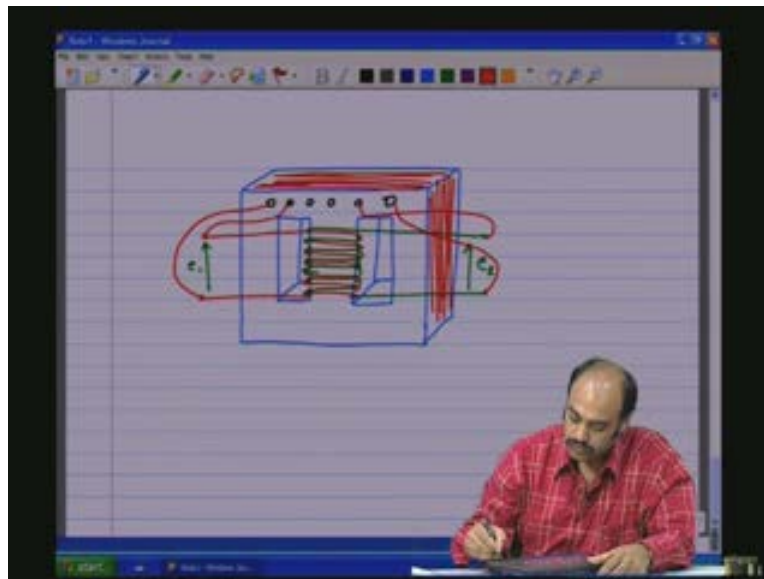
The core will be something like this. We have this rectangular block solid block and then we have a central limb (Refer Slide Time: 25:32). Now the winding is done on the central limb here in this case. So you see the winding is done on the central limb so let us say this is the primary and the secondary is also wound on to the same central limb like that so they are wound one above the other with an insulation layer in between so this would be the secondary; this is e_2 , this is e_1 .

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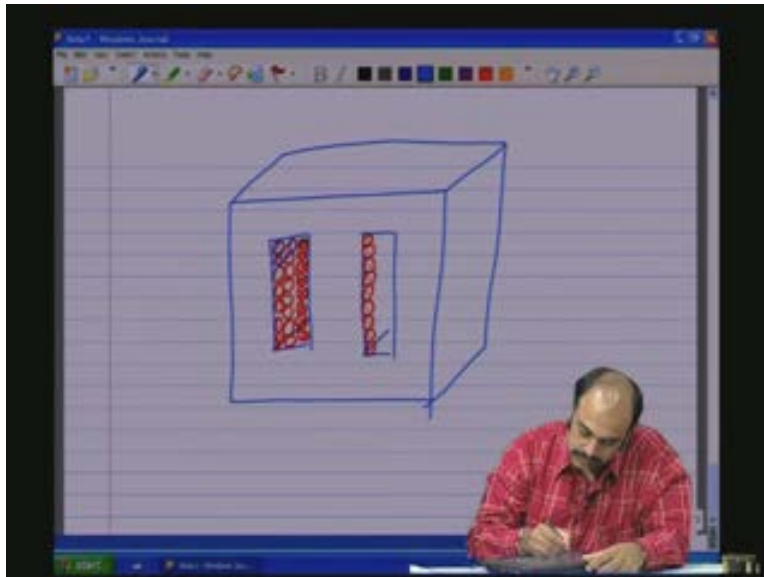
Now here of course if I make into an isometric 3D **this will look more like the....** this will look more like the transformer that we have here and we have the terminals which are brought out here and actually these points are soldered on to this so you could have these points which are soldered on to this and the secondaries which are soldered on to this and these are the laminations for which you will see the lines here which are the thin laminations of the grain oriented silicon steel.

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Now here you need to worry about few physical dimensions **which I will explain now**. We will take that same core, we have the central limb **we have the central limb**, this is the three dimensional view of the core. So we have the winding around the central leg. What is the maximum amount of windings that one can accommodate? If I cut you will see that this is the number of windings that can be accommodated both primary, secondary all put together. So what winding goes out will come out through this so it is the same loop (Refer Slide Time: 28:49) so what goes out through here will come out so they have the current direction as coming out here and going in there in this way.

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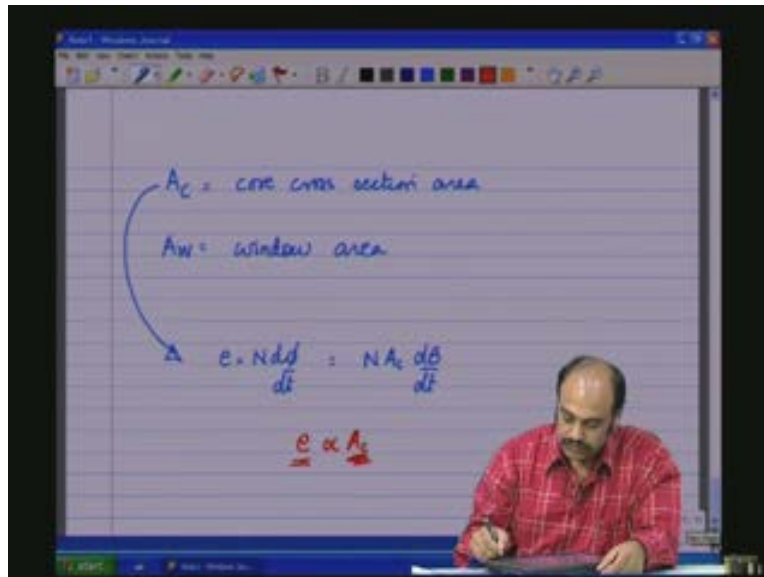


Therefore, effectively we have this area only available to accommodate the windings that is called the window area A_w . There is another important area that you need to bother about and that is **that is** you have this of the central leg ((Refer Slide Time: 29:49) or let me draw the other area which is also very important and that is the core cross-sectional area; now this core cross section area **let me show it in this different colour** in this 3D image here like that through which the flux are flowing the perpendicular cross section area that is the A_c or the core cross section area.

Therefore, we have two areas here: one is called A_c which is the core cross section area, another is called the A_w which is the window area. These are the physical dimensions of the core. How will you map these physical dimensions to the electrical property? One is by the Faraday's law. So this is linked this e is equal to $N \frac{d\phi}{dt}$ this is the Faraday's law this we know which is equal to $N A_c \frac{dB}{dt}$. Thus, if you see that for a given number of turns and for a core with a particular swing in the flux density e is proportional to A_c ; the core cross-sectional area determines the voltage withstanding capability for the transformer. How much voltage it can withstand beyond which the transformer saturates because beyond which the volt second product

will exceed the flux density capability of the core and the core will saturate. Therefore, A_c will tell you how much voltage a particular core can withstand.

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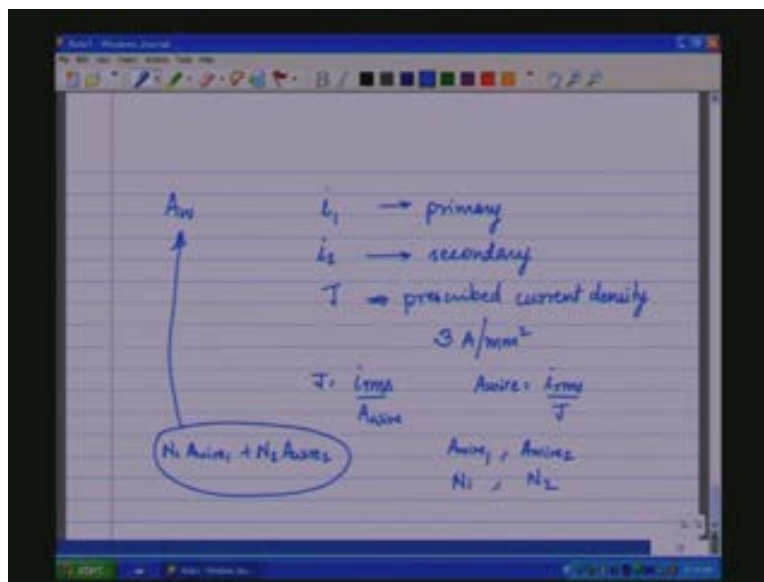
Then you have the A_w . A_w is related to a **number of** number of turns that can be accommodated in the window area both primary, secondary put together. Hence, now that is going to depend on the thickness of the copper wire that you are going to use to do the windings and the thickness of the copper wire is decided by what; it is decided by the currents flowing through the copper wire because there is i^2 or loss and we cannot have too much loss in to the copper and therefore **if we have i_1** if we have i_1 as the current flowing through the primary and i_2 as the current flowing through the secondary and let us say that J is the prescribed or the recommended current density that can be allowed in the copper normally this is around 3 amps per mm square; this is a typical value; it can vary from design to design, power rating to power rating but 3 amps per mm square is a typical value that one can expect in the copper wires of the transformer.

Therefore, what is current density J ?

Current density is i_{rms} by area of cross section of the wire **area of cross section of the wire**. So area of cross section of the wire is dependent on i_{rms} by J the recommended current density. So you will obtain the A_{wire} for the primary in the area of cross section of the wire for the secondary based on i_1 by J and i_2 by J .

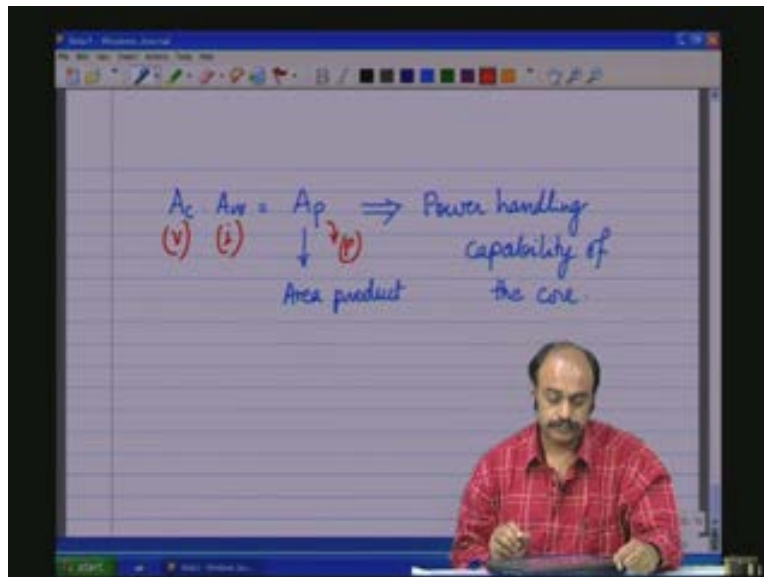
Now there are **N number of** N_1 number of windings for the primary wire, N_2 number of windings for the secondary wire. So N_1 into its cross section area plus N_2 into the wire's cross section area of the secondary all these must fit into the window area into the available window area of the core.

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Now A_{wire} is dependent on the rms value of the currents. So the current carrying capability is decided by A_w , the voltage capability is decided by A_c or the cross-sectional area of the core. Therefore, the product A_c into A_w should decide or it should give an indication on the power handling capability of the core. this is the area product A_p this is called the area product and this should decide the power handling capability of the core because this is deciding the voltage handling capability, this is deciding the current handling capability and therefore this decides the power handling capability of the core.

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This is the electrical parameter which is trying to map to the physical dimensional parameter of the core called the cross-sectional area of the core, another electrical parameter the current rms current which is trying to map to the window area of the core another physical dimensional quantity of the core; A_c , A_w and sometimes A_p are given the data sheets of the core, v and i are calculated from the electrical circuit and by this mapping one can choose what should be the size of the core for a given power handling capability for a given **power of the.....** power that has to be passed from the primary to the secondary the sizes of the core will be decided.

Now there is one more important deduction that we could do from order from the already known formulae that we have and that is E which is equal to $4.44 N A_c B_m$ into f . this is the standard formula the voltage induced formula that we derived for sinusoidal excitations for a transformer based on the Faraday's law of electromagnetism.

Now here for our mains which is 50 hertz **which is 50 hertz** we are using..... normally the most common types of core materials in the market would be..... especially for the 50 hertz frequency is the CRGO which is Cold Rolled Grain Oriented or CRNGO the Cold Rolled Non-

Grain Oriented which is slightly less expensive. So, if we are using let us say CRGO the Cold Rolled Grain Oriented this would be around 1 tesla the design value.

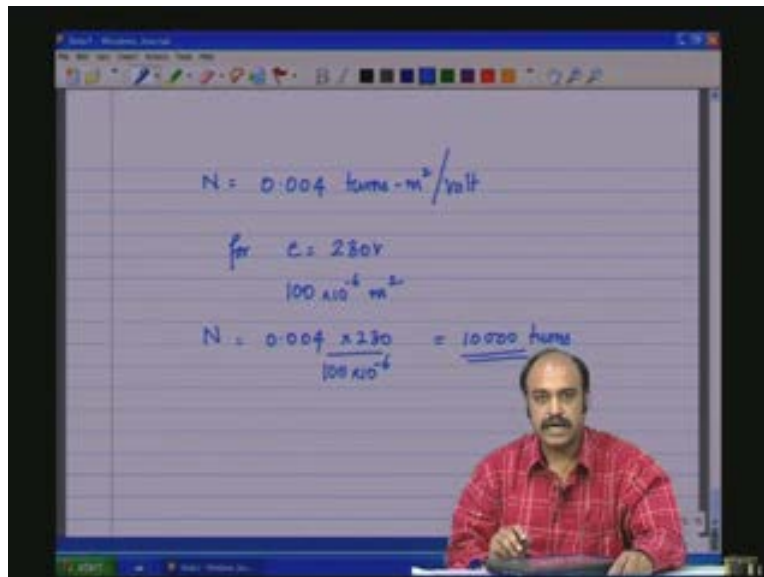
Let us say for 1 volt for an A c of 1 metre square we have 1 which is equal to 4.44 into 50 into 1 tesla into 1 metre square into N. So N is equal to 1 by 4.44 into 50 which is approximately 0.004 term metre square per volt this is for 1 volt so many turns for 1 volt the area comes in so there is a metre square per volt, this would be an approximate thumb rule which will give you at least a figure of what will be the order of magnitude of the turns that you would be applying.

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The image shows a whiteboard with handwritten equations and annotations. At the top, the equation is $E = 4.44 N A_c B_m f$. Red arrows point from the terms to their values: E is 1V, A_c is 1m^2 , f is 50Hz, and B_m is CRGO (Cold Rolled Grain Oriented) with a design value of 1T. Below this, the equation is rearranged to solve for N: $1 = (4.44)(50) \cdot (1) \cdot (1\text{m}^2) \cdot N$. The final result is $N = \frac{1}{(4.44)50} = 0.004 \text{ turn-m}^2/\text{volt}$.

So N is equal to 0.004 turns metre square per volt. So let us say for e is equal to 230 volts and a core area which is 100 into 10 to the power of minus 6 metre square this is the order of the magnitude of the core 100 to 1000 into 10 to the power of minus 6 metre square which you will see the core cross-sectional area. let me take a value of 100 into 10 to the power of minus 6 metre square easy for calculation it is a small core then what is the turns that one would get which would be 0.004 into 230 divided by 100 into the 10 to the power of minus 6 this is approximately 10000 turns.

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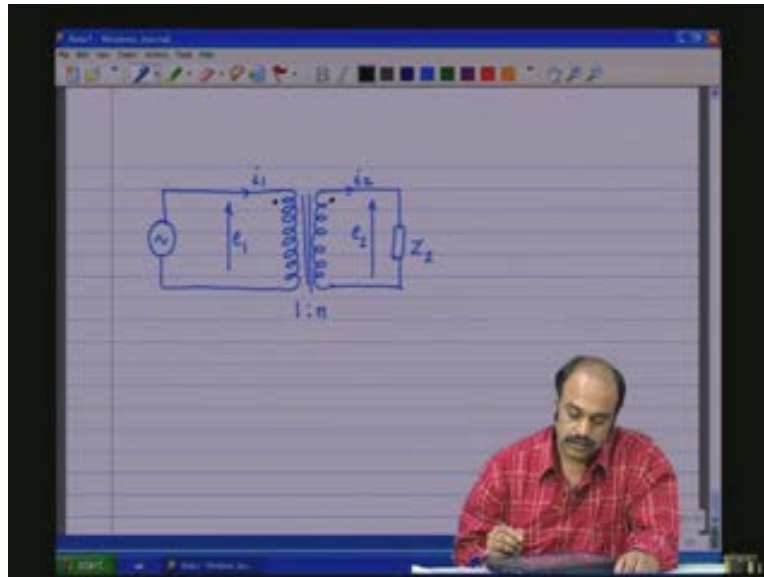


Therefore, a core of this size 100 into 10 to the power of minus 6 metre square core cross-sectional area if you want to apply 230 volts you will have to approximately put around 10000 turns so that it can withstand that voltage and deliver the power from the primary to the secondary.

Now there is another important concept that one need to know about the transformer. The transformer we said has some applications. It can be used for isolation and it can be used for power transfer with step up or step down to make the voltages and the current compatible between one part of the electric circuit to the other part of the electric circuit.

There is another important function the transformer can do, the impedance matching **impedance matching**. What I mean by this is if you have a transformer and we have the source, the core like that and let us say we have a impedance Z_2 here, the turns ratio is 1 is to n , the voltage across this was e_2 , current through that was i_2 , current through this primary is i_1 and the voltage here is e_1 or e_g and of course there is a dot polarity coming in to the picture.

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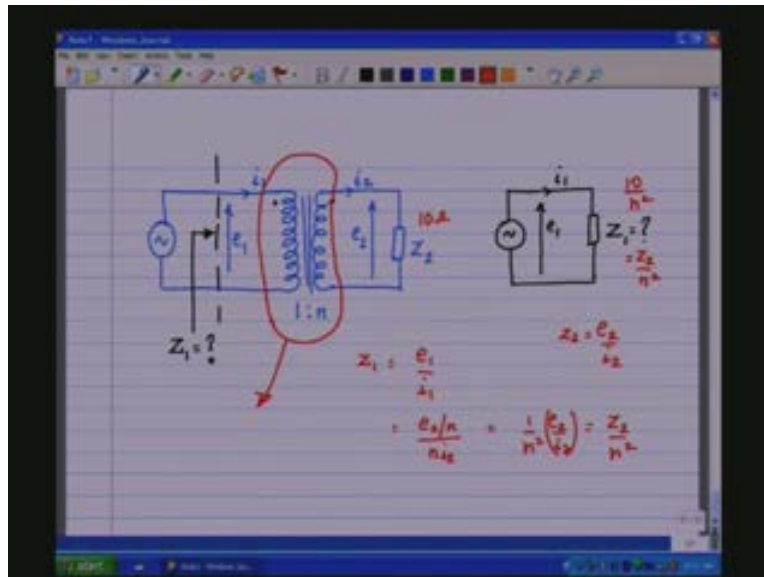


Now it is interesting to see what is the impedance seen by this source **what is the impedance seen by the source**? The source impedance which means what is it equivalently the value of Z_1 if we are to get the same i_1 and e_1 . So what is Z_1 such that we get the same i_1 and of course the voltage across is e_1 ? So source impedance Z_1 is equal to e_1 by i_1 but this is..... the Z_1 is supposed to be the reflected source impedance as seen from the primary. But the actual real impedance is on the secondary side and that is Z_2 and Z_2 is e_2 by i_2 .

So let us express e_1 and i_1 in terms of e_2 i_2 that is the secondary side. we know that e_1 is nothing but e_2 by n or e_2 is n times e_1 e_1 is equal to e_2 by n divided by i_1 in terms of i_2 would be $n i_2$ which is 1 by n square e_2 by i_2 which is Z_2 by n square. What it means is that if I have Z_2 here it is equivalent to having a Z_1 which is equal to Z_2 by n square at this point on the primary side. Therefore, if I have the resistance of 10 ohms here and I have a turns ratio of n then what appears here is 10 ohms divided by n square. Now if n is 2 it would be 10 by 4 which is 2.5 ohms on the primary side so it is a reflected resistance as seen by the source; a real 10 ohms load is seen as 10 ohms by n square by the source. Actually the transformer is doing a transformation on the impedance value (Refer Slide Time: 48:40) and that is a very powerful

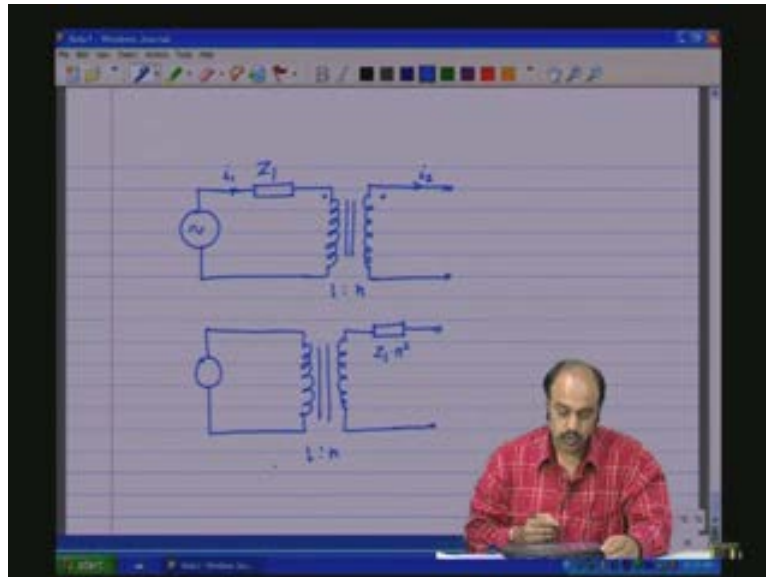
feature especially when you want to mash the impedances of the primary side circuit and the secondary side circuit.

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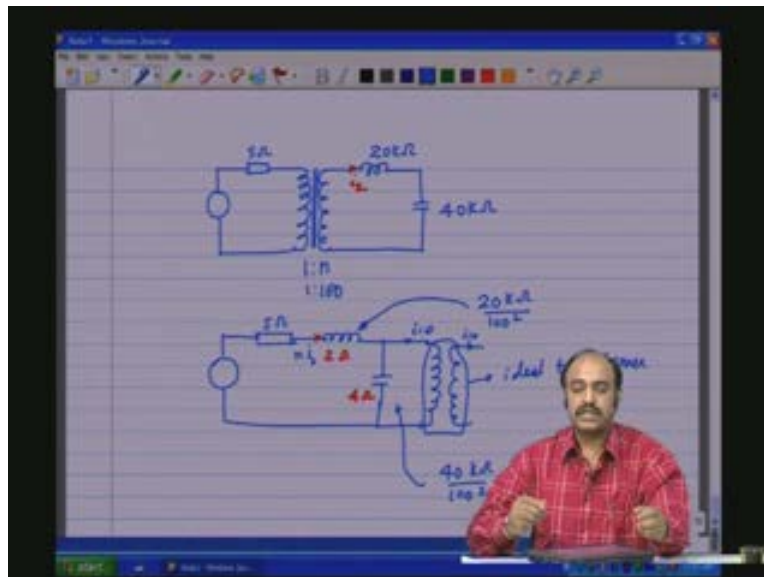
Likewise, any impedance on the primary side to be reflected on the secondary side it is **of dual**. So, if I have an impedance on the primary side, let us say I have Z_1 a dot polarity there is a current i_1 and there is a current i_2 this can equally be represented as an impedance on the secondary side which is Z_1 into n square where 1 is to n is the transfer ratio of the transformer or if you have an impedance on the secondary side it can be reflected on to the primary side by dividing by n square.

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Therefore, if I have..... so, if I have a transformer with a core 1 is to n and let us say I have a series resistance of 5 ohms and this is 1 is to 100, n is 100 and on the secondary side I have an inductance and a capacitance an inductive reactance of let us say 20 kilo ohms, capacitive reactance of 40 kilo ohms then I can equivalently represent it as looked from the primary side as a resistance 5 ohms, an inductance and a capacitance and a transformer which is like that. So, the amount reflected will be 20 kilo ohms divided by 100 square will be here, 40 kilo ohms by 100 square would be here which means this would be 2 ohms and this would be 4 ohms by n square. this is And the current through this one if this were i_2 the current through this one would be also i_2 by sorry $n i_2$ the current through this would be $n i_2$ and the current here is equal to 0 current here is equal to 0 this being an ideal transformer ideal transformer.

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So the transformer has very interesting properties of impedance matching. We will do few examples on this impedance matching issues in the next class and then look at discuss the practical transformer in slightly more detail, thank you.