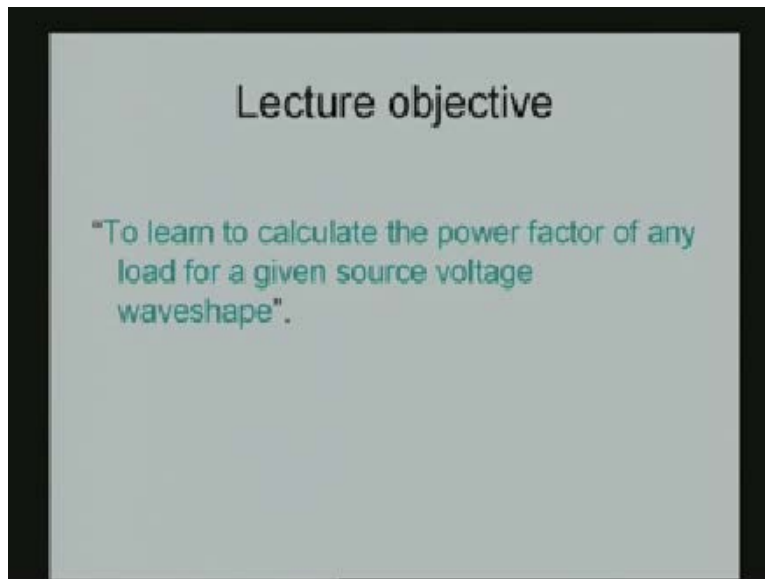


Basic Electrical Technology
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Department of Electrical Engineering
Indian Institute of Science, Bangalore

Lecture - 15
Power Factor

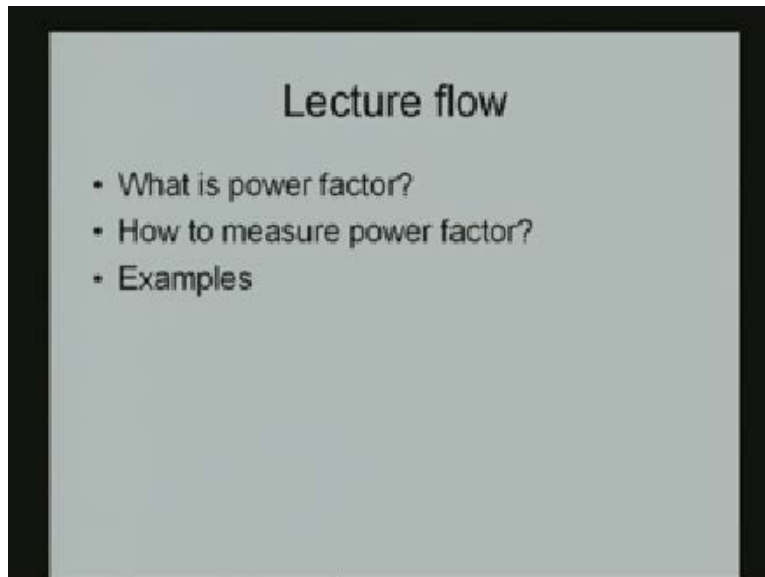
Good day to all of you. Today we shall learn about the concepts of power factor. The objective of this lecture is to learn to calculate the power factor for any load for a given source voltage waveshape that's what you will be able to do at the end of this lecture.

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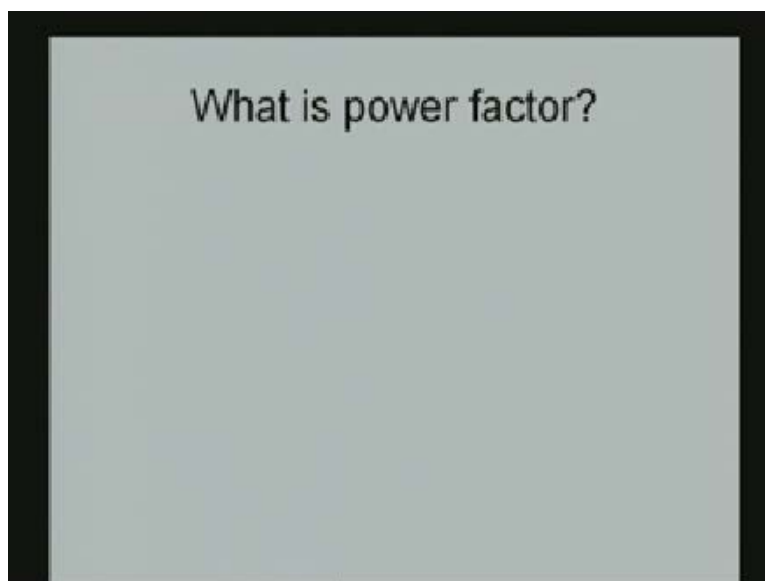
Based on the above objective, this is the lecture flow.

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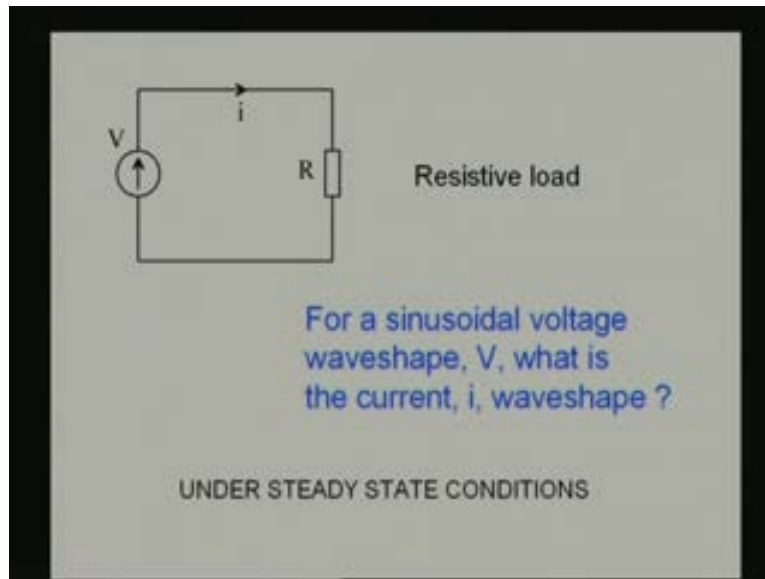
First we shall see what a power factor is, what is it all about. Next we should see how to go about measuring the power factor or calculating the power factor from the measured power and then we shall see few examples.

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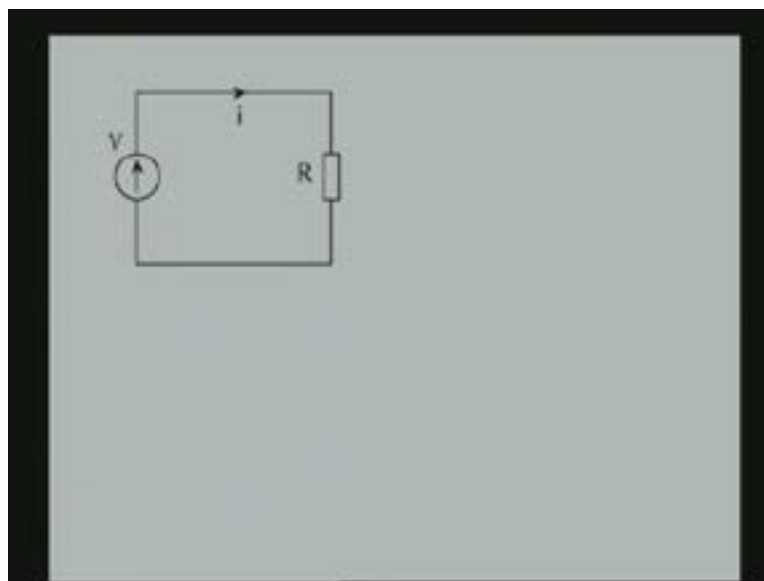
So the first topic which is what is power factor.

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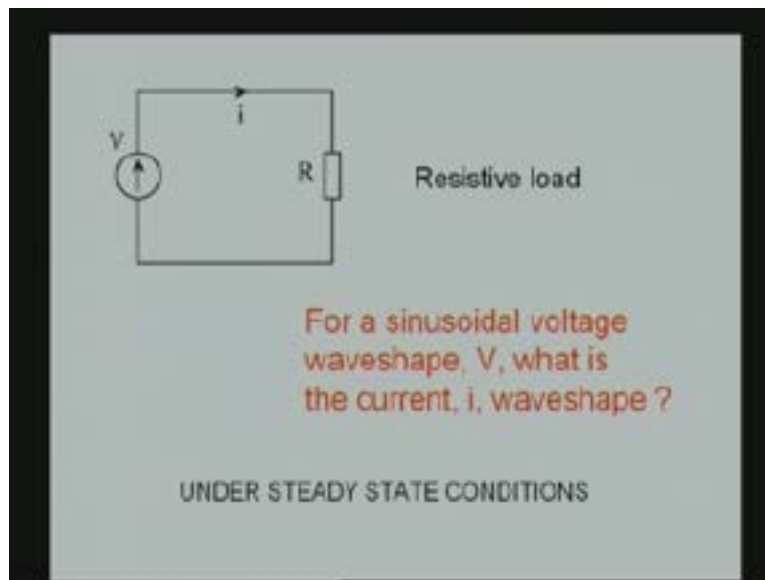
First consider the following circuit.

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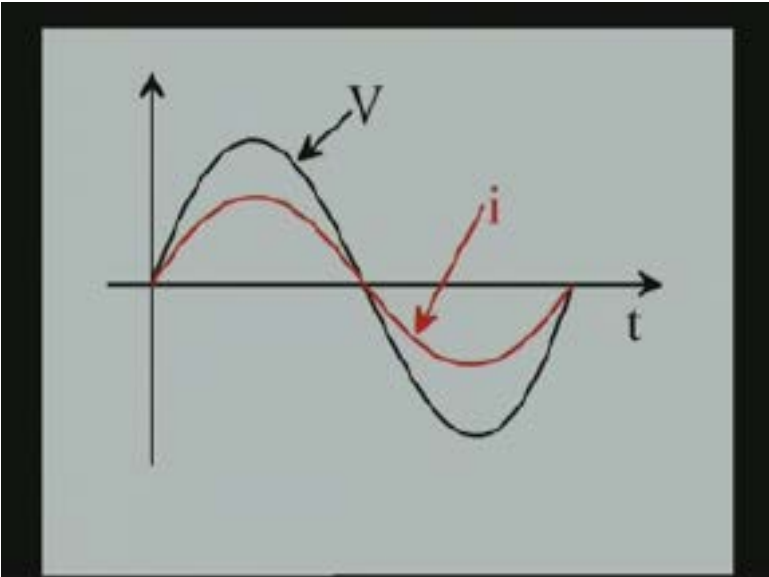
It is a simple circuit with a voltage source of an arbitrary waveshape and also consists of a resistance, it is a resistive load, and as shown in the diagram there is a current i also flowing through it.

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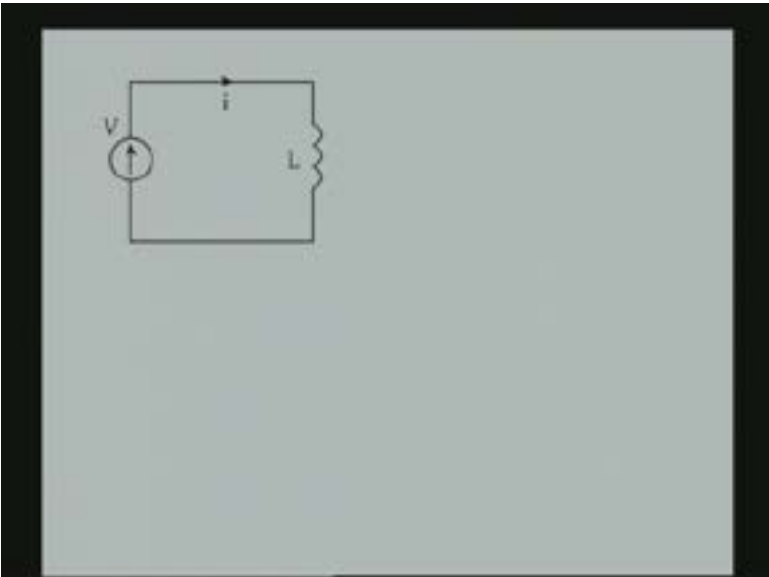
Now let me ask a question. For a sinusoidal voltage waveshape how will the source voltage waveshape and correspondingly the current waveshape look like under steady state conditions. So let us look at that.

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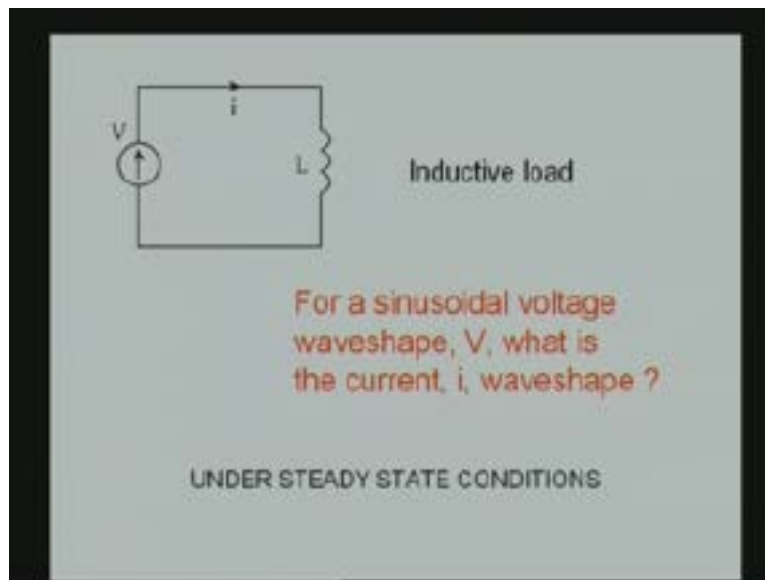
We have here the x axis which is the time axis and y axis the amplitudes, the black curve is the voltage waveshape which is the sinusoidal voltage waveshape, and it is a source voltage. The red colour as you see is the current waveshape. So, for a resistive load pretty simple, you see that the current is in-phase with the voltage because it is just V by R .

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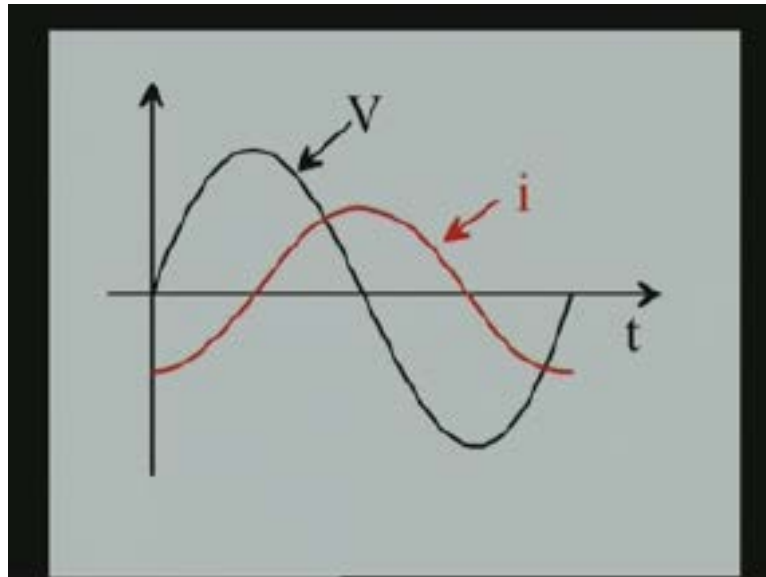
Now consider instead of a resistive circuit an inductive circuit, the load is inductive because the R has been replaced by L here as you see. Now what happens if we apply a sinusoidal voltage to this inductor under steady state conditions?

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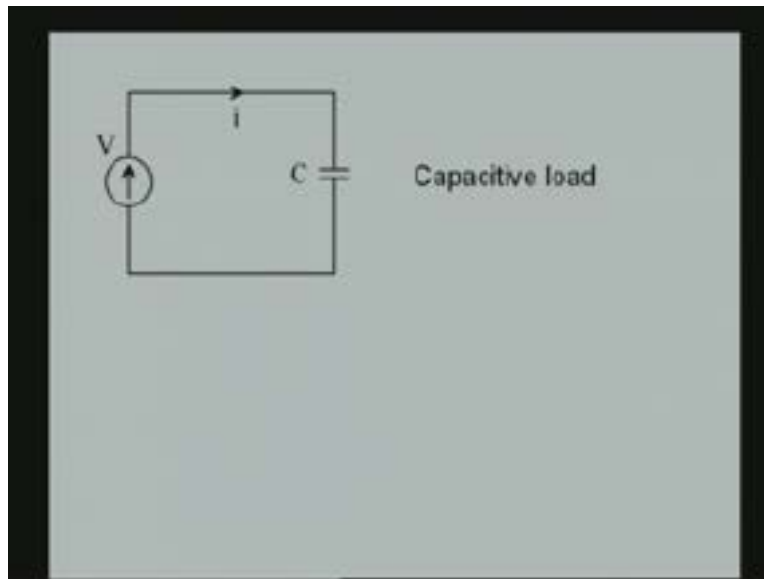
You see the same time axis that is the x axis, this is the time t and the y axis is the voltage, voltage being black in colour as usual and the current is red in colour.

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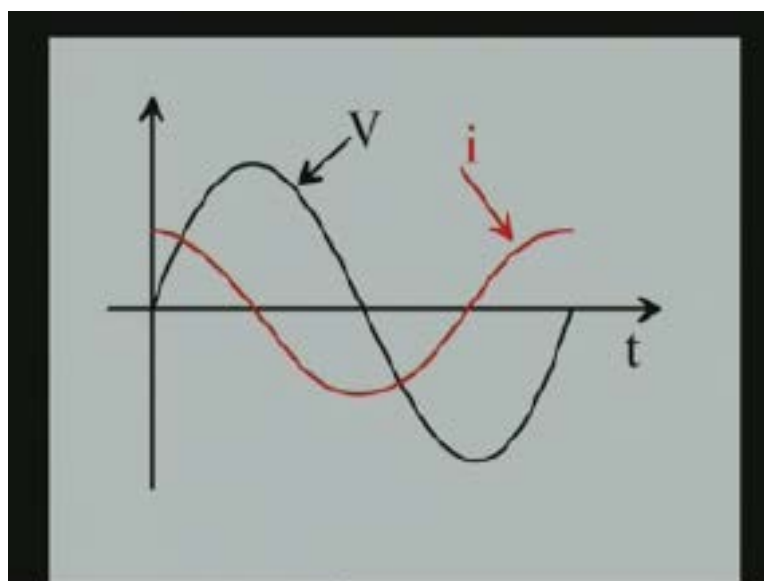
What do you see? What is the difference between the previous voltage waveshape in the case of the resistive load and that of now with the inductive load? You see that the current lags the voltage. The current crosses the zero and goes positive much later than the voltage waveshape which started going positive crossing zero right at the origin. So basically we see that in the case the inductive circuit the current is lagging the voltage. Now let us consider one more modification that is the capacitive circuit.

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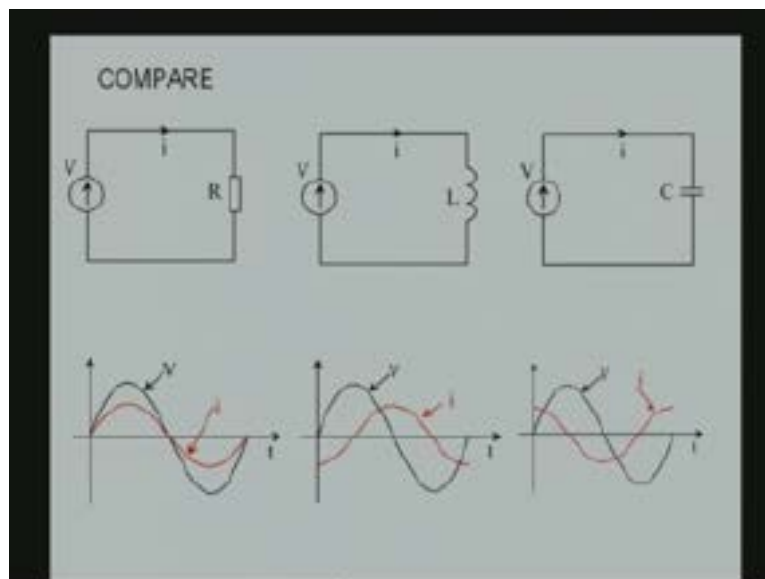
Here the load is a capacitive. Let us apply the same sinusoidal voltage waveshape through this capacitor on this capacitive load and see what happens to the current waveshape under steady state conditions again (Refer Slide Time: 05:15).

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So we see again here the same familiar axis, the time axis t , the amplitude axis with the source voltage being black in colour, the current being red in colour. See the difference here. Here the current is crossing the zero going positive before the voltage source goes positive towards the end of the circuit. Or in other words, if you look **towards the origin of the** towards the origin when the voltage is crossing zero and going positive the current is already positive which means the current is leading the voltage. So in all capacitive circuits as you know the current will lead the voltage waveform therefore they are called the lead networks and the inductors as the current lag the voltage they are called the lag networks. Let us compare all the three circuits here.

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You have the resistive circuit the R network, the L load that is the inductive load, the C load the capacitive load all three put side by side there and the corresponding voltage source waveforms which are all sinusoidal in all the three cases and the corresponding currents. Currents are all red in colour as usual and you see that the current for the resistive load is in-phase with the voltage, the current for the inductive is lagging the voltage and the current for the capacitive is leading the voltage.

Why have I taken only these three types of loads?

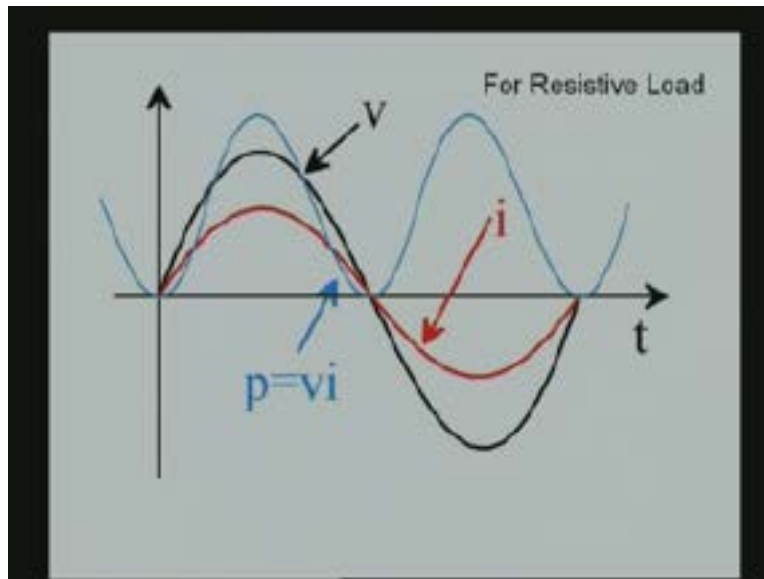
Generally if you see when energy is flowing from source to destination what all can happen to the energy. There are only four things that can happen. One: the energy while flowing from the source to the destination can get dissipated; meaning it can become unavailable to the destination. Second: the energy can get stored. It can get stored in two ways: one is it can get stored in the kinetic form. As an inductor where the energy is stored as $\frac{1}{2} L i^2$ where i is the flow flowing and therefore it is related to the kinetic form of energy storage.

The other form of storage is the potential storage like in a capacitor where the energy storage is in the form $\frac{1}{2} C V^2$, the effort or the voltage is the medium through which the energy gets stored. And there is also a fourth action that can happen that is energy can also get transformed from one domain to other. It can get transformed from electrical to magnetic, magnetic to electrical, electrical to mechanical, mechanical to hydraulic all these transformations can occur as energy flows from source to the destination.

But if you see the energy basically can get lost that is become unavailable that is resistive load or can get stored either as an inductive form, kinetic form or capacitive form potential form. That is why these three basic types of loads that is a pure resistance; a pure inductance and a pure capacitance are the ones that equivalently exist in any complex networks. And therefore these three are the ones that are basic to the understanding of the power factor also.

Now coming back here (Refer Slide Time: 09:26) consider the resistive load circuit. Now in the resistive load circuit we saw that the voltage source waveform and the resulting current waveform are like this that is you have as usual the black voltage source waveform sinusoidal and the current waveform which is red in colour. Now to this we shall apply a process that is we shall do a process and what is the process; we will multiply the voltage waveform and the current waveform. What do we get? The voltage is in volts, the current is in amps when you multiply V and i we get watts. So we will produce the power waveform in watts, instantaneous power waveform. so we are going to produce the waveform P which is equal to $V i$ by multiplying these two signals every point by point, instant by instant.

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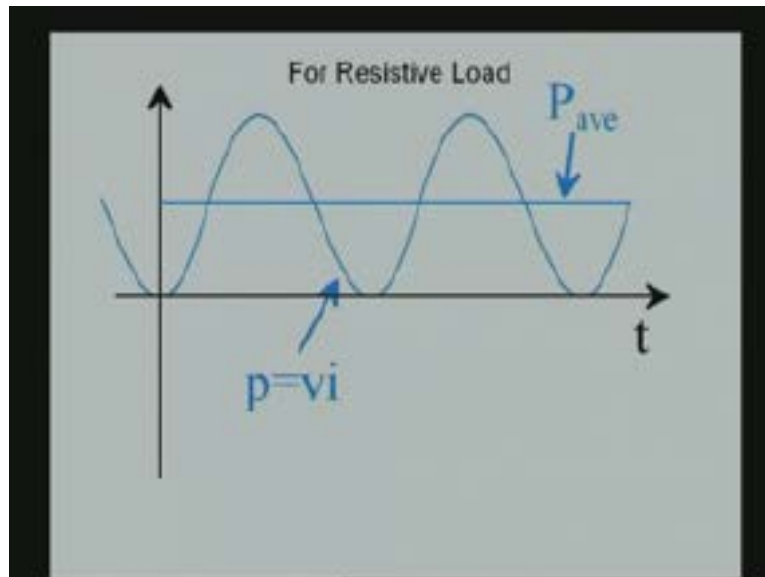


Therefore, on multiplication you see that there is one more waveform added on to the axis the blue coloured waveform which is the instant by instant multiplication of the voltage in the current waveforms. notice that at the origin and the current and the voltage both are zero, power is zero, gradually the power is positive rising up because both the currents and the voltages are positive and then again at the 180 degree point voltage and current have become zero, power has become zero and then further on both voltage and current becomes negative so product of that is still positive and therefore the power waveform continues to again rise up and become positive. so you see some kind of a sinusoidal it is a sinusoidal waveform, the blue coloured waveform, the power waveform; it is shifted up it does not have any negative component in this case it has an offset and it has a frequency which is double the original voltage and current waveforms. Of course we will come to the mathematics of that later on much later but you try to understand the basic concepts graphically first. So this is the power waveshape.

Now what is the average value?

So the average value let us unclutter the graph, we remove the source voltage and the current waveform; just keep the power waveform and the average. So we have a finite average as indicated by the thick line shown on the graph.

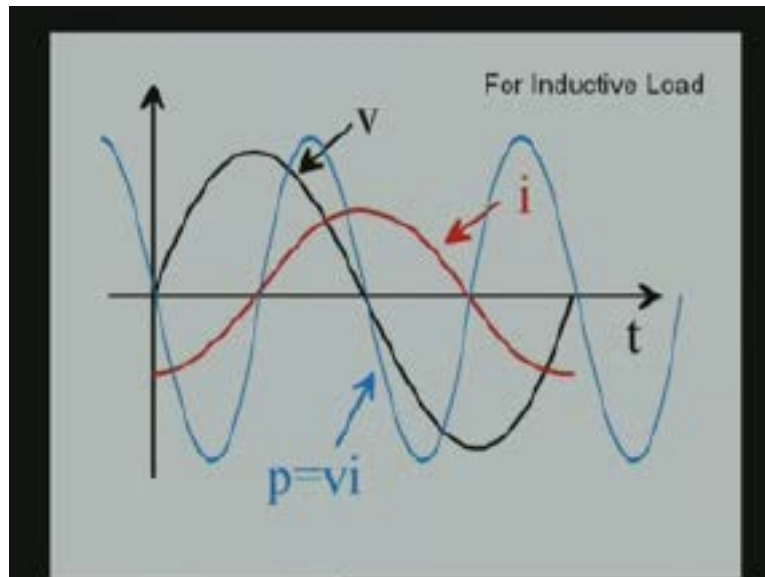
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Incidentally if we have V_m as the maximum source voltage and I_m as the maximum current then $V_m I_m$ by 2 will be the average.

Next consider the inductive load. In the case the inductive load we saw before actually this specific graph the current lags the voltage as you see. Now to this also we apply the same process the process of multiplication multiplying the voltage waveform and the current waveform to produce the power waveform p is equal to vi and that we will sketch it in blue colour. You see what happens, much different from the resistance waveform is it not?

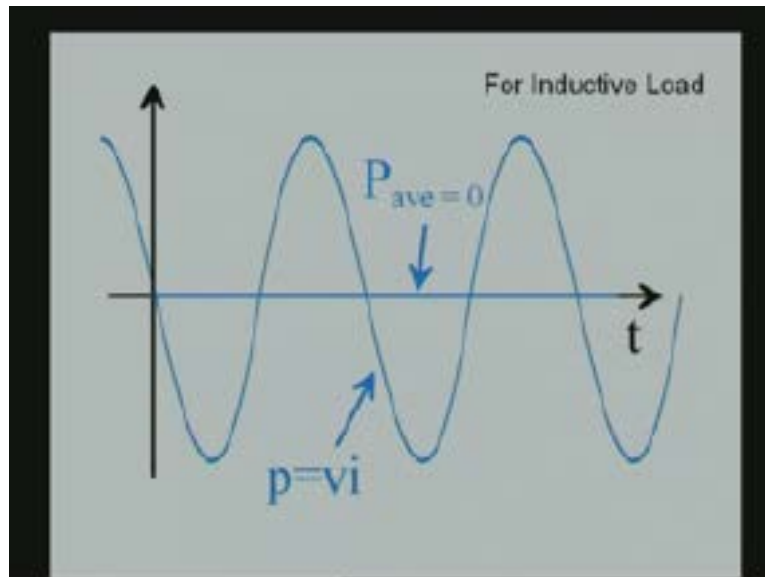
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Here you see at towards the origin that is at angle zero, the voltage is zero trying to go positive, already the current is negative because it is a lagging circuit. At that point the power is zero and the power starts going negative because voltage is positive current is negative and therefore the product has to be negative. So you have the sinusoidal first the negative half coming in and then at the 90 degree point corresponding to the voltage waveform you see that the current is zero at that instant, voltage is maximum at that instant and the product is zero at that instant as you see in the blue coloured waveform. And then further on the power waveform goes positive and traces the positive half of the sine power sine and so on as **you would see can** you can reduce from the voltage in the current waveform and then point by point multiplication. So this is the product curve or the power curve the blue waveform here.

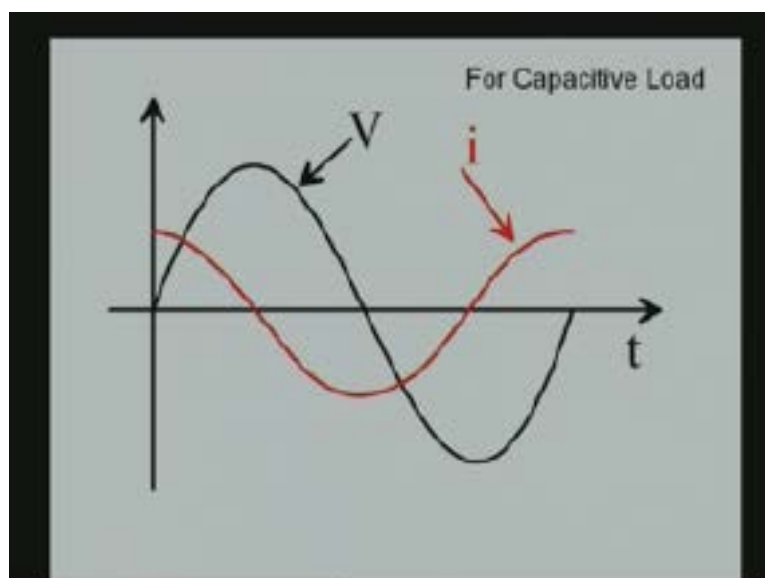
One difference with respect to the previous power waveform as in the case of the resistive load is that here you do not see a DC offset. What is the average value here? Let us unclutter the graph by removing the V and the i waveform and just keep the power waveform and you see that the average value is exactly zero for a true inductive load whereas in the case of the resistive load remember that there was an average value. In fact, that is the crucial difference between the two.

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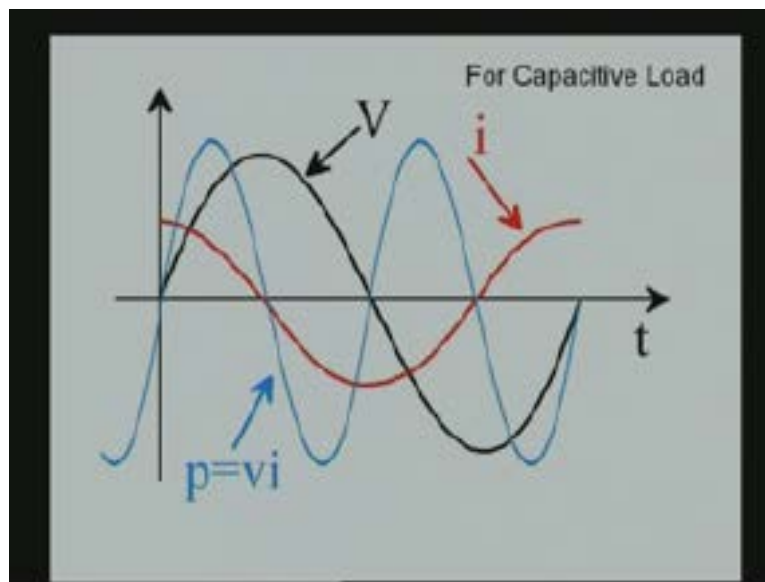
Now consider the capacitive load. We saw this waveform previously again. Again we are going to apply the same process as we did for the resistive set and the inductive set. So we are going to multiply the voltage waveform in black and the current waveform in red, what do you get? You get the power waveform.

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Here, actually the resultant power waveform in the case of the capacitive circuit is 180 degrees phase shifted with respect to the inductive circuit. This is because the current is leading the voltage waveform. So at the origin near the origin you see that the current waveform is positive, voltage waveform is positive so the upper half of the power **sine I V** is traced first and then the negative half of the sine wave is traced after the 90 degree point corresponding to the voltage source waveform and thereby you get this corresponding resultant p is equal to v_i .

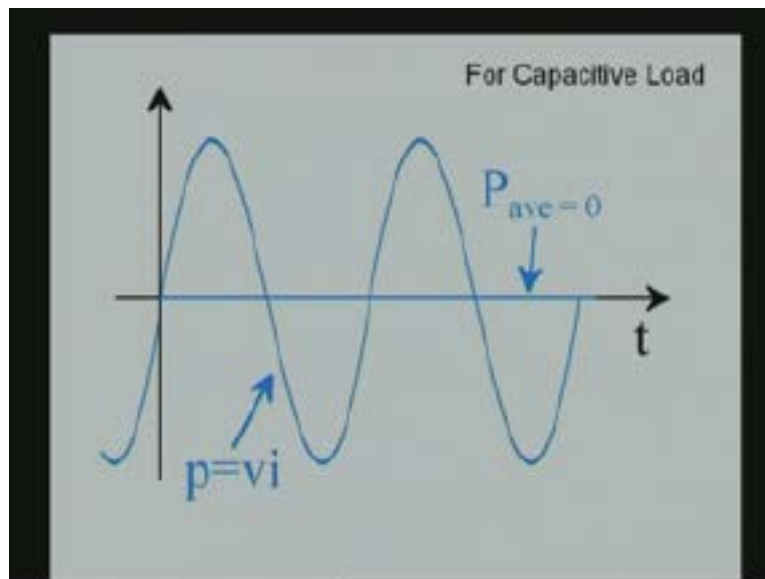
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What do you see here also?

We see that the average power is zero like in the case of the inductive circuit.

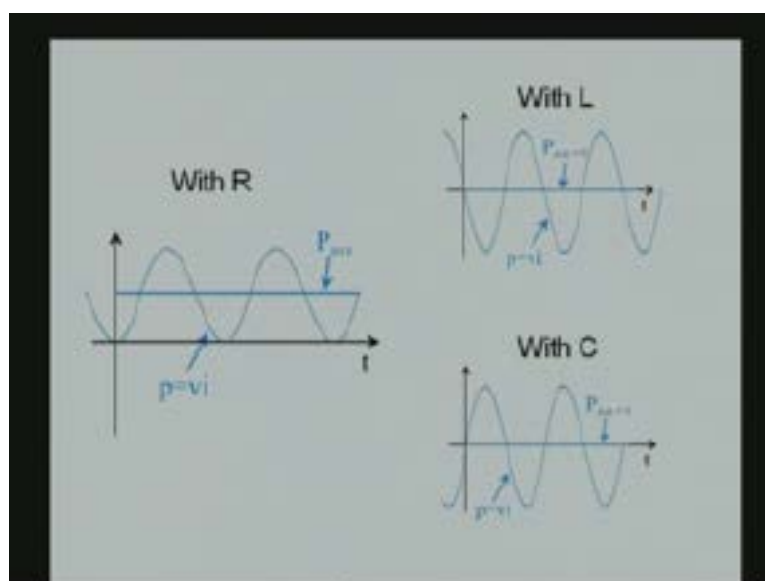
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The capacitive circuit also results in an average power equal to zero interesting is it not?

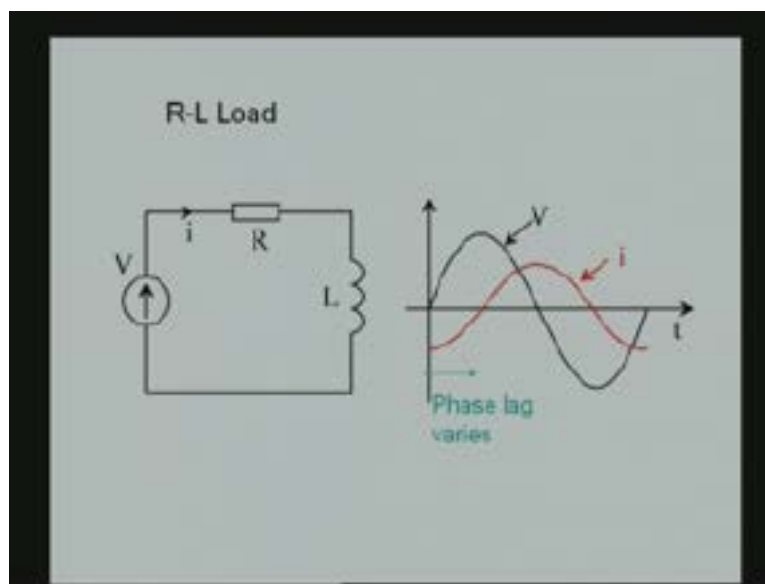
Only the resistive circuit gives a finite average, keep that in mind because we will be using it much later. So let us put them altogether and compare.

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You see that with R you have a power waveform twice the frequency of the input sinusoidal voltage waveform but it is shifted DC shifted as you see **and has a finite** and therefore as a finite average power non-zero average power. In the case of the inductive load and the capacitive load the average power is absolutely zero; only that the power waveform is having a 180 degree phase shift with respect to each other. So this is the comparison of the power waveform for the R, L and C circuits.

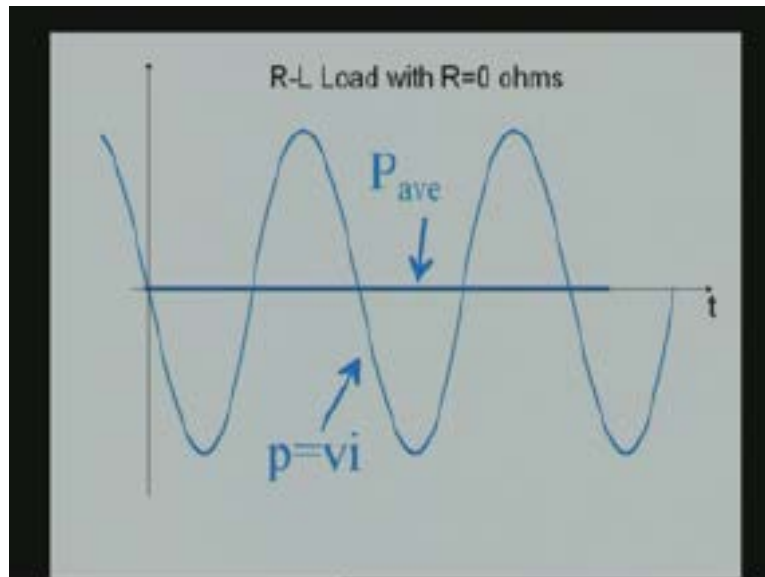
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What happens if we have an R-L load?

You see, you now have a combination of R and an L. and what happens to these source voltage waveform and the **current** source current i for such an R-L load. You see it alongside; again we have used the black colour for indicating the voltage and the red colour for indicating the current. We have a phase lag as usual like in the case of an inductive load. However, the phase lag is variable. that is when R is 0 you will have a phase lag of 90 degrees meaning the current will lag the voltage by 90 degrees and as R increases the phase lag starts decreasing ultimately when R goes towards infinity L becomes very small compared to R and then it will start going to **a unity** **sorry start going to** a zero phase lag situation like in the case of the pure resistive load.

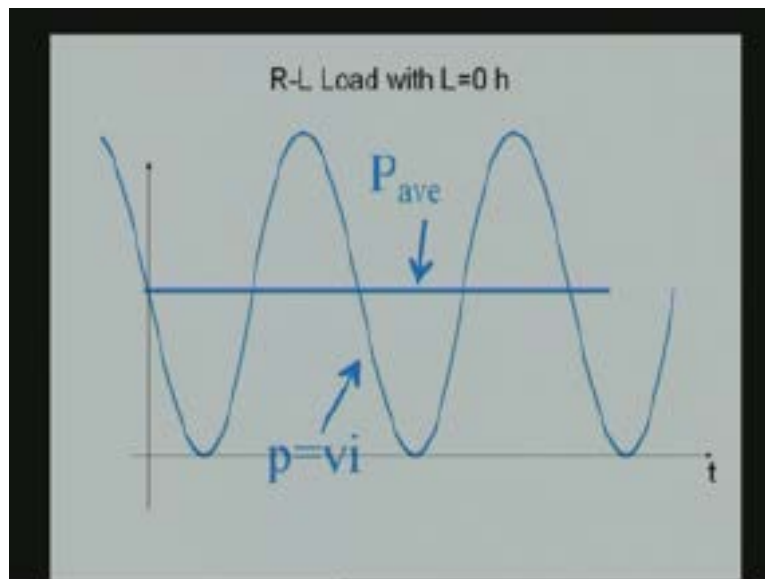
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Let us have a look at the effect of the R, effect of the resistance on the power curve. What happens to the power curve in the R-L circuit as R is varying. Now that would be interesting. Now let us have this graph again with the time axis or the x axis and the y axis now showing the power. Because we are taking only the power curve and for an inductive load, to start with an inductive load R is equal to 0 we saw that the average power was zero. So now what happens when R increases **what happens when R increases**, that is something that we need to look at.

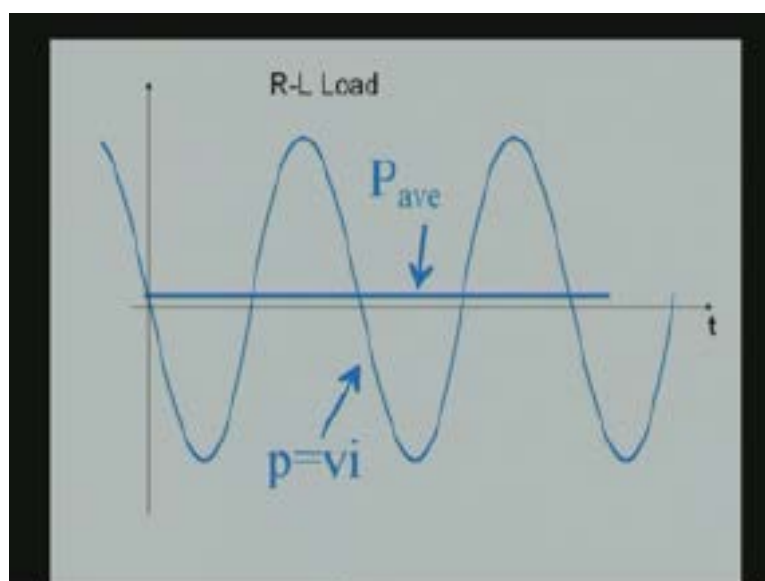
Now let us say R increases slightly from zero. We will see that the power curves shifts up; there is now a small finite average power as seen from the graph. Further you increase the R; you see that the average power gradually starts shifting up. On further increasing the R the average power keeps on shifting up till it reaches L is equal to 0 value something like your pure resistive load.

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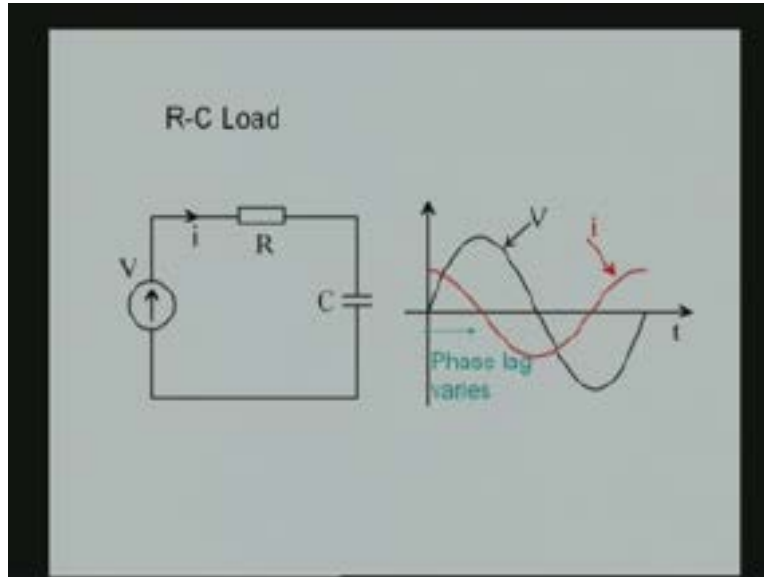
So when R starts increasing towards a very large value you will reach the condition like a pure resistive load or L is equal to 0 Henry load that kind of a situation. So you see the way the power curve transforms as you increase the load and this is as you decrease the resistance.

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Now this is as you increase the resistance. So keep this in mind.

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Now in the case of R-C load something similar happens. Have a look at this circuit in the waveform, you have the voltage source, R and C and we are interested in the voltage source waveform and the current. The black waveform again indicates the voltage and the red waveform indicates the current. Now in this case we see that the current leads the voltage by some phase angle and the phase angle varies according to the amount of R that you are going to put into the circuit.

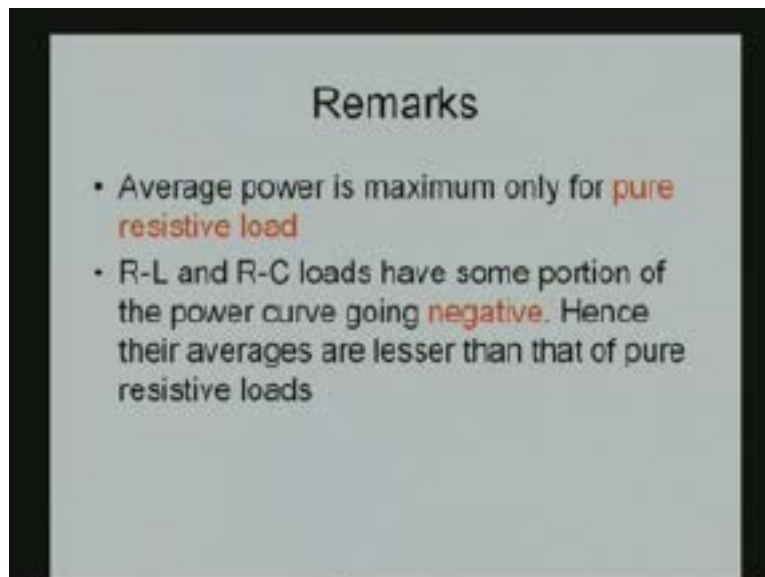
Now consider the power curve for an R-C circuit. You see that the power curve as we saw before, we have average power is equal to 0 for a capacitance pure capacitance circuit R is equal to 0 ohms. Now what happens if I start increasing R? It no longer becomes pure capacitance and gradually you will see that an average power starts developing. So as you start increasing the R the average power starts increasing gradually till at a point when the R value becomes so high that it swamps out the impedance provided by C that it emulates a pure resistive kind of a circuit as though there was no C. So here again you see (Refer Slide Time: 23:41) that when R is

decreased goes towards average power is equal to 0 and as R is increased you see that it starts going more and more towards having an average power.

So some remarks from these graphs. First of all the average power is maximum only for pure resistive load.

R-L and R-C loads have some portions of the power curve going negative and therefore that portion of the area **subtract** from the positive portion of the area and trying to find the average and therefore that average is going to be much lesser than what was for the pure resistive case and therefore the average power for R-L and R-C loads are going to be lesser than for a resistive case.

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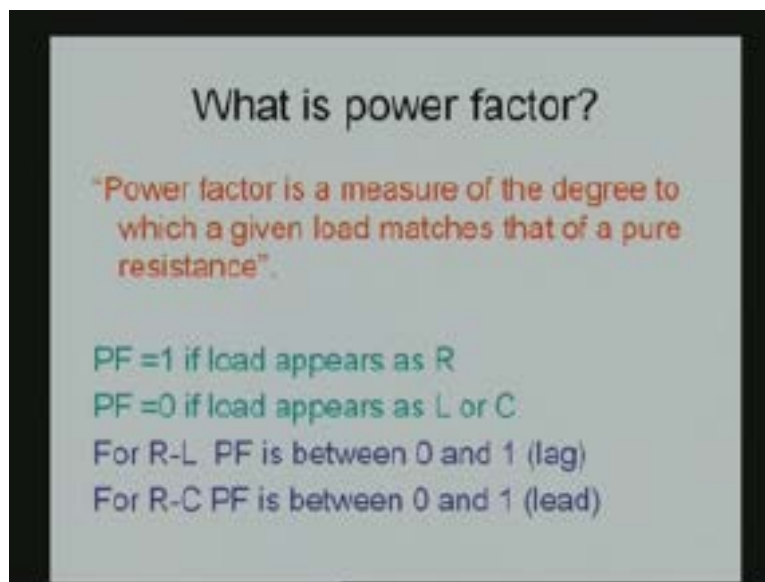


So we come back to our basic question, what is power factor. So let us now give a notional definition and then later on of course when we use the mathematical relations we can have a mathematical definition. The notional definition will in fact help you derive the mathematical relationship. So power factor is a measure of the degree to which any given load matches a pure resistance that is our notional definition. Any load should be either pure resistive or equivalently

capacitive or equivalently inductive or a combination of these, that load which results in a power curve or v_i waveform such that it matches as close as possible to the pure resistance or as close as possible to the R or an equivalent R is **called to have is** said to have a very high power factor.

In fact, if the load appears just resistive then we say power factor PF is equal to 1 and if the load appears purely inductive or purely capacitive then that is most away from the pure resistive case where the PF is equal to 0 and for all other cases that is R-L case where R is finite, L is finite and the PF is between 0 and 1 and we call it PF lagging and in the case R-C circuit R is finite, C is finite, PF is again between 0 and 1 but leading so we said PF lead PF lag. So PF lag is for R-L circuits, PF lead is for R-C circuits; however, the PF value can take values only between 0 and 1; 1 when it is pure resistive, when the load resembles exactly a resistor and 0 when it is purely like inductance or a capacitance that is purely reactive in such a case PF is 0 and for other intermediate cases you will see the power factor taking values between 0 and 1.

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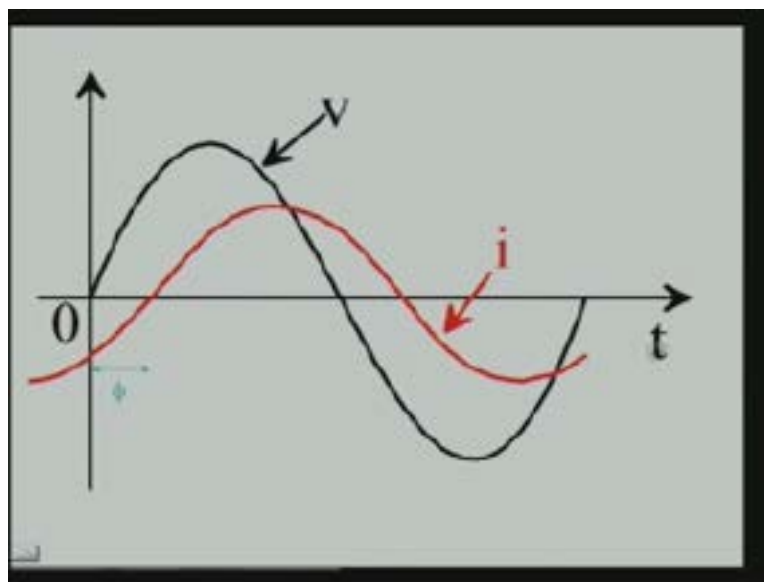


Now we have to learn how to measure power factor. Till now we had a glimpse of what power factor is. In fact, we made a notional definition saying that the power factor is a degree to which any given load matches a resistive load. Now we shall see how we go about measuring power

factor and quantifying power factor. Always remember that whatever be the load we will always be comparing it with an equivalent resistive load.

Coming back to our screen with the typical graphs of the voltage and the current waveforms with respect to time, we have the voltage waveform in black as usual; we have the current waveform in red as usual.

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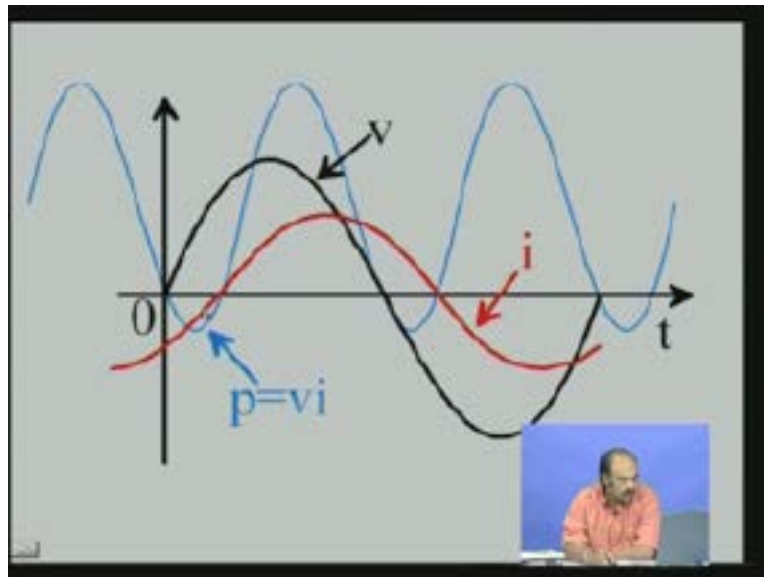


Here notice that the red colour current waveform is lagging the voltage waveform by an angle ϕ . This is an RL load (Refer Slide Time: 28:28). If the current waveform were in-phase at zero with a with respect to the voltage waveform then it could have been a pure resistive load and if it had been exactly at this 90 degree point then it is a pure L load and if it had been exactly leading by 90 degrees somewhere here it would have been a pure C load.

Now this zone from 0 to 90 degrees any current waveform corresponds to an equivalent RL load. So let us take an arbitrary RL load like this and try to see how the power waveform looks like and we saw before that the power waveform is got by undergoing a process of multiplication of

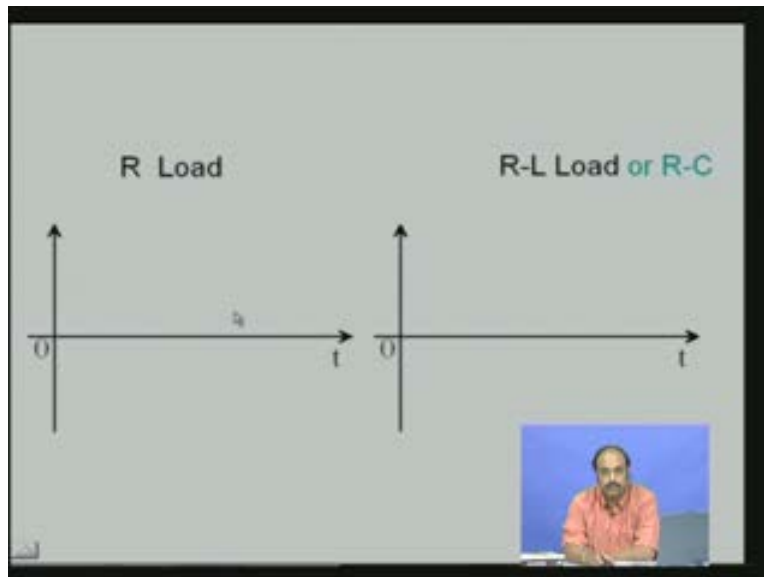
the voltage waveform and the current waveform point by point. So this is the power waveform that you would get.

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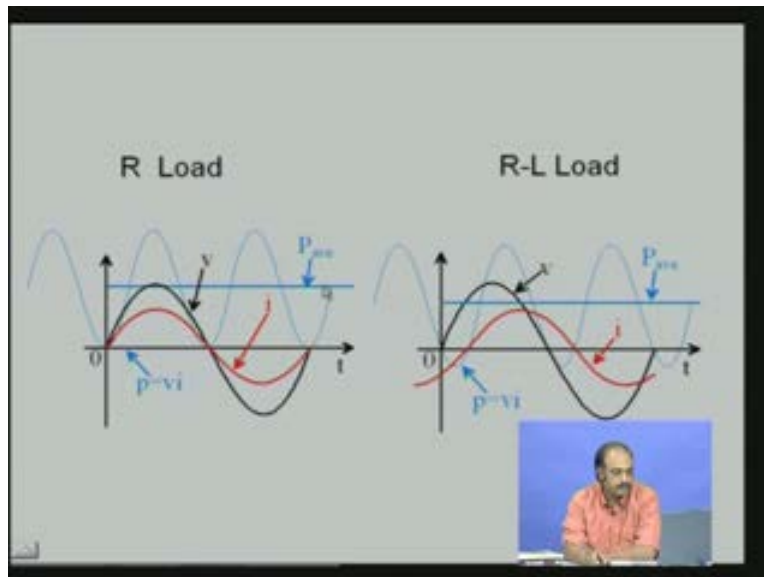
Look here (Refer Slide Time: 29:27), this portion contributes to the negative power. The power is negative in this region which means that the power is put back to the source and in this region the power is positive and it has an average power which is positive which says that that is the power which goes to the load actually, it is the active power. So this **power this** average power is the active power and this portion the negative portion which is put back to the load is called the reactive power. So this is only the power waveform along with the average and the reactive component.

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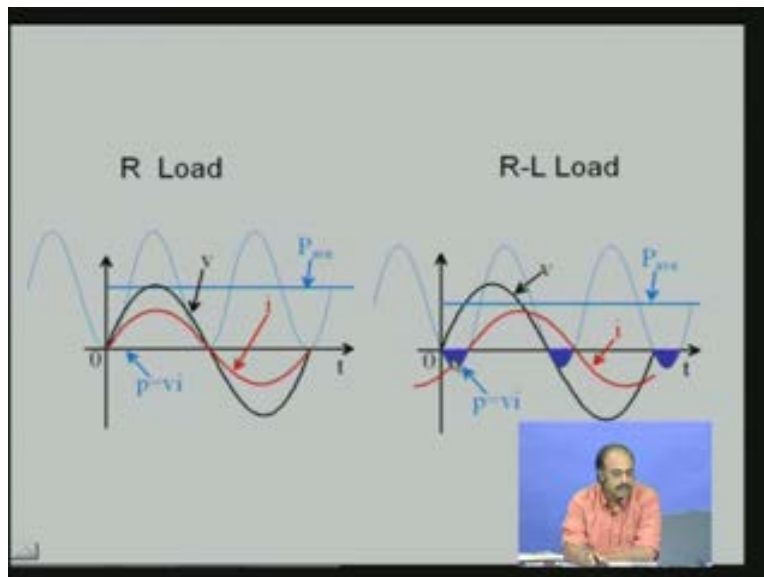


Now we have to always..... as I told you before, compare it with an equivalent resistive load. so let have two graphs: one for the resistive load or the R load; the other for the RL load or the RC load whatever the case may be. let us have the voltage waveform which is sinusoidal in shape, of course it can be any arbitrary shape but let us take the sinusoidal waveform and then you have the current waveform of similar amplitudes same amplitude; see the (n.....30:48....al) load it is in-phase, it is lagging in the case of RL load. You have the power waveform for the R load and the RL load, see there is a reactive power contribution some power goes negative and as a consequence the average in the case of an RL load is going to be lower than the average of the pure R load.

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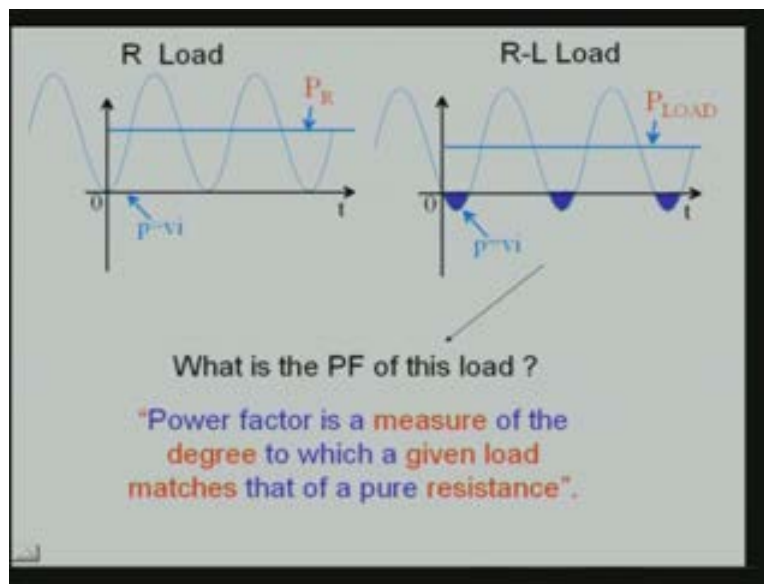
This (Refer Slide Time: 31:15) is the reactive power contribution which goes back to the mains but being negative means that it goes back to the mains. Now these two powers that is the instantaneous powers its finite average; along with the RL loads instantaneous power and its average we have to compare and arrive at a quantitative value of power factor. So let us clear up

the screen, let us move up unclutter the screen, now this average load (Refer Slide Time: 31:53) which corresponds to the resistive load we will call it as P_R and this average which corresponds to the average of the RL load power we will call it as P_{LOAD} .

Now the question is what is the power factor of this RL load what we have here?

We said earlier that the power factor is nothing but a measure of how close the given load is to a pure resistance or pure resistive load.

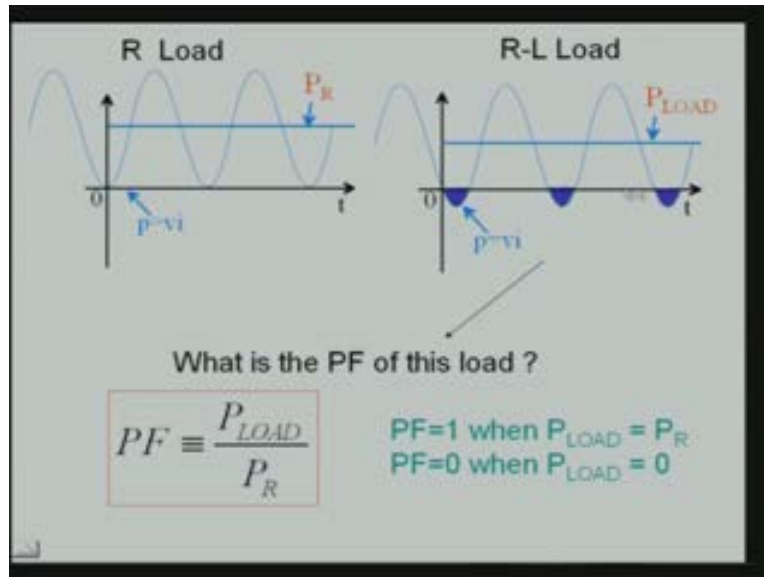
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Let us define PF as follows. PF is defined as P_{LOAD} that is the average load power of the RL or any other given load to the average power of an equivalent resistive load, P_{LOAD} by P_R this is the definition of PF. You see that here that when PF is equal to 1 which means P_{LOAD} is equal to P_R which means this becomes this raises up (Refer Slide Time: 33:12) and this average matches this average only when this raises up and becomes equivalent to this in which case this becomes purely resistive and when it is zero as we saw earlier when it is purely inductive, purely capacitive there is no average component in this case and PF will be equal to 0 and P_{LOAD} is 0, PF will be equal to 0. so once again we see that PF is defined as P_{LOAD} by P_R where P_{LOAD} is the average power of any given load, it may be a RL load, it may be an RC load or any other

arbitrary load and P_R is the average power of a resistive load for the same voltage magnitude peak and current magnitude peak. We will understand this much better as we do few examples.

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Now it should be noted here that the powers are calculated by using integrals. Integral of waveform is basically the area under the curve. So it is actually immaterial whether the shape of the waveform is sinusoidal or any other arbitrary waveshape. So the powers that we get here, the power values that we get here does not depend upon waveshape. So whatever may be the waveshape this definition is valid: PF which is equal to P_{LOAD} by P_R .

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$$PF \equiv \frac{P_{LOAD}}{P_R}$$

P_{LOAD} = average power of any given load

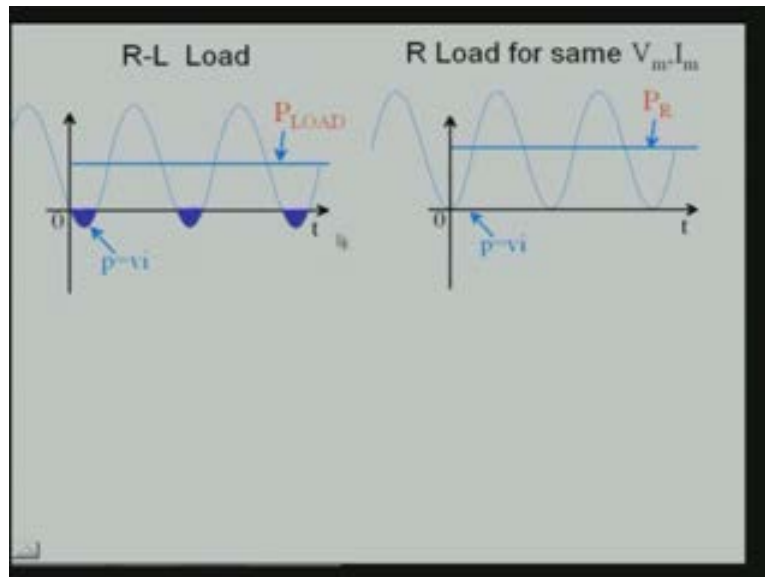
P_R = average power of a resistive load for same voltage and current magnitudes

As power is an average quantity, the above equation is valid for any voltage and current waveshape.

So let us consider some examples. Consider the same RL circuit and let us try to find the power factor of the RL circuit if we are giving a sinusoidal input. So let us give a sinusoidal input voltage of V_m amplitude and this will result in a current of amplitude I_m and phase shift ϕ . Now this will have a comparative voltage and the current waveform for the R with the same V_m and I_m as follows. See we should have a same V_m , same I_m but only thing is equivalently in the R case the phase shift is not there it is in-phase with V .

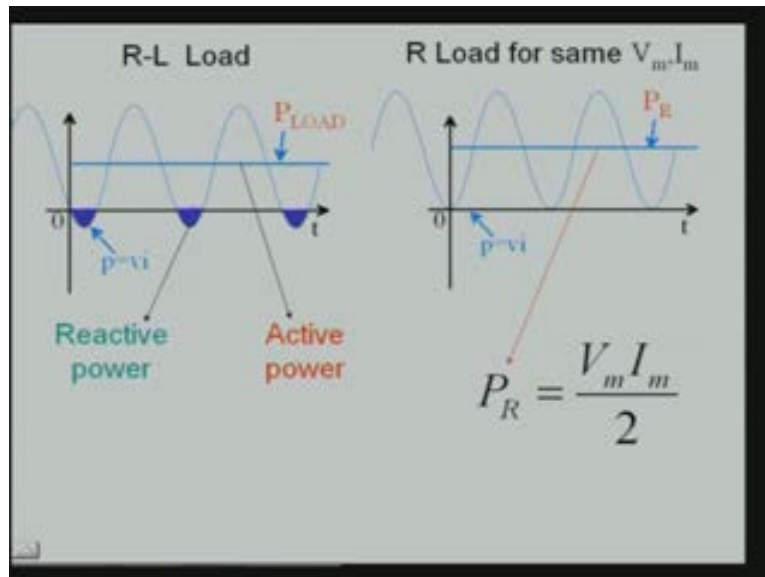
So you will see that the powers in the case of the RL load you see some negative portion here (Refer Slide Time: 35:49) and in the case of the resistive load there is no negative portion and the average is P_R here and the average p_{load} is slightly lower than this because of the negative contribution because the power wave is nothing but the area under the current. So this area will cancel out the area which is there in the positive side and therefore effectively P_{load} is going to be lower than P_R .

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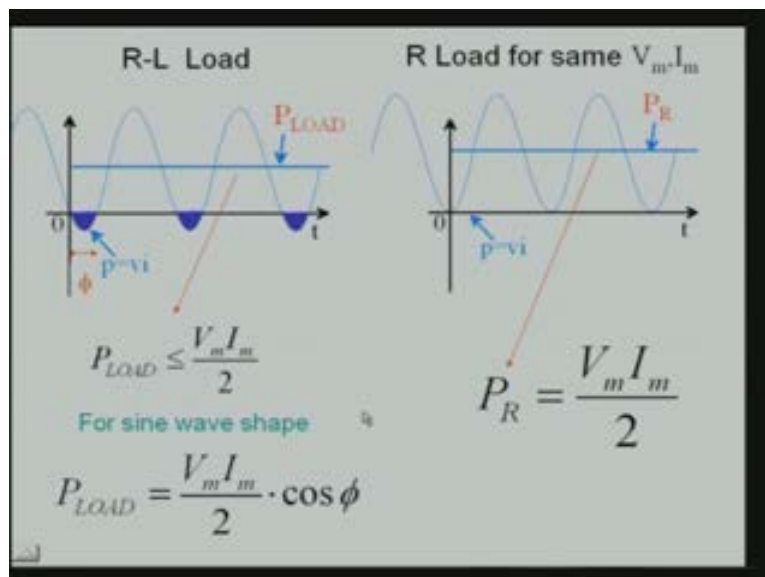
So let us see what is this P_R value for a sinusoidal wave. we know that this peak value in the case of R load is V_m into I_m , half of it is $V_m I_m$ by 2 therefore P_R is nothing but $V_m I_m$ by 2 and in the case of a RL load this portion is called the reactive power, this portion is the active power portion. So it is actually the active power portion that you have to consider for the power factor because our power factor says how close does the load match the resistive load or how much of the act portion of the input power is an active power.

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So P_{load} is definitely going to be less than $V_m I_m$ by 2. So, for a sine wavelshape of phase angle let us say ϕ of current with respect to the voltage, this P_{load} will turn out to be $V_m I_m$ by $2 \cos \phi$, how is this obtained; this is nothing but integral of $V_m \sin \omega t$ into $I_m \sin \omega t - \phi$ divided over by the period T . So this will result in $V_m I_m$ by 2 into $\cos \phi$.

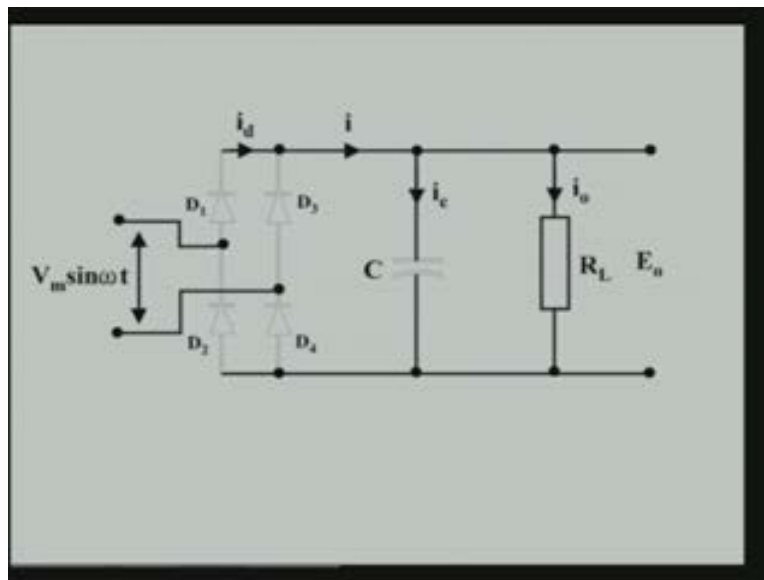
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Now what is power factor?

power factor as we defined is P_{load} / P_R so that turns out to be P_{load} / P_R which is $\cos \phi$. Therefore, for a sinusoidal voltage waveshape, sinusoidal current which is resulting from the load you will always see that the power factor is nothing but $\cos \phi$ or cosine or the phase angle between the current and the voltage, this is a special case only for sinusoidal. but the general definition of the power factor should always be, you should always go back to our original equation which is P_{load} / P_R where P_{load} and P_R are the average values of the power curves for any arbitrary waveshape.

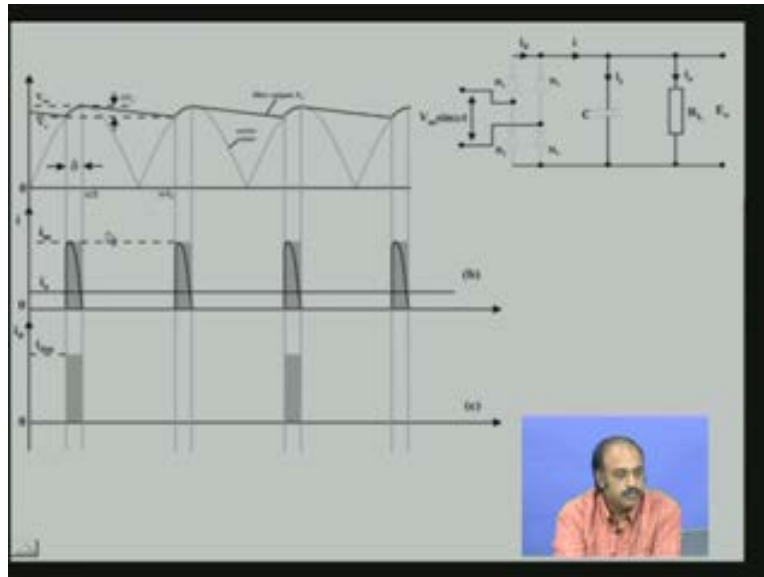
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Now consider this circuit. This is a very common circuit, this is a diode which..... you see the input is a sine wave here which we were applying and there are four diodes, this is a full bridge diode rectifier (Refer Slide Time: 38:57) this is followed by a capacitor which stores the energy and then followed by the load R_L . So this whole circuit is called a capacitor filter circuit. In fact, this is one of the most common front-end circuits that you would find in almost all power supplies, whether it be PC power supplies or whether it be any of the DC power supplies, converters so on and so forth all the front-ends will be of this nature.

Now do the various waveforms here look like?

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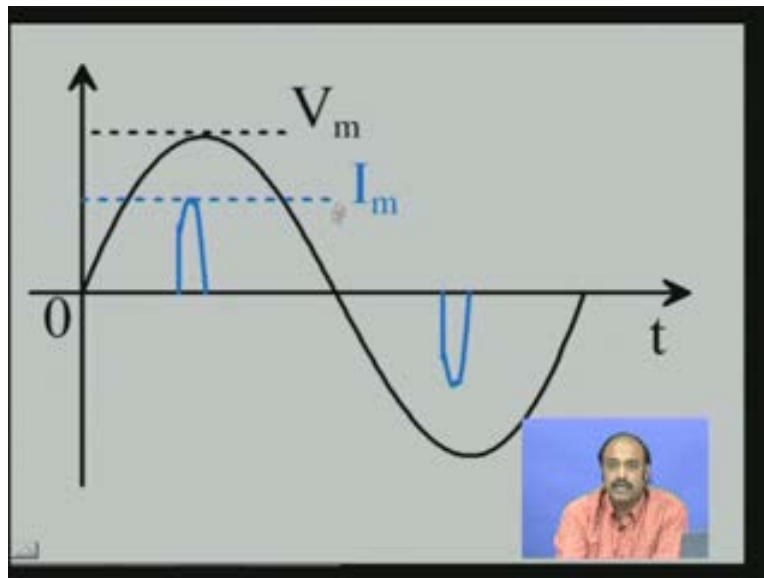


Just to get an idea for this capacitor filter circuit there is..... the output here, output is nothing but what you see here in dark, this sine wave which is the rectified sine wave is what you would obtain here as **I am showing here at the mouse points** (Refer Slide Time: 40:00) and only during this portion there is conduction and therefore the currents of the diodes here would be of this nature, sharp and there is no current **any of these** at the any of these points. **This circuit I think you will be familiar with, you would have learnt at some other course some other class with these kinds of waveforms.** Let us now find out what will be the power factor as seen by the input here for this entire circuit along with the rectifier capacitor filter, what would be the power factor.

So again we come back to our time grid. You have the time axis the x axis. Let us plot the voltage waveform which is having the amplitude V_m , let us plot the current waveform for this circuit. we saw the current waveform is a kind of a spiky current waveform as we saw back here, let me go back again; you see this is a spiky current waveform which is flowing here which will flow through the diodes which will also flow through the mains here (Refer Slide Time: 41:24).

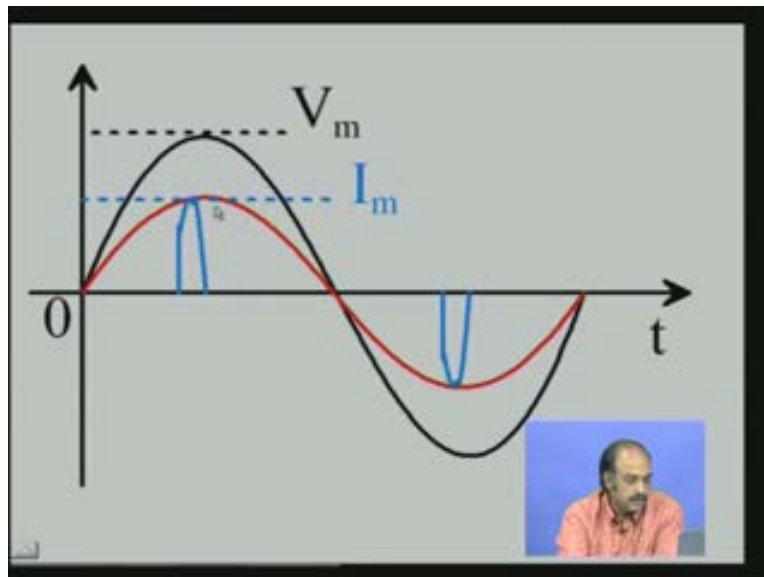
Therefore, this is the type of the current waveform you can expect at this point. So input current waveform is of this nature which has an amplitude I_m . How do we propose to go about finding the power factor.

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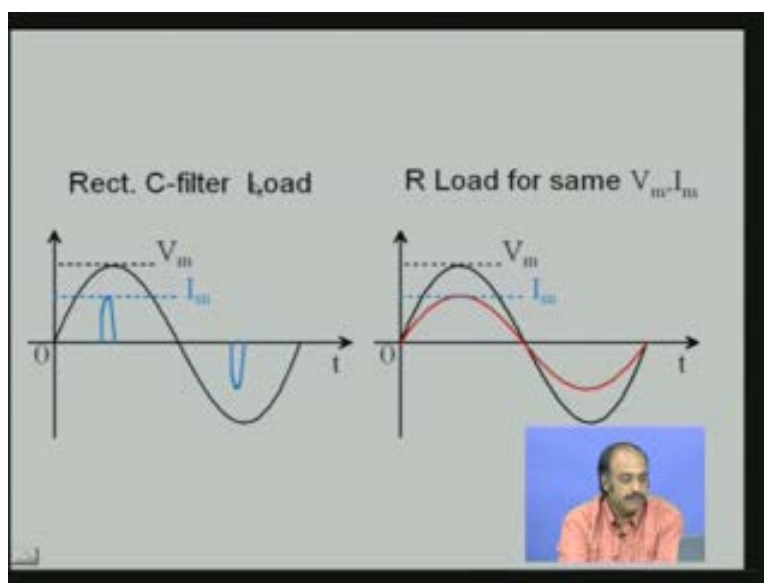
Again we go back to the basics; it should be P_{load} / P_R . So let us have an equivalent current which has a maximum value of current I_m only. But suppose if it had been a resistive load what will happen, you would land up with a waveform which is having the same waveshape as the voltage waveform and in-phase. So in this case this red line indicates the current waveform that you would have got of the same amplitude I_m if it had been a pure resistance.

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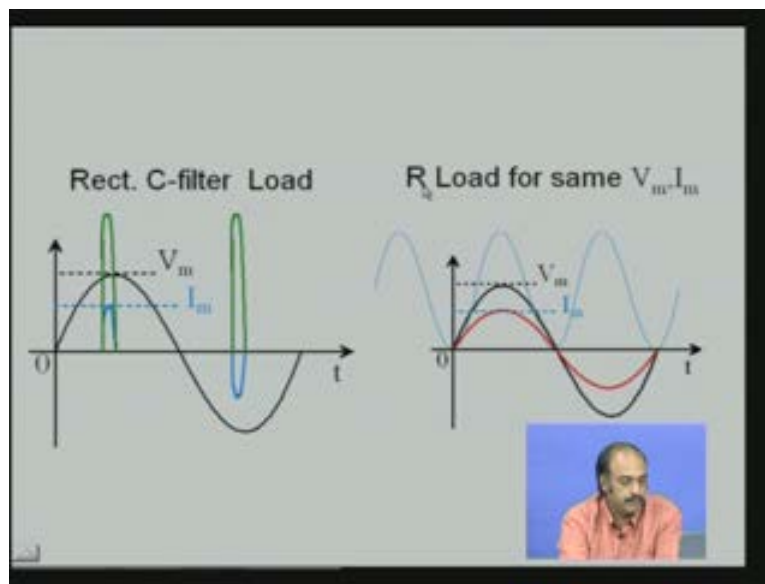
So let us compare V and the red coloured current waveform for a pure resistance, V and the blue coloured current waveform for the capacitive circuit.

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This is the rectifier C-filter load and this is the voltage and the current waveform and this is the pure resistive load waveforms for an hour load which has the same V_m and I_m that is important. Now let us find the power. Power is nothing but multiplication of the voltage curve and the current curve. So, multiplying the voltage and the current curve all these portions will be zero in the case of the C-filter load so this portion alone where there is a finite current and the finite voltage will have a power curve and in the case of the R load we have seen earlier power curve which is of this sinusoidal nature.

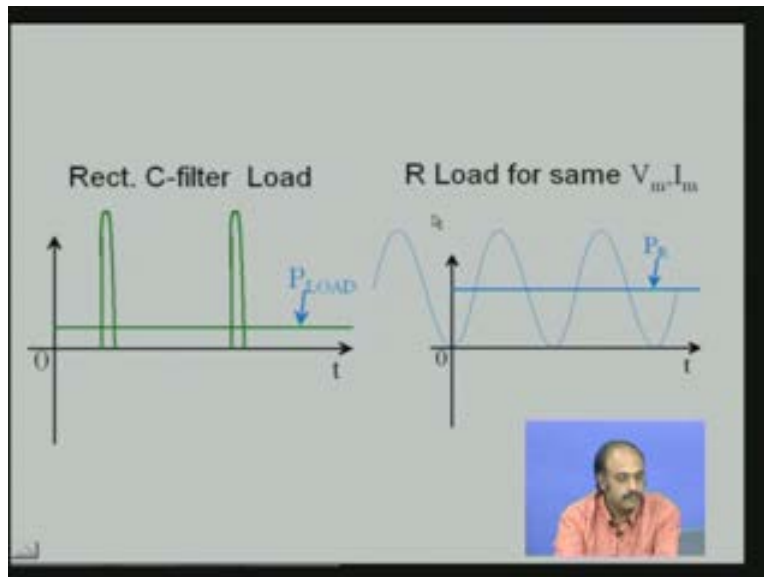
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Now what about the averages?

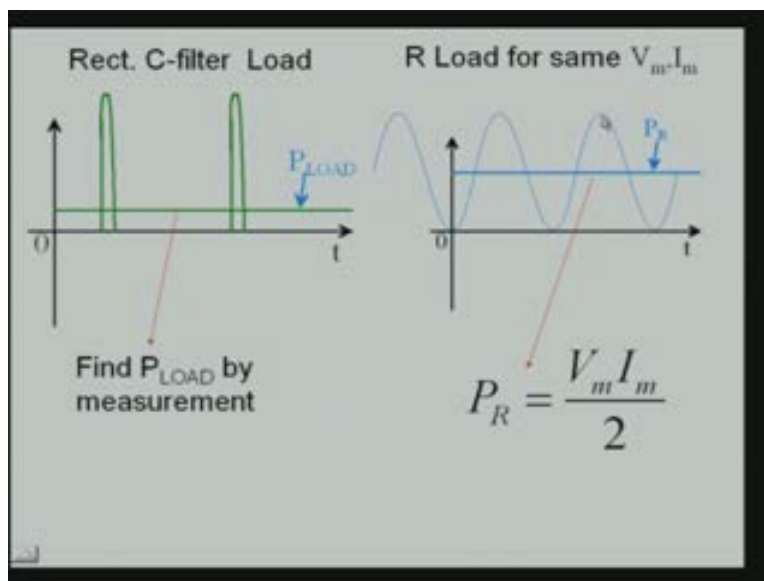
taking the average of the power curves we have P_R as follows here which is $V_m I_m$ so this is $V_m I_m$ by 2 (Refer Slide Time: 43:38) and P_C load which will be average of just this green waveform which you see here. Removing all other cluttering voltage and current waveforms you will see that this is the power curve instantaneous power curve and its average for the C load. We see the instantaneous power curve on its average for the R load.

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Let us move up, clear up the space and then see that P_R is nothing but as we saw earlier $V_m I_m$ by 2, how do you obtain this?

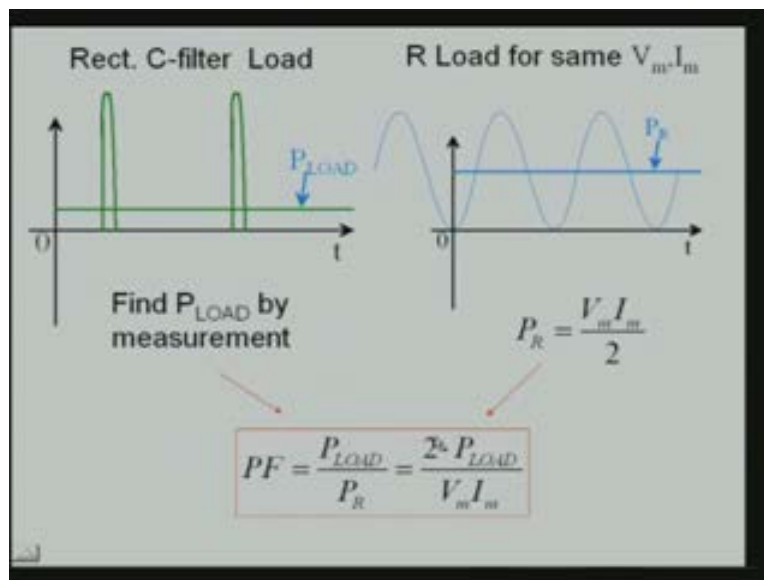
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This point here or the cursor point is $V_m I_m$ and this will be the average value which is $V_m I_m$ by 2 and this P_{LOAD} the average will of course will depend upon the actual load it will actually depend upon the area under these curves for which we can get the measurement **which I will tell later on.**

So, finding the power factor is nothing but P_{LOAD} this average value divided by this average value which is P_R which is $2 \cdot P_{LOAD}$ by $V_m I_m$. Therefore, for the case of a C-filter load this is the power factor equation where V_m and I_m is the maximum values of the voltage and the current respectively, P_{LOAD} is the average load power that you are applying and this 2 times P_{LOAD} divided by V_m by I_m will be the power factor.

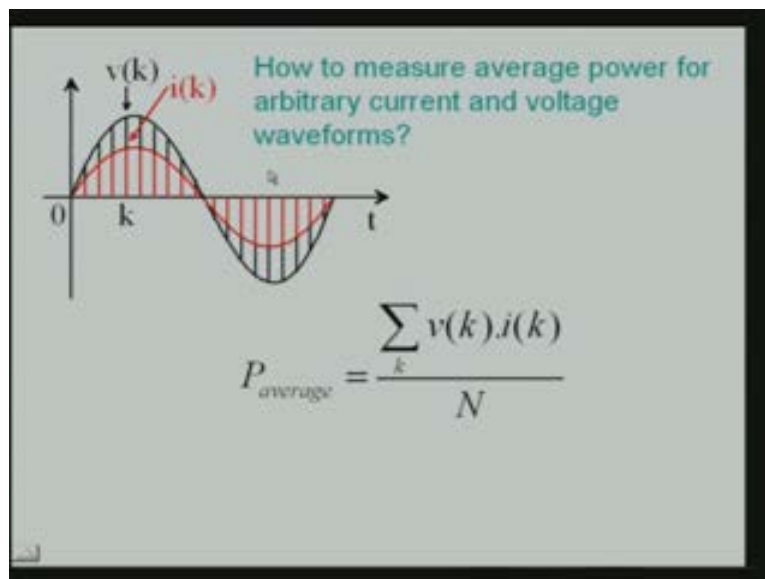
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Now let us see how we measure the power for an arbitrary current and voltage waveform on an arbitrary power waveform. Now let us say we have this voltage and current waveforms and you see these waveforms on the Oscilloscope. Let us take measurements that is measure the points these points on the voltage waveforms $V(k)$ at every let us say known instance of time all along the voltage curve for the whole period. Similarly, you measure the current waveforms that is the red points all along the current curve whatever may be the curve, it may or may not be sinusoidal

for of the whole period $i(k)$. Then the average is nothing but $V(k)$ into $i(k)$ the product the product which you obtained instantaneous product which you obtained for the voltage and the current waveforms, sum up all the products and divide by N that is the number of instantaneous values that you have taken for the whole period and that would give you the average for one period. This is how you can take the measurement of the average power whatever may be the waveform because these waveforms you can see on the scope and then note down the values. Let us say for example; some period you take some thirty thirty-five samples and then divide by thirty-five after having summed up the products $V i$.

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Again summing up we say that PF power factor is defined as P_{load} , P_{load} is the average power of any arbitrary load the active portion of the power that is given to the load divided by P_R which is the power that you would have delivered to the load if the load has been resistive. As power is an average quantity you need not actually bother about the waveshape. Whatever may be the waveshape if you are able to measure the average powers yes, then you have the values of P_{load} and P_R to measure PF.

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$$PF \equiv \frac{P_{LOAD}}{P_R}$$

P_{LOAD} = average power of any given load

P_R = average power of a resistive load for same voltage and current magnitudes

As power is an average quantity, the above equation is valid for any voltage and current waveshape.

One thing you should keep in mind here is what would happen **what would happen** if the power factor is low. So the power factor is low it means that P_{LOAD} is low and this P_{LOAD} is low which means that there is a quite a difference between P_{LOAD} and P_R for the same I_m V_m .

Now let us say you want to deliver the same active power. Let us say you have 10 watts of load and you want to deliver 10 watts of active power load with the same V_m and I_m as the resistive load. Then what could happen here? You see, this has to match this that this has to come to 10 watts then these value goes still higher to get that same average which means for the C-filter load the maximum I_m will be much much higher than what would be for a R load which means that the ratings of your diodes the ratings of your power sockets all those things must be rated much higher for this same output active power load delivery.

Therefore, low power factor loads are not recommended in fact you have to pay more if you are having low power factor loads and if you are having unity power factor loads which means close to resistive load then your optimally utilizing the mains of the grid and therefore you will be less penalised. Therefore, there is lot of scope for having front-ends which make any given load look like a unity power factor. Therefore unity power factor front-ends are very important piece of

equipment especially in today's condition where every equipment has to be certified for having power factors which are more than 0.98 or so.

Till now we have been looking at the power factor without making any assumptions on the waveshape even though we have been looking at the slides where the waveshapes are sinusoidal in nature there is no basic assumptions made that the waveforms need to be sinusoidal and therefore the power factor as we stated that it is the power that is delivered to the load divided by the power that is delivered to the load if the load had been a pure resistance.

Now you can find out the power delivered to the load as described just previously for the case of the capacitor filter type of load by measuring the various points on the waveshape in an Oscilloscope the voltage and the corresponding current, the voltage and the corresponding current along the time axis for the whole period. Let us say you take something like thirty pairs of points for one period, you multiply and then you average it by number of..... multiply V and I corresponding V and I, add them up and divide by N the number of sample points will give you the average power. And for the same peak value of current and I that is the same effective value of I and voltage if I had a resistance then it could have been $V_m I_m / 2$ or $V_{rms} I_{rms}$ so that is the absolute maximum that you can get. So this average which you found out with obtaining the various sample points on the Oscilloscope and then dividing it by the maximum value that can be provided with resistive load you get the power factor.

Now for a sine wave you can cross check and see that the power factor is P load that is given to the load by $V_m I_m / 2$ or this is P load by V effective I effective, these are effective or root mean square RMS values as we have discussed in the phasor diagram modelling of the last class last session. V into I would give you the power delivered **to a resistive load** to a resistive load and in the case of the sine wave V into I is the apparent power **which is delivered to the** which is taken from the source and deliver to the load.

So if we talk of the apparent power V I so V I is like that, this is VI this VA. Now the power delivered to the load P load this is watts (Refer Slide Time: 54:00) so watts is given this is watts or P load in terms of the phasor diagram and the vectorial difference P load with respect to the

VA apparent power and the P load this would be the VAR or the reactive power and this angle theta is the power factor angle. So if you see VI cos of theta will give you P load or P load by VI is cos of theta which is the power factor that we have been talking of in sinusoidal waveshapes all along.

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$$PF = \frac{P_{load}}{V_m I_m}$$

$$= \frac{P_{load}}{VI}$$

VI → power delivered to resistive load
 effective or RMS value

$$VI \cos \theta = P_{load}$$

$$\frac{P_{load}}{VI} = \cos \theta \text{ pf}$$

Now look at the comparison of these two equations; one and the same (Refer Slide Time: 55:08). The general equation and the specific case what we get for the case of the sinusoidal condition is the same.

(Refer Slide Time: 55:14)

Handwritten notes on a digital whiteboard explaining the derivation of Power Factor (PF) for a sinusoidal waveform. The notes show the formula $PF = \frac{2 P_{load}}{V_m I_m}$ and its simplification to $\frac{P_{load}}{VI}$. A phasor diagram shows a voltage vector V and a current vector I lagging by an angle θ . The real power P_{load} is the component of VI along the V vector, which is $VI \cos(\theta)$. The text notes that VI is the power delivered to a resistive load and that V and I are effective or RMS values. The final equation is $VI \cos(\theta) = P_{load}$, which simplifies to $\frac{P_{load}}{VI} = \cos(\theta) = PF$.

So in the case of the sinusoidal condition P load by VI the apparent power which is basically the product of the effective value of the voltage and current is equal to $\cos \theta$ which is equal to PF the power factor. PF is equal to P load by $V_m I_m$ by 2 this is the general equation. PF is equal to P load by VI that is V_m by root 2 I_m by root 2 for the case of sinusoid this is equal to $\cos \theta$ is the specific equation for sinusoids so the power factor for the sinusoidal waveform is $\cos \theta$.

(Refer Slide Time: 56:20)

The image shows a digital whiteboard with handwritten mathematical formulas. At the top, the equation $\frac{P_{load}}{VI} = \cos \theta = PF$ is written. Below this, two boxed equations are shown. The first box contains $PF = \frac{P_{load}}{\frac{V_{rms} I_{rms}}{2}}$ with the text "general eqn." written below it. The second box contains $PF = \frac{P_{load}}{\frac{V_{rms} I_{rms}}{\sqrt{2}}}$ with the text "sinusoidal." written below it.

$$\frac{P_{load}}{VI} = \cos \theta = PF$$
$$PF = \frac{P_{load}}{\frac{V_{rms} I_{rms}}{2}} \text{ general eqn.}$$
$$PF = \frac{P_{load}}{\frac{V_{rms} I_{rms}}{\sqrt{2}}} \text{ sinusoidal.}$$

So we stop at this point after about the discussion of power factor and continue in the next class.