Basic Electrical Technology
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Lecture - 13
Phasor Analysis

Hello everybody, today we shall discuss about another analysis called the phasor analysis. We have been seeing all along that there are lots of different analysis and different domains that we have been applying for the understanding of the circuits. We initially obtained the state equations, studied the dynamic behaviour of the R, RL, RC, RLC circuits; we saw how we go about obtaining information about the circuit and three major domains which is the time domain, the frequency domain and the pole-zero domain and we saw how we can extract from the state equation the transfer function and see how the picture evolves in the pole-zero domain and so on. There is one more analysis that you need to become familiar with before we start discussing the other major components and electrical circuits and networks which is the transformers, the induction motors, the DC motors and the others. And this analysis which is called the vector analysis or the phasor analysis gives you information about the relative phase differences between the various waveforms in the circuit. You could have the voltages, currents, currents in various branches; how are these various currents and voltages displace with respect to each other in a phase in a in a phase shift manner in a phase shift that is the phase shift between two waveforms and then what is the active component of the power, what is the reactive component of the power all those information can be obtained from the phasor analysis.

There is also another important thing that can be obtained from the phasor analysis which is the power factor for the sinusoidal. Of course all the analysis will be performed with sinusoidal waveshapes. The importance of the power factor will be discussed later in a later session. But however, there is also an important feature of a particular circuit which also will be obtained from phasor analysis. So phasor analysis is a pretty important analysis useful analysis for you to understand and use it in the circuits; especially for circuits with sinusoidal excitation. So all these applications all these analysis that is this phasor analysis, the sinusoidal steady-state analysis are applicable for sinusoidal excitation. However, the transfer function and the state space analysis are applicable for any excitation.

So let us discuss today the concepts of phasor analysis. In the phasor analysis a sine wave is represented as a space phasor or space vector. We saw earlier that a sine wave can be represented in space by a vector. Let me draw two axes. Now let us say this is a spatial a spatial coordinate system we will call this the alpha axis and the beta axis (Refer Slide Time: 5:15) so alpha and beta axis are always orthogonal which means 90 degrees. And then we have of course the temporal coordinate system which is with respect to time. This is the temporal coordinate system which is with respect to time. Now we have to map the temporal waveshape i mean the time waveshape the waveshape that is evolving with time into the spatial coordinates and then we obtain the space vectors or the space phasors.

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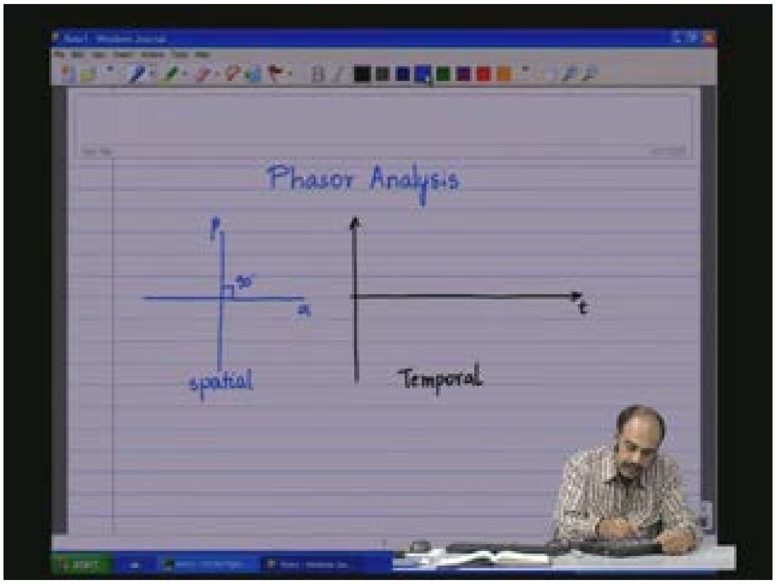

So let us take a sine wave. We have a sine wave here (Refer Slide Time: 6:17). Now this sine wave starts with time equal to 0, this is time equal go T by 2 and this is T where frequency f is equal to 1 by T. Or if it is in terms of angle we say that one whole period is completed from 0 to T and then again it repeats for every period which means this could be considered as 0 degrees, this is pi that is in radians, let us fix it all in radians, so 0 radians pi radians 2pi radians with this point here. So of course this point the peak the where the peak occurs will be pi by 2 radians and this would be 3pi by 2.

Now equivalently let us take a vector. Now this is a vector; let us say this is spatial vector in space it could be a pole I mean a stick; let us create some space here, let me erase out this. Now stick has a this vector has an amplitude V m that is its length. It is anchored at this point and this is same as the peak of the sine wave V m in the waveshape in the temporal coordinates system.

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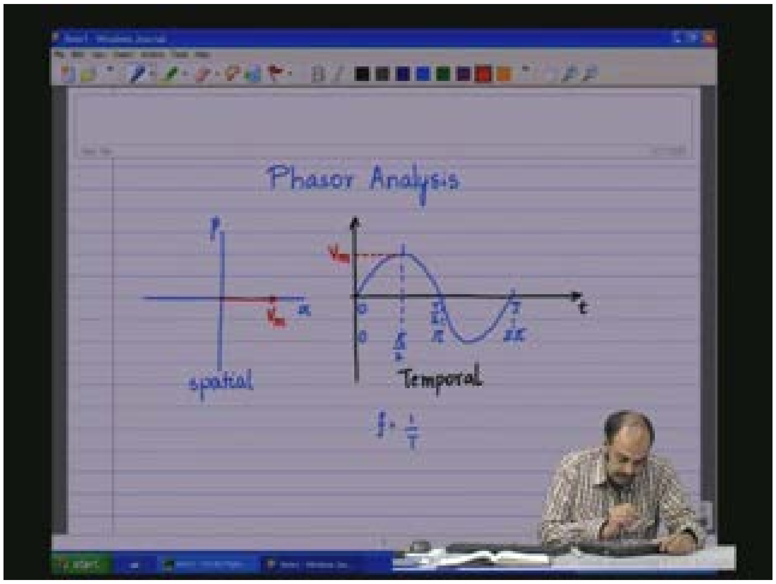

Now this V m is made to rotate being anchored at the centre here and it starts rotating like this (Refer Slide Time: 8:45) and completes one circle for one period. That is it starts from 0 goes through a 90 degree shift so there would be a pi by 2 if the vector was at this point then it is 3 by 2 3pi by 2 sorry it is pi and then a vector at this point which is 3pi by 2 and then you have 2pi; it comes to 2pi. So a vector which rotates like that completes one complete circle is equivalent to a sine wave which has completed one period. And amplitude-wise you see that vector at this position at that corresponding angle shift theta will have a projected amplitude on the on the vertical axis at exactly the corresponding angle theta from the 0 or the reference. So, likewise every point on the circle will map onto points on this sinusoidal. So when it comes back again to 2pi this would have come back to 0, the amplitude would have come back to 0. So at every position of the vector you take the projection of the tip of that vector on to the vertical axis and then take the angle and then take the corresponding angle on the time axis and then place a point there. So you will see a sine wave evolving which means this sine wave is equivalent to this vector of this amplitude rotating at that frequency; that is rotating at a frequency of f or 2pi f or omega which is equal to 2pi f at the radian frequency of omega.

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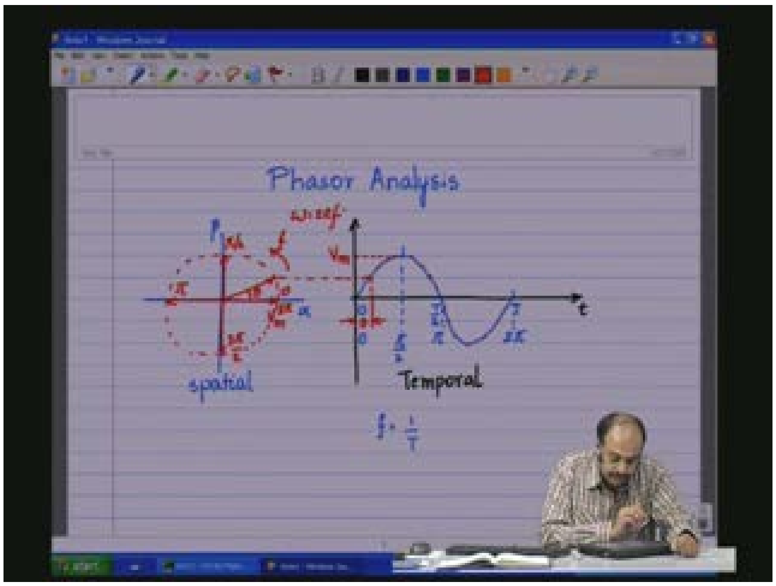

So now that we have an idea that a sinusoidal waveshape can be represented in this spatial coordinates by a single vector, at any given point of time the vector is in that special coordinates anchored at the centre at any position or angle corresponding to the time which is 0 to 2pi in the case of the spatial coordinate system.

Now if I am having in the temporal coordinate system one waveform which is shown in the blue line like that and let here be another waveform and let me put it phase shifted phase shifted like that, let me draw the sine wave and the amplitude is also different sorry it was like that (Refer

Slide Time: 12:37) and here and this can be extended like that so this is another……… so this is……… let me call this one as A and let us call this as a waveform B.

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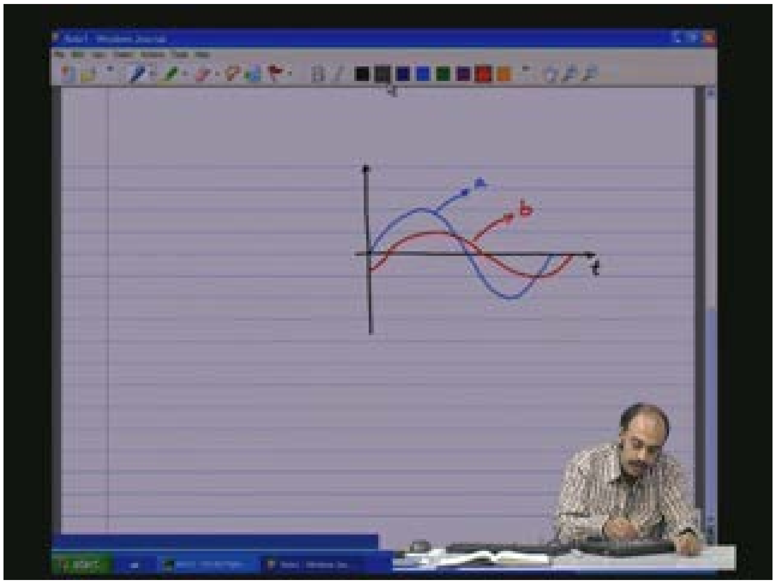

Now let us try to project this onto the spatial coordinate system that is alpha and the beta axis system. This is the alpha axis (Refer Slide Time: 13:20) and this is the beta axis, orthogonal axis, this is the centre 0. Now these two sinusoids are having two different amplitudes and different phases, they start with different phases so how do you represent them. So let us take first the blue the blue waveform. The blue waveform has an amplitude let us say Am and it is starting with 0 this is the….. and this has amplitude of Am and let it reflect on to the let it be projected on to the vertical axis so we see the peak to peak amplitude here and the red has a peak to peak amplitude of this magnitude as shown here like this which means the blue equivalence space vector will inscribe a circle within this and the red waveform will inscribe the circle within this (Refer Slide Time: 15:04). So the blue waveform is starting from zero because this is starting from zero so the projection of this vector at time t is equal to 0 is 0 so you could take the alpha axis as the reference axis.

Now the red waveform is starting with some value which is not zero at time t equal to 0 so relatively the position of……… so this is the d waveform that is the b space vector and the blue space vector. So the blue space vector on the red space vector are displaced like that. So this would be a and this would be b.

Now this unit as a whole together they start rotating and they complete a cycle because this together at every point of time preserving the same phase difference between the blue space vector and the red space vector you get. Or in other words, if we have a space vector like blue and a space vector like this red as shown here with a space difference and if they rotate at some given frequency both rotate at the same frequency then on projecting on to with a temporal coordinate system they will evolve into the sine wave as shown here where the red sine wave is lagging the blue sine wave which means the red phasor here is lagging the blue phasor; this is the meaning of this particular wave. Now how do we apply it for the case of the circuits.

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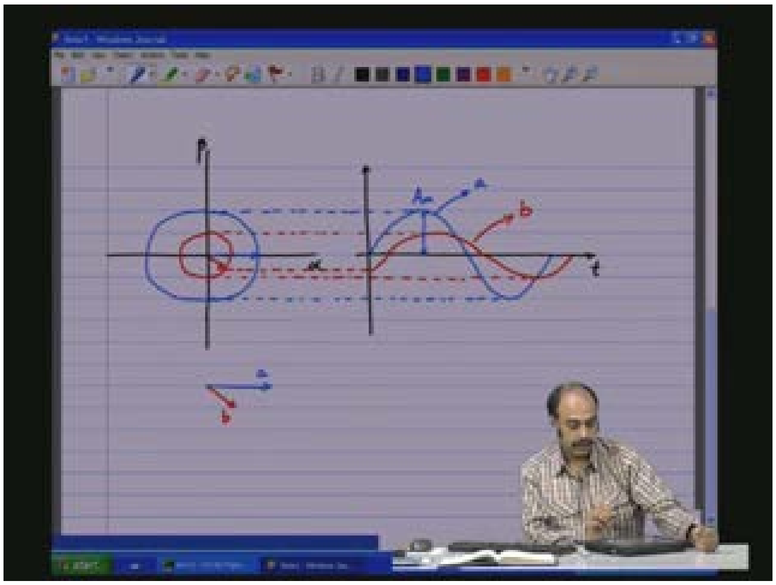

Let us take now only alpha and the beta axis. So let me take the beta axis and we have the alpha axis. Of course this is the alpha axis and the beta axis. Normally it is good to have a reference. So the starting reference is something that we apply. So we know that we apply the excitation voltage which is sine wave the input voltage so it is a good starting point to have a reference. So let us say that we have along this alpha axis we say that V i or the input or the V in; this is the space phasor or the phasor of the input waveshape or the input waveform. Now this is going to be rotating with an amplitude V m whatever it is and it is going to rotate with a frequency of omega radians per second and this when reflected on to the temporal axis will result in a sine wave which is going to start from zero like that (Refer Slide Time: 18:55) and when this completes one complete circle you will see a sine wave having……

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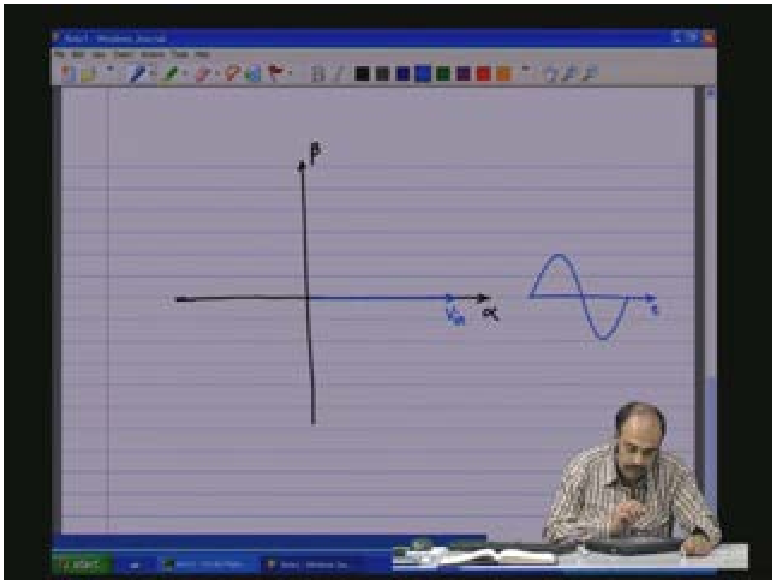

Now the currents….. I could probably have a current which is in phase so this is my i phasor or the vector or one could have a current let me call this one as i 1 or a current which is lagging let us call this i 2 or a current which is leading let us call this i 3. So this current i one could be in phase with the upper voltage which is this or it could be lagging by some angle theta 1 or it could be leading by some angle theta 2. So, if it was i 1 you would see the current waveform like this that is i 1; if it was i 2 i 2 is lagging V in so you will see that it lags like that so this would be i 2. If it were i 3 it is leading therefore you see like that this would it implies that this is starting somewhere there this would be i 3. So this way you could have these three positions that is you could have an in-phase that is you could have a current which is in-phase with V in along V in or you could have a current which is lagging V in where the applied voltage or you could have a current which is leading the applied voltage V. So these are the three possibilities that you could obtain. But how do these three possibilities come?

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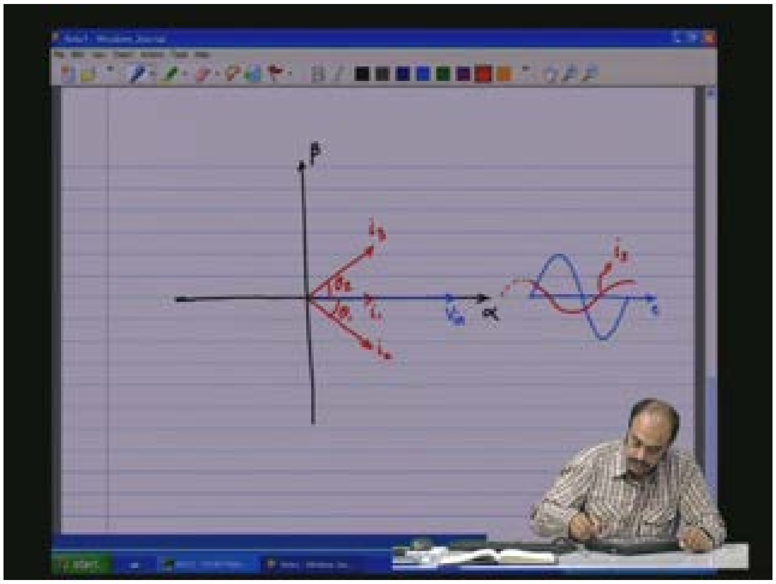

So, if I take for example; a purely resistive circuit so you have let us say a voltage source V i, this is V in and then this is a resistance R. Then you have i and there is a current which is flowing through this one we will call that one as i and i is equal to V in by R; R is just a constant it is an algebraic constant there is no phase associated with it there is no phase shift associated with this one and therefore i will be having the same phase whatever it was with V in because i is a variable, V is a variable, R is a constant. Therefore, instant by instant if I have V in which was like that with respect to time that is the temporal coordinates then i would be in-phase with V only this peak and if this peak were V m this would be V m by R and this is the i waveform, this is the V waveform.

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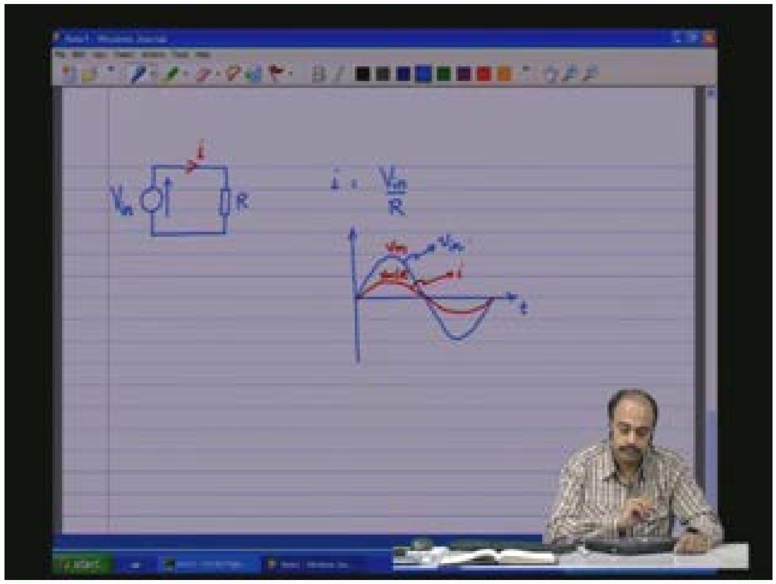

Now if I project this into the spatial coordinates the alpha beta coordinates let me start by taking V in as my reference. So V in is something a vector with an amplitude V m then another vector we will call that one as i vector which is having an amplitude V m by R and that is also in-phase because both are starting from zero that is also in-phase with some value and that would be the i vector.

So if you see I have one vector with amplitude V m and I have another vector with amplitude V m by R. Now this whole combination is rotating with frequency omega as determined by the frequency here: T is equal to 1 by f and omega is equal to 2pi f which is equal to 2pi by T with that angular frequency the radian frequency. So this whole set is rotating at that frequency but we want to know the relative measures or the relative differences between the different or the various phasors therefore we take a picture at an instant. So this is that picture at an instant at any one particular instant and it gives the picture of the current and the voltage phasors relative to each other of course in this case it is in-phase.

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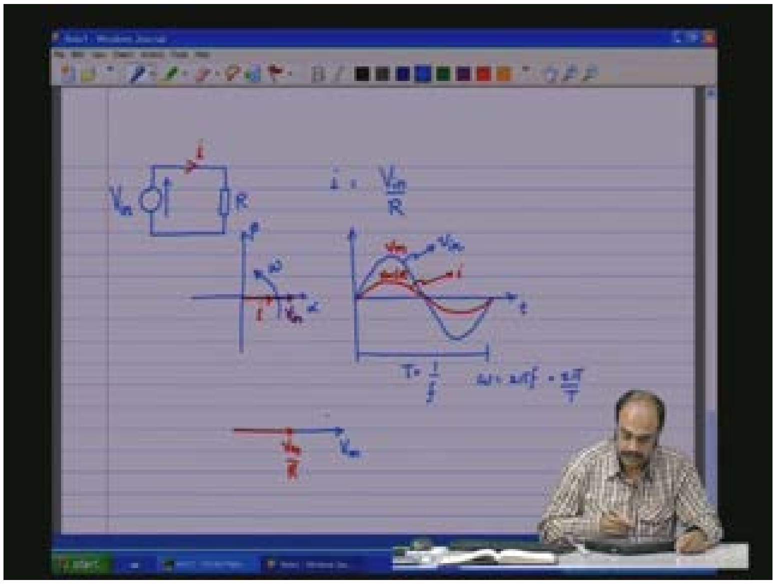

Now if you take the example of the capacitance connected across the AC source V in C now there is a current here i flowing. This is going to offer, capacitance is going to offer an impedance capacitive reactance which is j minus j into X c minus j into X c this of course has a frequency component that is nothing but 1 by omega c. So what is i? i is a complex quantity which is V in by the reactance which is X c. But there is this j operator which indicates that this is going to introduce a phase shift of 90 degrees and because it is minus there is going to be a minus 90 degrees phase shift to the voltages that is going to come across C which means the voltage that is going to come across C which is same as V in is going to be minus 90 degrees behind the current vector or the current waveform which means current is leading the voltage.

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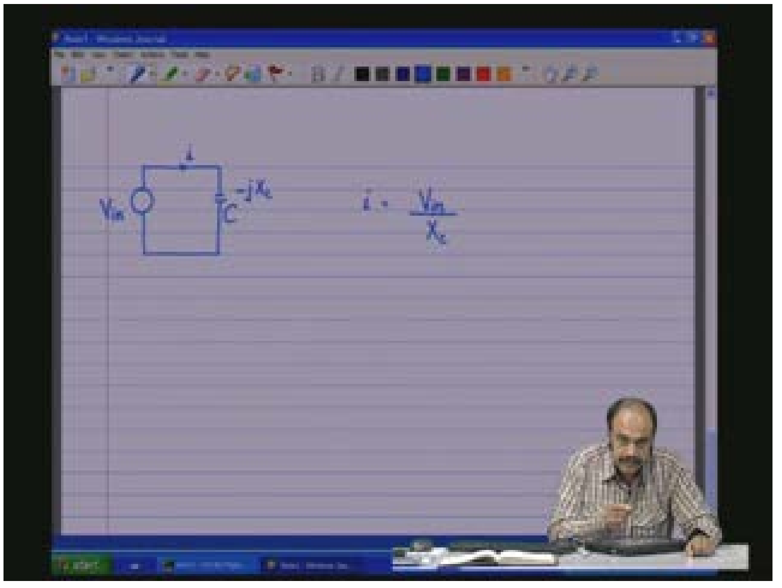

Let us just have a look at the simple simulation of this. I will use the spice for simulating this. We will have a reference ground the node we will call that one as 0, there is one node 1 here so V in is connected between 1 and 0, C is connected between 1 and 0 and let us give a sine wave here of 50 hertz and then simulate and see what happens. So, going to this Pspice editor we have here V i V input between node 1 and 0, a sine wave of source 0 DC offset 10 volts amplitude 50 Hertz 0 rise time 0 fall time and no phase shift. And then a capacitance is also connected between 1 and 0 node 1 and 0 and it has a value of 1000 Microfarads 1000 e minus 6 and let us do a transient analysis with a step of maximum step of 1 millisecond, 20 millisecond is the maximum time that is one period and then let us probe and see the waveforms how they look like.

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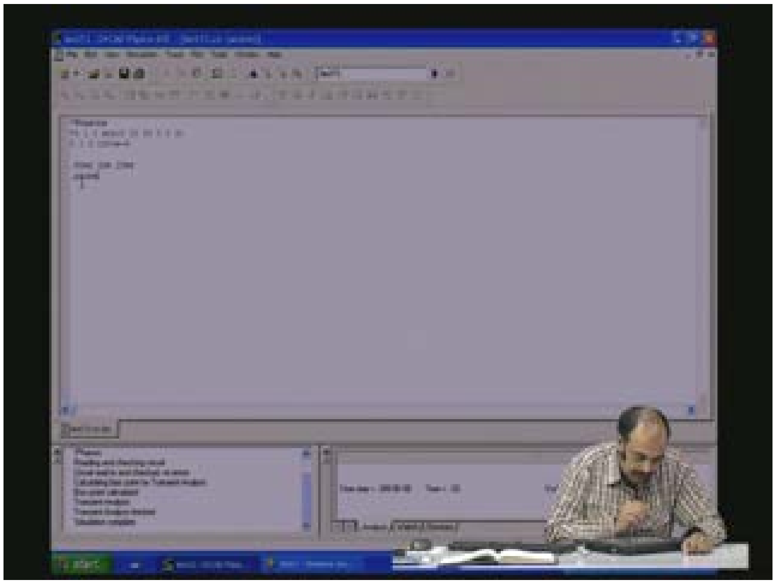

So let us simulate this circuit in Pspice. I am simulating and you see the I C and the voltage across this one.

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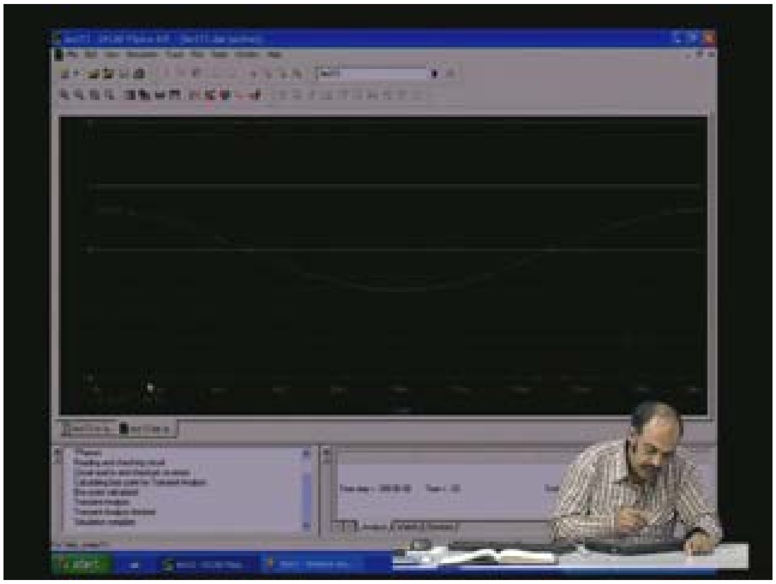

So the red one is the voltage that you are applying you see that you are starting from zero and the current which the currents with capacitance is already at the positive peak and then it cuts goes down cuts 0 at 90 degrees corresponding to the voltage and then becomes again 0 there is negative peak when the voltage is 0 and so on. So you see that there is a 90 degrees phase shift between the current and the voltage. So if you see the voltage is 0 current is positive maximum which means the current is leading the voltage. So if you go back to our notes here so we have the temporal waveshape in this form t (Refer Slide Time: 30:48). We saw that the voltage is our reference of course and therefore we are writing the voltage waveform like that. So when the voltage was at zero the current was already at its peak and then it became zero when the voltage was so it goes like that, like this……….. so this is your i, this is your V in.

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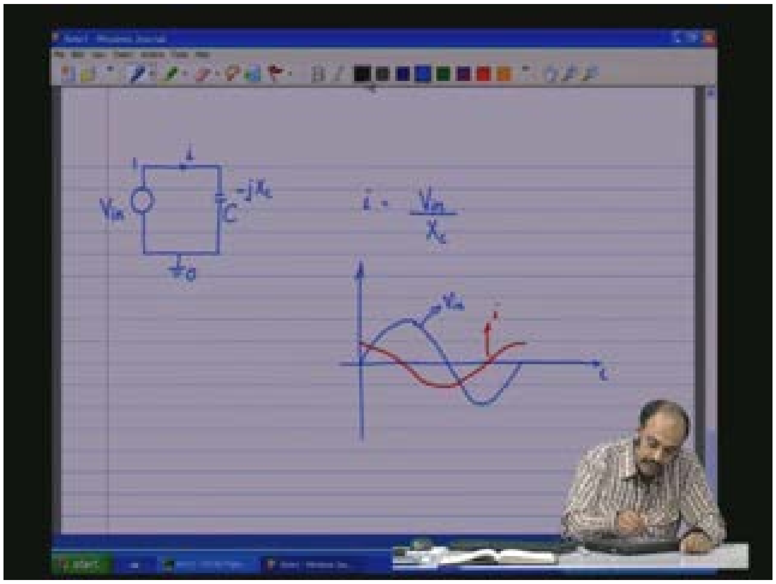

So now let us see how it gets projected on to the alpha beta axis alpha beta axis. What is our reference?

Reference is the input voltage starting at zero it has an amplitude V m. So this is our V in. Now the other phasor is starting this is starting at this point. What is the angle; the angle between these two is 90 degrees 90 degrees leading so the current is leading. Now this whole unit is rotating with a frequency omega. Now if you see that as it is rotating you see that this will now decrease the current is decreasing because when it goes on this side the projection decreases and as it comes to this point this is 0 as far as the current is concerned and that is at maximum 90 degrees so this will be the i (Refer Slide Time: 32:58). So if you see relative to the voltage V in I now have an i which is leading the input voltage by 90 degrees by 90 degrees. This would be the phasor relationship between the voltage and the current in a capacitive pure capacitive circuit like this.

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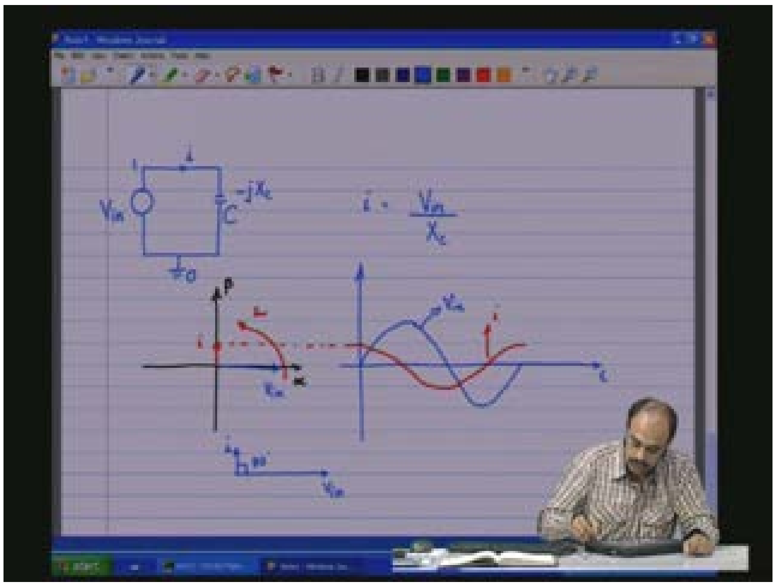

How does it look like for the inductive?
Now in the case of the inductive circuit we have resource and we have the inductor we have the inductor L, we have the source V in and this gives an inductive reactance or impedance which is j X L; X L is omega into L.

Now how does this give you the current waveform in relationship to the voltage wave?
In this case when a voltage is apply because of the inductance effect it is not going to allow immediate change in current so the current is going to be slower in raising when the voltage is raising which means the current will lag the applied voltage. And also if you see from the j X L term the voltage across the inductance is going to be multiplied by j that is j X L into i this is the

voltage across the inductance which is the applied voltage which means there is going to be a rotation of 90 degrees for the voltage which means that the current the voltage is going to be ahead by 90 degrees with respect to the current or in other words, the current is lagging the voltage. Therefore, this is what would happen in the case of in the case of the inductive circuit.

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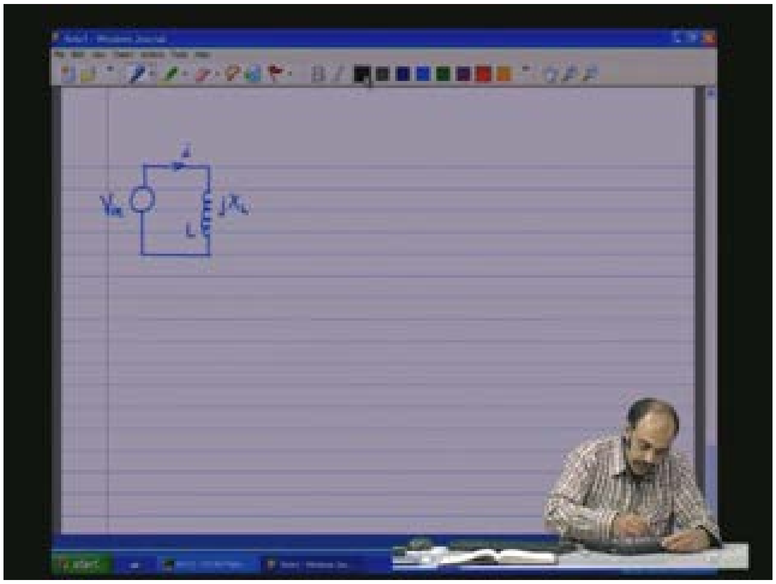

Let us look at the temporal waveshape pattern of the voltage and the current for this pure inductive circuit. The voltage is considered as the reference waveform starting from zero like that and this is the 90 degrees point (Refer Slide Time: 35:55) or shall we say pi by 2 point then the current waveform is going to lag the voltage by 90 degrees so it will reach at peak when the voltage is zero then starts coming down and so on like that which means and it start coming like that. This is the current waveform i and this is the voltage waveform V in.

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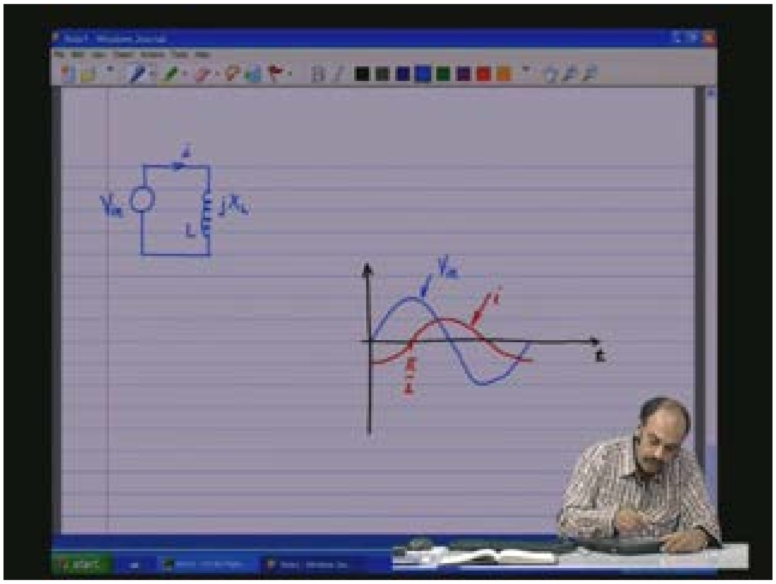

Now let us look at how it looks like in the spatial coordinate system; the alpha beta coordinate system. So again we are taking V in as the reference starting from zero let that be this vector, this is the V in vector. Now the current it is starting at the peak minus peak and with the phase shift of 90 degrees so this is the current vector. Now this too as I said rotates with a frequency of omega which is 2pi by t or 2pi f radians per seconds. Now you see here the current is lagging the voltage vector. Therefore you say that the current lags the voltage and this is the lag circuit.

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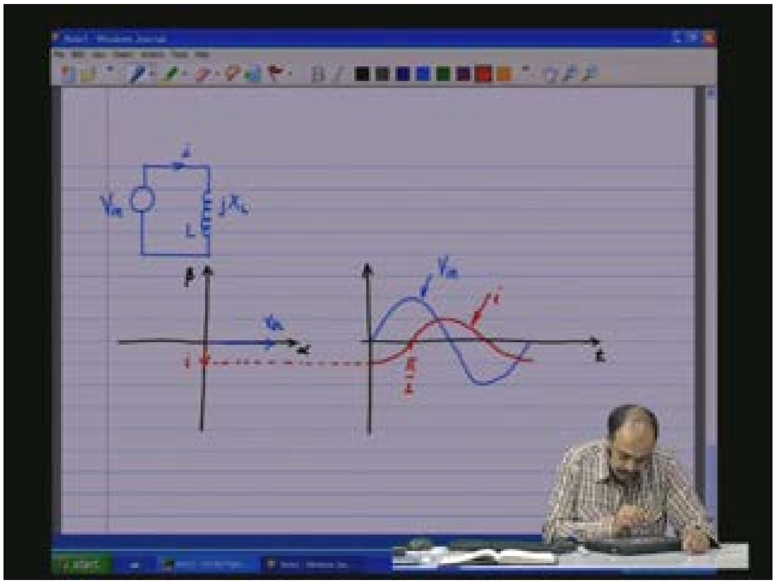

Now we come to three important types of vectors/vector combinations. I have the voltage vector V in and the current vector which is in-phase is the i or I have the voltage vector V in and the current vector which is let us say leading the voltage vector by 90 degrees or I have the third option we saw an inductive circuit for a voltage vector as shown, the current vector is lagging by 90 degrees. So this is for a pure resistive circuit, this is a pure capacitive circuit and this is a pure inductive circuit.

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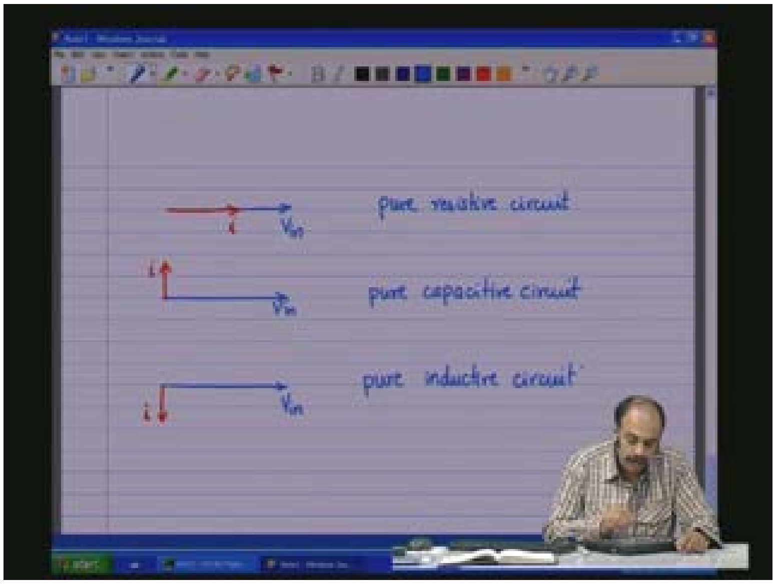

However, in actual practice and real circuits we will not see pure resistive or pure capacitive or pure inductive circuits; it will generally be a mix up piece so we do not get the current leading or lagging by exactly 90 degrees so it will be somewhere in-between depending upon the values of the resistance, the values of the capacitance, values of the inductance and so on.

Now let us take the RC circuit which we have been seeing for quite some time and we are now quite familiar with this circuit RC. This is our RC circuit RC, this is V in, so V in is the voltage that we are going to use as the reference. Just by looking at this we see that there is of course going to be a current which is flowing through this and that we call I; if there had been no R it is going to be a pure 90 degrees lead, the current is going to lead the voltage by 90 degrees, R does not introduce any phase shift which means that using an R C it is going to be somewhere between between 0 and 90 degrees lead depending upon the value of the R and Cs. This of course is going to give you a capacitive reactance j X c minus j X c.

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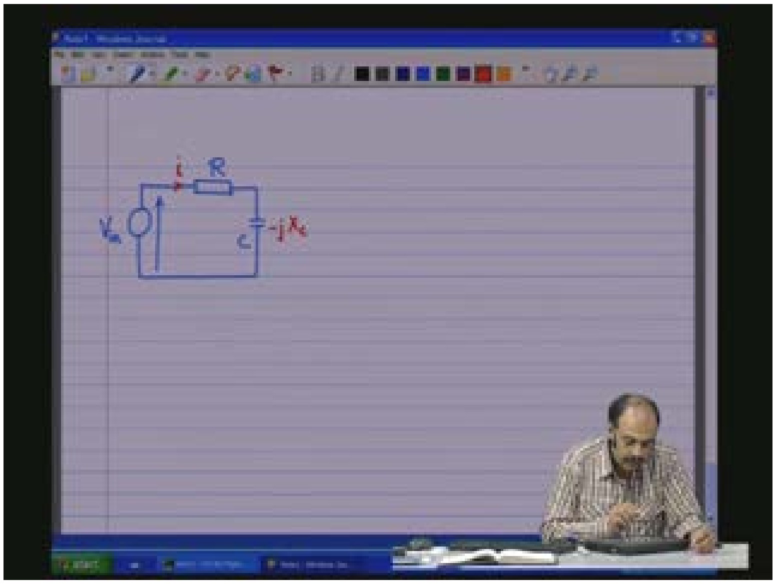

Now let us let us draw here directly the spatial coordinate system the alpha beta. This is directly the spatial coordinate system (Refer Slide Time: 42:05). Now here let us take our reference vector as V in, so V in is the reference. Now I know that the current here b is going to lead the voltage; by what amount of course we could see later. Therefore, a current is going to be somewhere in this let us say this direction because it is leading. the current i is leading the voltage by some angle theta, this implies that there are two components to this current; there is one component which is in-phase so we call that one as let us say i alpha and there is another component which is 90 degrees phase shifted with respect to V in and we call that one as i beta. So i alpha is the in-phase component of V in and i beta is the component of the current which is 90 degrees orthogonal to V, now this whole unit is rotating at frequency omega and this is a picture at an instant of time when V in is equal to 0 that is the V in projection is equal to 0 on the temporal axis.

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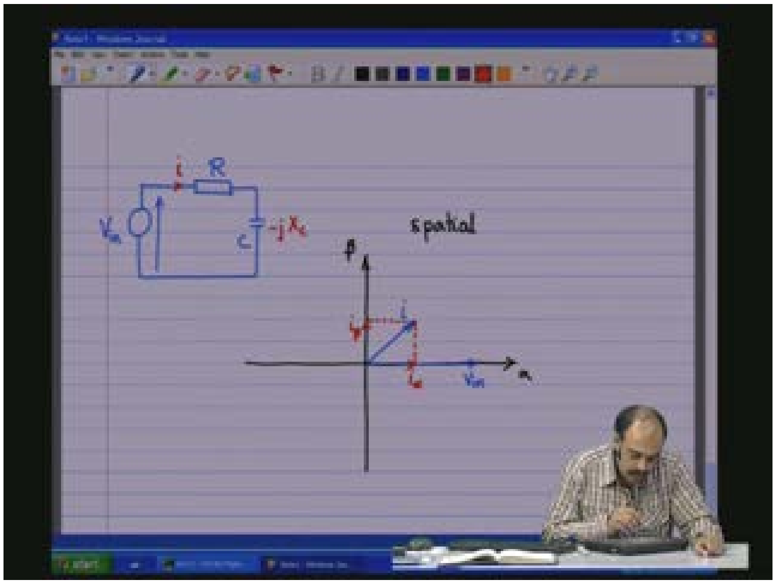

So, given this situation here, the in-phase component when multiplied by R is going to give you i alpha into R which is the voltage across R because V and the i beta is due to the capacitive portion which is causing this one. So i alpha into R is the voltage portion which is going to be in phase like that and i beta is due to the capacitive reactance and this is the resultant current (Refer Slide Time: 44:43) due to the in-phase component of the current and the out of phase component of the current. So this is how the this is how the vector diagram will look like for an RC circuit.

Now there is an angle theta here; what is the phase difference; what is the phase shift?
Now the voltage across the voltage across the capacitance is minus j X c which means whatever may be the current that is flowing through this current vector which is flowing through this gets multiplied by X c and gets rotated minus 90 degrees. So let us say we have this current which is flowing through X c; i is flowing through X c that now gets multiplied by X c and then gets rotated 90 degrees and let us say I put it in a different colour; gets rotated 90 degrees and comes like that which means this is 90 90 degrees and this amplitude would be X c into i and whatever i is flowing into R will be along that so the voltages-wise we have let us say R was unity it is i into R, if R is equal to 1 then it is the same value there and then R need not be 1, in that case let us say if I write V in and this is the i direction so we talk of the i direction which is in this direction

and let us say we have i into R and this is 90 degrees and this is going to be X c into i voltage-wise. So this and this (Refer Slide Time: 47:55) vectorially add up to V in and this is an angle theta with this has the hypotenuse so the opposite side divided by this side will give you the tangent or the tan of the angle and that is going to give you the phase angle, so tan inverse X c by R will be theta.

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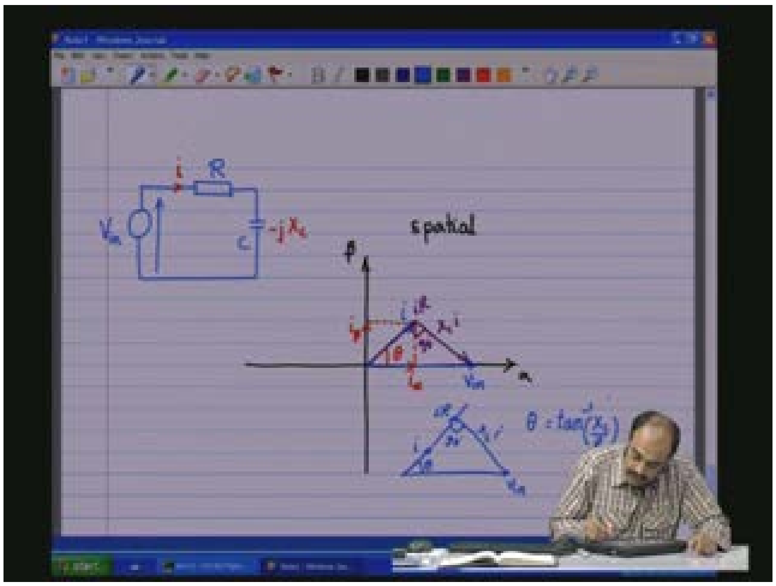

Therefore, knowing the value of C here knowing the value of omega knowing the value of R here you can estimate the phase angle theta which this vector is going to be displaced. Therefore I know that the current is going to lag lead the voltage by this angle theta as decided by the frequency, the capacitance value and the resistance value. So that is the important of this space vector concept.

Now here we see one important thing. There is a voltage across this V c and there is a voltage across this which is V R (Refer Slide Time: 49:34). So we can now extend the KVL. We said that the algebraic sum of the voltages in a loop should add up to zero; that is Kirchhoff's voltage law and we can extend it now by saying, in the steady-state if you are doing sinusoidal analysis with vectors the vectorial sum of voltages across in a loop across the components in a loop will add up to zero or the voltages of all the components, the vectorial sum of voltages of all the components should add up to the input vector which is what is happening here. We have the input vector, this is the input vector (Refer Slide Time: 50:41) we have the voltage across at V R and we have the voltage across the capacitance V c. So we have V in, we have V R and we have V c; this is 90 degrees; they are all adding up to the resultant V in which is what Kirchhoff's voltage law would state in a normal sense that you need to have the voltage to be zero in a loop vectorially also.

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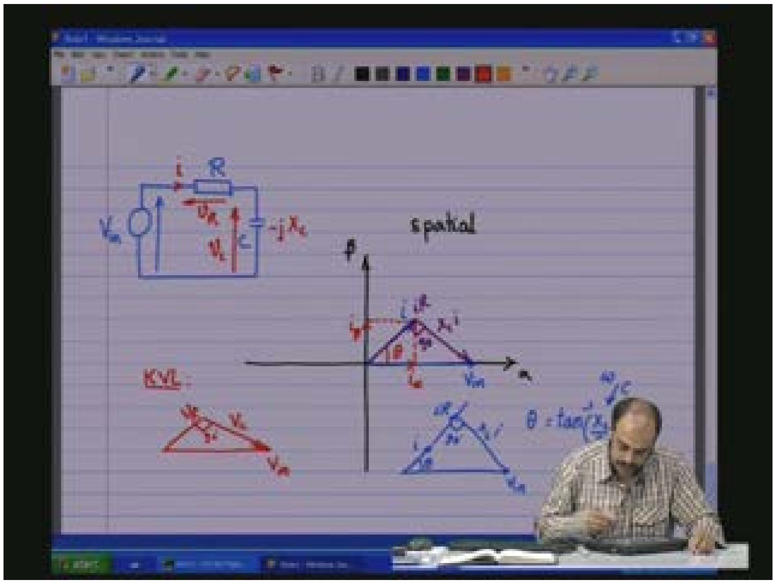

Now if you look at an RL circuit, an RL circuit is written like this. This is also familiar to you. We have gone through it many times. You have L, you have R, you have V in and this is the voltage we want to take as our reference and there is going to be a current i in the circuit, this has a reactance j X L.

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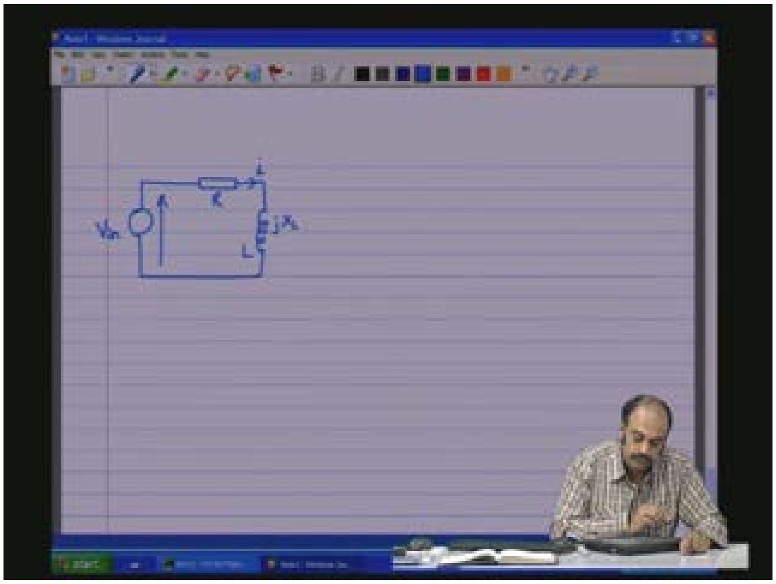

So let us take the alpha beta coordinate system or the spatial coordinate system directly; alpha, beta this is the spatial coordinate system (Refer Slide Time: 52:28). We have the reference which is V, so we start with V and we know that the current is going to lag so let us write down the current is lagging the voltage and this is the current direction, this is the current direction.

Now i into R which is going to be in the same direction so let me let me say that this is iR this is going to be in the same direction. Let me put this as i into R. Now the voltage across the inductance is X L into i but rotated to 90 degrees rotated from here rotated 90 degrees in the positive that is anti-clockwise direction so you have i into X L which is rotated at 90 degrees; this is i X L so there is an angle theta here, so theta is equal to tan inverse X L by R which is equal to tan inverse omega L by R. So you get the phase angle or the phase lag of the current space vector with respect to the voltage space vector.

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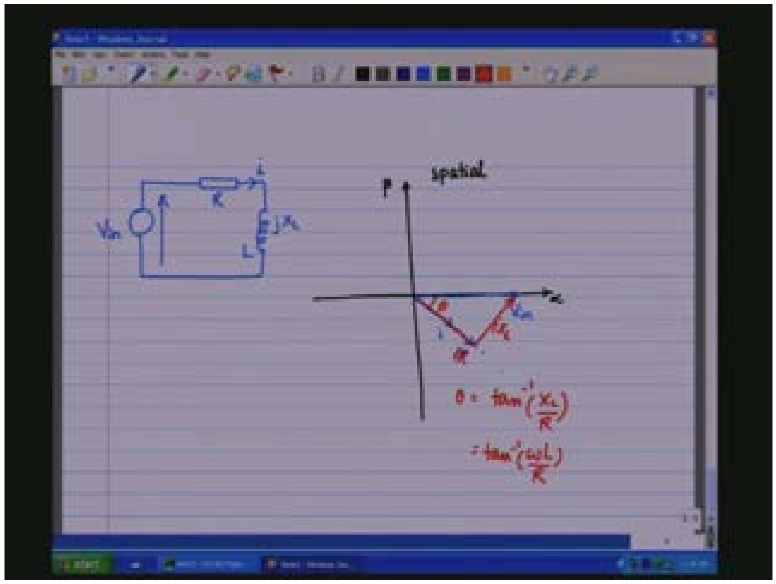

Here also you see that the voltage across R V R, the voltage across L V L vectorially add up to equal V in they vectorially add up to equal V in, such that the voltage across the loop is zero, Kirchhoff's voltage law is valid and thereby upholding the principle of energy conservation.

Now this whole unit (Refer Slide Time: 54:38) set of vectors are going to rotate with a frequency omega given by 2pi f which is equal 2pi by T and this is a picture taken at an instant where V in projection on the temporal axis is zero.

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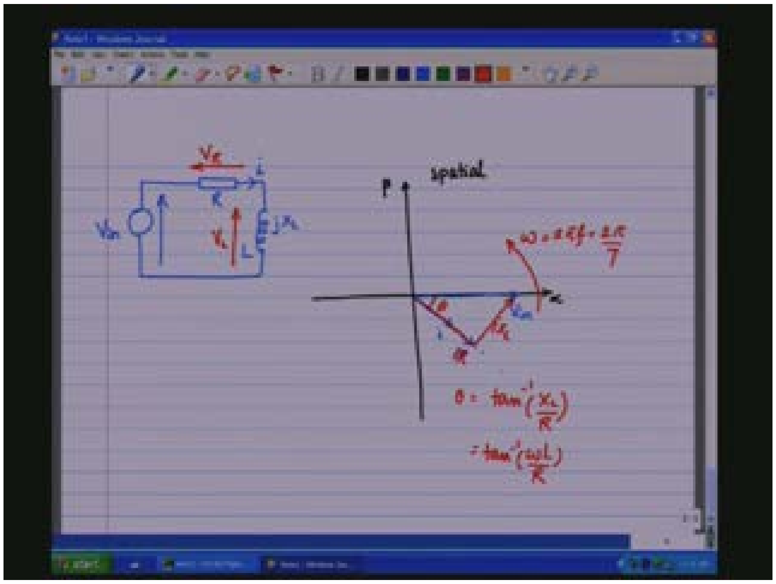

You can apply it now to an RLC circuit. You have R L C, so you have RLC, V in the input source and a reference R, this is L and C, this is going to give an impedance j X L, this is going to give an impedance minus j X c and in the spatial coordinate system alpha beta coordinate system so we have V in reference. So you always start with V in reference because you are applying V in and you know at what point it is going to cross zero and let us say we take that one as the reference. And depending upon the values of L and C the current could lead or lag, so we do not know whether it could be in-phase lead or lag; if the capacitive reactance of these two are same then it is going to be in-phase because R is the only impedance or if the inductance dominates then the current is going to lag, if the capacitance dominates then the current is going to lead.

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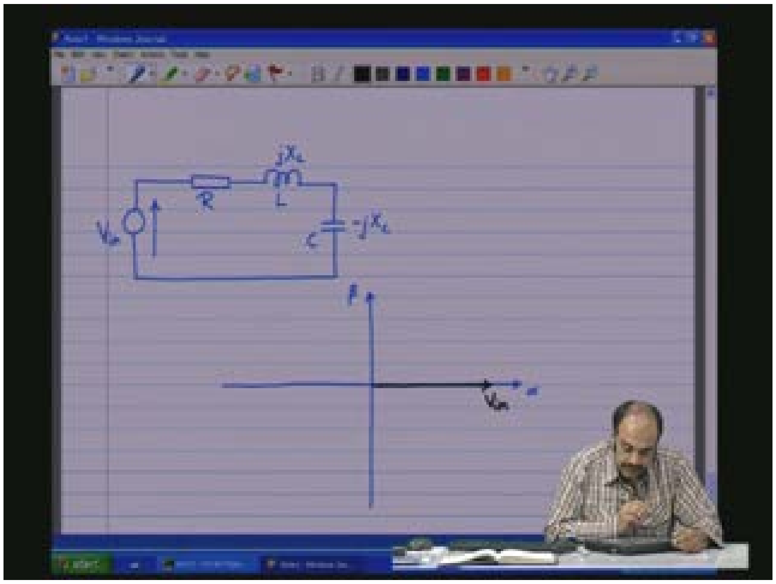

Now let us say for example the capacitance dominates which means the current is going to lead. So now to apply the Kirchhoff's voltage law how do the currents look like; how do the voltage using this current how will the voltages across look like?

We have the current which means the voltage across R is going to be in-phase along this line along this line, the voltage across the inductor is going to be orthogonal to this and so also the voltage across the capacitance which means they have to be orthogonal to this, this is 90 degrees. So iR i into R let us say is this vector i into R, now, along this rotating by j along this rotating by j would be j X L (Refer Slide Time: 58:15) and along this rotating by……… would be the j X c because along this minus j rotates it by minus 90 degrees, j X L rotates it by plus 90 degrees so along this it is going to be……………

Now these two being 180 degrees in opposition, the resultant of this is what is going to come across this one and fill up the gap here and that would be X L minus X c that would be X L minus X c; of course in this case when we say X c dominates meaning X c value is more then we take X c minus X L, we take it as X c minus X L. So in such a case the X c value is dominating and this will cancel out XL and then that completes the vectorial sum for V in. if the current were lagging which means inductance dominates the current would be here and then X L would dominate X c and it would be X L minus X c.

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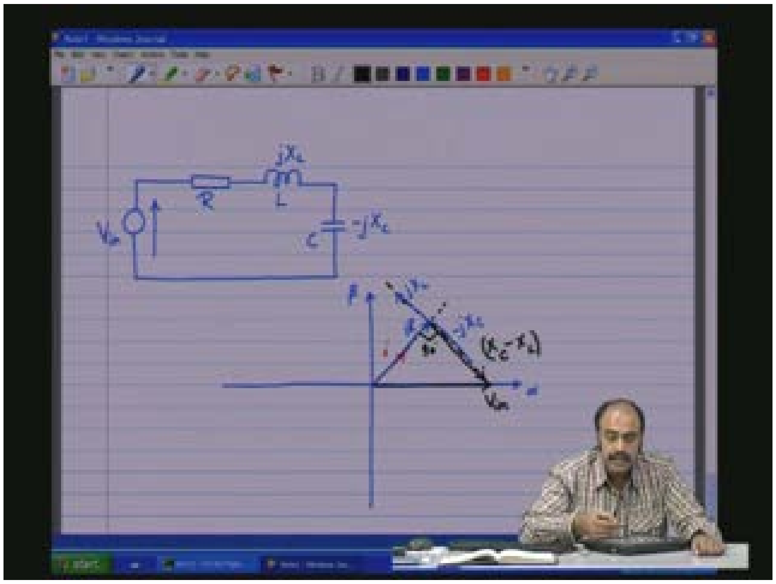

We will look at this in detail more in the next class and few more other circuits to understand to consolidate the process of phasor analysis much better. Thank you.