

**Basic Electrical Technology**  
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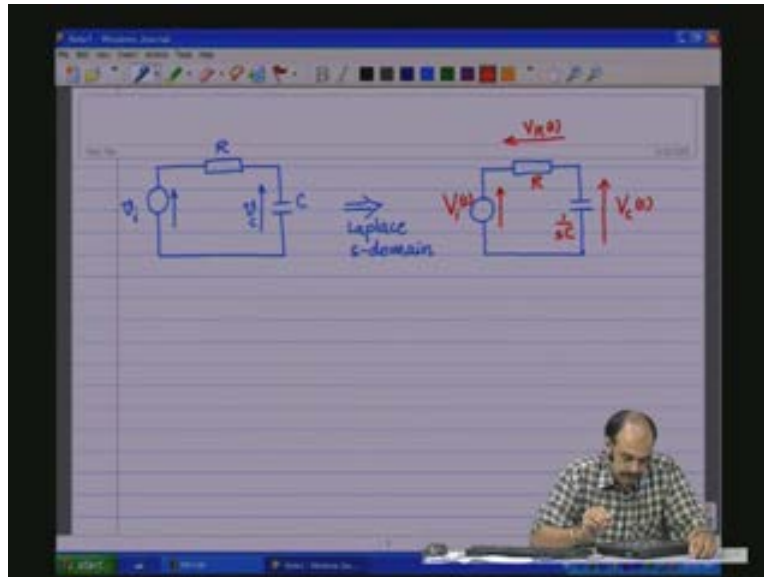
**Lecture - 11**  
**Transfer Function & Pole Zero Domain Part – II**

Hello everybody, in the last session we had quite some detailed discussion on the transfer functions in the pole-zero domain; modelling of circuit in the pole-zero domain. We saw that the transfer function is a special case of the state equation where the initial conditions are zero and then towards the end of the last session we also saw that there is a procedure, a method to extract the transfer function from the state equation. Basically the transfer function is  $C(sI - A)^{-1}B + D$ ; that is using the  $A, B, C, D$  matrices as the input matrices you can obtain the transfer function. We will see some of the examples in MATLAB today.

Further today we will try to consolidate the concept of pole-zero in the pole-zero domain using the transfer function model. We shall start by the simple circuit which is the RC circuit and then take up the RL circuit followed by the RLC circuit. So this RC circuit by now you all are familiar with this particular circuit and its state equation and also its transfer function equivalence. Now this has a  $V_i$  source voltage, there is a resistance  $R$ , and there is a capacitance  $C$  which has a voltage here  $V_c$  which is the state variable in this case because  $C$  is the energy storage element.

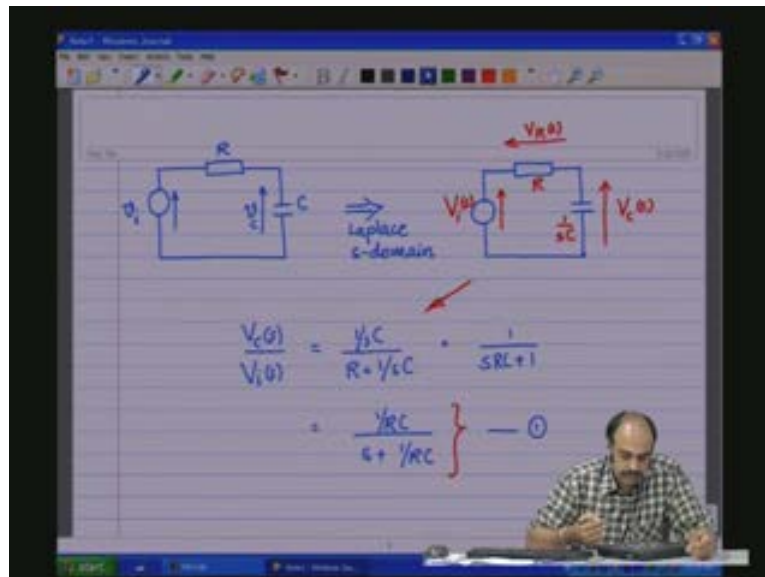
Now this let us shift or represent it in the Laplace domain or the  $S$ -domain or even the pole-zero domain as we have been calling it. So in the Laplace domain we still have the same topology of the circuit, we have the  $R$ , we have the  $C$ ; now here the variables are represented slightly differently; the lower cases are made upper cases and now they are functions of  $(s)$ ,  $V_i$  is a function of  $(s)$ ,  $V_c$  is also a function of  $(s)$  and  $V_R$  will also be a function of  $(s)$ ,  $R$  is the parameter and the capacitive reactance in the  $S$ -domain is given by  $1/sC$  **this we saw in the last class that** we just need to replace  $j\omega$  terms in the reactance that is  $j\omega$  with  $s$  which means  $1/j\omega C$  becomes  $sC$  the capacitive reactance in the Laplace domain and  $j\omega L$  becomes  $sL$  in the Laplace domain.

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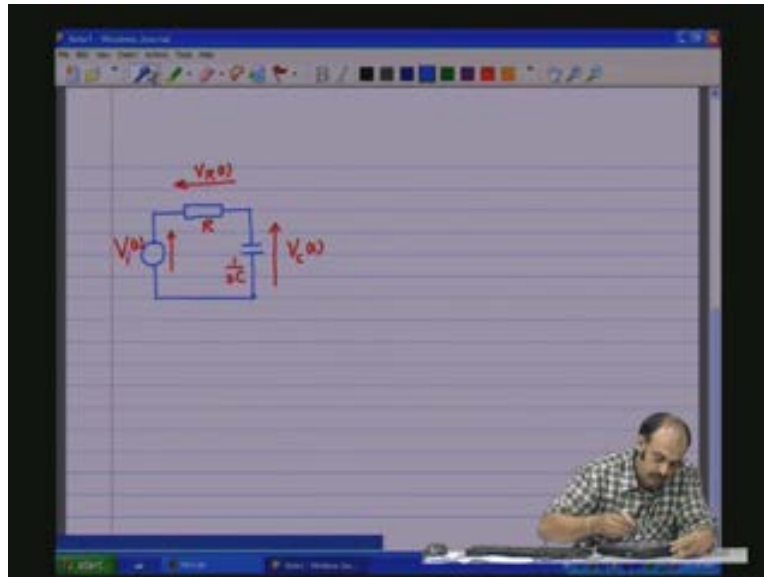
Now this circuit that is this circuit which we have transformed to the Laplace domain let us obtain a transfer function and we know that we also have seen that **obtaining the output transfer function by the input** obtaining the transfer function of the output by the input, Laplace transform output by the input as it is defined is just like in any other resistive network, in this case it is the capacity reactance  $1/sC$  divided by  $R$  plus  $1/sC$  and this can be written as  $1/sRC$  plus  $1$  or still further simplifying it you have  $1/RC$  divided by  $s$  plus  $1/RC$  this is one transfer function that we have obtained for this circuit wherein the transfer function basically in this case is defined as the Laplace transform of the output variable  $V_c$  divided by the Laplace transform of the input variable  $V_i$ .

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But one need not look at the transfer function just as the output variable  $V_c$  by the input variable  $V_i$ , it could be any variable, it could be the branch currents or it could be a voltage across any other component in this circuit in the same circuit. So if you take the same circuit let us say we would like to see the voltage across  $V_R$ ; what is the transfer function of the voltage across  $V_R$  with respect to the input  $V_i$ . So if we look at that **let me just have a copy of this circuit in our next page.**

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So we saw that we had  $V_C(s)$  by  $V_i(s)$  equals one by  $RC$  by  $s$  plus 1 by  $RC$ . Here the numerator does not contain the  $s$  terms which means there are no zeros, the denominator contains one order that is just to the power of 1 so there is only one pole and that pole is at  $s$  is equal to minus 1 by  $RC$  **this we saw in the last class.**

Now what will you get if you want a transfer function of  $V_R(s)$  by  $V_i(s)$ . You would like to have a transfer function of the voltage across  $R$  with respect to the input voltage. Now here it is voltage across  $R$  divided by  $R$  plus 1 by  $sC$  so this could be the ratio in which they divide the resistance divided by the resistance plus the capacitive reactance. So this on simplification would give  $sRC$  divided by  $sRC$  plus 1 slight further simplification will give you  $s$  divided by  $s$  plus 1 by  $RC$ . You see this is a little slightly interesting result. Here you have a numerator polynomial  $s$  which is of the order of 1 which means there is a zero at  $s$  equals 0.

There is of course a pole at  $s$  equals minus 1 by  $RC$ . So you see for the same circuit I can have different transfer functions depending upon what is the output variable that I want to choose and what is the input variable that I want to choose.

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$$\frac{V_c(s)}{V_i(s)} = \frac{1/RC}{s + 1/RC}$$
$$\frac{V_r(s)}{V_i(s)} = \frac{R}{R + 1/sC} = \frac{sRC}{sRC + 1} = \frac{s}{s + 1/RC}$$

zero at  $s = 0$   
pole at  $s = -1/RC$

Now these two transfer functions for this circuit that is  $V_c(s)$  by  $V_i(s)$  which is equal to  $1$  by  $RC$  by  $s$  plus  $1$  by  $RC$  and  $V_r(s)$  by  $V_i(s)$  which is equal to  $s$  by  $s$  plus  $1$  by  $RC$  these are the two transfer functions that we just obtained this output variable  $V_c$  across the capacitance with respect to the input the voltage across the resistance with respect to the input.

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$$\frac{V_c(s)}{V_i(s)} = \frac{1/RC}{s + 1/RC}$$
$$\frac{V_r(s)}{V_i(s)} = \frac{s}{s + 1/RC}$$

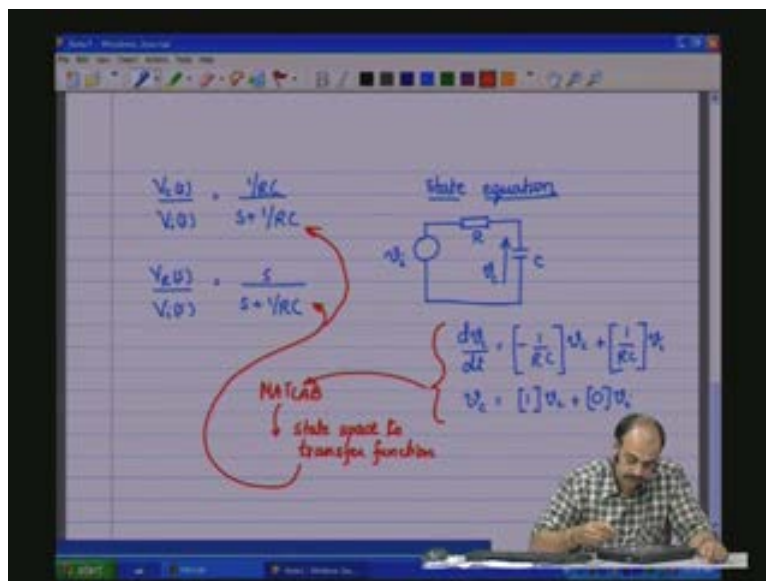
Now here if you see the poles are the same they have the same pole locations; only in this case there is no zero, there is a zero here in this case, a zero at  $s$  is equal to 0. How do they affect the responses we will shortly have a look at it in the MATLAB.

Now we could also get the same transfer function by..... as I said in the last class by the process of converting the state equation to the transfer function form using the conversion matrices or the conversion process that we discussed in the last session.

So for this particular circuit we also saw that the state equation; state equation for this circuit we already know we have done this many times but still for the sake of clarity I will just quickly write that down:  $V_c$  is the state variable so  $dV_c$  by  $dt$  is equal to minus 1 by  $RC$  which is the A matrix into  $V_c$  plus 1 by  $RC$  the B matrix into  $V_i$  this is the dynamic part of the state equation and of course output equation we saw  $V_c$  equals  $[1] V_c$  plus  $[0] V_i$  this is the output equation.

Now we shall do a small process that is we take this state equation model, input it to MATLAB. So in MATLAB let us make a conversion state space to transfer function model and obtain these models depending upon what we want to give as the output variable that we want to see.

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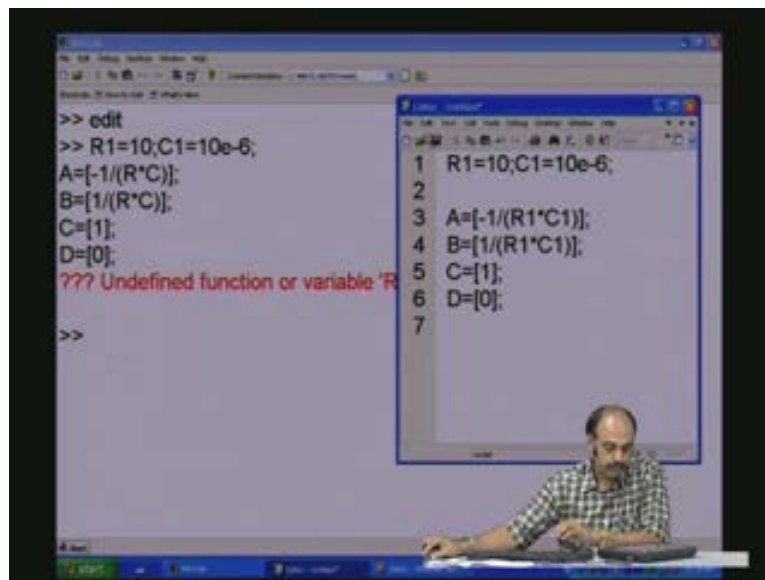


If it is V c then we would like to get this transfer function and if it is VR then we would like to get this model (Refer Slide Time: 13:42). So there are few functions in MATLAB which we can use to do these conversions. I will just briefly describe to you about these things.

So first we go into the MATLAB domain. In the MATLAB domain let us open an edit window. So we have the edit window here. Let us first give the parameters, there are two parameters let us say R1 equals let us say 10 ohms for now; C1 equals 10 e minus 6 or 10 Microfarads.

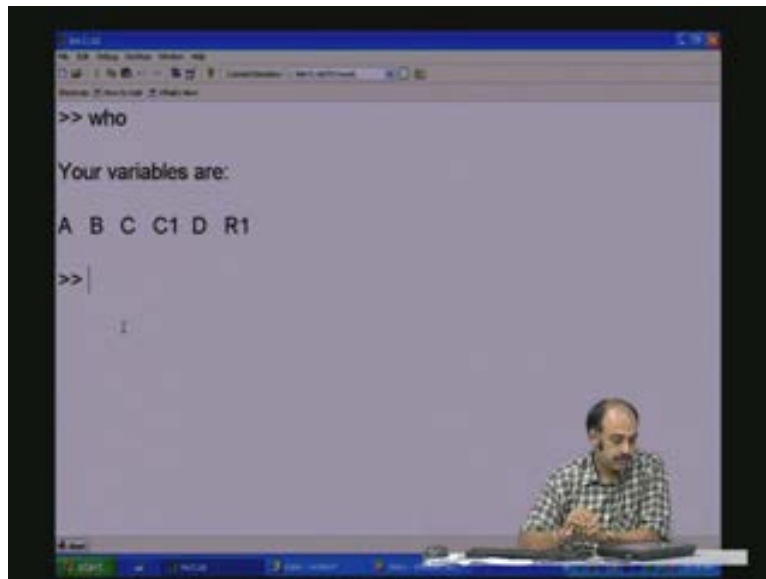
Now the A matrix is given by minus 1 by R into C we saw those in the earlier session also, B matrix equals 1 by R into C and if you want to see the output voltage as VC then we give the C matrix equals 1 and the D matrix equals 0. This is the model of the **RL circuit sorry** RC circuit which we shall input to MATLAB **sorry** I will go back to the editor these are R1 C1 R1 C1.

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Now we copy this, paste (Refer Slide Time: 15:43) and we have the variables.

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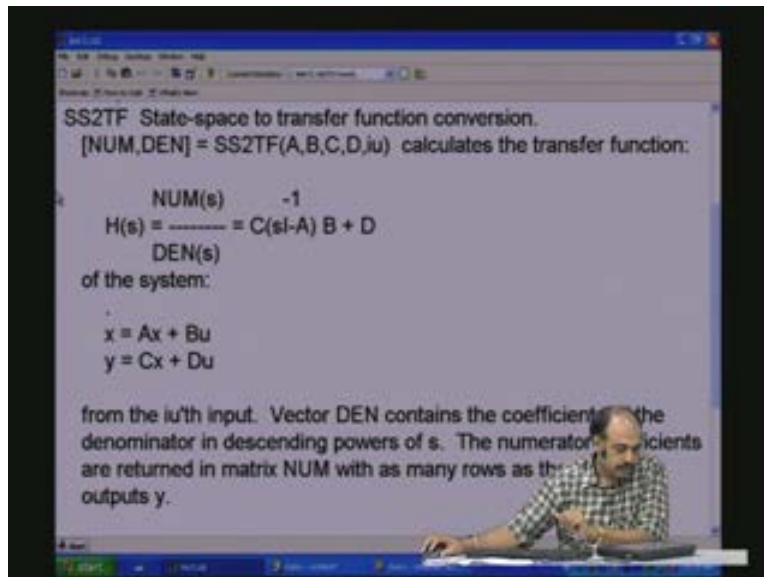


Now what we want to do with these variables?

These variables are in state-space form. Now we want to convert it into the transfer function form. so we use one function in MATLAB called state-space to transfer function form that is ss2tf this means state-space to a transfer function form. The syntax is very simple; in fact MATLAB is very friendly; if you want to know to the syntax of this you just have to type help ss2tf so you get the syntax of the particular function command.



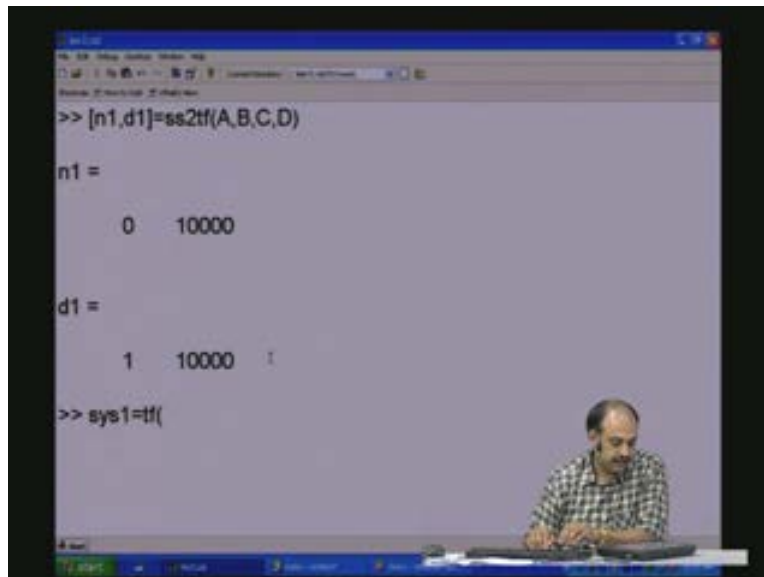
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So what does it say; state-space to transfer function conversion. A numerator polynomial denominator polynomial, this is the output is equal to the state-space to transfer function for which the parameter that you are inputting are A B C D and initial conditions and for the transfer function which we are trying to calculate we are going to give usual condition zero and **this will** basically we use this kind of an algorithm  $C(sI - A)^{-1} B + D$ . in fact this is what **we derived in the last class if you remember** of this particular state-space former A B C D matrices or as defined for a general state equation.

So let us do this particular conversion. Let me have a numerator polynomial n1, a denominator polynomial d1 of the transfer function which should be obtained after the conversion ss2tf state-space to transfer function and we have A B C D matrices. So this A B C D matrices should get converted to the numerator polynomial denominator polynomial. So this is the numerator polynomial function of s this is denominator polynomial function of s, of course it does not look like a transfer function so **what do** what we can do is we can represent the system in the transfer function for using this numerator polynomial and denominator polynomial in a more friendly form using the tf function.

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```
>> [n1,d1]=ss2tf(A,B,C,D)

n1 =

    0   10000

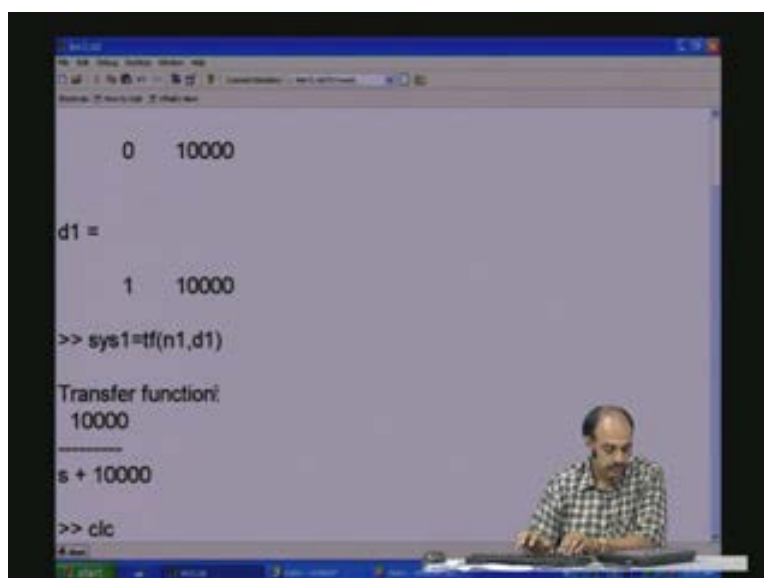
d1 =

    1   10000

>> sys1=tf(
```

So if you input the numerator and the denominator polynomials n1 d1 so that will give you this. So you see the transfer function now is in our familiar notation, this is 10000 divided by s plus 10000 and 10000 is 1 by RC is it not? 1 by R was 10 and C was 10 e minus 10 Microfarads so 1 by RC is going to be 10000 s plus 1 by RC.

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```
    0   10000

d1 =

    1   10000

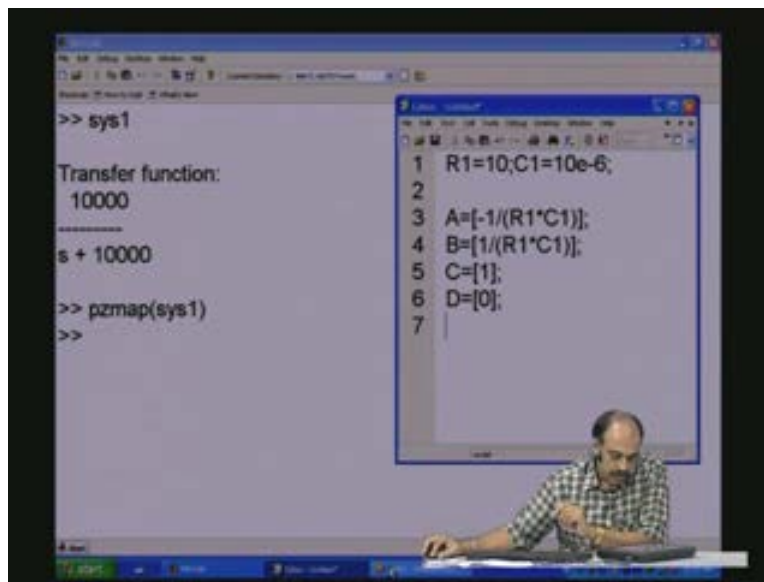
>> sys1=tf(n1,d1)

Transfer function:
 10000
-----
s + 10000

>> clc
```

Now sys1 is this (Refer Slide Time: 19:15); what will you get for the pole-zero map of sys1. You see that in the earlier session when we did the pole-zero map we had given the state equation as the input the A B C D or you have the transfer function model you can give the transfer function model also. So sys1 is the transfer function model and we had given what is the pole-zero map of the transfer function model; so you see here there is one pole at minus 10000 which is minus 1 by RC no zeros.

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Now let us try to find out what will be the transfer function for the voltage across R to the voltage that is your input voltage that you are supplying. (Refer Slide Time: 00:20:16) So, going back here to this portion I would like to see what would happen to the output equation when you want an output here as  $V_R$  instead of  $V_c$ . You see that  $V_R$  here, this is  $V_R$ ; what is  $V_R$ ?  $V_R$  is nothing but  $V_i$  minus  $V_c$ . So how does it reflect here? Now I am going to modify this, I will erase the C's here and I will put that one as R now I want  $V_R$ , now the state reflector of course will remain the same that is now minus  $V_c$  and therefore I got minus 1 here and  $V_i$  is a plus 1 so **I erase this 0** and put a plus 1 here so this is our output equation.

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The slide contains the following content:

- Transfer functions:
$$\frac{V_c(s)}{V_i(s)} = \frac{1/RC}{s+1/RC}$$
$$\frac{V_R(s)}{V_i(s)} = \frac{s}{s+1/RC}$$
- Circuit diagram: A series RC circuit with input voltage  $V_i$ , resistor  $R$ , and capacitor  $C$ . Output voltages  $V_c$  (across capacitor) and  $V_R$  (across resistor) are indicated.
- State equations:
$$\frac{dv_c}{dt} = \left[-\frac{1}{RC}\right]v_c + \left[\frac{1}{RC}\right]V_i$$
$$V_R = [R]v_c + [0]V_i$$
- Text: "state equations" and "state space to transfer function".
- Reference: "MATLAB" with an arrow pointing to the state equations.

Now C is minus 1, D is 1 this is the change that you have to do to obtain V R. So let us do that modification in the MATLAB editor. We make D C as minus 1 and D as plus 1.

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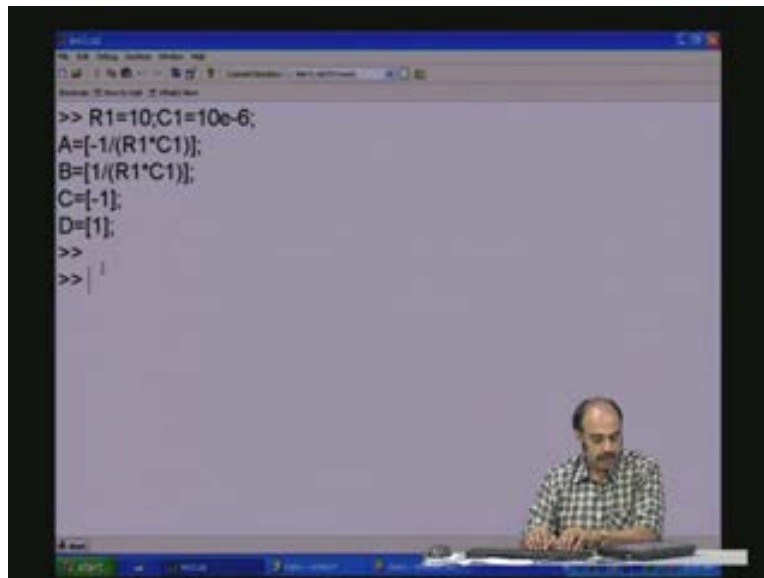
The slide contains the following content:

- Transfer functions (same as the previous slide):
$$\frac{V_c(s)}{V_i(s)} = \frac{1/RC}{s+1/RC}$$
$$\frac{V_R(s)}{V_i(s)} = \frac{s}{s+1/RC}$$
- Circuit diagram (same as the previous slide).
- State equations (same as the previous slide):
$$\frac{dv_c}{dt} = \left[-\frac{1}{RC}\right]v_c + \left[\frac{1}{RC}\right]V_i$$
$$V_R = [R]v_c + [0]V_i$$
- Text: "state equations" and "state space to transfer function".
- Reference: "MATLAB" with an arrow pointing to the state equations.
- MATLAB code window:

```
1 R1=10;C1=10e-6;
2
3 A=[-1/(R1*C1)];
4 B=[1/(R1*C1)];
5 C=[-1];
6 D=[1];
7 ;
```

Now with this state equation this represents  $V_R$  as the output which means we can now obtain the transfer function. **Let me clear everything**. We can now get the transfer function of  $V_R$  by  $V_i(s)$ .

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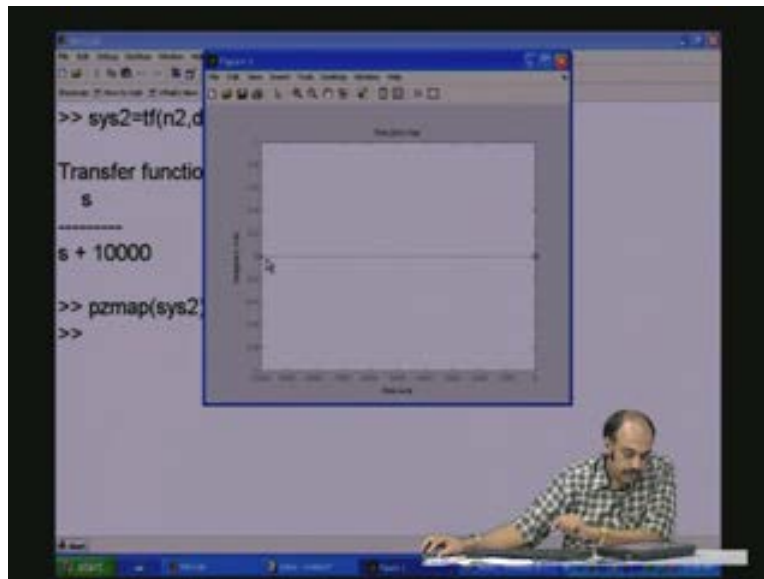


```
>> R1=10;C1=10e-6;
A=[-1/(R1*C1)];
B=[1/(R1*C1)];
C=[-1];
D=[1];
>>
>> |
```

Now we perform the same operation that is we now get the numerator polynomial of the second transfer function, denominator polynomial of second transfer function equals state-space 2 tf conversion of A B C D matrix. Now this is going to give you the numerator and denominator polynomial. now this let us view it in a more friendly manner that is we define the transfer function of the system for the voltage across R by the input voltage with tf numerator polynomial denominator polynomial of what you just obtained and this is what you see. This is  $s$  divided by  $s$  plus 1 or  $s$  divided by  $s$  plus 1 by  $RC$  10000 is  $RC$ .

How does this look in the pole-zero map? `pzmap sys2`.

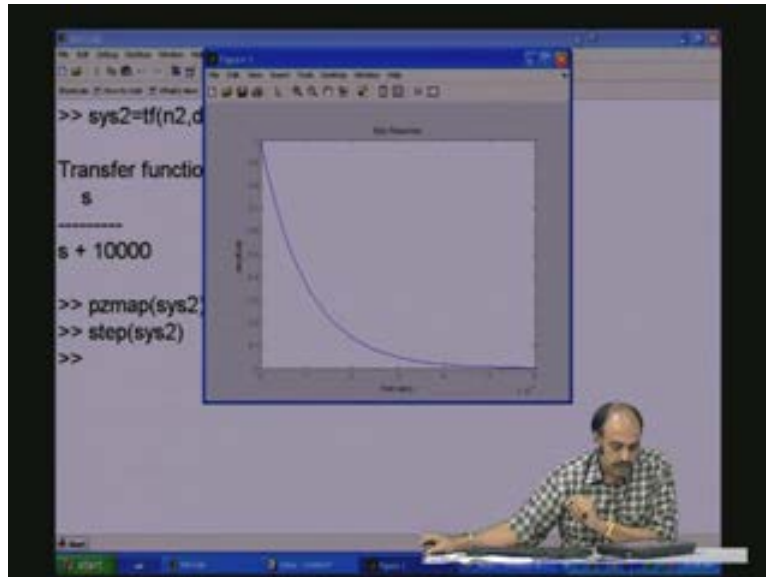
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You see there is a pole here at minus 10000 same as in the earlier transfer function and there is one more thing there is a zero here at  $s$  is equal to 0 which corresponds to the numerator. The **numerator the** roots of the numerator of the zeros that is what corresponds to the zero here, the roots of the denominator of the poles that corresponds to the pole here.

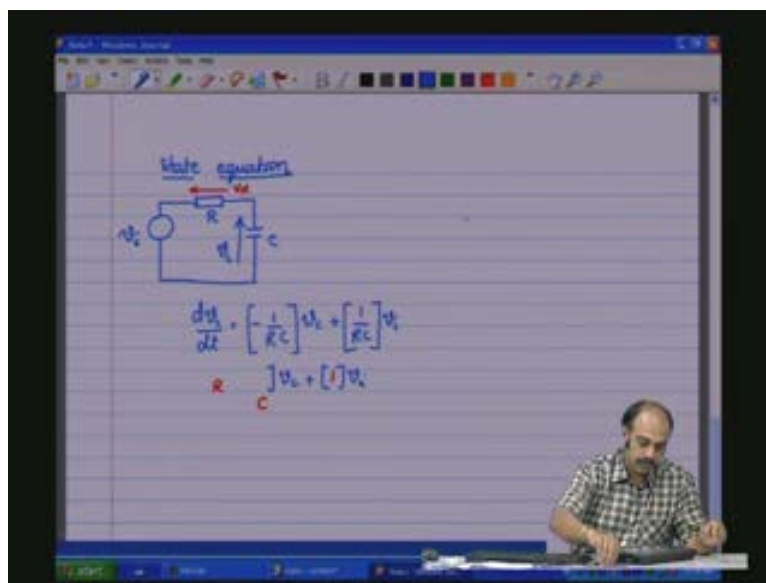
Of course we could also see the step response. Also, if you just plot `step(sys2)` you would obtain the step response. You see here what is happening, decaying towards zero whereas in the case of the earlier transfer function it would have been increasing. Now let us see both together. Transfer functions are single input single output systems. They can be defined only for one input one output, the numerator is  $V_i$  or  $V_s$  or  $V_R$  or any other parameter or a current and the denominator is another variable, you cannot have multiple outputs you have to look at them separately. But in the case of the state equation I can have multiple inputs multiple outputs you can look out many outputs simultaneously. So let us see simultaneously both the outputs.

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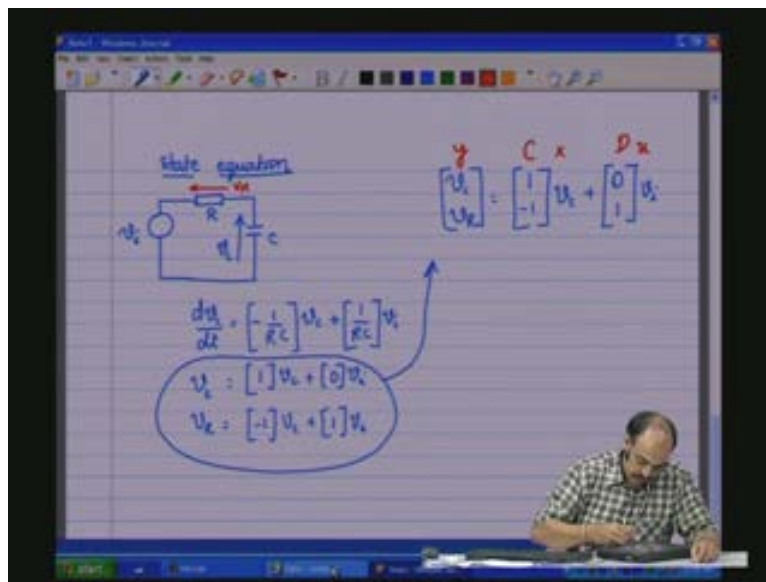
Let me go back to the notepad here and show to you..... let us take this, let us go the next page (Refer Slide Time: 25:40).

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Now here if you see that..... **let me erase some of these things** we wanted  $V_c$  and therefore we had put this as 1 and 0 (Refer Slide Time: 26:16) and when we wanted  $V_R$  we put this as minus 1  $V_c$  plus 1  $V_i$ . I could as well put the output equation as that is this output equation let us put it as  $V_c$   $V_R$  equals 1 minus 1  $V_c$  plus 0 1  $V_i$ . You see **this is** this is our  $Y$ , this is our  $C$  matrix, this is our  $D$  matrix, this is our  $x$ , this is our  $Dx$ ,  $y$  is equal to  $Cx$  plus  $Du$  this can be easily inputted in MATLAB and then you could look at all the outputs simultaneously.

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So let us go and make that change. So what do we do here? You see that  $V_c$  the  $C$  matrix is 1, semicolon is next row minus 1  $D$  0 semicolon is next row 1. So, by just modifying  $C$  and  $D$  matrices now you are able to see multiple outputs so this becomes a single input multiple output system which is very easy to define in the state equation format and the transfer function. So let us look at what this will do and then we have the model.

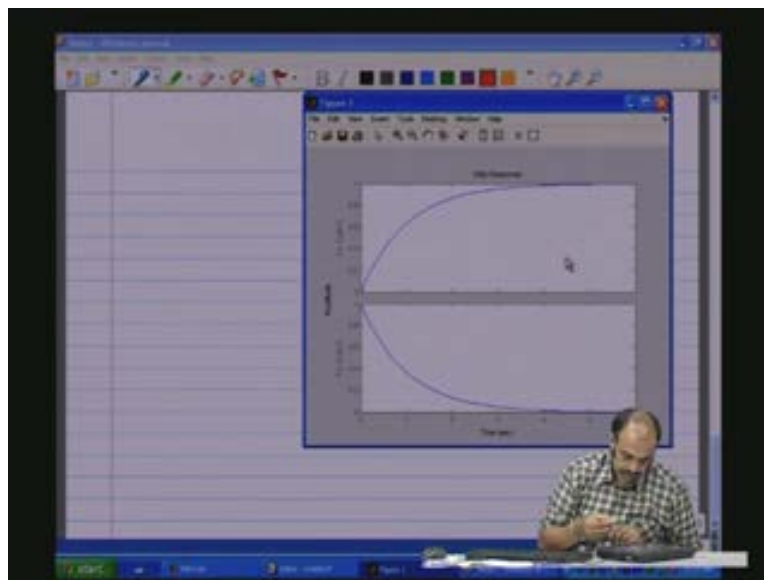
Now let us see the step response of the system. You see that both the outputs are now printed this output one which is  $V_c$ , this is output two which is..... **let me go to the windows here notes let us take that so** this is  $V_c$  and this is the voltage across  $R$   $V_R$  (Refer Slide Time: 28:54).



Now notice that there are some interesting features here. In the voltage across  $V_R$  you see that the output is growing and settles down at 1, the input is the step a unit step, it is growing and settles down at this particular value 1 here and in the case of  $V_R$  it is decaying and settles down at around 0 or exponentially it decays to 0.

So here even if there is input there is an input which is not zero the output is zero which means the gain for the transfer function is zero. That is what the concept of zero explains. Zero means that at some points of the time domain curve you will see that the output is zero even when the input is present that is zero again. And here (Refer Slide Time: 30:03) of course we see that this is rising and now we are removing the input; when you make it zero you will see that this will decay and the output will exist meaning it will not be zero even when the input is zero and therefore the gain will be infinite so there is the pole, the existence of the pole and the zero.

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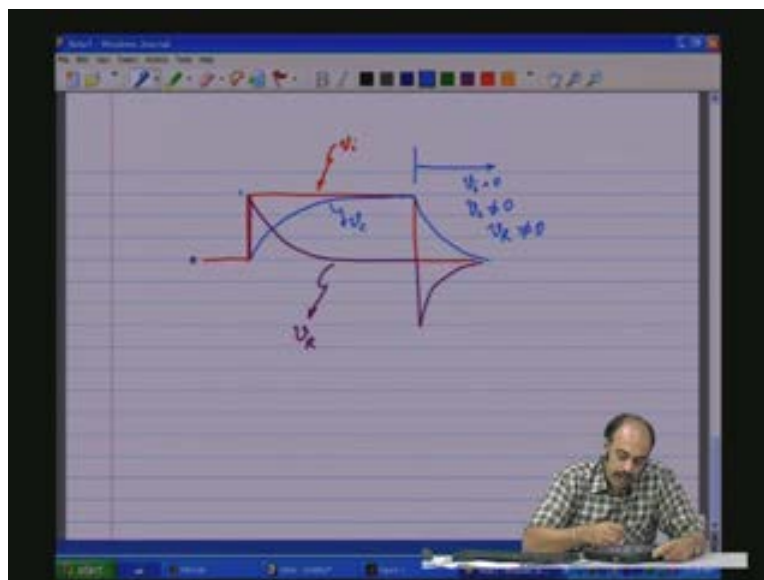
So let us just have a bit more consolidation on this pole-zero concept. You see that we give a step; we give a step as the input, what happened to the output? Output you see exponentially increased and then merged with the step there so this is value 1. What happened to the voltage

across  $V_R$ ? It was a value which started from here (Refer Slide Time: 31:20) and then starts going gradually down to zero.

Now let us say we close the step that is we bring the step back and then make it zero. So  $V_i$  becomes zero. So what happens to the voltage across the capacitance? It starts decay like that gradually to zero and what happens to the voltage across  $V_R$ ; you will see that there is a reverse current the reason being if you look at the circuit **sorry** circuit here (Refer Slide Time: 32:13) the moment the input voltage is made zero there is a charge here, this will pump a reverse current throughout and the voltage across  $R$  becomes instantaneously negative. That is what we are seeing it becomes instantaneously negative and then as the charge decays it will decay to zero, so this is the zero value.

Therefore, you see here the various waveforms; this is  $V_i$ , (Refer Slide Time: 32:45) this is  $V_c$  and this is  $V_R$ . Look at this particular waveform combination. If you take a voltage across  $C$  or a voltage across  $R$  during this portion or during this portion because during this portion of the time I know that  $V_i$  is equal to 0; because  $V_i$  has become 0 so when  $V_i$  is equal to 0 in both the cases the  $V_R$  and the  $V_c$  voltages are non-zero  $V_c$  is not equal to 0  $V_R$  is not equal to 0 what does this mean; this means that the input voltage is zero, output is non-zero therefore the gain is infinite so this implies the existence of a pole.

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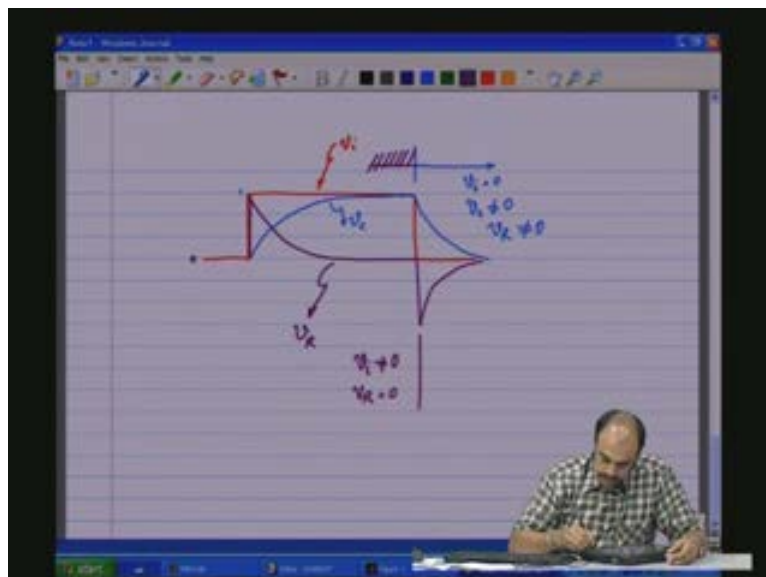


How fast or how slow it decays gives you the position of the pole in the S-plane so that is the major concept you should understand. Now just before when the input is existing so during this that is during this portion of the time axis here  $V_i$  is non-zero that is  $V_i$  is not equal to 0 however you see that  $V_R$  goes towards 0 and  $V_c$  is not going towards 0 so it is..... therefore **the transfer function** the  $V_c$  transfer function does not have any zero gain it is not having any zero whereas in the case of  $V_R$  it goes to zero at some of these durations and therefore there is a zero gain possibility which implies the occurrence of a zero in the circuit and which reflects as zero in the pole-zero plane which in this case occurs at  $s$  is equal to 0.

This basically is the physical concept that could be there in any dynamical system. For example; if you look at a fan rotating or if you look at the wheels of a vehicle rotating, you switch off the engine which means there is no input but the vehicle is still coasting which is basically this region (Refer Slide Time: 35:56) where  $V_i$  is made zero but the vehicle speed is coasting gradually decaying to zero which means the output is there even though the input is zero that is the existence of the pole and the decay depends upon the position of the pole in the S-plane.

Likewise for the zeros if any particular variable decays to zero like a differentiator then there is existence of a zero. This concept you should get it clear.

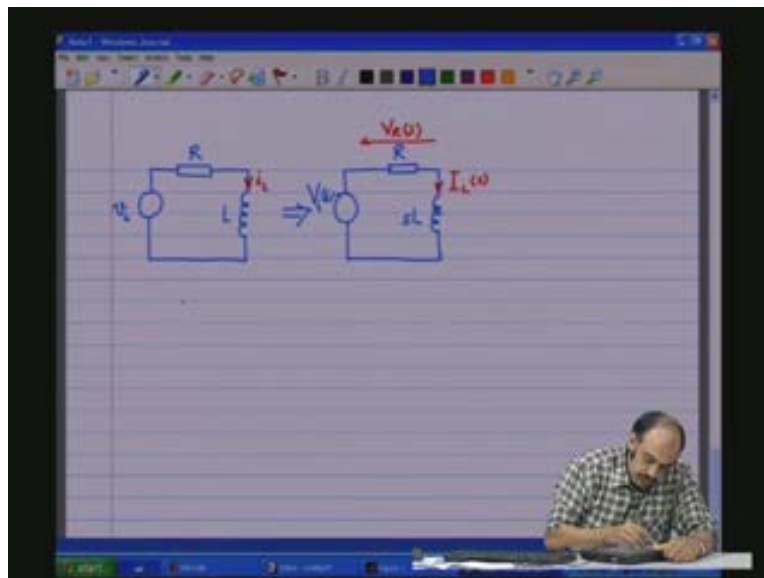
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Now **this procedure** we shall use the procedure of analysis that is obtaining the state equation and then from there the sinusoidal steady-state equation and then from the state equation again the transfer function which can be obtained and then taking it to MATLAB suitably, appropriately converting the state equation to the transfer function and then doing the analysis in the three major domains which is the time domain, the frequency domain and the pole-zero domain would be the standard package for other.....; standard process to follow for analysis. So let us strengthen our feel for this by doing an analysis on the RL circuit.

We have seen till now the RC circuit, let us have a look at RL circuit along similar lines you will not have much problems. So we have  $V_i R L$ . Of course this has a state variable which is  $i_L$ . Now this we take it to the Laplace domain the circuit by a simple process of replacing  $j\omega$  in the inductive reactance with  $S$  and we also capitalise all the variables and they become functions of  $s$ , this is  $R$  and then you have  $L$  and now the reactance would be  $sL$  and we have  $I_L(s)$  as another variable here and if you want to have the voltage across  $R$  it is  $V_R(s)$  all in the Laplace domain you will have the Laplace variable which is the  $s$  variable coming to the picture.

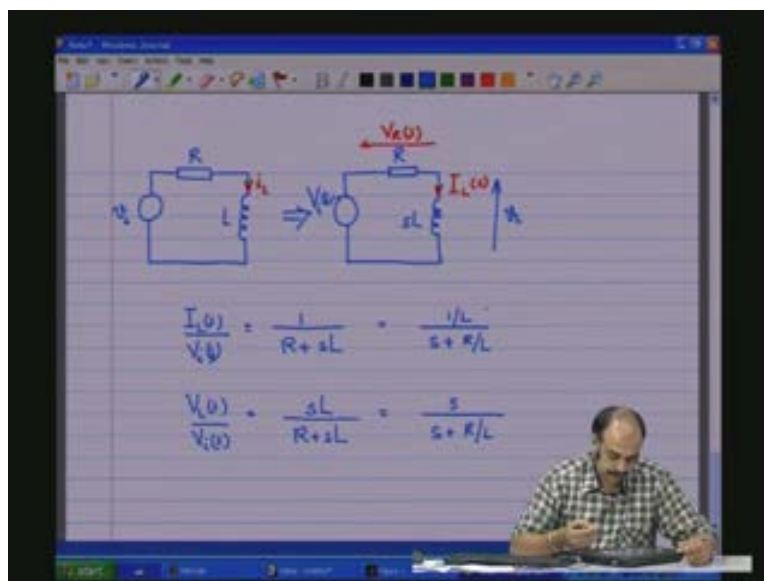
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Now if you want to have a transfer function of  $I_L$  as the output with respect to the input  $V_i(s)$  then what do you get? It is nothing but  $\frac{1}{R + sL}$  divided by that is voltage divided by these impedances which is  $R$  by  $sL$ ; plain and simple. And by small simplification you have  $\frac{1}{s + R/L}$  that is if I divide throughout by  $L$ ,  $1$  by  $L$  divided by  $s$  plus  $R$  by  $L$ . So we have pole at  $s$  is equal to minus  $R$  by  $L$  **this also we saw in the last class.**

Now if you want to get the voltage across  $R$  by **sorry we will not do that**, let us take the voltage across because we want to see a zero across the inductance  $V_L$ . So what is  $V_L(s)$  by  $V_i(s)$ ? That is equal to  $sL$  divided by  $R + sL$  this is equal to  $s$  divided by  $s + R/L$ . You see this is  $1$  by time constant just similar to the capacitor circuit  $1$  by  $RC$   $1$  by time constant there also, you have a zero in the numerator  $s$  is equal to  $0$  and one pole in the denominator at minus  $R$  by  $L$  these two are similar in terms of the poles but in terms of the zeros the different; this is decaying in nature, (Refer Slide Time: 40:57) this is not decaying in nature this is settled down at some value which is whatever the input value would be.

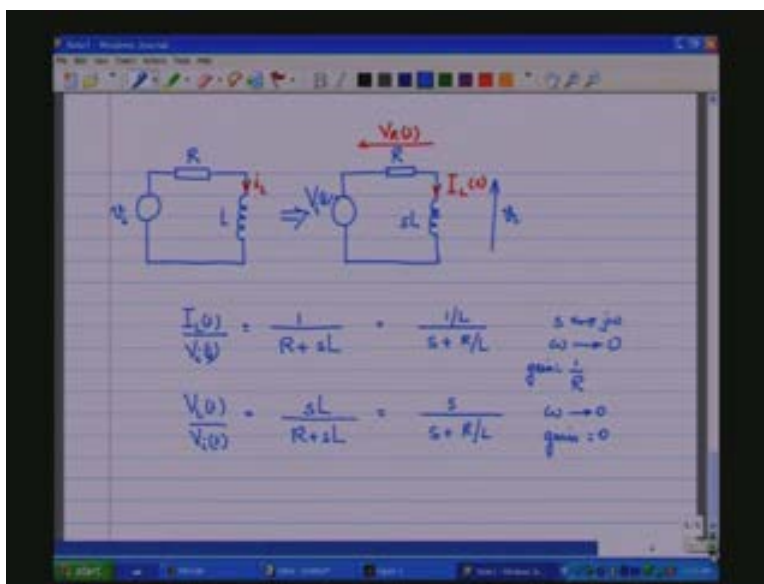
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Now if you look at the RLC circuit..... before that where will this settle down you see; as you should see that  $s$  was representing the  $j\omega$  in frequency terms. So as the step, when we give an input step as it starts going towards the steady-state the  $\omega$  starts tending towards  $0$  that is

pure DC away from the transients so this would start going towards 0 and then you will have **1 by R as the** 1 by R as the scaling in this case that is  $i_L$  by  $V_i$  would be just 1 by R or  $V_i$  that is whatever the input  $V_i$  is divided by R would be the gain so this is the gain. And in this case as  $\omega$  starts tending towards 0 this will also tend towards 0 and then you will see that the gain tends towards 0 **gain tends towards** 0 so that is basically the idea that you will have to get from this pole-zero analysis, the transfer function analysis.

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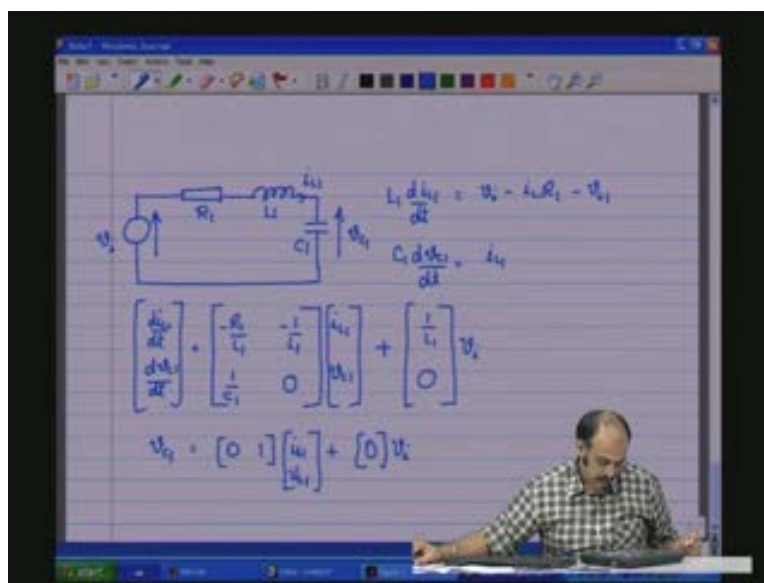
Let us to one more circuit which is the R L C circuit which combines both together. So you have the R L and C all these are combined together. Now I will call this one as R1 L1 C1 V i and there is a V c the state variable V c 1, there is a current  $i_L$  through this. So let us quickly write the state equation. So  $i_L$   $\frac{d i_L}{dt}$ ; you take across that one which is  $L \frac{d i_L}{dt}$  is the voltage across the inductor which is  $V_i$  minus  $i_L R$  minus  $V_c$ . You see that this is expressed only in terms of the state variables and the input variables and  $C \frac{d V_c}{dt}$  equals..... **we have done this in our previous session I am just repeating this for clarity** so  $C \frac{d V_c}{dt}$  is the current through the capacitance which is  $i_L$ . So, putting it as a state equation form so you have  $V_i L \frac{d i_L}{dt} - V_c \frac{d V_c}{dt}$  this is the  $\dot{x}$ ;  $x$  being the state vector you have  $i$

$L$   $1$  and  $V$   $c$   $1$  as a state vector plus an input matrix that we need to fill up here (Refer Slide Time: 45:06).

Now i  $L$   $1$  look at the equation here, there is a minus  $R$   $1$  by  $L$   $1$  coming in to the picture, now there is a  $V$   $c$   $1$  which is minus  $1$  by  $L$   $1$  which is coming in to picture and  $1$  by  $L$   $1$  which is coming in to the picture for the  $B$  matrix that is  $V$   $i$ .

Now the other equation we see that there is a  $1$  by  $C$   $1$  there is nothing here and nothing here (Refer Slide Time: 45:47). So this is the dynamic equation or output equation what is it that you would like to see. So let us say we would like to see  $V$   $c$   $1$  then in that case you have  $0$   $1$  i  $L$   $1$   $V$   $c$   $1$  plus  $0$  into  $V$   $i$  so this is the output that you obtain. Let us have a look at the transfer function of this.

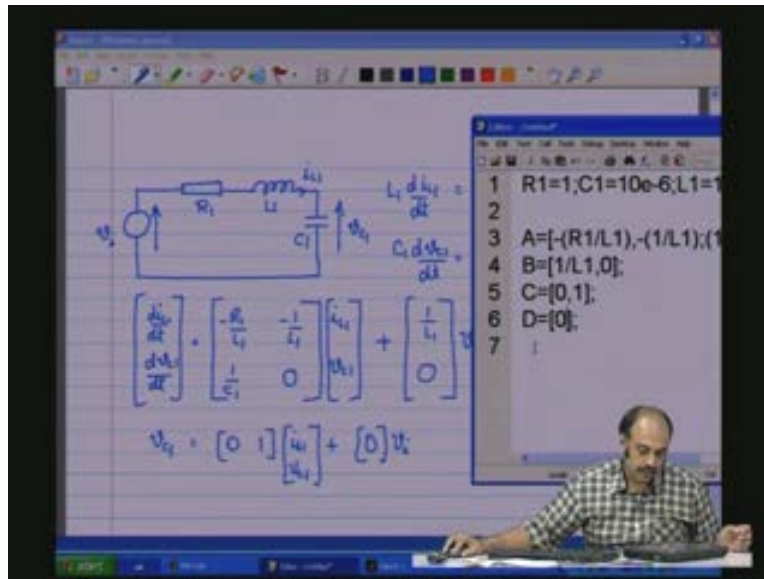
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So, going into the MATLAB editor we have  $R1$  which will define as may be still lesser let us say  $1$  ohm,  $C1$  is  $1$  Microfarad,  $L1$  equals  $10$  Millihenry minus  $3$ . Now we have to define the  $A$  matrix;  $A$  matrix contains  $R1$  by  $L1$  that is  $1$ , minus  $1$  by  $L1$  the next row which means semicolon  $1$  by  $C$   $[1, 0]$  so that is the  $A$  matrix. Then for the  $B$  matrix we have  $1$  by  $L1$ , then  $0$  is

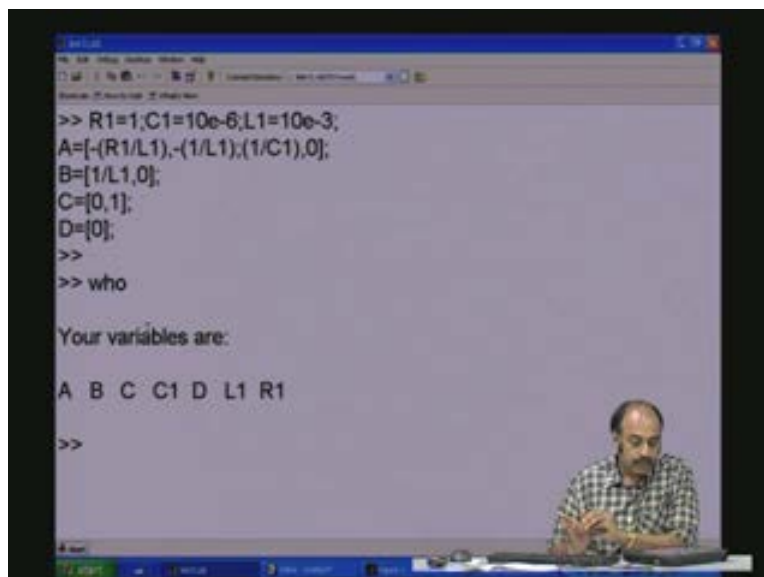
it not? 1 by L1 0. Now for the C matrix we have [0, 1] and for the D matrix just 0. This is the definition of this particular circuit.

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Let us copy that, go to MATLAB, clear the screen, paste and we have the.....

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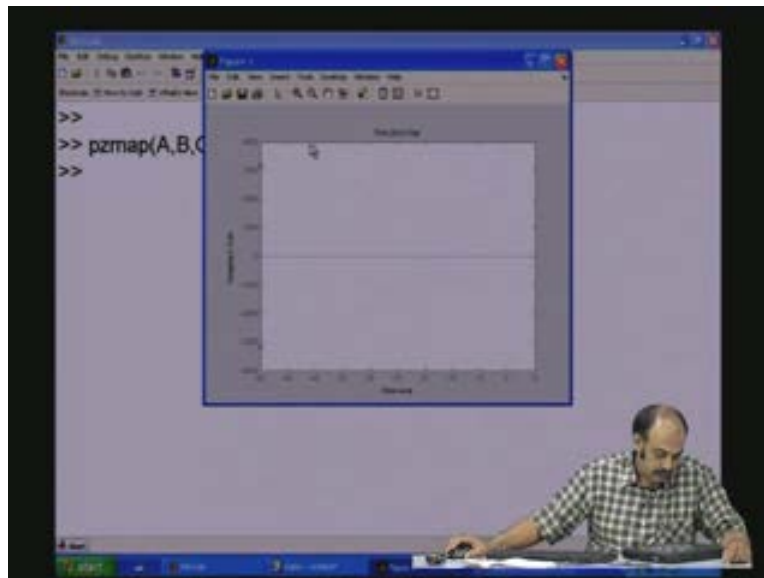




What are the variables?

A B C D R1 C1 L1. Let us have a look at the pole-zero map of A, B, C, D. Let us see how the transfer function pole-zero map looks like in the pole-zero domain.

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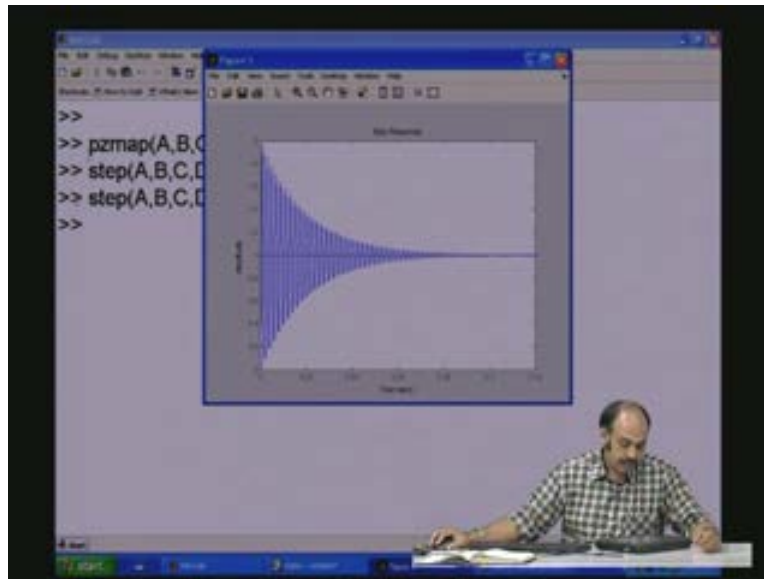


So you see you get a pole-zero map of the domain S-plane, it is the real axis. You see that at minus 50 you have one pole two pole there are two poles; one pole at the **positive side of the** positive side of the omega axis and one pole on the negative side of the omega axis they are exactly at the same position as far as the projection on the real axis is concerned and they mirror each other. So whenever you get a complex pole you should have another mirror image complex pole on the other side of the real axis.

Now this implies that there can be some oscillation and damping **which will be** which will be normal in most LC circuits tank circuits; and LC tank circuit will give rise to oscillations because the energy gets stored only in kinetic form then in potential form and then keeps going back and forth and because of the presence of the resistance some energy will gradually get lost in the resistor and that is what will cause damping and the amplitude of the oscillation will start coming

down. So let us have a look at the step response to this particular transfer function. Step A, B, C, D. So let us see the step waveform.

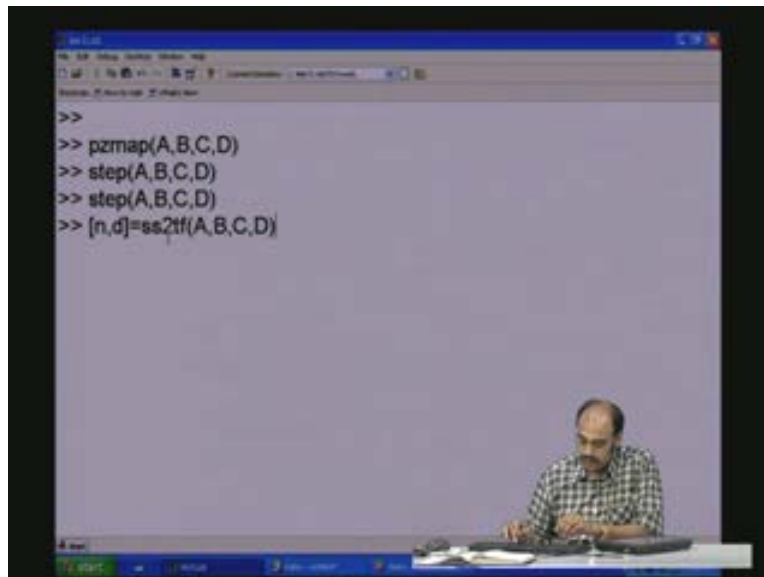
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As I have been telling you that for a unit step input you see the output voltage across the capacitance; you see the oscillatory nature of the voltage here and then gradually damps down. Since the amplitude is gradually decreasing this implies the loss in the resistor and there is a time constant involved in that. And this damping (Refer Slide Time: 51:25) is actually related to the position of the poles **with respect to the  $j\omega$  axis** how far it is with respect to the  $j\omega$  axis and this oscillatory nature as we were discussing the frequency will depend upon how far it is from the real axis that is what is the projection on the  $\omega$  axis.

Now let us have a look at the transfer function which we saw in the pole-zero and the step response figures. So let us define two variables  $n$  and  $d$ ;  $n$  is the numerator  $d$  is the denominator of the transfer function that we want to get let us use two transfer function function `2tf` and we want to get the transfer function of (A, B, C, D) the system which is the RLC circuit.

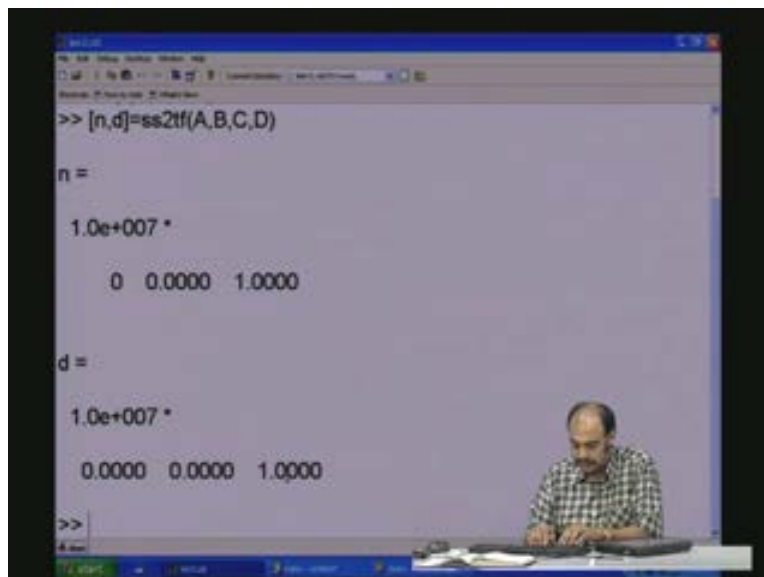
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```
>>  
>> pzmap(A,B,C,D)  
>> step(A,B,C,D)  
>> step(A,B,C,D)  
>> [n,d]=ss2tf(A,B,C,D)
```

So this results in numerator polynomial and the denominator polynomial.

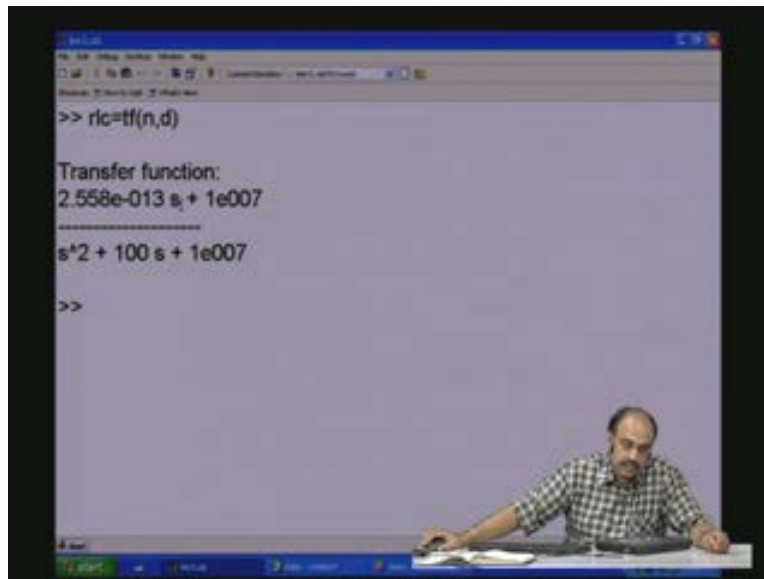
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```
>> [n,d]=ss2tf(A,B,C,D)  
  
n =  
1.0e+007 *  
0 0.0000 1.0000  
  
d =  
1.0e+007 *  
0.0000 0.0000 1.0000  
  
>>
```

Let us try to present this in a more friendly fashion. Let us use the tf function; I will define a system LC or shall we say RLC equals tf numerator and the denominator polynomial.

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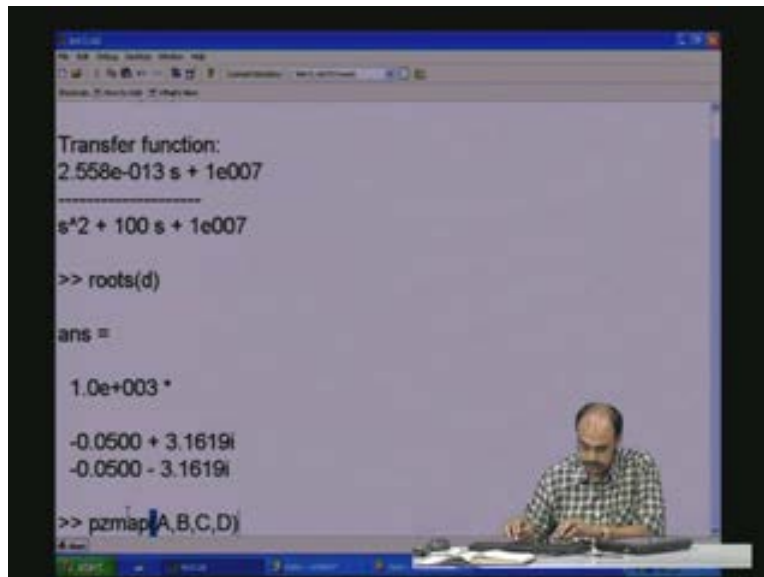
```
>> ric=tf(n,d)

Transfer function:
 2.558e-013 s + 1e007
-----
 s^2 + 100 s + 1e007

>>
```

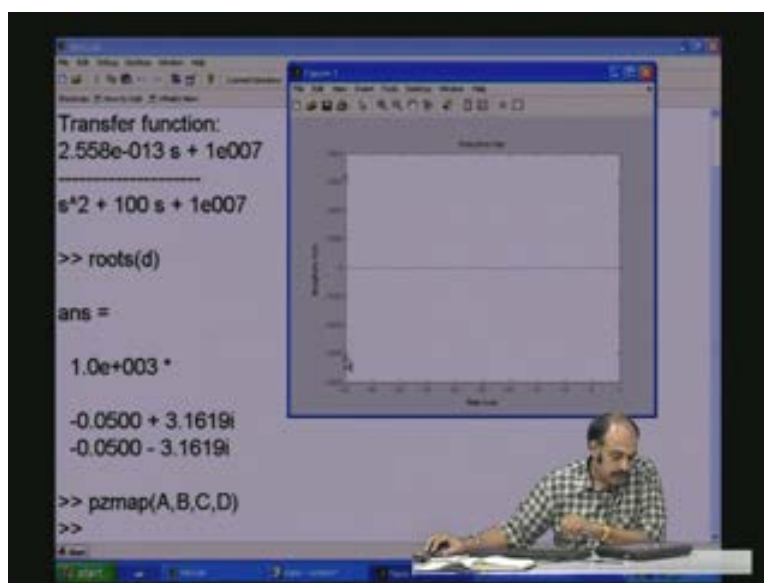
Now you see here, interesting, there is a numerator polynomial with an  $s$  of order 1 which means 1, 0; there is a denominator polynomial is of order 2 which means there needs to be two poles. Now the roots of this is going to be..... the roots of the denominator is going to be the pole locations. So if I take the roots of  $d$  the denominator this will be the pole locations; you see the pole location that if you multiply by this factor it will be 50 plus 3000 and odd in the complex; was it not the same thing you got, you look at the pzmap (A, B, C, D).

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You see that at 50 you have 1 plus 2000 and odd and another 3000 plus (Refer Slide Time: 54:30) something that is at the negative omega axis. (54:38.....) about the poles.

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Now there is also a zero location which is basically the roots of the numerator. So roots of the numerator will give you a zero and zero is located at this portion quite far away on the negative real axis. This can also be obtained from the state equation that is the A matrix. A matrix contains the information of the pole zero. Let us say you have eigenvalue of A. Eigenvalue of A gives the pole location.

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```

>> roots(n)

ans =

-3.9094e+019

>> eig(A)

ans =

1.0e+003 *
-0.0500 + 3.1619i
-0.0500 - 3.1619i

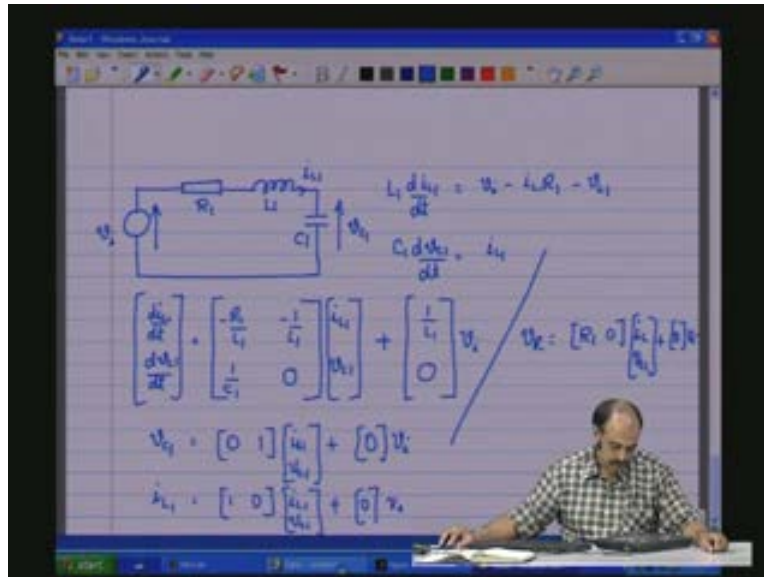
>>

```

Basically the roots of the denominator of the transfer function is same as the eigenvalue of the A matrix which gives you the pole and that defines the dynamics of the system or any system for that matter. We can get any output to any input as I was saying before.

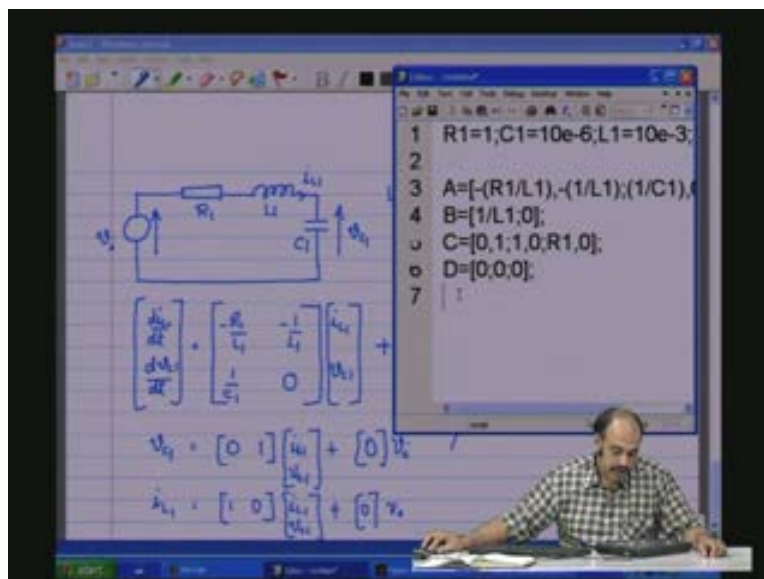
Going back to the notebook. Not necessary that we need to get only  $V_c$ . It could as well have been  $i_L$  for example (Refer Slide Time: 00:56:05) if I had wanted  $i_L$  you would say  $[1, 0] i_L + 0 V_c$ . Or if you had wanted  $V_R$  it would be  $V_i$ , the voltage across R would be the state variable  $i_L$  passing through R and therefore R into  $i_L$ . So if you had wanted  $V_R$  which is equal to  $[1, 0] i_L + 0 V_c$  plus  $0 V_i$ . But this should be multiplied by R that would give you  $i_L$  into R which is  $V_R$ . So you see that you could see any variable to any variable. So let us just modify the output equation as follows.

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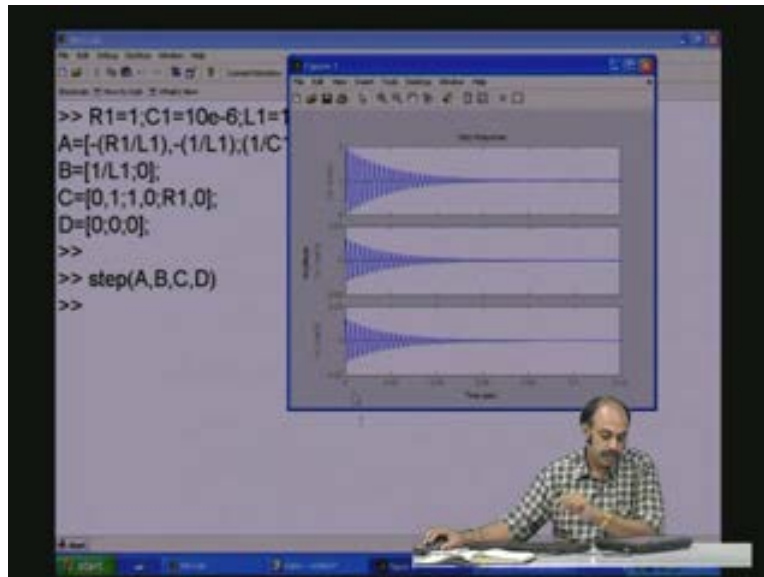
So the output equation would be [0, 1] if you want to see  $V_C$ , it would be [1, 0] if you want to see  $i_L$  and it will be  $R_1/L, 0$  if you want to see  $V_R$ . And in the case of the D matrix it is 0 0 0 all throughout in this particular case.

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So this is the model where we want to see the multiple outputs with respect to a single input. Let us input it in MATLAB; control paste, yes and see the step response (A, B, C, D).

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So you will be seeing the step response of all three. The first one is  $V_c$ , the second one is  $i_L$  the current  $i_L$ , the third one is the voltage across  $R$ . In both these cases it is decaying to zero and in the case of  $V_c$  it is going and settling to a value which is 1. So in this manner we go about doing the analysis of any given circuit whatever may be the complexity. Thank you.