# **Switched Mode Power Conversion Prof. Ramanarayanan. V Department of Electrical Engineering Indian Institute of Science, Bangalore**

# **Lecture - 9 Energy storage-Capacitor**

Good afternoon to you all. In this session today, we will continue with the problems which we had seen in the last session. We have a set of few problems in this session which are relating to the devices used for power conversion. In the last problem session, we looked at the electronic switches which are used in power conversion namely transistors, diodes, their conduction models, switching models, power loss and so on. We will continue in this session on the reactive devices or energy storage devices and see a few problems related to the energy storage devices namely capacitors, inductors and transformers.

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For example, we had seen that for storing energy we use capacitors or inductors in power converters and many times these capacitors also have parasitic resistances. In this particular example, we have a capacitor whose value is 10 microfarad which is what you see here, 10 microfarad capacitance. But, it also has a parasitic resistance of 0.2 ohm which is in series with the device as shown here and this capacitor has a time t equal to 0; initially, a voltage of 0 voltage across the terminals of the capacitor. And what you see here, as the current waveform I c of t is shown here has a function of time. It is a periodic function whose period is from here to here which is a total of 30 microsecond is a period of this waveform and it has a positive half of current and the negative half of current.

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So, what you see here is a positive half consisting of a flat 15 amperes and on top of that a half sinusoid totaling upto 65 amperes. And in the negative half, we do not know the magnitude I star but, it is for duration of 10 microseconds. The first exercise here is to find out what is I star? What is the value of the negative current I star flowing through this capacitor? To solve this problem, we invoke the condition that in a capacitor under steady state condition the charge balance exists. That is over one period. The charge that is given to the capacitor is the same as the charge that is taken from the capacitor. So, this is the principle that is used in order to solve the value of the negative current I star.

So, the strategy is to find out, what is the total charge here which is the area of this area under this curve in the positive half, find out the area under this curve in the negative half, equate both of them and we will get I star. This is the strategy of solving this problem. So, let us solve that and see.



So, there is one section of current which is 15 amperes and 10 micro second. So, this area is 150 micro coulomb, 15 into 10 micro second into ampere is micro coulomb. So, this is the first area. Then, we have a negative area which is minus I star and for again 10 micro second. So, this area can be written as minus I star into 10 microcoulomb.

So, this is the third part. This is the first part. In the second part that we have here is the 50 amperes half sinusoid. And this area can be found out by finding out what is the average value multiplied by the time duration, time is 10 micro second multiplied by average of a half sinusoid is 50 into 2 times divided by pie.

So, for a sinusoidal waveform the average is given by for half sinusoid peak multiplied by 2 by pie. So, this area is our second area, this can be written as 1000 by pie micro coulomb. So, if we add the positive areas and equated to the negative areas you will be able to get I star.

So, this positive areas add upto 150 micro coulomb plus 1000 by pie is 318.3 micro coulomb and that total is 468.3 micro coulomb the negative area is I star into 10 micro coulomb. So, these two are equal.

So, we can write that I star is 46.83 amperes so, what we have seen right now is that in a capacitor under steady state condition the positive charge that flows into the capacitor is a same as the negative charge that flows out of the capacitor. Using that principle we

equate the charging coulomb equal to the discharging coulomb and find out the unknown quantity of I star here.

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In the same problem if we go, we might also like to find out what is the voltage across the capacitor under the steady state condition. We know that during the positive charging capacitor voltage will build up and during the negative charging capacitor voltage will decay. We start with an initial voltage of zero volts so it is possible to find out how this voltage will.

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For example, if we see the current waveform so, if we draw these we can see that initially the voltage across the capacitor is 0. So, during this time the capacitor will charge up because we are applying coulomb into the capacitor so it would charge up following a waveform which is integral of this.

Then, during the time when no current is flowing capacitor voltage will remind constant and during the time the current is in the opposite direction the capacitor will discharge all the way to 0 and then the next cycle will start.

So, the question is what is this peak voltage? We see peak this can be formed out by integrating this area and dividing by c or integrating this dividing it by c because both are equal this one is simple and we might say that total quantity of charge that is transferred is 46.83 into 10 micro coulomb divided by the value of c which is 10 microfarad so this micro, 10 and 10. So, what we get is 46.83 volts so, this peak voltage that is seen on the capacitor will be 46.83 volts and it will be a linear discharge here.

And echo sin as integral of sin is nearly integral of sin plus constant, it is echo sin plus ramp and that is what you will see here at this pickup in voltage is the same as the drop in voltage here. So 46.83 volts is a peak voltage that is obtained across the capacitor.

So, we might go further and then see what will be the loss in the resistor in this particular capacitor. For that the current that is flowing in the capacitor will produce loss in the resistor and I square R is the resistive loss. So, you have to find out the rms current of this and multiplied by this R C the square of the rms current.



We again see through some the current consists of three parts and so, we might call this current as A and this current as B and this current as C. So, we can find out the rms current by finding out what is the period 0 to T A which is this part plus B which is the sinusoidal part plus C which is the constant part whole square d t and when we expand this. This will have because A exists only from 0 to T 1 to T 2. C exists only from T 3 to T 4. We might say that T 1 to T 2 A plus B whole square plus T 3 to T 4 C square d t.

The other products like A multiplied by C, B multiplied by C exectra will be 0 because A, B and C do not simultaneously exist so, these are 2 sections. This portion is a very simple square wave so, we might simply say that this would be 46.83 square multiplied by the period 10 microsecond for that micro divided by total period is 30 micro so, if we find out I rms square on account of the C component, that will be 46.83 whole square divided by 3 so, this is the first part of this portion of the rms current. The next portion of rms which has A and B so, we might probably restart again, erase and restart this part.

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This is 50 and this is 15. This duration is 10 micro and total is 30 micro so, we are now interested in A plus B whole square. So, that can be found out A square is very simple which is 15 square and that exists for a time of 10 micro and total time is 30 micro. So, this portion is because of A alone. B portion is a half sin. So, the rms of half sin is peak divided by root 2. So, if you square it, it is 50 square by 2 and here also it exists for 10 micro and total period is 30 micro so, this is the portion which is because of B alone and then there is a third term which is integral of A plus B with 2 and d t.

So, that portion will give you A B in constant 2 times 15 and integral of the sin so, this portion can be written as 2 into 15 into the average of sin is 50 into 2 by pie on its own base multiplied by 10 micro second divided by 30 microsecond to get the total value so, this is the third part which is the two A B part.

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So, now we can put all these things together to have a first term which is 15 square into 1 by 3 plus second term is 50 square by 2 into 1 by 3 plus a third term is 30 into 100 by pie into 1 by 3 plus the portion corresponding to this which is 46.83 square into 1 by 3 so, this is I rms square so, this turns out to be 1533.7 ampere square so, the loss is I rms square into R C which is 1533.7 multiplied by 0.02 which is 30.67 watts.

So, in this resistor what we have in this capacitor what we have seen there is a parasitic resistor and on this resistor on account of the current that is flowing, we have a total loss of 30.67 watts.

If we try to find out what is the peak volt ampere of this capacitor because for any reactive element, the product of peak current and peak voltage is what is known as the reactive power. In this particular case the peak current is 50 plus 15, 65 amperes and we had seen the voltage was 46.

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So, we can roughly say that the volt ampere, peak volt ampere of this capacitor is 65 into 46. Approximately is about 3000 V A. So, the power loss in the resistor is about 1 percent of the V A rating of the capacitor. So, this parasitic resistor is not a very significant resistor. The V A is the product of volt and current in the capacitor. I square R is the loss in the capacitor. We notice in this problem the total loss is about 1 percent of the V A rating of the capacitor which is not a very significant quantity.

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We will move on to understand the capacitive circuit elements in a little more detail. What we see here is a capacitor whose value is as before, 10 microfarad and this capacitor also has a parasitic inductance of 75 nano henry and parasitic resistance of 30 milliohm and in a circuit schematic it is shown as the capacitor that is desired and the parasitic value of inductance which is part of this capacitor and the parasitic value of resistor which is part of this capacitor.

So, in order to understand the significance of the unwanted parasitic inductor or resistor in a capacitance or a parasitic capacitance and resistance in a desired inductor, it is good to plot the inductance plot the impedance diagram of the capacitor and try to get some insight into the unwanted components. For example, in this particular example we have a capacitance which is 10 microfarad.

So, it will have an impedance of Z C is 1 by omega C and it has a parasitic inductance of 75 nano henry. So, there will be a Z L of omega L, where omega is the frequency that is being considered and L is the parasitic value and the impedance of the resistance is 30 milli ohm. The resistive component is independent of the frequency, this portion is independent of the frequency but, the capacitive component 1 by omega C and the parasitic inductance omega L these are functions of impedances. Functions of frequencies so, what we try to do is to plot the total impedance Z C which is Z L plus Z C plus this R C in the series combination in the form of an impedance as a function of frequency.



For example, if we try to find out Z C as 1 by omega C, magnitude of this this can be plotted as a function of omega in a logarithmic omega scale we see that every time omega increases by a factor of 10 the capacitive impedance as drops by a factor of 10. This can be shown on a d B ohm scale, an impedance which is a straight line dropping at 20 d B ohm per decade and this will cross 0 d B line or 1 ohm line at a frequency which is 1 by C for a C equal to 10 microfarad this will be 100,000 radian per second.

So, we would say that this is the magnitude of  $Z C$  as a function of omega. It is a straight line which has a slope of minus 20 d B per decade and it is crossing 0 d B ohm line at 1 by C, which is 100,000 radian per second for a value of 10 microfarad.

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In the same way it is possible to find out the impedance for L. Z L magnitude is omega times L. So, we will see that the impedance will increase with frequency. So, this impedance will be, if we plot it as d B ohm and log of frequency with 0 d B line here 0 d B ohm line here it will have a plus 20 d B ohm per decade. What this means is, every time the frequency increases by a factor of 10 on this scale, the impedance will increase by a factor of 20 d B and the cross over the frequency at which the impedance is 1 ohm or 0 d B ohm is 1 by L and in this particular case for an L of 75 nano henry, this will be 13.33 mega radian per second. So, this would be the impedance of L as a function of frequency.

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Next we may plot the resistance R as a function of frequency. R is independent of the frequency whose value is 30 milli ohm. So, if we represent this in d B ohm this will be at minus 30.5 d B ohm, this is 20 log to the base 10, 30 milliohm is .03ohm. So, if you evaluate this it turns out to be minus 30.5 d B ohm. So, we see that our capacitance has really three behaviors. At very low frequency it behaves like a capacitance. At very high frequency it behaves like an inductance. And it also has at all frequencies, a resistance value of thirty milliohm.

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So, if we put all three and try to find out the overall behavior R C L C this is 100,000 radian per second, this is 13.33 mega radian per second and this is the 30 milliohm or minus 30.5 d B ohm and because this is a series circuit R C and L at any frequency. Whichever component is the highest value will dominate.

So, you will find that the actual impedance will follow the capacitance at the low frequency because it is the highest of the three and it will follow the inductance because it is a highest of the three and at the resonant frequency which is 1 by 2 pie root L C. In this particular case it is 1.154 mega radian per second at the resonant frequency, the capacitive impedance and inductive impedance cancel.

So, you have what is left is only resistance so, what you will find is the impedance will follow a curve like this which is a standard resonance curve that, at this frequency this circuit has a resonant frequency at 1.154 the capacitance behaves like a good capacitance in this range, but this capacitor behaves almost like an inductance in this range and in this range it is almost like a resistance.

So, this capacitor can be used satisfactorily in this frequency range which is probably one or two decades less than the 1.154 mega radian per second. So, probably upto about 100,000 radian per second. This capacitor is a good capacitor beyond that there is a resonance and even beyond that it is pure inductors. So, this type of impedance diagrams are very useful in evaluating circuit elements along with all its parasitic quantities.

So, from this it is possible to identify which is a frequency range in which the circuit element behaves close to the ideal and which are the frequency regions where this circuit element is dominated by the parasitics and it is better to avoid those areas where the parasitics dominate the overall performance.

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Now, we will move on to one or two more problems we had seen in the theory of inductors, transformers etcetera. Mostly we were using square wave type of voltage for excitation to a transformer or square wave for excitation of inductor and so on.

But a lot of applications are there where the transformers are excited with the sinusoidal voltage and in such situations the relationship between the voltage across the winding of the transformer and the size of the transformer or a little different from what we had seen before.

For example, what you see here is the voltage applied to the winding of a transformer and this voltage because V is related to the rate of change of flux through the number of turns, the flux is really integral of  $1$  by N V d t from initial condition  $0$  to t from whatever phi naught.

So, what we see here is the flux waveform is an integral of the voltage waveform or alternately the voltage waveform is the derivative of the flux waveform with the scaling factor of N.

For example, in this case where the flux has 0 slope the voltage is 0 and where the flux has a maximum slope, voltage is maximum and where the flux again reaches the maximum value positive the voltage has come to 0. The flux is a negative cosine waveform and the voltage is a positive voltage waveform. See in such situations we would like to know, what is the relationship for the design of such transformers? Let us look at the situation here.



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We can say that the flux as a function of time, in the previous example is a minus cosine waveform with some peak which is a phi M. You can see from the previous curve there is a phi M and it is a cosine waveform and you can see a time t equal to 0. Flux is minus phi and at when t equal to pie flux becomes plus 5 so, if we find out what is V of t it is N d phi by d t, which is N multiplied by the derivative of this which is phi M into omega into sin omega t this is V of t. This can be written as 2 pie f phi M N sin omega t. See if the flux is a sinusoidal waveform the voltage is also a sinusoidal waveform.

But, with the phase difference flux is a cosine minus cosine waveform and voltage is a sin waveform. So, if we write the rms value of the voltage that is equal to 2 pie divided by root 2 phi can be written as maximum flux density multiplied by the area of the cross section because in the magnetic circuit of the transformer the maximum flux density multiplied by the area of cross section of the core will be the maximum flux multiplied by N because we are talking about rms, rms is nothing but, the peak divided by root 2 this can be written as V rms is 4.44. This first term 2 pie by root 2 is 4.44 f B m A c and number of turns. This is the famous voltage equation for a transformer when we excite the transformer with sinusoidal voltage.

The rms voltage of the winding is a related to the winding number of turns, the core area of cross section. The maximum flux density and the frequency of excitation. The rms voltage is directly proportional to every one of these quantities and the constant of proportionality is 4.44. So, this is one of the relationships that we will use in order to find out what is the size of the transformer.

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In this particular example, we can go a little further to see supposing if the transformer has a core and the winding primary and secondary are arranged like that. So, the area of the window in which the winding is organized is the window area and that has the primary multiplied by the area of cross section of the primary conductors plus secondary multiplied by the area of cross section of the secondary conductors this is the total copper area, but only a fraction of the window is utilized for this.

So, we might say that the window area that is utilized for accommodating the copper in the conductors is a fraction of the window area that is the sum of the primary copper area plus the secondary copper area. Primary copper area is primary turns multiplied by primary area of cross section. Secondary copper area is secondary turns multiplied by secondary cross section area.

We had also seen that the voltage ratio and the current ratio are related to the number of turns. So, this can be easily seen to be 2 times N 1 A because the cross sectional area of the secondary will be smaller if the voltage is more, but in which case current will be less and the number of turns will be more so this  $N$  1 by  $N$  2 is same as V 1 by V 2 is the same as I 2 by I 1 with that relationship, it is possible to find out that this total areas 2 times the primary area.

This can be written as 2 times N 1 and primary area is primary current rms divided by current density, the maximum current density that can be passed for that. Now the same steps we had followed for square wave, but the sinusoidal kind of excitation is more common in distribution type of power supplies.

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And this example can be seen to have V rms is 4.44 f B m A c N 1, it is for primary. And then we had also seen that 2 times  $N$  1 I rms by J is equal to K w A w. So, if you multiply both these equations you will get V rms I rms this quantity is the V A rating of the transformer, volt ampere rating of the transformer. It is related to A c A w, are the product of core area and window area these two terms. All the other terms this N 1 and N 1 will cancel when we multiply both of them. All these other terms like 2.22 J B m f k w. So, these are the other terms are.

They might say that the size of the transformer is related to current density, flux density, maximum frequency of operation, window space factor. In the square wave excitation, we were getting two here in this place and in sinusoidal excitation that number 2.22. This is the difference between sinusoidal excitation and square wave excitation. Now using this relationship it is possible to design transformers for sinusoidal excitation.

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Let us see an example where we want to design a transformer whose V A rating is 150, that is rms voltage multiplied by rms current is 150 and it is a 1 is to 1 transformer. That is primary voltage and secondary voltage are equal. Such transformers are used mainly as isolation transformers the purpose is to electrically isolate, but not modify the frequency or voltage.

Such isolation transformers will have a turns ratio of 1 is to 1 and this particular example the transformer that we want is a 50 hertz transformer and it has a primary rating of 230 volts and because it is a 1 is to 1 transformer secondary voltage is also 230 volts.

So, this transformer is excited with 230 volts rms 50 hertz sinusoidal voltage and the secondary is also 230 volts 50 hertz single phase. In such a transformer we would like to know how do you design this transformers. What is the core to be used? How many turns are to be used for primary? How many turns are to be used for secondary? What will be the cross section of wire for primary? What will be the cross section of wire for secondary? This is what we mean by designing a transformer.

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In this particular example, we might go through this process and see that A c A w is the product which is V rms I rms and that product is 150 V A which is given to us, divided by  $2.22 f J B m$  and  $K w$ .

And this is, we might take frequency is 50. The current density can be taken as 2.5 into 10 power 6 ampere per meter square. It is 1 meter square of copper conductor can carry a current as highest 2.5 into 10 power 6 amperes. And the iron used for the core can withstand without saturation, a magnetic flux density as high as 1.2 tesla and we might say that the window space factor, we are able to utilize 35 percent of the window effectively in the transformer.

So, we know all the quantities here and this A c A w, we calculate this number. It turns out to be 1287001 millimeter power 4, 12 lakhs 87 thousand 1 millimeter power 4. How do we select a core for this application?

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For that, we have here a a table which shows different types of magnetic cores listed for each. For example, that type L202 core has an iron area of 12.3 m m square and window area of 27.7 m m square. So, that the product is the product of these two will be the millimeter power 4. In this, these are all organized in increasing area product increasing values of there.

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And the number that we wanted is 12 lakhs 87 thousand, the immediate next size available here is the T.16 which has 15 lakhs 85 thousand 900 and 13 millimeter power 4. So, this is a suitable. The next one is too big and the one before that is too small. So, we might select the size T.16 for this application.

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**Switched Mode Power Conversion Problem Set 02** Problem No. 4 AcAN  $= 1452 \text{ mm}^2$  $\sqrt{2}c$  $1093 \, \text{mm}^2$  $\sqrt{A_{nd}}$  $\sim$  $4.445012145210$ 230 30 4-44 50 1-2 1452 10 TVRNS  $N_1 = N_2 =$ 595 Wire Data & Core Data

So, we say that A c A w tells us T.16 can be used for this particular application. And for that core, the iron area is 1451 and the window area is 10 92. So, we might also write down that quantity here 14 51 and 10 92. A c is 14 52 m m square and A w is 10 91 10 93 m m square. So, this is the core area that is valid for this type T 16 and this is a window area. Now it is very easy to select N 1 rms voltage of 230 volts is equal to 4.44 into frequency which is f B m is 1.2 tesla, A c is 14 52 into 10 power minus 6 meter square into number of turns. So, from this relationship it is possible to find out our N 1 230 divided by 4.44 50 1.2 14 52 10 power minus 6 and that is 500 and 95 turns. And because this is a 1 is to 1 transformer, the primary and secondary are equal. N 1 is equal to N 2 that is 500 and 95 turns.

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We might go to the next step and see, what is the rms current is 150 V A divided by 230 volts and that number is 0.65 amperes. So, with the current density of 2.5 ampere per millimeter square, this leads to wire cross section of 0.65 divided by 2.5 that is 0.26 millimeter square. So how do we select this cross section of the wire? We again go back to the table that we had here. So, if I open this wire data here which is given.

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So, this is a table which relates the cross section of the wire with the size in standard wire gauge.

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So, what we need is 0.26 millimeter square. As you come on this table, what you see here as 0.29 size 23. What you see here is a little more than what we want in terms of cross sectional area. So, the last column is millimeter square of cross sectional area and the first column is the standard wire gauge number.

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So, what we can now do is for this, we might say that 23 SWG wire is what is suitable.

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So, if we go back we might say that the transformer design is with T 16 core which is E I core which has an A c of 14 52 m m square, then A w of 10 92 millimeter square. In this we need to put primary a turn of 595 turns and a secondary of 595 turns and then cross section of primary will be 23 SWG and the secondary cross section also is 23 SWG. That is because this is a 1 is to 1 transformer. So, this transformer is an isolation transformer with 150 V A 230 volts 50 hertz with the core of T 16 with 595 turns for primary and secondary with the 23 gauge wire for the cross section of primary as well as secondary.

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Now, the design of the transformer followed the similar kind of procedures that we had seen earlier then we were explaining about the transformer, that find out the core product, select a core, then for that core find out A c. Find out the number of turns N 1 N 2 etcetera and then find out the currents I 1 I 2 etcetera.

Then, find out the cross sectional area a w 1 a w 2 and then find it out in the standard wire gauge. So, this is the sequence in which we had seen in theory how the design is done. We had seen a small example, where we calculated just the basic design the first level design which is core plus the wire size plus the number of turns. It is also possible to find out the mean length of a turn, total length of primary, total length of secondary, the parasitic resistance and so on.

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So, what we will do in the next few, in the next session is to look at one or two more of these inductor designs where we come across coupled inductor as well as non-linear inductor. So, right now I will stop with this. But in the next session, we will go into design of coupled inductors as well as design of non-linear inductors. Many applications you may have to tailor your inductor in order to meet certain non-linear characteristics as well. So, we will see in the next session some of these aspects and after that will get back to power converters.

Thank you very much.