

Switched Mode Power Conversion
Prof. Ramanarayanan. V
Department of Electrical Engineering
Indian Institute of Science Bangalore

Lecture - 6
Transformer

In this session, we will continue with what we have seen before on the devices that are used for efficient power conversion. We had seen in some of the past sessions the devices such as the electronic switches and the inductors which are the devices, lossless devices used for efficient power conversion. In today's session, we will look at transformers which are used in power conversion circuits for the purpose of voltage matching, impedance matching, isolation, and so on.

Transformers also come under the same category as electromagnetic elements just like inductors. The transformer has an electrical circuit and also a magnetic circuit, and electrical circuit and the magnetic circuit are coupled to each other. So we will see in today's session some of the basic principles of operation of transformers, the electrical circuit modeling, the magnetic circuit modeling, and the mathematical relationship between the different electrical quantities as well as the magnetic quantities to the geometric properties of the magnetic circuit.

Then we will look at some sample and specifications of a transformer, how to designs such a transformers? What is the meaning of designing a transfer? How to design such a transformer? And so on. And having design what kind of non-idealities existing the transformers in comparison with the ideal transformer? How to quantitatively evaluate this non-ideal properties of the transformer and so on.

(Refer Slide Time: 02:44)



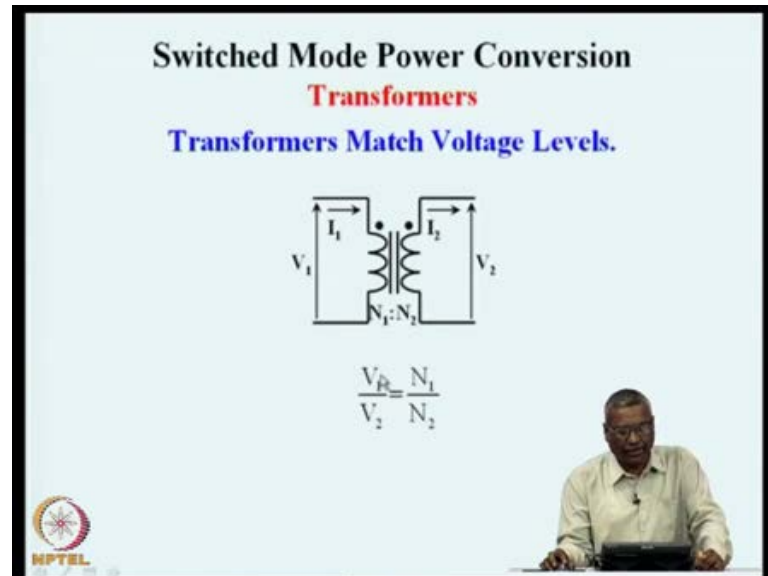
The devices, the transformers several purposes in the power conversion circuits in one of them is to match voltage levels in power conversion circuits from one part of the circuit to another part of the circuit. Ideally transformers do not store energy, this is the fundamental difference between a transformer and an inductor. We had seen in the previous session, the inductors are used in power conversion circuits in order to store energy, in order to smoothen the flow of energy which is necessary, because we control the power flow using switches which are discontinues elements. So, the basic purpose of using inductors in a power conversion circuit is to smoothen the flow of energy.

Inductors have the property of storing electrical energy and this storage of energy helps us to smoothen the flow of energy in a power converter. In comparison with the inductor, the transformer is also an electromagnetic circuit, but the fundamental difference is that they do not store energy. Ideally the transformers do not store energy. In converters, transformers also used for another purpose and that purpose is to provide electrical isolation between one part of the circuit in another part of the circuit.

If you wish to transfer energy from one part of the circuit which is a electrically isolated from another part of the circuit, transformers are absolutely essential in such situations. So, we see two reasons for the presence of transformers in power conversion circuit, one of them is to transform voltage levels and the other is to isolate circuit ports, circuit

portions or the circuit parts two parts of the circuit where we wish to transfer energy, but they have to be isolated electrically, in such cases also transformers are used.

(Refer Slide Time: 05:07)



As a schematic, this is the representation of a transformer. It has two parts which normally we call is the primary and the secondary and with the lines that are indicated here is the electromagnetic, the magnetic core, the electrical circuit primary, the electrical circuit secondary. This primary and secondary have a characteristic turns N_1 turns for the primary and N_2 turns for the secondary. From one part of the circuit, the transformer is excited with a certain voltage and this voltage sets up a flux in the core and the rate of change of flux determines the voltage across the winding primary winding as well as the secondary winding.

So, without any electrical connection between one part of the circuit to the other between the primary and the secondary through the electromagnetic coupling in the core of the magnetic transformer, energy is transferred from one part of the circuit to the other part or from the primary to the secondary. We can transfer electrical energy through the intermediate core by the property of electromagnetic induction. Now, one of the fundamental relationships which determines the ratio of voltage on the primary to the voltage on the secondary V_1 by V_2 is relate to N_1 by N_2 . By this relationship, V_1 by V_2 is equal to N_1 by N_2 .

This is the primary of the transformer with N_1 turns, you have the secondary of the transformer with N_2 turns and both this primary and secondary are coupled to each other through a magnetic core which is part of the same circuit. The excitation is from one part of the circuit in this case we call V_1 as the primary and excitation is from V_1 and a secondary voltage is induced on the secondary winding which is V_2 and a load can be connected across V_2 and I_2 will flow through the load. Energy can transfer from this part of the circuit to that part of the circuit without having electrical connection in between. We can transform the voltage levels by simply changing the ratio of N_1 and N_2 if N_1 and N_2 has the ratio of 1 is to 2 then V_2 will be two times V_1 .

It is possible to step up the voltage on the transformer from the primary to the secondary or if we have N_1 by N_2 with a ratio of 1 is to 0.5, it is possible to step down the voltage from a higher voltage to a lower voltage. Accordingly transformers we also be called as step up transformer or step down transformer or you could have simply a 1 is to 1 isolation transformer where the voltage levels are same, but electrical isolation between the primary and the secondary is provided by the transformer. So, in this particular situation we see that the transformers can match voltage levels between V_1 and V_2 by the appropriate selection of the ratio of number of turns

(Refer Slide Time: 08:27)

The slide features a title 'Switched Mode Power Conversion' in black, followed by 'Transformers' in red, and 'Transformers Provide Electrical Isolation' in blue. Below the text is a schematic diagram of a transformer with a rectangular magnetic core. The primary winding on the left has current I_1 flowing upwards and voltage V_1 across it. The secondary winding on the right has current I_2 flowing upwards and voltage V_2 across it. The number of turns for each winding is indicated as N_1 and N_2 respectively. Below the diagram is the transformer equation:
$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$
 In the bottom right corner of the slide, a small inset shows a man in a light-colored shirt sitting at a desk, likely the presenter. The NPTEL logo is visible in the bottom left corner of the slide.

The transformers are also used for providing electrical isolation between one part of the circuit another part of the circuit as I had said, the primary excitation sets up in the core a

certain amount of flux in the relationship between the primary voltage and the flux follows the Faradays law of V is n times $d\phi$ by $d t$. The same flux linking the secondary winding produces a voltage, which is secondary turned times the same $d\phi$ by $d t$ and you get secondary voltage, which is V_1 times N_2 by N_1 with an isolation between the primary and secondary. Electrical isolation is desired in many cases for the purpose of safety from one part of the circuit to another part of the circuit. If you do not want electrical connection which may lead to hazards, in such a case isolation can be provided using transformers

(Refer Slide Time: 09:30)

Switched Mode Power Conversion
Transformers
Efficiency of Power Conversion is Unity

$$V_1 I_1 = V_2 I_2$$

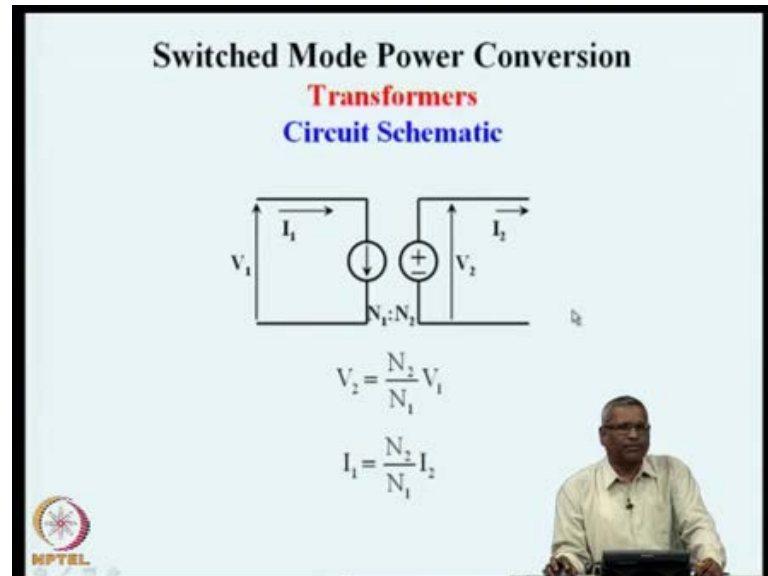
$$\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

The ideal property of the transformer is that it transforms energy without any loss in the transformer. Ideally the transformer converts power with unity efficiency. Efficiency of power conversion in a transformer from the primary to the secondary is one. There are no losses in the transformer. We might say that the primary power V_1 into I_1 is equal to the secondary power V_2 into I_2 and because of this ideal property of efficiency being 1, the turns ratio is the ratio of the voltages as well as the inverse ratio of the currents for example, V_1 by V_2 is N_1 by N_2 and in the same ratio is also I_2 by I_1 .

We would see that by cross multiplying these terms will find $V_1 I_1$ is $V_2 I_2$ and that is the property of lossless power transformation. So this forward voltage conversion ratio and the reverse current conversion ratio in a transformer ideally are the same as the ratio of the number of turns. So what you notice here is V_1 by V_2 is I_2 by I_1 which is the

same as the ratio of the number of turns on the primary and the number of turns on the secondary.

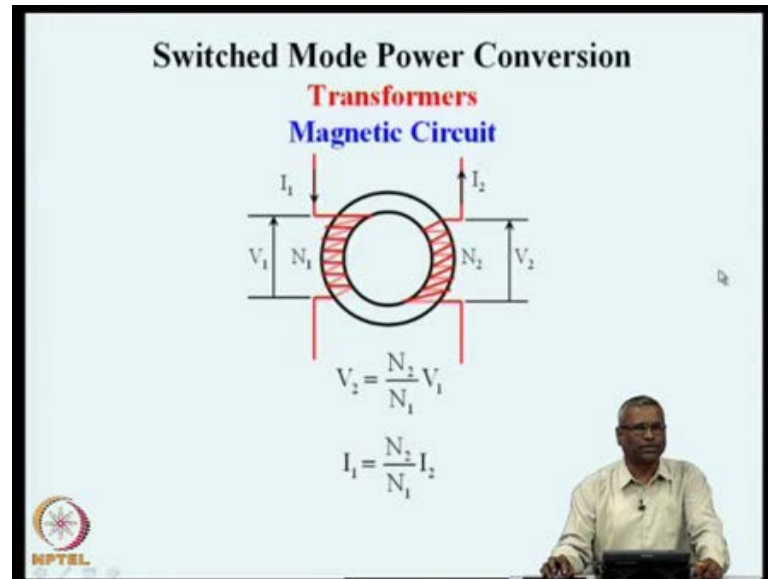
(Refer Slide Time: 10:56)



The circuit schematic of a transformer in many situations will be represented in this form. So, this has a dependent current source and dependent voltage source and the turns ratio being N_1 by N_2 , we can indicate that V_2 is a voltage source which is dependent on V_1 by the conversion ratio N_2 by N_1 or V_2 is N_2 by N_1 times V_1 . So, this is a dependent voltage source and this voltage source when a load is connected here will result in a secondary current and the secondary current will set up a dependent current source. Here where I_1 is N_2 by N_1 times I_2 and so the primary current is decided by the secondary current multiplied by N_2 by N_1 and the secondary voltage is determined by the primary voltage multiplied by N_2 by N_1 .

These relationships are through the turns ratio transfer transformation and the primary and secondary can be shown as a dependent voltage source on this side and dependent current source on this side. This dependency is the same as the ratio of the number of turns ratio of the secondary turns to the primary turns V_2 is N_2 by N_1 times V_1 and I_1 is N_2 by N_1 times I_2 .

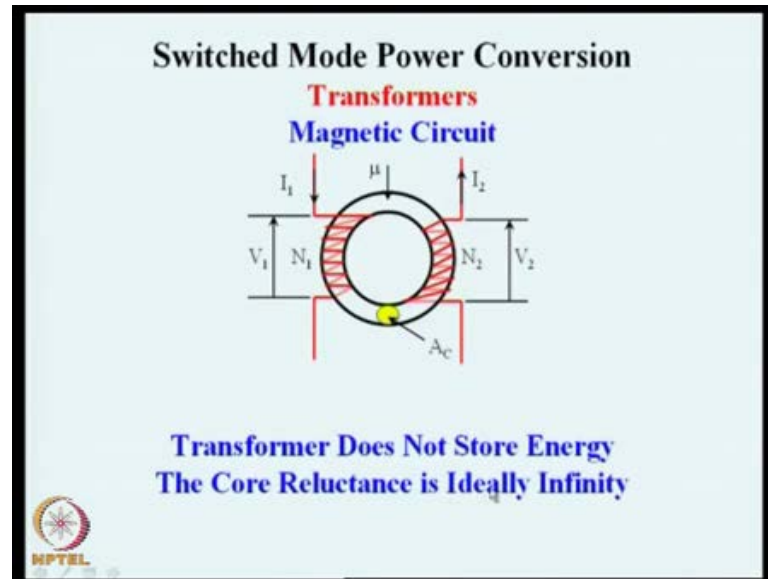
(Refer Slide Time: 12:27)



Physically a transformer has a magnetic circuit and electrical circuit. The electrical circuit in this particular situation consists of two parts, a primary the red winding shown here and the secondary winding shown in red color here and the same schematic I have now shown physically through a toroidal core. The shape is not very important we will see later. There are many preferred shapes are available we will see from the standard data sheets the different kinds of core shapes are available.

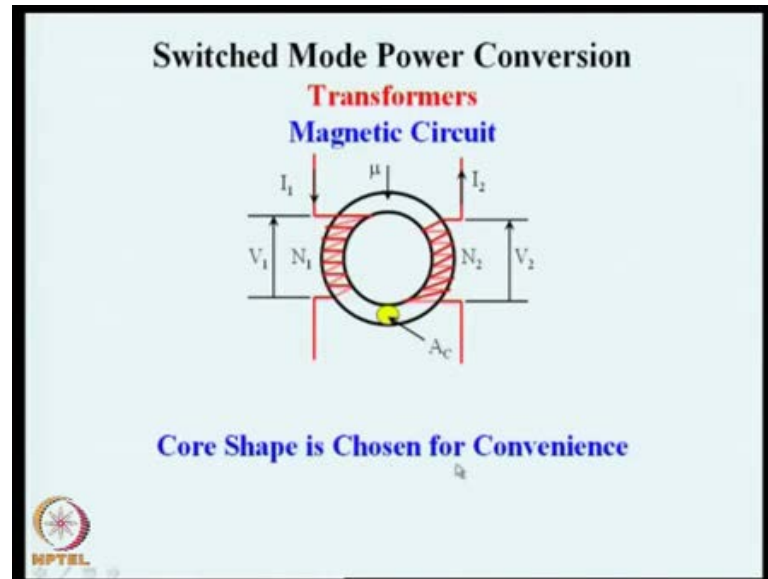
So, here I have shown a schematically a magnetic circuit which a toroidal core a ring core. On that I have a primary winding which has N_1 turns and which is being excited by a voltage V_1 and drawing a current of I_1 . On the same core we have the secondary winding also wound, which has N_2 number of turns and an account of the fact that they are both coupled, the primary excitation sets up a flux in the core and sets up a secondary voltages which is V_2 and V_2 if you connect an external load on the output site will result in a current of I_2 . V_2 is N_2 by N_1 times V_1 , I_1 is N_2 by N_1 times I_2 to the same relationship as we have seen. We have now explicitly shown the magnetic circuit and also the windings which are wound on the magnetic circuit so that they are coupled to each other.

(Refer Slide Time: 14:08)



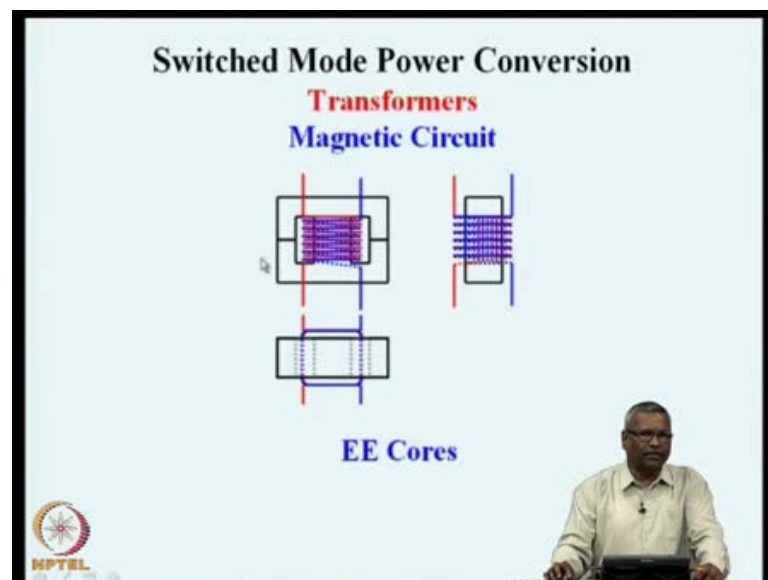
Now, in this I have indicated a few more quantities which will be of interest to us. The ring has a cross-sectional area of A_c . This is normally referred to as the core area of the transformer. The magnetic circuit has a core which is the circular cross-section and it has an area of A_c millimeter square or meter square. This core is made up of a magnetic material whose permeability is μ . This μ is a magnetic permeability of the core. Good transformers can be built with core materials which have a very high permeability. The ideal transformer will have a core whose permeability is infinity so that you will be able to set up a magnetic field in the core with a minimum amount of magnetization effort. The transformer does not store energy as we have seen. So, as a result the core material is made up of ideal magnetic material whose permeability is infinity. The core reluctance is ideally $1/\mu$. The reluctance must core reluctance I think is marked here as infinity we should say core permeability is infinity or core reluctance is 0.

(Refer Slide Time: 15:32)



The core shape is chosen more for convenience. In this case in order to understand the concepts I have shown it has a ring core, but we will see later that there are other shapes available.

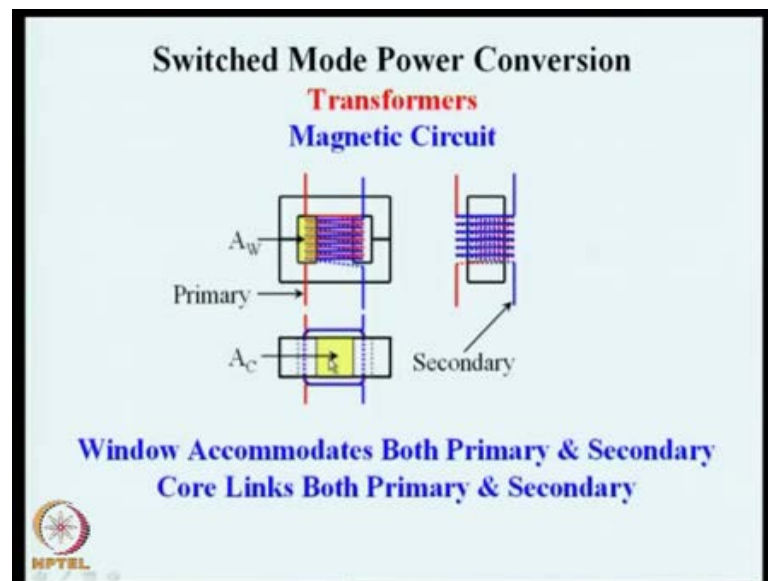
(Refer Slide Time: 15:48)



Now, the kind of the cores that we had used for inductors, EE cores are also suited for transformers. In this circuit instead of a ring core, I have e shape core 2 2 pieces of e shapes, they are put against each other so that you have a close to magnetic circuit and in the central limb both the primary winding the red color winding and the secondary

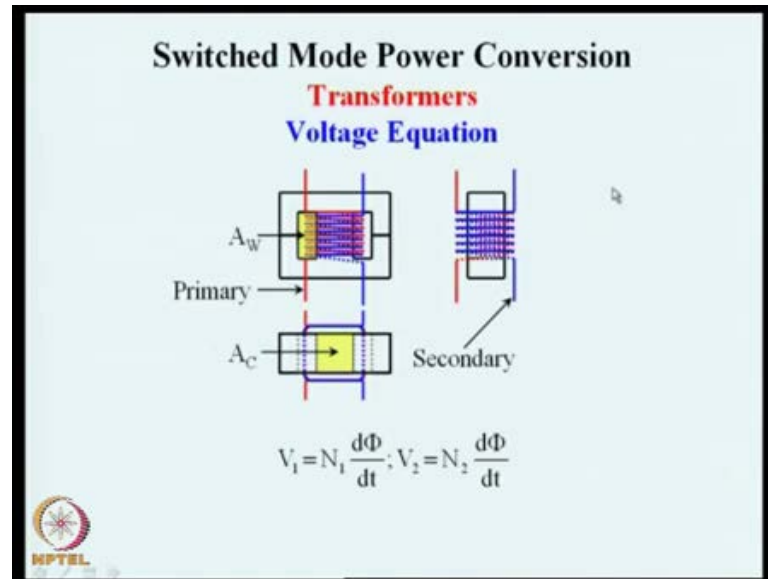
winding which is the blue color winding are bound. These other views or of the same assembly from the top and from the side, but the important point of notice is at the core does not have a gap the core is closed and there is no air gap. This is because the core does not store energy and the ideally property of the core material is a high permeability and on account of that the core reluctance has to be 0 in an account of that, the flux set up in the core can be done with minimum magnetizing effort.

(Refer Slide Time: 16:55)



Again, just like in the case of inductor electromagnetic circuit, there are a few interesting geometrical shapes that we try to identify here. This cross-section that is shown here is the core area is the area through which the magnetic field is set up and you see here the cross-section that is shown here is the rectangular part is the window through which the winding is wound. So the total copper conductors made making the primary and the secondary are wound in the central limb and they have to be accommodated in the window that is shown here. The total flux in the core has to pass through this cross-section. We can say that the window A_w accommodates the primary and the secondary conductors, the core links both to the primary and the secondary and the core area supports the total flux in the magnetic circuit. The blue winding is the secondary winding and the red winding here is the primary winding.

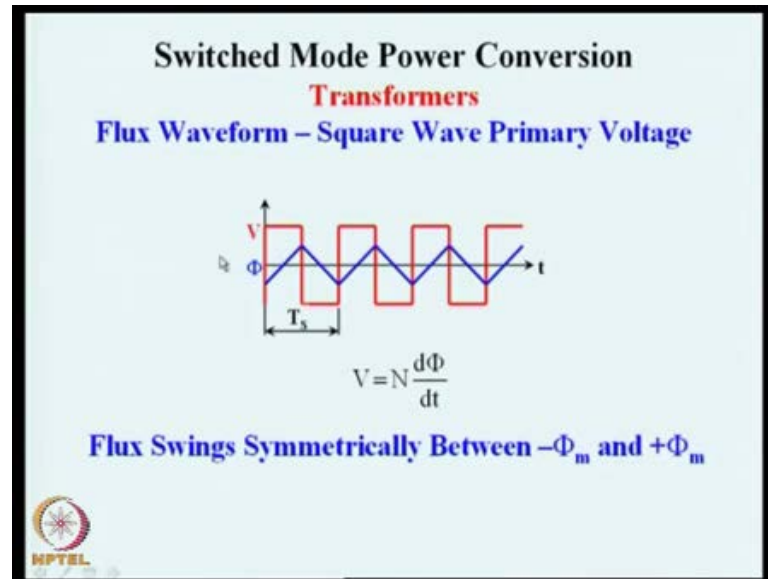
(Refer Slide Time: 18:15)



The Faradays relationship of the voltage and the flux in an electromagnetic circuit is given here. We see that the core links both the primary and the secondary. All the flux in the core are linking the red winding which is the primary winding and they are also linking the blue winding which is the secondary winding. So, in the central limb of the magnetic circuit, we have a flux which is denoted by the symbol phi. So, we can say that the primary voltage is primary turns multiplied by the rate of change of flux in the core and the secondary voltage is the secondary turns multiplied by the rate of change of the flux in the core.

The flux in the core is same because both primary and secondary are linking the same flux the same limb that is in the middle limb the winding primary winding is also wound round the same core and secondary wound winding is also wound round the same core. So, the flux that is linking the primary winding which is phi is the same as a flux that is linking the secondary winding. The primary voltage is N_1 times $d\phi$ by $d t$. Secondary voltages is N_2 times $d\phi$ by $d t$. This relationship immediately relates to the property that we have seen that V_1 by V_2 is same as N_1 by N_2 because the flux in the core is same common for both of them.

(Refer Slide Time: 19:53)

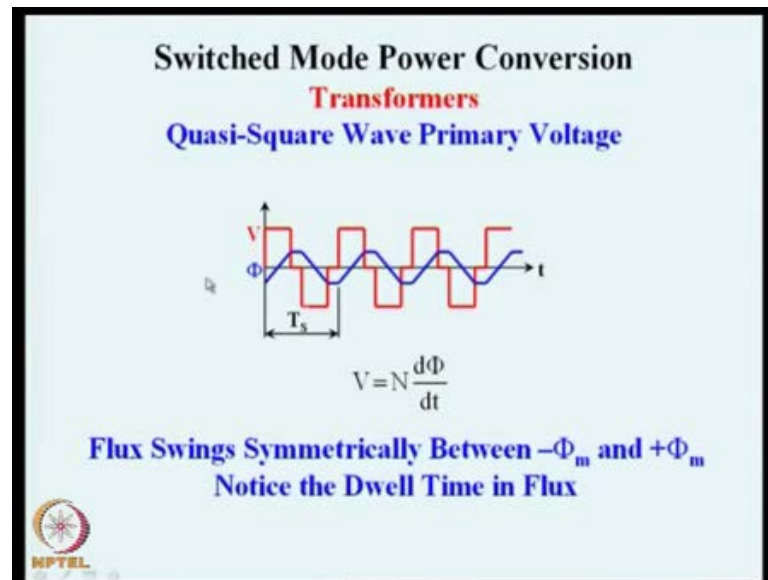


Supposing if we excite such a magnetic circuit what we have seen if we excited such a magnetic circuit on the primary winding with a square wave voltage, what kind of flux will be set up in the core is a question which we are interested in. When we excite the primary winding with a square wave voltage what we see here which has a certain period T_s , which has a shape of a square wave and which has a magnitude of V , in such a situation we would like to understand what will be the flux in the core. The relationship between the flux and the voltage is $V = N \frac{d\Phi}{dt}$ or we can say that flux is integral of voltage divided by the number of turns. If you want to get flux from this equation, this has to be integrator on both sides so we might say that the flux in the core will be an integral of the voltage to some scale, scaling factor being the number of turns. This property can also be seen graphically here.

If a square wave voltage is applied to the winding, the integral of a square wave is nothing but a triangular wave. When the voltage applied to the winding is constant and if that constant is integrated, you will get flux. So, you get a flux which is a triangular kind of flux. Whenever a positive voltage is applied to the primary, the core in the the flux in the core is increasing linearly to reach some maximum value in the positive direction. When the voltage reverses polarity, it becomes negatives the core flux starts decreasing linearly and goes towards the negative maximum and this goes on. So, under steady-state when a square wave voltage is applied to the transformer primary voltage, the core flux the flux in the core will be a triangular shaped waveform. Here we have taken the

frequency of the voltage excitation to be one over the period $1/T_s$ is the frequency. Assuming symmetry in the flux, we might say that flux is swinging symmetrically between minus ϕ_m to plus ϕ_m under steady-state situation. So with a square wave excitation on the primary voltage, the flux waveform will be a triangular shape.

(Refer Slide Time: 22:40)

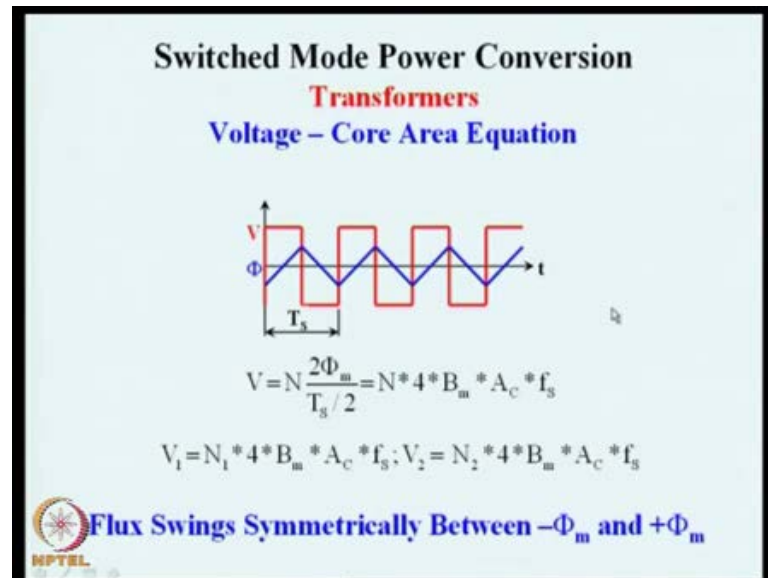


Supposing, if we give a quasi square wave. Quasi square wave is slightly different from is square wave. In the half period for a certain section for a certain duration the voltage is full V and for some duration voltage is 0. Then in the negative half, for a certain part of the negative cycle the full voltage minus V is applied and for a short duration it is 0. So, this 0 periods again will be symmetrical. You will see that during positive half, a plus voltage is applied for certain fraction of the positive half cycle and during negative half for the same fraction a negative voltage is applied to the primary winding.

By doing the integration of the graphical shapes here, you will see that during the time plus voltage is applied, flux is constantly increasing and during the small duration when the voltage is 0, flux remains constant because when the applied voltage is 0, integral of $d\phi$ by $d t$ is integral of 0 which will be a constant quantity. Then again when negative voltages applied, flux starts decreasing and it keeps all the way down to some negative maximum and during the dwell period here, during the 0 period here, the flux remains again constant and instead of a triangle wave we have a kind of a trapezoidal waveform. That is because of the 0 duration in both positive half cycle and negative half cycle. Here

also the flux is swinging symmetrically between some minus maximum and plus maximum. There is a small dwell time on account of that, the flux waveform is not a triangle, but it is a trapezoidal waveform.

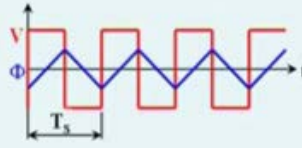
(Refer Slide Time: 24:32)




Now, if we wish to find out the design equations for the transformers, it is possible to the extent possible to see this quantitative relationships and relate them to the various geometrical properties of the magnetic circuit. So V is N into $d\phi$ by dt and the change in or this slope is $d\phi$ by dt and because it is a straight line we can take it as maximum flux minus minimum flux divided by the duration which is half of t_s . So this relationship tells the voltage is not four times $B_m A_c$ this is the total flux maximum flux maximum flux density multiplied by the core area and this t_s is one by f_s and if we substitute those relationships we find that the voltage is n times four times maximum flux core area and frequency. V_1 will be the same relationship with N_1 and V_2 will be N_2 times four times B_m times A_c times f_s .

(Refer Slide Time: 25:45)

Switched Mode Power Conversion
Transformers
Voltage – Core Area Equation


$$V_1 = 4N_1 B_m A_c f_s$$
$$V_2 = 4N_2 B_m A_c f_s$$

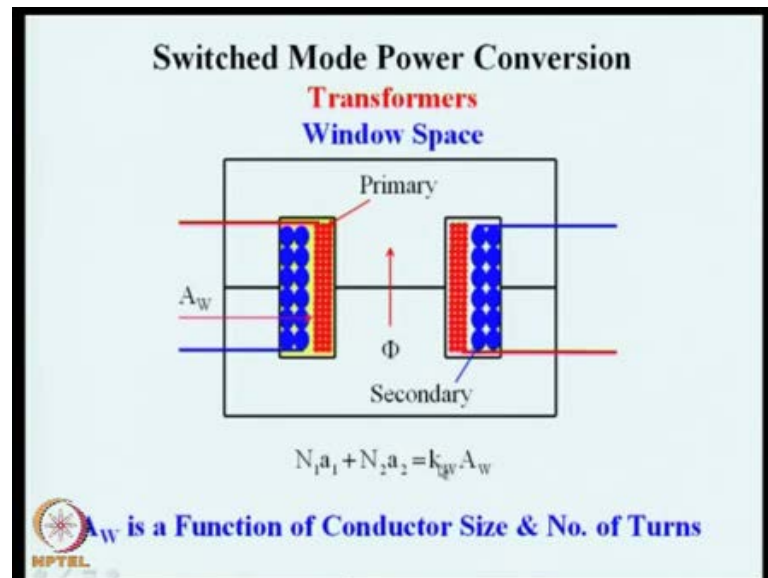
A_c is a Function of Voltage and Frequency



So I am writing both of them in a simple relationship as V_1 is four times $N_1 B_m A_c$ into f_s and V_2 is four times $N_2 B_m A_c$ into f_s . The important point here is the core area of the transformer is a function of the voltage and the frequency. V_1 is A_c and f_s . All other quantities are a fixed number of turns of the winding and the property of the magnetic material is maximum flux densities B_m so you would say that V_1 is proportional to A_c and proportional to f_s or we could say that A_c is directly proportional to V_1 and inversely proportional to f_s .

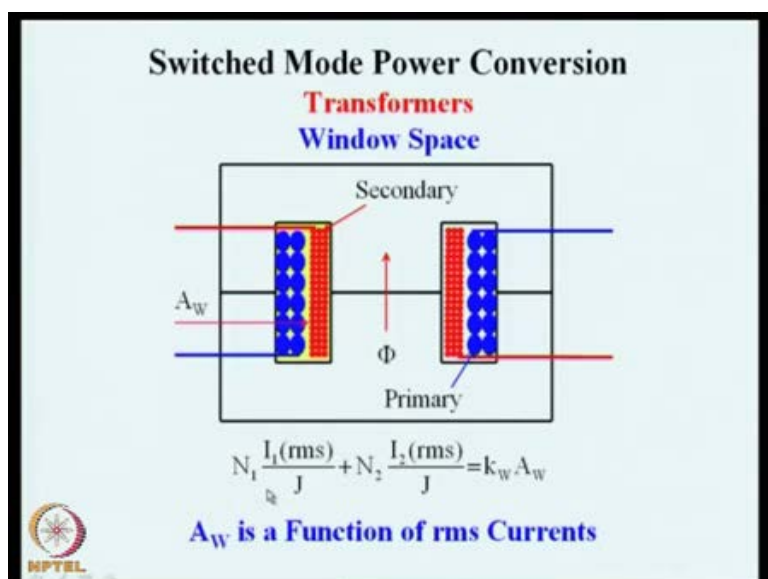
This is the reason many times when you want excite a transformer at constant flux it is necessary to maintain the relationship of V by f as constant. You would have come across in many places V by f constant excitation of transformer that relates to constant flux operation of a transformer. The important point is core area is now a function of voltage and frequency.

(Refer Slide Time: 26:55)



Now, if you look at the window area, this is the second relationship we see that the total window area accommodates the primary number of turns multiplied by the cross-section and secondary number of times multiplied by the cross-section to a fraction of the window area. We can say that $N_1 a_1$, that is primary number of turns multiplied by a_1 and the secondary number of times multiplied by a_2 is equal to a fraction of the window area or A_w , the window area that we write here is a function of the conductor size and the number of turns.

(Refer Slide Time: 27:38)



The conductor size can be replaced by whatever is rms current divided by the maximum current density allowable for the material. For copper, it is about 3 ampere per millimeter square, for aluminate may be about 1.5 ampere per millimeter square, for some other material it could be something different. So we might say that this N_1 times I_1 by J is the primary area of cross-section. N_2 times secondary area of cross-section is a fraction of the window area A_w or the window area is a function of the rms currents.

(Refer Slide Time: 28:15)


Switched Mode Power Conversion
Transformers
Design Equations

$$V_1 = 4N_1B_m A_c f_s$$

$V_1/2f_s$ is the Volt-Sec Integral
 $2N_1B_m A_c$ is the Flux Linkage

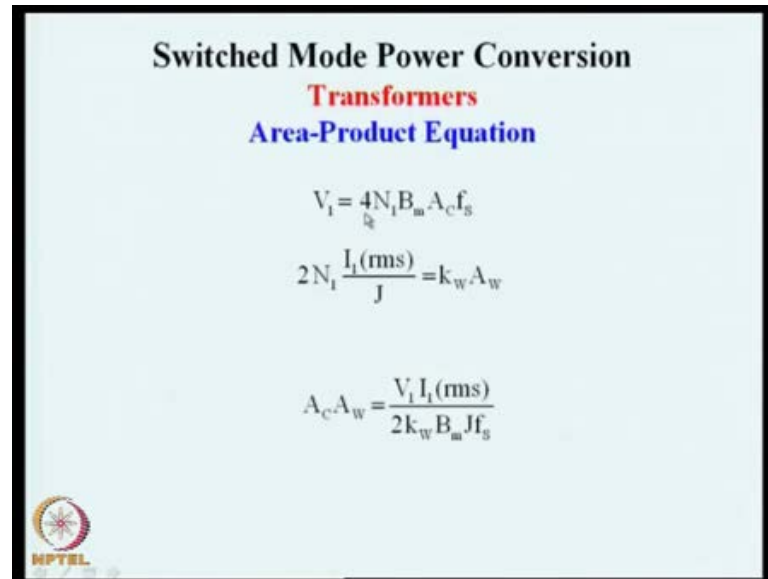
$$2N_1 \frac{I_1(\text{rms})}{J} = k_w A_w$$

Primary Ampere-Turns Equal Secondary Ampere-Turns



Now, if we combine these equations design equations, where the total number of turns multiplied by the cross-section area of each of the turns is related to the window area and the voltage of excitation is related to the core area and frequency, if these two equations are combined together,

(Refer Slide Time: 28:37)



The slide contains the following text and equations:

Switched Mode Power Conversion
Transformers
Area-Product Equation

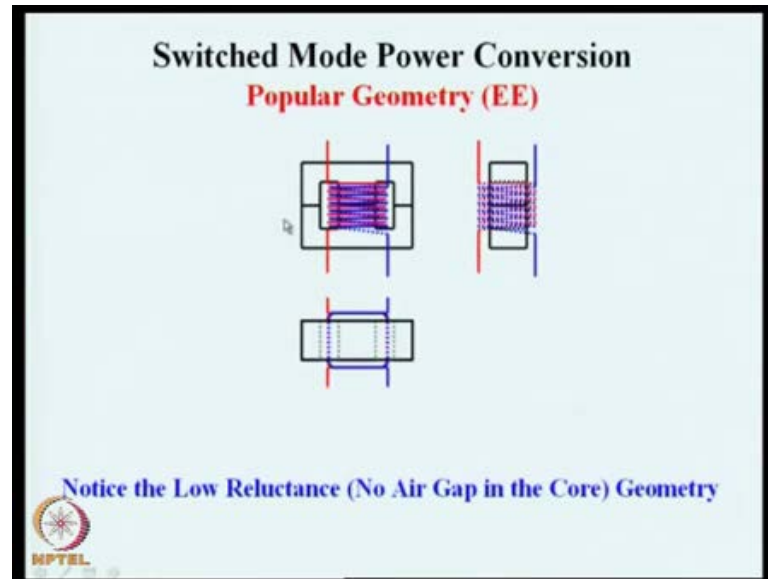
$$V_1 = 4N_1 B_m A_c f_s$$
$$2N_1 \frac{I_1(\text{rms})}{J} = k_w A_w$$
$$A_c A_w = \frac{V_1 I_1(\text{rms})}{2k_w B_m J f_s}$$

NPTEL

we get the most important design equation for a transformer which is called the area product equation. The area product of the core, core area multiplied by the window area, the area which accommodates the flux and the area which accommodates the current. This product, the unit of this will be metro power four or millimeter power four is the product of V_1 and A_1 or the volt ampere rating of the transformer divided by several quantities, the window space factor, maximum flux density for the core material, the maximum current density for the conducting material and the frequency of operation.

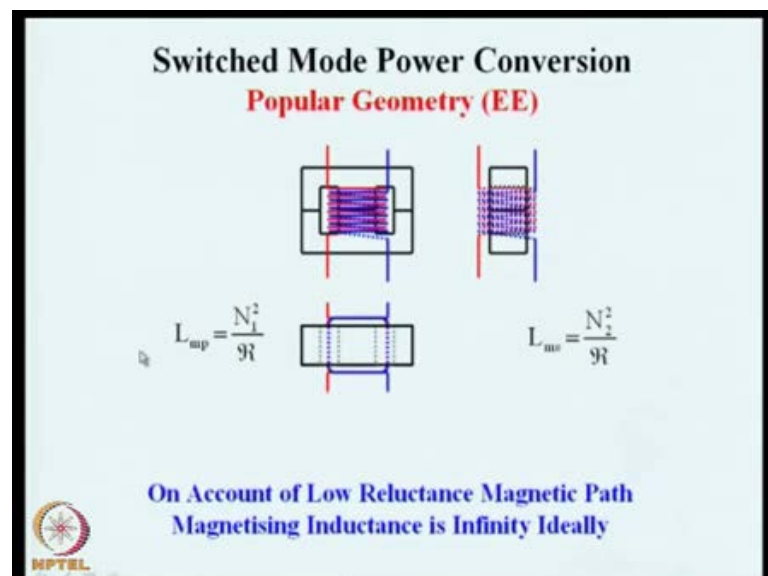
So, we see that the core has to be bigger if you want to transfer large power V into I . The core also is inversely proportional to the window space factor, inversely proportional to the flux density of the core, inversely proportional to the current density of the core, inversely proportional to frequency. This is an important observation. You can see that because f_s is in the denominator, if you build a transformer for a certain frequency if the frequency becomes double the same transformer can be used for nearly double the volt ampere rating. This is one of the motivations to process power at high frequency so that all the magnetic elements will be smaller in size.

(Refer Slide Time: 30:06)



The popular geometries of transformers are E E geometry. Notice that there is a very low reluctance in the gap because; there is no air gap in that. The reluctance in the magnetic circuit is practically 0. On account of that, magnetizing inductance of the transformer will be practically infinity. Ideally, the transformer does not draw any magnetizing current to set up a flux or the magnetizing inductance of a transformer will be infinity. We will see that in a few moments how to evaluate those quantities.

(Refer Slide Time: 30:45)



So, we see here that the magnetizing inductance seen from the primary is N^2 divided by the reluctance of the magnetic circuit. In the earlier lecture, we had seen that in an electromagnetic circuit N^2 divided by reluctance is the inductance of the winding. The primary has N_1 turns and secondary has N_2 turns, but the magnetic circuit is common for both primary and secondary so it is possible to find out the magnetic magnetizing inductance and express it as a primary magnetizing inductance or a secondary magnetizing inductance.


It is possible to magnetize a core either from the primary or from the secondary in each case, the appropriate magnetizing inductance will be applicable. On account of this lower reluctance magnetic path, the magnetic inductance is really infinity. So ideally it will not draw any current to magnetize the core or if you draw a finite amount of current, the flux magnetic flux in the magnetic energy storage is practically energy stored in the transformers is practically 0.

(Refer Slide Time: 31:56)

Switched Mode Power Conversion
ETD Core Series Data

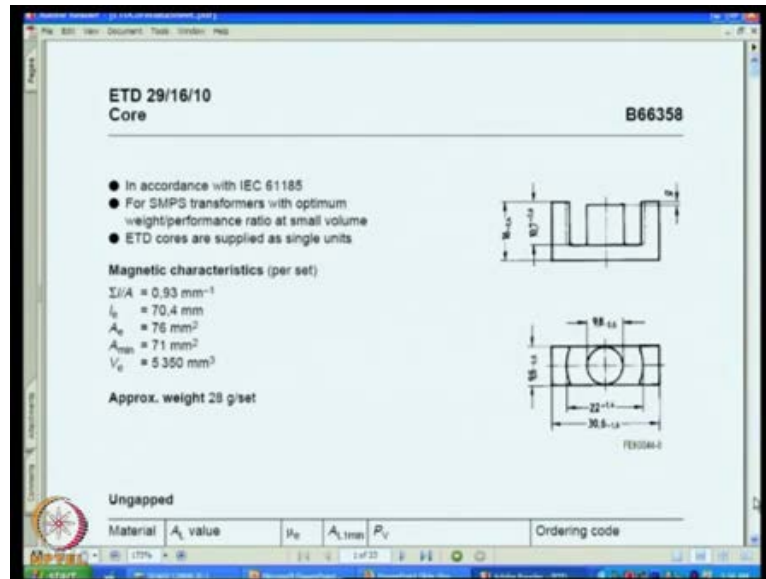
Type Number	A_c mm ²	A_w mm ²	$A_c A_w$ mm ⁴
ETD 29/16/10	76	128	9728
ETD 34/17/11	97	171	16587
ETD 39/20/13	125	234	29250
ETD 44/22/15	173	279	48267
ETD 49/25/16	211	343	72373
ETD 54/28/19	280	442	115360
ETD 59/31/22	368	473	174064

Standard Cores
ETD



This is one of the points that we have seen. So what I have shown here is one particular type of core called E T D core, so several of them are available in this family; 29, 34, and so on. So all of them I have arranged in a certain ascending order where the core area in millimeter square and the window area millimeter square in the product of $A_c A_w$ in millimeter power four are organized like this for example.

(Refer Slide Time: 32:28)



if we open the magnetizing the data sheet of this type of cores, this is E T D 29, 16,10. Let me expand the size. You can see that this core has a shape of e, the front view and the top view. This is the central post which is around which the winding disorganized and these are the two side limbs. We put two such cores another e inverted and put on top of that will make the complete magnetic circuit. In the central limb is the primary and secondary wound and these are the side limbs and the dimensions are given here. You will see that the central limb has 9.8 diameter circular cross-section and this has a cross-section area of about 76 millimeter square.

You can see that the window consists of two such things, the dimension on this site and the dimension on this, the product of that is the window area and these quantities are expressed here as 76 millimeter square of core area and 128 millimeter square of window area the product of that is 9728 millimeter power four. Now, if we have such a core data available for any particular design, it is possible to select the suitable core by making a sample design. We will be able to understand the process. There are several other geometries we will come back to that a little later.

(Refer Slide Time: 34:07)

Switched Mode Power Conversion
Sample Core Selection

$V_1 = 48 \text{ V};$
 $V_2 = 400 \text{ V}; I_2 = 3 \text{ A};$
High Frequency (50 kHz) Application

$$A_c A_w = \frac{400 * 3 * 10^{12}}{2 * 0.35 * 0.2 * 3 * 10^6 * 50000} = 57142 \text{ mm}^4$$

NPTEL

Let us look at a sample design where these core selections are worked out. This is a design where V_1 is 48 volts primary has 48 volts, secondary has 400 volts, secondary current is 3 amperes, total secondary power is about 1200 watts, 400 into 3200 volts. It is used for a high frequency application, 50 kilo hertz frequency and let us assume that this is a square wave operation. So, in such a situation we had seen that $A_c A_w$ is related to the V into I product divided by two times K_w times B_m times the current density and frequency. All these numbers are put what you get here is 57142 millimeter power four. Now, if we go back in our core, the core size is set in the ascending order of area product so 57 is more than this 48 and less than this 72000.

(Refer Slide Time: 35:22)

Type Number	A_c mm ²	A_w mm ²	$A_c A_w$ mm ⁴
ETD 29/16/10	76	128	9728
ETD 34/17/11	97	171	16587
ETD 39/20/13	125	234	29250
ETD 44/22/15	173	279	48267
ETD 49/25/16	211	343	72373
ETD 54/28/19	280	412	115360
ETD 59/31/22	368	473	174064

Select ETD 49/25/16
N67 Material

So, we could select E T D 49 as a suitable core for this application for this 57000, we could select a core which has an area product which is more than the required 57000 millimeter power four. Such a core has an area of core A_c 211 millimeter square and a window area of 343 millimeter square and the product is 72000 which is more than what we need. Now, this core has been selected 49, 25, 16 and the dimensions are mentioned here.

(Refer Slide Time: 35:57)

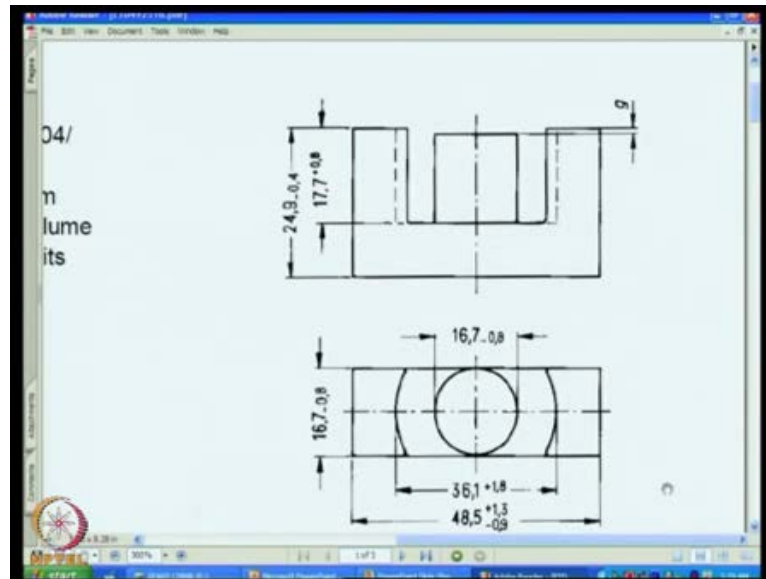
- In accordance with IEC 61185
- Quality assurance per UTE 83313-004/CECC 25 301-004 (material N27)
- For SMPS transformers with optimum weight/performance ratio at small volume
- ETD cores are supplied as single units

Magnetic characteristics (per set)

$\Sigma/A = 0,54 \text{ mm}^{-1}$
 $l_e = 114 \text{ mm}$
 $A_e = 211 \text{ mm}^2$
 $A_{min} = 209 \text{ mm}^2$
 $V_e = 24\ 100 \text{ mm}^3$

You can see that it has an area of 211 millimeter square.

(Refer Slide Time: 36:01)



And the other dimensions are central post of 16.7 and the window height for half of this window height is 17.7 and so on and end to end, from this end to this end is about 48.5 in fact and that is a number which is given in the core has the first quantity e e E T D 49, 49 refers refers to the full width of the core. This is made up of n 67, material it is available as we see on different types of materials and we have selected n 67 material. It is available as n 87, n 67 which are capable of having different types of losses. Losses on magnetization.

(Refer Slide Time: 36:50)

Material	A_L value nH	μ_e	A_{L1min} nH	P_V W/set
N27	3700 + 30/- 20 %	1590	2910	4,59 (200 mT, 25 kHz, 1
N67	3700 + 30/- 20 %	1590	2910	15,50 (200 mT, 100 kHz,
N87	3800 + 30/- 20 %	1630	2910	12,40 (200 mT, 100 kHz,

Gapped			
Material	g	A_L value approx.	μ_e

This can be used at a 200.2 Tesla at 100 kilohertz the n 27 is suitable up to 25 kilohertz, n 87 can go to 100 kilo hertz, but with less loss compared to the n 67. In this particular design we have selected n 67 material.


(Refer Slide Time: 37:10)

Switched Mode Power Conversion
No. of Turns

Type Number	$A_c \text{ mm}^2$	$A_w \text{ mm}^2$	$A_c A_w \text{ mm}^4$
ETD 49/25/16	211	343	72373
N_1	6		
N_2	47		

$$N = \frac{V}{4B_m A_c f_s} = \frac{48}{4 * 0.2 * 211 * 10^{-6} * 50000}$$

Select $N_1 = 6$ Turns ; $N_2 = 47$ Turns



The next part in the design is to find out the number of turns. Number of turns, it is very easy to calculate. It is related to voltage divided by $4 B_m A_c f_s$. We know now the area of cross-section for the selected core which is 211 millimeter square and B_m is 0.2 Tesla or 200 mille Tesla. All other quantities, 50000 is the frequency so the number of turns for the primary when you put 48 volts is 6 turns and when this replaced by 400 volts that will give you a secondary turns which is 47 turns.



(Refer Slide Time: 37:49)

Switched Mode Power Conversion

Wire Size

Type Number	A_c mm ²	A_w mm ²	$A_c A_w$ mm ⁴
ETD 49/25/16	211	343	72373
N_1	6		
N_2	47		
a_1	8.33 mm ²		
a_2	1 mm ²		

$$a_{w2} = \frac{3}{3} = 1 \text{ mm}^2 \qquad a_{w1} = 8.33 \text{ mm}^2$$

The next a selection is how big should be the size of each of these conductors. This is again very simple to calculate because; the cross section of the conductor has to support the current. In this particular case, secondary carries a current of 3 amperes and if we use a current density of 3 ampere per millimeter square, the secondary cross-section of conductor will be 1 millimeter square, primary cross-section of conductor will be 8.33 millimeter square. This has a current of 25 ampere divided by 3 ampere roughly 8.33 millimeter square.

(Refer Slide Time: 38:25)

Switched Mode Power Conversion


Wire Size – Skin Effect

Type Number	A_c mm ²	A_w mm ²	$A_c A_w$ mm ⁴
ETD 49/25/16	211	343	72373
N_1	6		
N_2	47		
a_1	8.33 mm ²		
a_2	1 mm ²		

$$\text{Skin Depth} = \frac{1}{\sqrt{\pi \mu \sigma f}}$$

$$\mu = 4\pi 10^{-7} \text{ H/m} \qquad \sigma = 59.6 \cdot 10^6 / \Omega\text{-m}$$

Copper @ 50 kHz, Skin Depth = 0.29 mm : 0.26 mm² (24 SWG)



There is one other important point to see. Whenever we process power at high frequency, the standard conducting material is not capable of carrying current across its entire cross-section. This particular property is known as skin effect. As you go to higher and higher frequency, the current is limited to the surface of the conductor and it does not flow through the entire cross-section. It is as if only the skin of the the conductor is carrying the current and the core of the conductor is not carrying the current and this effect is known as skin effect.

There is a mathematical relationship between the skin depth and the various properties of the conducting material. μ is the permeability of the conducting material which is normally $4\pi \times 10^{-7}$ Henry per meter, σ is the conductivity of the conducting material, for copper it is 59.6×10^6 per ohm meter and f is the frequency at which the conductor is used. From this you can see that if the frequency is infinitely, then the skin depth become 0 the conductor is not capable of carrying any current inside the section ,all the current will go through the surface only. At other frequency determined by the skin depth, the current will be limited to a portion of the cross-section.

For our application at 50 kilohertz for copper this permeability and conductivity, the skin depth is 0.29 millimeter and this is equivalent to 0.26 millimeter square of cross-section so 0.29 millimeter skin depth you can approach this skin from either site so that this will mean I can use a conductor whose diameter is double this 0.58 millimeter and such a conductor will have a cross-sectional area of 0.26 millimeter square and this cross-sectional area is easy equal to an appropriate standard wire gage for that.

(Refer Slide Time: 40:39)


Switched Mode Power Conversion
Primary & Secondary Conductors

Type Number	A_c mm ²	A_w mm ²	$A_c A_w$ mm ⁴
ETD 49-25/16	211	343	72373
N_1	6		
N_2	47		
a_1	8.33 mm ²		
a_2	1 mm ²		

Q_1

For Secondary 4 wires of 24 SWG in Parallel

For Primary 32 wires of 24 SWG in Parallel



So, if we limit the size of the wire to that wire gage 24 standard wire gage for 1 millimeter square, it is necessary to use 4 wires in parallel in order to make the total cross-section the each conductor has only 0.26 millimeter square cross-section. For the primary which carries a current of about 25 amperes we have to have 32 wires in parallel such parallel connection of wires are also called litz wires. Litz wires are isolated sectionalized wires. Several of them in parallel, in order to overcome the skin effect. So the secondary and primary will be organized 24 S W G wire 4 in parallel for the secondary winding which is 3 amperes and 24 S W G wire and 32 wires is in parallel for the primary which carries a current of about 25 amperes.

(Refer Slide Time: 41:36)

The slide features a central diagram of a transformer with two windings on a core. The primary winding is on the left and the secondary is on the right. Below the diagram, the text reads: 'Mean Length of a Turn: 83 mm', 'Primary Resistance: 10 $\mu\Omega$ ', and 'Secondary Resistance: 629 $\mu\Omega$ '. In the bottom right corner, there is a small inset video of a man in a light-colored shirt sitting at a desk with a laptop. The NPTEL logo is visible in the bottom left corner of the slide.

The parasitic resistance, see the conductor has an inductance which is responsible for the flux. In the flux is responsible for transferring the energy from primary to secondary, but these conductors also have a finite conductivity and because of that, it has a resistance. The primary winding has a resistance, secondary winding has a resistance. This is related to the mean length of the turn, number of turns and then the property of resistivity and cross-section of the wire. In this particular example if we calculate, it turns out to be primary resistance is 10 micro ohms and secondary resistance is 629 micro ohm.

This is a parasitic resistance. Ideally a transformer should not have any resistance, on account of this resistance the 25 amperes of the primary current going through this 10 micro ohm will result in a certain loss. Ideally, transformer neither stores energy nor loses energy, but because of this primary resistance and secondary resistance there will be some loss.

(Refer Slide Time: 42:44)

The slide features a central diagram of a transformer core with two windings. The text on the slide is as follows:

Switched Mode Power Conversion
Transformers
Core Loss

Loss at 0.2 T at 100 kHz: 15.5 W
Loss at 0.2 T at 50 kHz: 7.75 W

Core Loss Resistance: 297 Ω

ETD 49/25/16
N67 Material

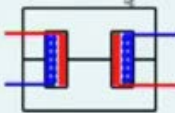
NPTEL logo is visible in the bottom left corner.

Let us look at the magnetization part. We assumed that the permeability of this material will be infinity that is really not true because most magnetic materials have a finite permeability it may be 1000 or 2000, but it is not infinity. On account of that, it is possible that this transformer, I think before the permeability lets come to core loss. So, this magnetic material is being or flux is being set up to the level of 0.2 Tesla at 50000 cycles per second on account of that, the material has a magnetization loss which is known as the core loss.

This core loss in materials consists of Eddy current loss and hysteresis loss, but in the high frequency course the Eddy current loss is negligible because the material has practically very very high resistivity and only the hysteresis loss is present and that is given in the data sheet. As we see here, the data sheet it is given as for n 67 material at 200 milli Tesla at 100 kilohertz it is about 15.5 watts per core set and if we operate at 50 kilo hertz this loss will be one half of that because hysteresis loss is proportional to the frequency. When operating at 50 kilo hertz, the loss will be about 7.75 watts and this can be representing the equivalent circuit with an appropriate resistance we will see that a little later.



(Refer Slide Time: 44:35)

Switched Mode Power Conversion
Transformers
Magnetising Inductance



$l_e = 114 \text{ mm} ; A_c = 211 \text{ mm}^2 ; \mu_r = 1590$
Reluctance = 270405 /H

Primary Inductance: 0.133 mH
Secondary Inductance: 8.17 mH





The magnetizing inductance of the transformer ideally has to be infinity so that there are no there is no stored energy in the core, but in a real transformer there is a finite magnetizing inductance. On account of this there is a magnetization current magnetization flux and so on. In this particular core, the core length is 114 mm, the core area is 211 mm square and the relative permeability of the core we had seen for the from the data sheet you can see that the permeability relative permeability is 1590 for n 67 material, μ_r is 1590 and the reluctance is given by l divided by $\mu_r \mu_0$ that quantity is 270405 per Henry is the unit for reluctance. The primary inductance is n^2 square 6 turns for the primary 6 square divided by this secondary inductance is 47 square divided by this quantity that is 0.133 milli Henry and secondary inductance is 8.17 milli Henry. Ideally, this should have been infinity, but in practice this is a finite quantity.

(Refer Slide Time: 46:09)

Switched Mode Power Conversion
Transformers
Primary Magnetising Current Peak

$$I_m(\text{peak}) = \frac{48}{2 * 0.133\text{m}} * 10\mu = 1.8\text{A}$$

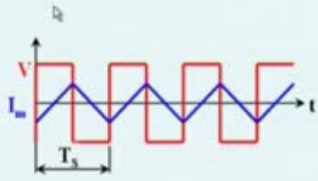
Magnetising Current Peak << Primary Current



Now, because of this magnetizing inductance, when we apply the given voltage in this case there is a magnetizing current which is a finite quantity not 0. This is related by the peak current is 48 volts divided by the magnetizing inductance and multiplied by the duration of this when we are switching it at about 50 kilo hertz t_s is 100 microsecond, t_s is 20 micro second, t_s by 2 is 110 micro second, the voltage is 48 and the slope of this is I_m peak both minus and plus is one half of that 48.133 into 10 micro is about 1.8 amperes. The rated current for this transformers was 25 amperes and you can see that magnetizing current is a small fraction less than about one-tenth of that. Magnetizing current peak is very much less than the primary current which is about 25 amperes in good designs this will always be true, in very good designs this inequality will be even better satisfied.



(Refer Slide Time: 47:25)

Switched Mode Power Conversion
Transformers
Stored Energy in the Core



$E_m = \frac{L_m}{2} I_m^2 = 0.5 * 0.133 \text{ m} * 1.8^2 = 0.22 \text{ mJ}$

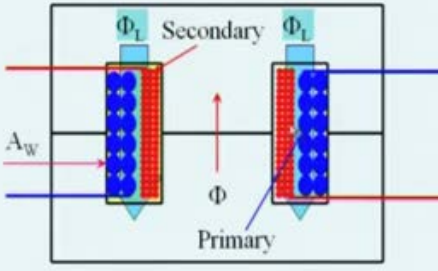
Stored Energy in the Core = 0.22 mJ




The stored energy in an ideal inductor ideal transformer was 0, but because of this magnetizing current there is a finite stored energy one half of $l_m I_m$ square and that is 0.22 milli Joule. This is another non-ideality ideally this should of been 0, but in real transformer the stored energy is a finite quantity, but it is a small quantity.

(Refer Slide Time: 47:53)

Switched Mode Power Conversion
Transformers
Leakage Energy in the Window



Leakage is Another Non-ideality in the Magnetic Circuit



The second important non-ideality in the transformer is the leakage field in the window area. Ideally, the field magnetic field has to be only in the core and not in anywhere outside the core. That will be true if the permeability of the magnetic material is perfect

is infinity that we now leakage field in the air in the outside the core or in the window, but in a real are transformer where the the permeability is not infinity, there will be some leakage field in the core, that leakage field is not linked by the both the primary and the secondary it is only partially linked by the primary or the secondary.

This is a non-ideality in most transformers and it is possible to evaluate the quantity through the geometrical properties of the core. In this particular situation, the blue wire blue cross-section is the primary winding, the red cross-section is a secondary winding, the window is seen here as a rectangular window and apart from the flux switch goes in the core, there will also be a leakage flux shown as ϕ_l both in this window and also this window and this ϕ_l if we can find out, it is possible to find out what is the leakage inductors. The method to find out this ϕ_l is to identify understand the magnetic circuit here. This is a purely nonmagnetic material, window is a completely nonmagnetic material, copper conductors and air and the top portion is the highly magnetic material, bottom portion is the highly magnetic material, but the entire window is a non magnetic material so it is possible to model this in a very simple way that the highly magnetic material at the top in a highly magnetic material at the bottom.

(Refer Slide Time: 49:50)

Switched Mode Power Conversion
Transformers
Leakage Energy in the Window

$$H(x) = \frac{N_1 I_1 x}{h b}$$

$$B(x) = \frac{\mu_0 N_1 I_1 x}{h b}$$

$$L_c = \frac{\mu_0}{I_1^2} \iiint H^2(x) dv$$

$$dv = 2h w dx$$

NPTEL

The magnetization ampere turns because of the red winding or primary winding and the magnetization magnetic turns or the other winding which is a secondary which is a increasing ampere turns because as the current is building up a any line that you take

from here is enclosing the current that is to the right-hand side of that so you might say that a distance x from the left hand side if you consider a vertical line it is possible to find $h f x$ along this vertical line.

This will be the total ampere turns divided by this h because magnetizing intensity is ampere turn divided by h and because it is linearly increasing from 0 to $N_1 I_1$ dropping to 0 at the end of the window. This relationship at any cross-section x the magnetizing field h is the ampere turns multiplied by this ratio x by b . At b it is it is maximum $N_1 I_1$ at any other distance x it is $N_1 I_1$ into x by d that is ampere turns that divided by the leakage path length, h is the magnetizing field. At the flux density is the field multiplied by μ .

If this is $h \mu$ into h is the field and the inductance can be simply calculated by volume integrating h square, in this volume of this window if you integrated x square multiplied by this μ not by I square that will be the leakage inductance. The volume that you have integrate is the volume of this complete window h multiplied by w which is this width and then $d x$ the the small incremental distance at we have considered is $d x$ and there are two such windows on this side and on the right-hand side. So, you get $2 h w d x$ is the volume that we want to integrate this and if disintegration is carried out.

(Refer Slide Time: 52:17)

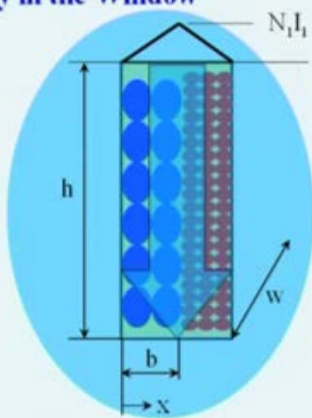
Switched Mode Power Conversion
Transformers
Leakage Energy in the Window

$$L_l = \frac{\mu_0 N_1^2}{h^2} \frac{2}{b^2} h w \int_0^b x^2 dx$$

$$L_l = \frac{2\mu_0 N_1^2 b w}{3 h}$$

0.07 $\mu\text{H} \ll 133 \mu\text{H}$

$L_{pu} = 6.1 \mu\text{H}$

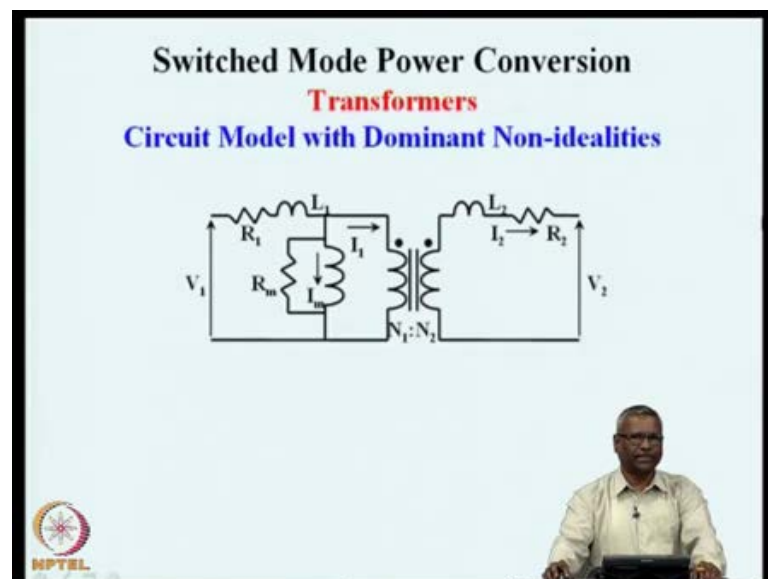


We find that for this particular example the leakage inductance is 0.07 micro Henry and this has to be compared with magnetizing inductance which is 133 micro Henry, the

magnetizing inductance will be very much higher compare to leakage inductance. We can also see, for this particular situation, 48 volts and 25 amperes and 5000 cycle per second, it is possible to find out what is the normally inductance. Normally inductance is a same as the rated current into omega l is equal to rated voltage. That inductance is 6.1 micro Henry.

You can see that leakage inductance will be very small fraction of per unit inductance and magnetizing inductance will be a very large multiple of per unit inductance. If we as we go to the ideal situation, leakage inductance will go to 0 and magnetizing inductance will go to infinity and per unit inductance is related by v divided by ω times i . In this situation, you can see that the leakage inductance is almost 100 times less than the per unit inductance. The magnetizing inductance is about 20 times more than the per unit inductance which is a very good situation

(Refer Slide Time: 53:41)



The transformer can be now modeled with all these non-idealities we saw that about 10 micro ohm and 629 micro ohm here and leakage inductance off about 0.07 micro Henry, a similar kind of secondary side leakage inductance which will be the same inductance multiplied by the square of the number of turns ratio and the magnetizing inductance of 133 micro Henry and the loss resistance which we had seen from the core loss we converted it into an equivalent resistance of something like 271 ohm or so. This is the circuit model of the transformer.

Now, whatever we had ideally designed that transformer is there, but there are several non-idealities. Magnetization, leakage, conduction, but this is our ideal transformer all my non-idealities will be normally very small compare to the ideal properties which we designed for.

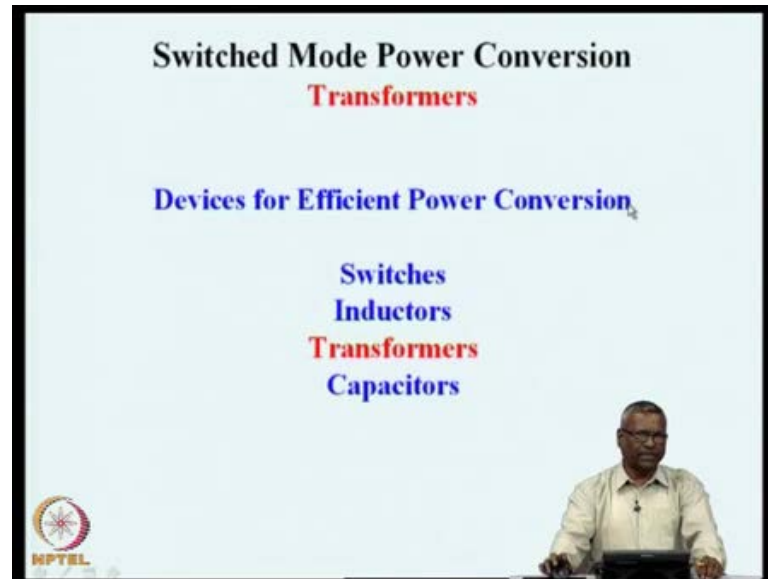
(Refer Slide Time: 54:45)

The slide features a title 'Switched Mode Power Conversion' in black, followed by 'Transformers' in red, and 'Measurement of Circuit Parametes' in blue. Below this is a circuit diagram of a transformer with primary resistance R_1 , primary inductance L_1 , primary current I_1 , primary voltage V_1 , magnetizing branch with R_m and L_m , secondary resistance R_2 , secondary inductance L_2 , secondary current I_2 , secondary voltage V_2 , and turns ratio $N_1:N_2$. Below the diagram is the text 'Open Circuit & Short Circuit Test' in blue. In the bottom right corner, a presenter is visible, and the NPTEL logo is in the bottom left.

How do you measure these quantities? Measurement of the circuit parameters is very simple all, of you must be studied in the transformer theory that by simply conducting an open circuit test and a short circuit test, it is possible to evaluate all these quantities in a transformer. If you short circuit the secondary and apply a small voltage and pass rated current and measure the power that is given to the transformer and a similar way open circuit, the secondary give the rated voltage to the primary, find out what is the current drawn and power supplied power supplied to the primary side, it is possible to separate all these quantities.

Resistance of the primary and secondary can be measured with appropriate dc voltage and dc current method. So, standard method of making suitable test dc resistance measurements then on open circuit test and a short circuit test. This is a standard method of evaluating the parameters of a transformer. After having designed evaluating the parameters will completely verified that our design method is good.

(Refer Slide Time: 55:58)



So, what we have seen in this section is about the transformers. What is the basic principle? How the primary and secondary or linking to each other through the magnetic circuit in between? What are the various relationships of the electrical quantities as well as a magnetic quantities? How does one design a transformer starting with the voltage current frequency ratings?

Having designed the ideal transformer, how do you go about finding out all the non-ideal features of the transformer such as the primary resistance, secondary resistance, leakage inductance, primary leakage inductance, secondary leakage inductance, magnetizing and inductance, the core loss and so on. This could be a good starting exercise to understand about transformers. Later on as we get to know about power converter, we will design very specific designs getting into even more detail so that.

Thank you very much.