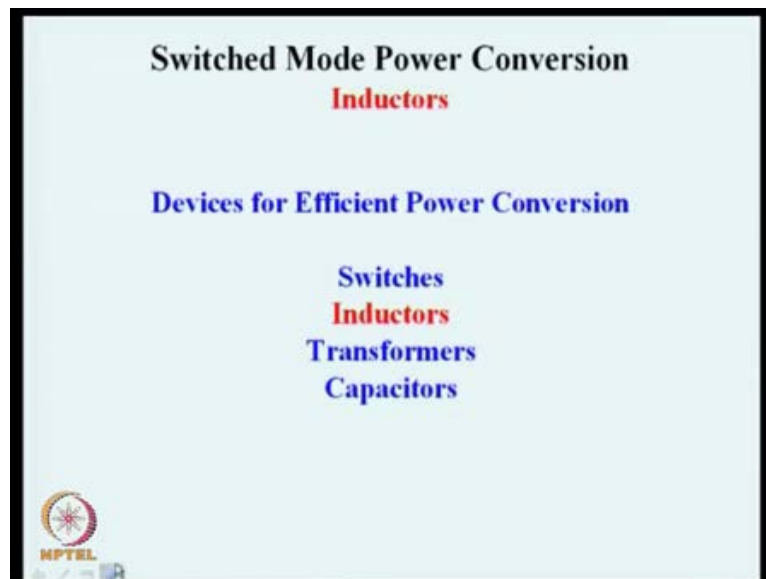


Switched Mode Power Conversion
Prof. Ramanarayanan. V
Department of Electrical Engineering
Indian Institute of Science Bangalore

Lecture - 5
Inductor

(Refer Slide Time: 00:20)




We have seen that the efficient power conversion employees devices, which are lost less. In the last session we had spent some time on the switches, the various electronic switches their idealized models, their performance characteristics and so on. In this session, we will learn about another device, which goes into power converters namely the inductors. Inductors are energy storage devices along with switches transformers, and capacitors, they are also one of the preferred devices to be used in power conversion, mainly because they do not dissipate energy. Just like the other components switches, transformers and capacitors inductors are used only to store energy and they do not dissipate energy.

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Switched Mode Power Conversion
Inductors

Inductors Store Energy
Inductors Store Energy in a Magnetic Field

In Power Converters Energy Storage
Elements Smoothen Power Flow

 MPTEL


Inductors are capable of storing energy the store energy in a magnetic field. In power converters the switches that we employ result in discontinuous power flow. The power flow is interrupted by the operation of switches an account of that in many converters. The power flows the interruption of the power flow results in discontinuities in the power flow, in such a case when we want to handle power in a smooth fashion there are many storage elements inductors and capacitors that are used to smoothen the power flow.


(Refer Slide Time: 01:55)

Switched Mode Power Conversion
Inductors

Inductors are also used as Switch
Protection Elements

In Switching Aid Networks Inductors
Provide Turn-On di/dt Protection

 MPTEL



The main function of inductors in power converters is to smoothen the power flow, they are also used as protection elements for the switches. We had seen that whenever a switch is turned on or turned off there are high stress in terms of voltages and currents imposed on the electronic switches. Therefore, it is necessary to appropriately protect the switches against over voltages and over currents the switching aid networks that are used to enable stress free turn on of the switches employ inductors. In switching aid networks inductors provide the di by dt protection or turn on di by dt protection. So, there are two functions for which inductors are used in power converters one of them is to smoothen the power flow and the other one is to protect high current stresses on the devices.

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Switched Mode Power Conversion
Inductors

$$V = L \frac{dI}{dt}$$

Electrical Circuit Element Equation
V, L, I are Electrical Circuit Quantities

Schematically, an inductor is represented by the symbol that is shown here just as any other electro electrical network element. The inductor has two characteristic variables associated with that one of them is the across variable, which is a voltage across the inductor and the other is the through variable, which is the current through the inductor. The inductor L in this schematic has a current flowing through it, which is I and the voltage across the inductor is V .

And the characteristics of the inductor is represent represented by the relationship between the V and I and this is a deferential equation, which relates the voltage across the inductor through the current flowing through the inductor through the element value,

which is L . V is equal to L into dI by dt or the voltage across an inductor is proportional to the rate of rise of current through the inductor and that proportionality constant is what is defined as the inductor. This is the electrical circuit equation of an inductive element where V and I are the two electrical variables one is the variable across the voltage across the inductor. And the other is the current through the inductor and they are related through the inductance value L . The unit for L is Henry voltage unit is volts and the unit for current is ampere.

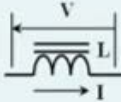
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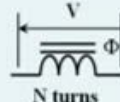
The slide is titled "Switched Mode Power Conversion" and has a sub-heading "Inductors" in red. It features a diagram of an inductor with a voltage V across it and a magnetic flux Φ through it. Below the diagram, it says "N turns" and the equation $V = N \frac{d\Phi}{dt}$. At the bottom, it states "Electromagnetic Equation – Faraday's Law" and "V is Electrical Circuit Quantity" and "N, Φ are Magnetic Circuit Quantities". There is a small logo for NPTEL in the bottom left corner and a photo of a man in the bottom right corner.

There is also another way of looking at an inductor, which is through the magnetic part of the inductor. The inductor is having a magnetic circuit, which has certain number of turns organized around a magnetic circuit and it is possible to relate the voltage across the inductor through the magnetic quantities. Namely the number of turns and the flux in the magnetic circuit this relationship is the famous Faradays law, which is the electromagnetic equation. Where the voltage across the inductor is equal to the number of times the rate of change of flux in the inductor or this is also defined sometimes as a rate of change of flux linkage in the inductor. The voltage V is the electrical circuit quantity, which is the voltage across the inductor and the number of turn N , which is in the inductive circuit and the flux that is in the magnetic circuit or the magnetic circuit quantities

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Switched Mode Power Conversion
Inductors



 $V = L \frac{dI}{dt}$


 $V = N \frac{d\Phi}{dt}$

$LI = N\Phi$

Relationship Between
L, I & N, Φ

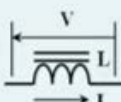
$N\Phi$ is also defined as Flux Linkage in the Inductor

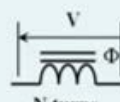


So, it is possible to define an inductor either through the electrical circuit as V is equal to $L \frac{dI}{dt}$ or through its magnetic circuit. Where the voltage is number of turns multiplied by the rate of change of flux with respect to time from this it is possible to see this identity, which is $L I = N \Phi$. This is also defined as the flux linkage in the inductor L and I are the quantities, which are defined on the electrical side of the inductor and N and Φ are defined on the magnetic circuit of the inductor. And inductor as we know is an electromagnetic circuit it has an electrical part as well as a magnetic part.

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Switched Mode Power Conversion
Inductors


 $V = L \frac{dI}{dt}$


 $L I = N \Phi$

$V = N \frac{d\Phi}{dt}$

How to Relate the Electrical Circuit V, L, I
and
The Magnetic Circuit of the Inductor N, Φ, I & R



The relationship between the to the electrical product L and I is also the same as the magnetic product N and ϕ . Now, how do we relate the electrical circuit quantities V L and I to the magnetic circuit quantities namely the number of turns the flux the current through the coil and the reluctance of the magnetic circuit. These R here is the reluctance of the magnetic circuit.

(Refer Slide Time: 06:46)

The slide is titled "Switched Mode Power Conversion" in bold black text. Below the title, "Ohm's Law" is written in red. A circuit diagram shows a resistor with a zigzag symbol, with a voltage V across it and a current I flowing through it. Below the diagram, the equation $V=IR$ is displayed. At the bottom, "Bulk Ohm's Law" is written in blue. In the bottom left corner, there is a logo for "MPTTEL" and a small inset image of a person in the bottom right corner.

Now, we might take a small detour to come back to the magnetic circuit. This is the Ohms law, which relates the voltage across a resistor to the current through the resistor through the value of the resistance V is equal to $I R$. And this is the bulk Ohms law where the terminal voltage is related to the current through the bulk resistance or V is the voltage across the resistor it is the product of the current and the resistance of the element.

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Switched Mode Power Conversion
Ohm's Law at a Point

$V = IR = JA \frac{\rho l}{A} = J\rho l$

$\frac{V}{l} = J\rho$

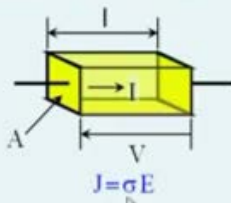
$J = \sigma E$

Now, if we try to define Ohms law at a point it is possible to modify these numbers in a slightly different way. These I may be defined as the current density in any region multiplied by that area of that region as the current is flowing through the conductive element. The current I can be replaced by current density and the area of that region and the resistance may be in any particular region may be replaced by the resistivity or the specific resistance multiplied by the length of the conducting path divided by the area of the conducting path.

So, when we modify this equation thus this results in another relationship, which relates the electrical field intensity which is voltage divided by the length of the conducting path to the current density and the resistivity of the material in this. This also can be rewritten as current density at any point in a conducting circuit is related to the electrical field intensity at that point and the proportionality constant is the conductivity of the material. This particular form of Ohms law is defined as a Ohms law at any particular point because this can be defined in any point in space. Current density at any point is equal to the conductivity of the medium at that point multiplied by the electrical field intensity. This is a very useful relationship because this is not based on the bulk properties, but this is based on the properties at any point in a conducting medium.



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Switched Mode Power Conversion
Ohm's Law at a Point



$J = \sigma E$

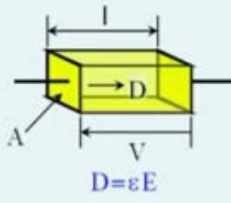
**In a Conducting Material
Electrical Current Density (J)
is Proportional to (σ), and
Electrical Field Intensity (E)
 σ is the conductivity of the material**



We can extend a similar kind of relationship for many of the other electrical circuit quantities for example, what we had just now seen is the Ohms law at a point, which can be stated us in a conducting material. Electrical current density J is proportional to the conductivity σ and the electrical field intensity E σ is the conductivity of the material and J is σ times. The electrical field intensity we can see this as a cause and defect relationship that in a conducting material if a field intensity of E is set up and if the material has a conductivity of σ there will be a resultant current density, which is given by J as amperes per meter square.



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Switched Mode Power Conversion
Dielectric Ohm's Law at a Point



$D = \epsilon E$

**In a Dielectric Material
Dielectric Flux Density (D)
is Proportional to (ϵ), and
Electrical Field Intensity (E)
 ϵ is the permittivity of the materi**

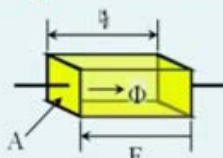


This same law which is true for conduction can be extended for magnetic circuit element as well as dielectric circuit element for example, the magnetic Ohm's law at a point can be defined as the magnetic field intensity or magnetic field density. B is proportional to the magnetizing field intensity H and it is related through the constant of proportionality μ . μ is the permeability of the material in any magnetic material whose permeability is μ if a magnetic flux density B is to be established, then it is necessary to set up a magnetic field intensity H or alternately in a magnetic material whose permeability is μ .

If a magnetic field intensity of H is established there will be a resultant magnetic flux density B and that is the relationship of magnetic Ohm's law at a point and we might also look at the dielectric Ohm's law at a point as the dielectric flux. Density D is the product of the permittivity of the material ϵ multiplied by the electrical field intensity E and this product D is equal to ϵ times E is for a dielectric materials. It can be stated as in a dielectric material of permittivity ϵ an electric field intensity of E will result in a flux density of D ϵ is the primitive the permittivity of the material. These three rules what we had seen for a dielectric Ohm's law, the magnetic Ohm's law and the conductivity Ohm's law these are the mathematical relationships of conduction induction as well as static electric field in various components that we see as electric circuit element, which are resistors inductors and capacitors.

(Refer Slide Time: 11:52)



Switched Mode Power Conversion
Magnetic Ohm's Law



$$BA = \mu A \frac{NI}{l} = \Phi = \frac{NI}{l/A\mu} = \frac{F}{R}$$

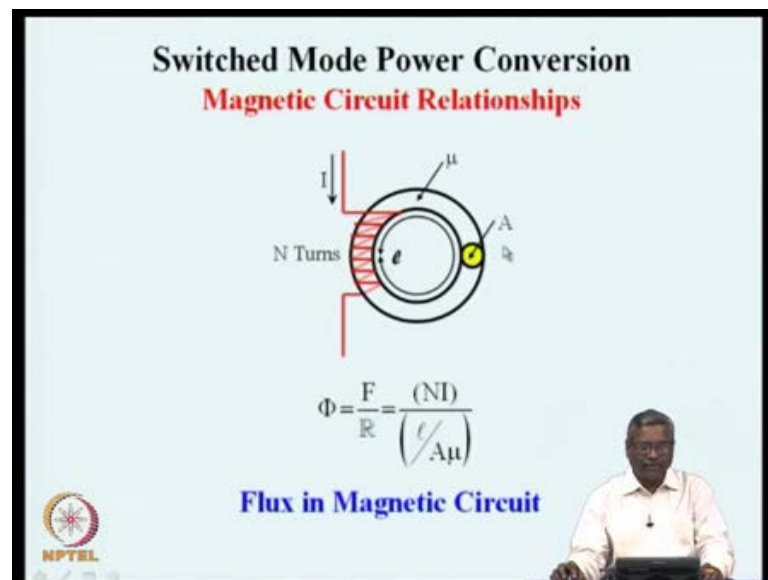
$$\Phi = \frac{F}{R}$$

Bulk Magnetic Ohm's Law

Now, we can go backwards taking the magnetic Ohms law, which is at a point can be extend at to a magnetic Ohms law which is true for bulk. For example, we had seen that B is equal to μ times H we multiplied by the area of the magnetic path both the sides the resultant is the flux in the magnetic circuit. In the flux is now related to the ampere turns divided by the quantity that we see here L over a μ . This quantity is defined as the reluctance of the magnetic circuit and we see that the flux in a magnetic circuit is related to the ampere turns and the reluctance flux is equal to ampere turns, which is setting up the field divided by the reluctance of the magnetic circuit. This particular equation is known as the bulk magnetic ohms law and this is one of the equations, which we use in order to design a suitable magnetic circuit.

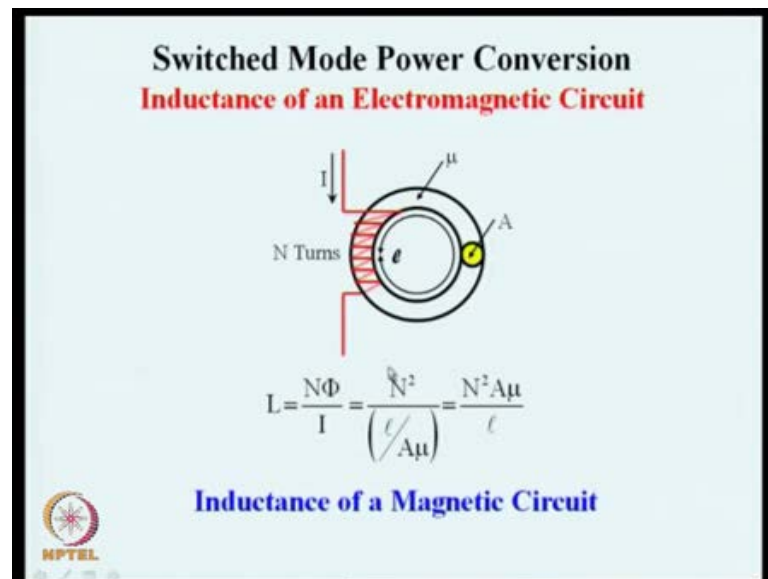
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What we see here is magnetic circuit where these relationships are explicitly shown, which will be useful for designing an inductor eventually in the process of our power conversion. Now, this is circuit where there is a toroidal magnetic ring here whose magnetic path length is L and the permeability of the material is μ . And what we see here is the cross section of the magnetic path is A is toroidal magnetic path has a cross section, which is circular and it is in the form of a ring. The magnetic path has an area of cross section of A the magnetic material has a permeability of μ and the mean magnetic length is L and we have N number of turns, which are wound on the magnetic material which carries a current of I .

This is our magnetic circuit and in this circuit if we wish to find out the flux. Flux is related to the ampere turns or magneto motive force, which is the same as the product of number of turns multiplied by the current through that. So, the total ampere turns which is responsible for setting up a magnetic field is $N I$ and the reluctance of this path is the length of the magnetic path divided by the area of the magnetic path and the permeability of the material. So, the flux in this circuit is given by this relationship $N I$ divided by the reluctance, which is L over a μ .

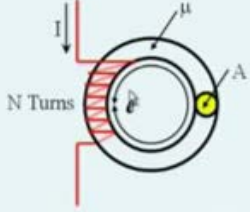
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Now, if you try to relate what is the inductance of this circuit. The inductance is defined as the flux linkage per ampere and if we substitute for Φ is $N I$ by reluctance. The inductance turns out to be N square by reluctance or N square a μ by length of the magnetic path. This is an important equation and this is our starting point if we wish to design an inductor. Inductance of a magnetic circuit is proportional to the square of the number of turns it is proportional to the area of cross section of the magnetic circuit. It is proportional to the permeability of the material that is used for the magnetic circuit and it is inversely proportional to the length of the magnetic circuit.


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Switched Mode Power Conversion
Conceptual Design of an Inductor



**For the desired L and I ,
To select a magnetic core (A, μ, ℓ),
number of turns N
and conductor size a**

 MPTEL

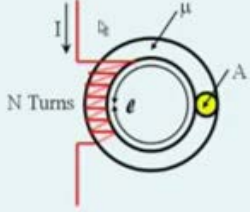


Now, conceptual design of an inductor fall is as follows for a desired value of inductance and current if you wish to build an inductor L Henry capable of carrying a current of I . Then it is necessary to build a magnetic circuit and this magnetic circuit has a core, which is called the magnetic core and the core has typically an area of cross section material property, which is permeability and length. Then we must also know how many turns of wire to be put and what will be the conductor size of this wire a . So, what is involved in design of an inductor conceptually or given inductance L in Henry and I in ampere. What will be the core that is to be used how many turns of conductors are to be put in the inductor and what will be the size of the conductor.



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Switched Mode Power Conversion

Design of an Inductor



μ of a magnetic material varies widely.
 l is not finely controllable.

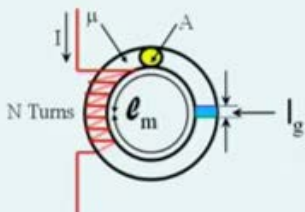


This essentially is the design of an inductor, but in practice there are a few difficulties for example, the permeability of a magnetic material is not constant and varies widely. It can vary between numbers like a 1000 and 2000 the relative permeability of magnetic material could vary by a ratio as much as 1:3 or 1:4. Similarly, the length of the magnetic path is not very finely controllable because in any magnetic path the reluctance is $L/\mu A$ all these quantities are not finely controllable. So, normally the design is done with a little bit of flexibility are shown here.



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Switched Mode Power Conversion

Practical Design of an Inductor



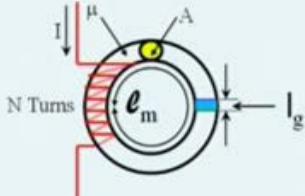
Magnetic path is made of:
Large Path of High Permeability and a
Small Controlled Path of Low Permeability (A)



What we do is in a practical design of an inductor we take a particular core. We introduce in the path of the reluctance an additional controllable path for example, in the total magnetic path now there is a large path of magnetic material almost the same as l_m . And a small controllable path of low permeability, which is air typically and this gap or this small length we call as air gap or l_g the length of the gap. The area of cross section is the same the length of the magnetic path is nearly the full toroidal rings length and a small controllable path of low permeability. The reason for introducing this will become very clear in the next slide.

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Switched Mode Power Conversion
Practical Design of an Inductor



$$L = \frac{N\Phi}{I} = \frac{N^2}{\left(\frac{l_m}{A\mu_0\mu_m} + \frac{l_g}{A\mu_0}\right)} \cong \frac{N^2 A \mu_0}{l_g}$$

Inductance is Independent of Core Material (μ)
Inductance is Independent of Core Shape (l_m)

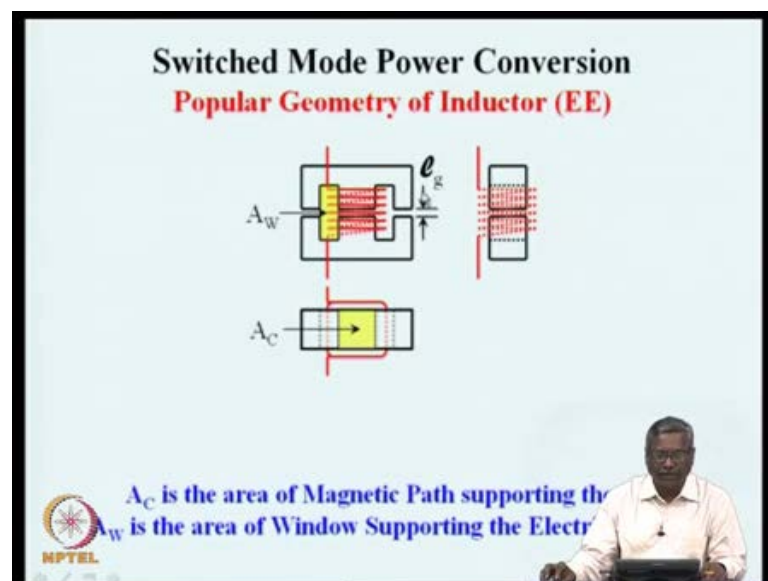
MPTEL

What we now see is that the inductance is now $N \phi$ by is just as before, but the reluctance now has two components. One of them is the magnetic path length divided by the area of the magnetic path and the μ of the material, which is the material relative permeability multiplied by the absolute free space permeability plus the gap permeability, which is the length of the gap divided by area of cross section multiplied by the free space permeability or the μ_0 permeability of air. Now, in this particular quantity that is shown in the denominator the relative permeability of the magnetic material will typically be of the order of 1000 or 2000 or 3000 as a result is first term will become negligible in comparison with the second term and effectively. So, this inductance is now multiplied $N^2 A \mu_0$ by l_g .

So, what we notice is that inductance is independent of the core material μ_r or μ_m it depends only on the air permeability, which is this part. And the area of cross section of this path and the length of this path μ_0 is a constant, which is $4\pi \times 10^{-7}$ Henry per meter. Number of turns we will be introducing by winding on this part area of cross section is relatively finely controlled and l_g can be controlled as fine as we want the gap length. Now, this relationship has much better controlling features and so this is the normal method of designing an inductor.

Where in the inductive path you put a controllable small air gap whose reluctance is totally under our control and the reluctance of the rest of the path is negligible compared to the reluctance, which is in the air gap. Now, when this happens the important point is inductance is independent of the core material and inductance becomes also independent of the shape of the core because the entire inductance is decided by the gap here. So, whatever shape that you have here whether it is a toroidal shape or u shape or a e shape all those become immaterial.

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So, normally what we try to do is we try to use popular geometry of inductors, which are manufactured by several companies and they are available the marketing in different sizes. One of the popular geometry is what is known as EE core there is one E and another E both are stacked one against the other and in the space in between the coil is wound. And as we had seen before if the permeability of the magnetic material is very

large relative permeability of the magnetic material is very large compared to the air relative permeability, which is one than the shape of the core. The material of the core does not determine the inductance value and so the designs become much simpler.

This is an EE type of core in which certain number of are turns are wound and you see the views as a cross section end view and also the top view. This type of cores are called EE core it is also possible to have several other geometries we will see it a little later. Now, in this particular geometry the area through which the flux is set up you know the coil is setting up a flux that is set up in this particular area and that is defined as a core area or A_c . A_c is the area which supports the magnetic path and the magnetic flux. Now, there is another area where the conductor is house and this area what is shown here is A_w is the area of the window and it supports the entire electric current. So, in any core you will find the two quantities that are defined window area A_w and the core area A_c and in the design eventually to get the required inductance, we will find out what is the required value of l_g .

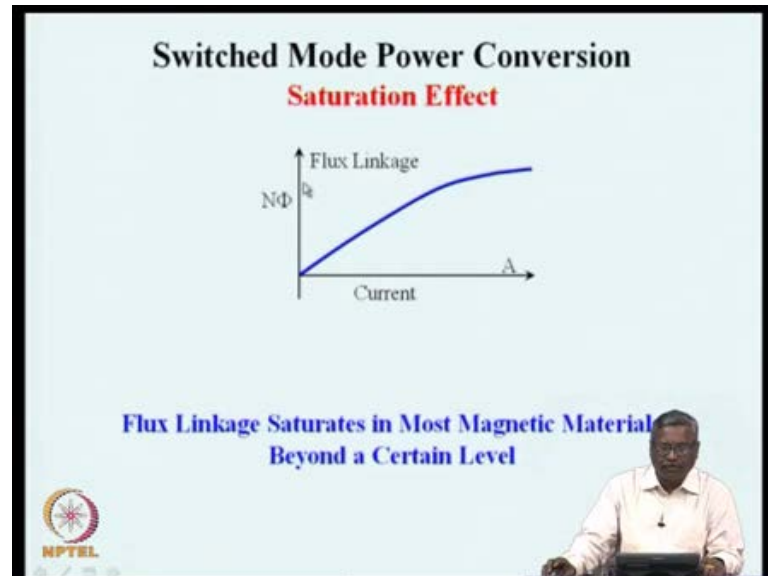
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The slide is titled "Switched Mode Power Conversion" and "Definition of Inductance". It features a graph with "Flux Linkage" on the y-axis (labeled $N\Phi$) and "Current" on the x-axis (labeled A). A blue straight line starts from the origin and slopes upwards. Below the graph, the text reads: "Inductance is Defined as Flux Linkage per Amper" and "Slope of $N\Phi$ vs I Curve is Inductance". In the bottom right corner, there is a small video inset of a man in a white shirt sitting at a desk. The MPTEL logo is in the bottom left corner.

Now, the definition of inductance as we had seen before is the relationship, which is $N\phi$ by I . So, if you draw a characteristic which is flux linkage on the y axis and current through the coil on the x axis and plot the flux linkage characteristics verses current. You will find this to be ideally a straight line the inductions are defined as the flux linkage per ampere or it is the slope of this $N\phi$ versus I curve. This would be the ideal kind of

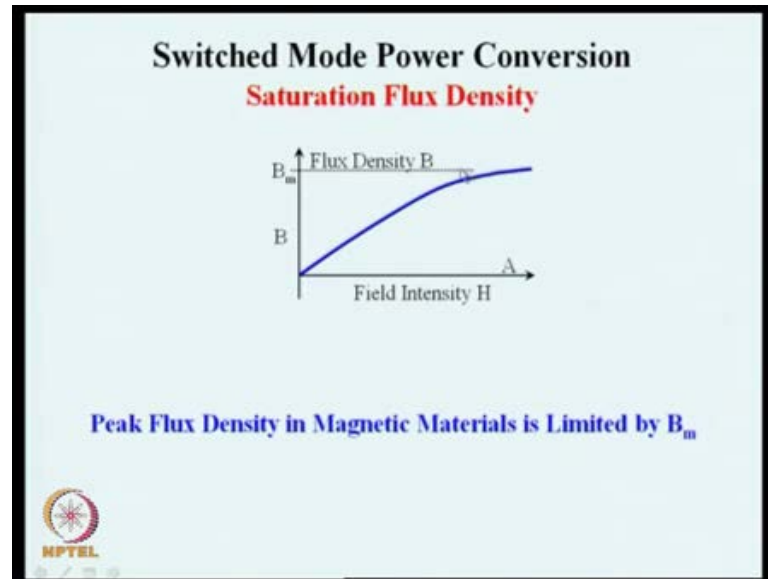
inductance characteristics where the slope of this $N\phi$ flux linkage versus current characteristics determined the inductions.

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Now, in real magnetic materials as the flux linkage increase. As the flux linkage increases the magnetic flux density in the material is not able to increase forever at the same rate and it reaches what is known as a saturation level and this effect is known as saturation effect in any magnetic material. So, as the current is increased the flux linkage does not linearly increase forever, but it reaches some plateau and it reaches a saturation level. And most magnetic materials this flux linkage saturation takes place at a certain level for ferrites it takes place at about 0.2 Tesla for iron it may take place at about 1.5 Tesla.

(Refer Slide Time: 23:46)



Now, the peak flux density beyond which the flux is not increasing proportionately is known as the peak flux density and it is defined by B_m . In this particular example we see that B_m is the level beyond, which flux density does not change and the as the current is increasing a magnetizing field intensity is ampere turns per unit length, which will be same as $N I$ by L . So, if we appropriately scaled that curve we can plot a $B H$ curve and in $B H$ curve the saturation effect will be seen as B_m . The peak flux density in most magnetic materials is limited by a number, which is known as the maximum flux density or saturation flux density.

Now, the saturation limit put some kind of a constraint on the various physical dimensions in a magnetic circuit. In this particular EE core we see that the product $L I$ same as $N \phi$. Now, maximum current will result in maximum flux, maximum flux is the product of maximum flux density multiplied by the core area. So, we might say that L into I_m the maximum current multiplied by the inductance will be the product of $n B_m$ and A_c of this. So, this is one of the magnetic path constraints beyond, which if we try to the increase the current this equation will not be valid because B_m cannot increase more, so this is one of the constraints of the magnetic path.

(Refer Slide Time: 25:24)

Switched Mode Power Conversion
Heat Produced in the Conductors

$Q = I^2Rt$

I²R Heating in the Conductors

The slide features a red waveform representing current over time. Below the waveform is the equation $Q = I^2Rt$. At the bottom, the text reads "I²R Heating in the Conductors". The NPTEL logo is visible in the bottom left corner.

There is another constraint in the inductor, which is a heat produced in the conductors. The conductor has some resistance and if a current is flowing through that is I^2R will produce heating so many watts every second or the heat produced in joules will be I^2R into t . This kind of heat has to be removed from the conductor otherwise conductor will become hot and eventually is damage the insulation or melt the wire.

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Switched Mode Power Conversion
Thermal Limit

$NI_{max} = k_w A_w J$

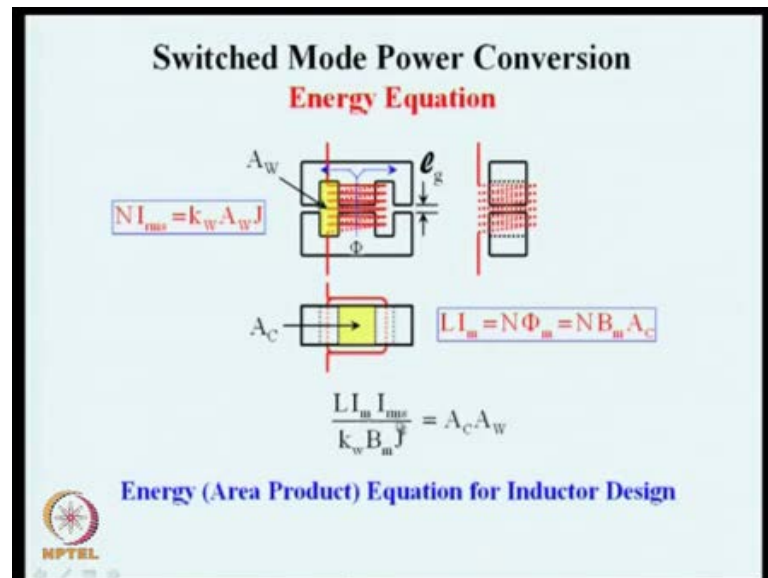
Current Carrying Capacity Constraint

The slide shows a schematic of a transformer with a core, windings, and a cooling path. Labels include A_w for winding area, A_c for core area, Φ for flux, and θ_g for temperature rise. Below the diagram is the equation $NI_{max} = k_w A_w J$. At the bottom, the text reads "Current Carrying Capacity Constraint". The NPTEL logo is visible in the bottom left corner.

So, the two constraints that we have one of them we had seen as the thermal the saturation constraint. The second constraint is known as the thermal limit and this

indicates that the r m s current of the conductor. And the number of conductors that are in the window area they have to be in such a way that the maximum current density in the material in the copper or the conducting material is not exceeded. So, we can save that the total ampere turns in the window is $K_w A_w$ multiplied by the maximum current carrying capacity.

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So, this gives the thermal limit and if we see both of them together we have two limits in the inductor design. One of them is the saturation limit or the thermal limit and the other is the saturation limit and if these two are combined together. If you multiply both of them you give you get a relationship which is called the energy equation for inductor design. What it indicates is that for any inductor that you wish to design, which has a maximum current rating of I_m and an r m s current rating of $I_r m s$. Then it will require a core whose area of the magnetic path and area of the copper path the product has to satisfy this relationship.

So, if you wish to store more energy you need a bigger core if you need to store less energy you can manage with a small core. How smaller how big is related by what is the maximum flux density for the material that we are using for the magnetic path. What is a maximum current density of the material? That we use for the conducting path and how good the window is packed with copper this K_w is called the window space factor. So, if these three numbers are known window utilization factor a maximum flux density, the

maximum current density and the inductor design current values and inductance values it is possible to find out how big a core one has to use.

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Switched Mode Power Conversion
Size of Magnetic Core

$$\frac{L I_m I_{max}}{k_w B_m J} = A_c A_w$$

The Magnetic Core has to be Big Enough to Handle the Stored Energy in the Core

The Core Size is Inversely Proportional to Saturation Flux Density B_m
Peak Current Carrying Capacity J
Window Space factor k_w

MPTEL

The size of the magnetic core is related to all this quantities. We have seen that the core size is inversely proportional to maximum flux density of the material peak current carrying capacity of the conductor and the windows space factor K_w .

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Switched Mode Power Conversion
Typical Design Constraints

The Core Size is Inversely Proportional to Saturation Flux Density B_m
 $B_m = 0.2 \text{ T}$ for Ferrite Cores

Peak Current Carrying Capacity J
 $J = 3 \text{ A/mm}^2$ for Copper, Natural Cooling

Window Space factor k_w
 $k_w = 0.35$ for round Conductors

MPTEL

A typical design constraint could be for ferrites type of course, which are normally common for all power converters. Power conversion equipment where we will be

switching at high frequency ferrite cores are used and they have typical maximum flux density of 0.2 Tesla and copper. Invariably is the conducting material and in natural cooling where you do not use a fans or any other medium for cooling. The peak current carrying capacity will be about 3 ampere per millimeter square or 3×10^6 amperes per meter square.

If you want to use the correct units and the window space factor typically could be about 0.35 what this indicates is that only about one third of the window can be effectively used by copper. The rest of the space will be used for insulation between layers insulation of the conductor itself, insulation between one winding and another winding and so on. So, typical design constraints are B_m about 0.2 Tesla for ferrites current density about 3 amperes per millimeter square for copper and the window space factor of about 0.3 or 0.35.



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Switched Mode Power Conversion
Sample Core Selection

**L = 20 μ H; DC Current of 5 A;
High Frequency (20 kHz) Application**

$I_m = 5 \text{ A}; I_{rms} = 5 \text{ A}; L = 20 \mu\text{H}$

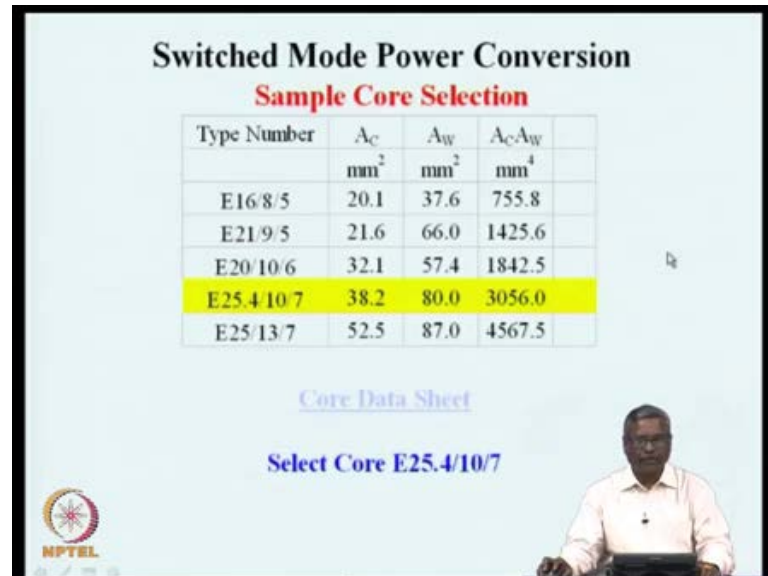
$$A_c A_w = \frac{20 \mu * 5 * 5}{0.35 * 0.2 * 3 * 10^6} = 2381 \text{ mm}^4$$

Let us take a sample design now it is a I want to design an inductance of 20 micro Henry carrying DC current of 5 amperes for an application. Where the switching is done at high frequency, which is about 20 kilo hertz. So, in this case the maximum current and the r m s current both are same, which is same as 5 amperes and inductance is 20 micro Henry and if we use a 0.2 Tesla for flux maximum flux density and 3 ampere per millimeter square for the current density. A copper and a window space factor 0.35 and this core area product are the product of core area and window area for this particular design is

2381 millimeter power 4 in this units. Now, if we wish to use a suitable core it has to have a cup core area and copper area product to be as high as 2381.

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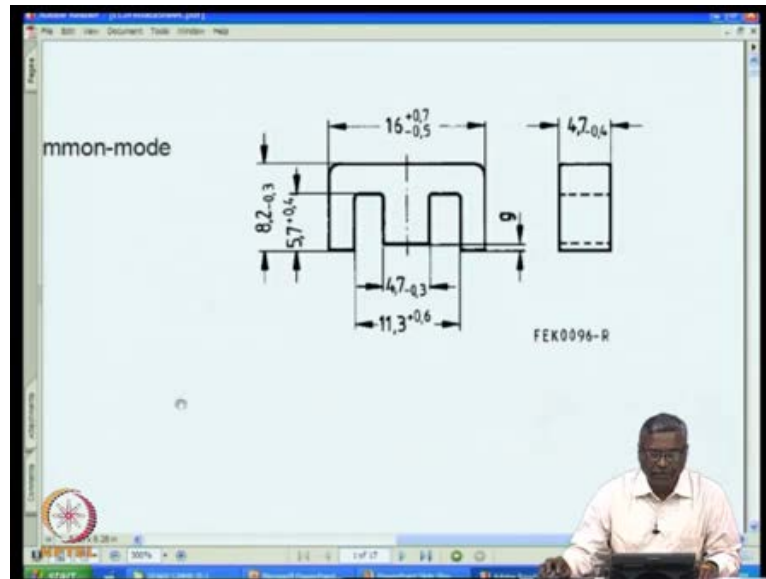
Type Number	A_c mm ²	A_w mm ²	$A_c A_w$ mm ⁴
E16/8/5	20.1	37.6	755.8
E21/9/5	21.6	66.0	1425.6
E20/10/6	32.1	57.4	1842.5
E25.4/10/7	38.2	80.0	3056.0
E25/13/7	52.5	87.0	4567.5

Core Data Sheet

Select Core E25.4/10/7

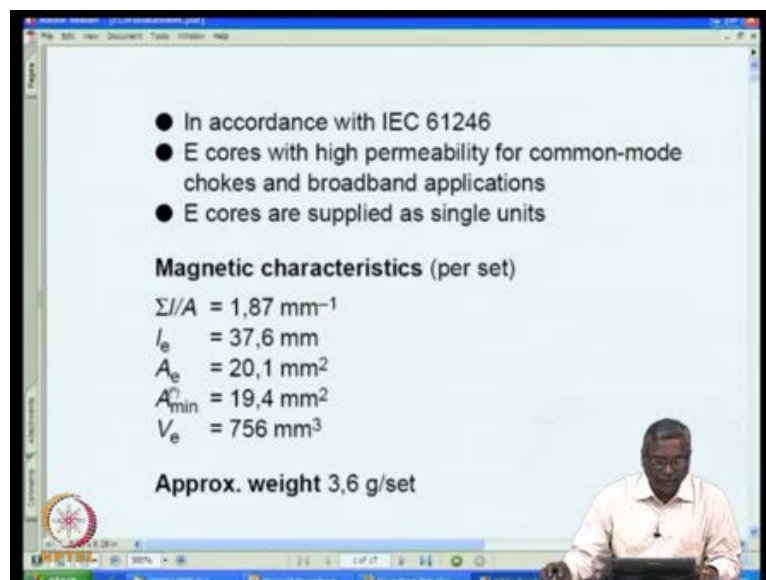
So, let us look at some of the cores that are available. What I have here is a spreadsheet for different type numbers of course, the core area window area and the product or all listed and the examples that we have seen. We require 2381 millimeter power 4 and if you see this table here this core has a area product, which is less than what we want this also has something less than what we want this also has something less than what we want. What we need is about 2381 and this is more than what we want this is even more than that quantity. So, probably this is a suitable core because it has a product of $A_c A_w$ which is more than this 2381, which we have calculated here so as a result we can say that this is the core that has to be selected. So, let us look at how this core looks like and what are the other important characteristics of this core what I have here is a data sheet for the core it is open it and see the data sheet.

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Now, this is a made by (()), so let me so you can see that this core has a shape of E where the central limb has 4.7 millimeter and the overall width is about 16 and the height is about 8.2.

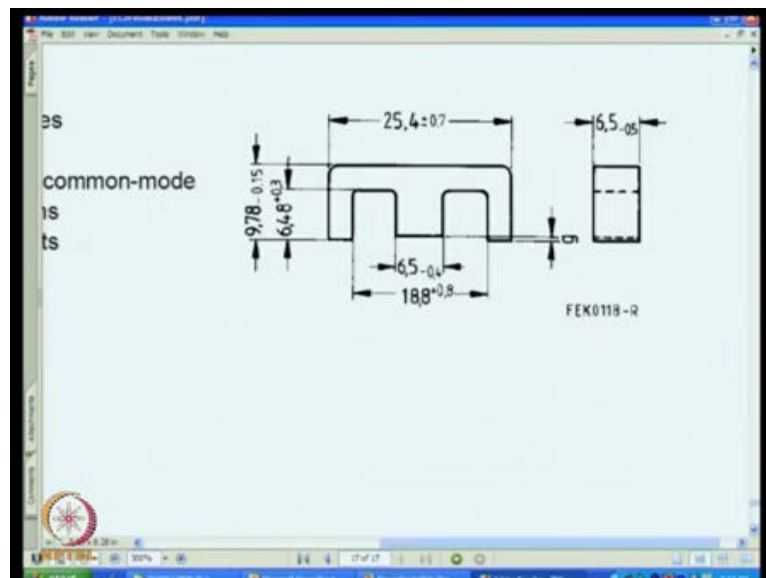
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So, two such E is can be put to form a core, but this particular core if you see here it has a core area of just about 20 and the window area which is this quantity, which is 5.7 plus 5.7 it is 11 and then 11 and about 3 about 30 or 40 millimeter square. If you go back to our 40 millimeter square, 37 millimeter square window, 20 millimeter square area of

core the product is only 755. So, this is not suitable for us if we come back again to the next core this is E 16 85. So, let me go to the next size of the core 16 this also has a area product which is less than what we want we had selected the core, which is E 25.4. So, let me look at that particular core will see that this is the one can see that it has core area of 38.8 mm square, which is this quantity and a window area of 80 mm square. The product of that is 3056 and this is a suitable core and that is what we select go back again. So, this core has dimensions 25.4 from N to N two such is if you put it will have 9.7 and 9.7 roughly 20 millimeter. So, those is the core and if you go to the let me expand the size.

(Refer Slide Time: 33:57)



You can see that how big this core is and what are the various dimensions and when we go to the next page you can see that there are suitable format switch or to be mounted on, which you can put the winding. So, these are in general the core details which will be useful for you to select the suitable core.

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Switched Mode Power Conversion

No. of Turns

Type Number	$A_c \text{ mm}^2$	$A_w \text{ mm}^2$	$A_c A_w \text{ mm}^4$
E25.4/10.7	38.2	80.0	3056.0
L	2.00E-05		
I	5		
N	13		

$$N = \frac{LI}{B_m A_c} = \frac{20 \times 10^{-6} \times 5}{0.2 \times 38.2 \times 10^{-6}}$$

Select 13 Turns

So, after selecting this core let us go back go further with the design you can see that this is a core that has been decide our required inductance is 20 micro Henry. The current is a 5 amperes and the relationship for the number of the turn is $L I$ by $B_m A_c$ L is 20 micro Henry I is 5 and d_m is 0.2 Tesla and A_c for the selected core is 38.2 when we calculate this quantity the number of turns is 13 turns. So, it can be arrange in the form of a spread sheet were this 13 is the number of turns calculated as per this formula with all the quantities that are here.

(Refer Slide Time: 35:17)



Switched Mode Power Conversion

Wire Size

Type Number	$A_c \text{ mm}^2$	$A_w \text{ mm}^2$	$A_c A_w \text{ mm}^4$
E25.4/10.7	38.2	80.0	3056.0
L	2.00E-05		
I	5		
N	13		
$a_w \text{ mm}^2$	1.67		

$$a_w = \frac{I}{J} = \frac{5}{3} = 1.67 \text{ mm}^2$$

Select a_w 1.67 mm² which is 16 SWG

So, we might select 13 turns for this inductor and if we go to the next step we would like to know how big the conductor size has to be. We know that the current density is 3 ampere per millimeter square for a 5 ampere current this will require a conductor whose cross section is 1.67 millimeter square. Current divided by a low able current density 5 divided by 3, which is 1.67 millimeter square and this size of wire is available with 16 S W standard wire gage wire. So, if we use a 16 standard wire gage 13 turns that will satisfy the requirements.

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Switched Mode Power Conversion

Air Gap

Type Number	A_c, mm^2	A_w, mm^2	$A_c \cdot A_w, \text{mm}^4$
E25.4 10/7	38.2	80.0	3056.0
L	2.00E-05		
I	5		
N	13		
a_w, mm^2	1.67		
l_g, mm	0.41		

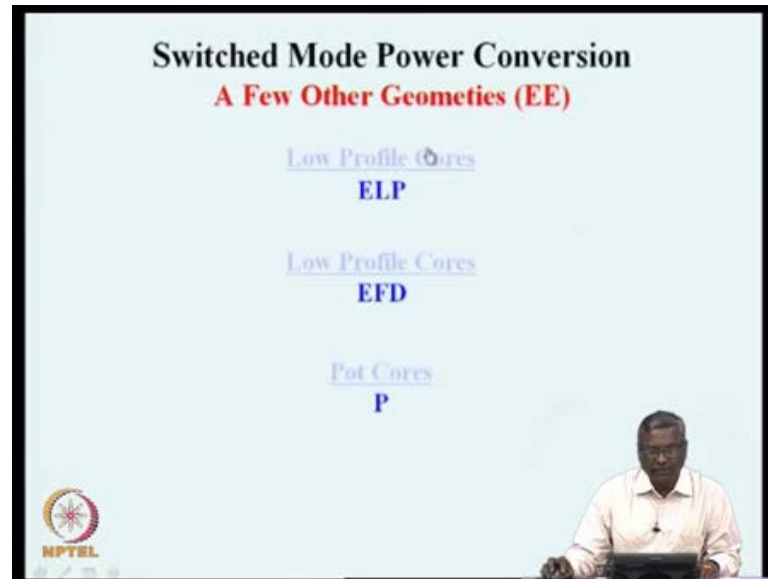
$$\hat{\rho}_s = \frac{N^2 A \mu}{L}$$

Select Airgap of 0.205 mm on each limb



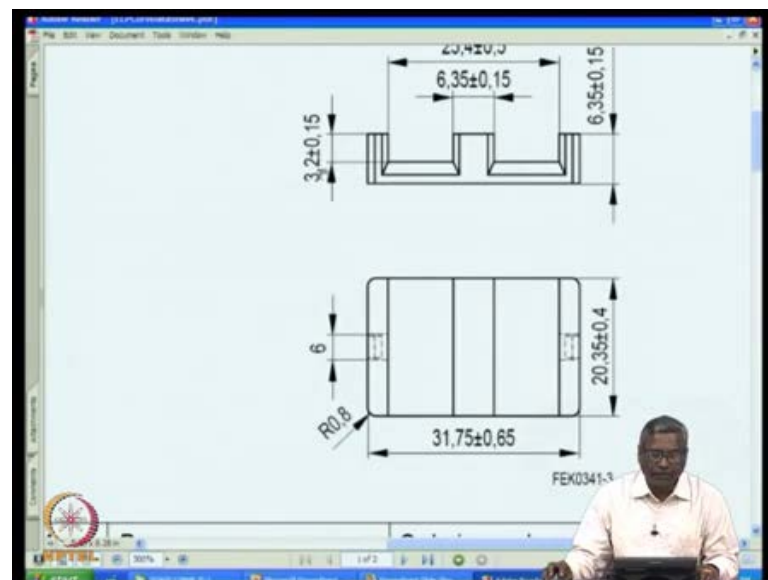
We further go and we need now what should be the gap air gap to be put in the a core and that is calculated by N square a mu by L what we have seen the inductions is N square by reluctance. So, on the reluctance is predominantly destroyed by the air gap so you can say that air gap is N square by this quantity and if you calculate this from the same spreadsheet it is 0.41 mm so in the E there are two gaps both extremes. So, whatever is the calculated one half of the value is the air gap to be employed everywhere in the core so we would select an air gap which is 0.205 mm on each limb.

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There are many other geometries of course, available for example this is another geometry, which is known as low-profile geometry.

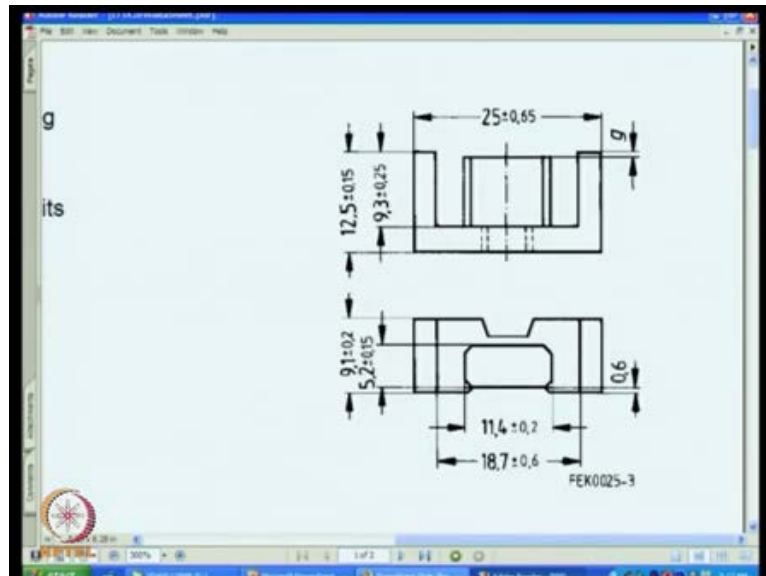
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You can see that this also has the same shape as E I shape, but the total height is only about 6 millimeter from this end to this end. So, when you put two such cores one above the other the total height will be only 13 millimeter roughly half an inch. So, your entire magnetic circuit now has a height of half an inch, which is a low profile such course are

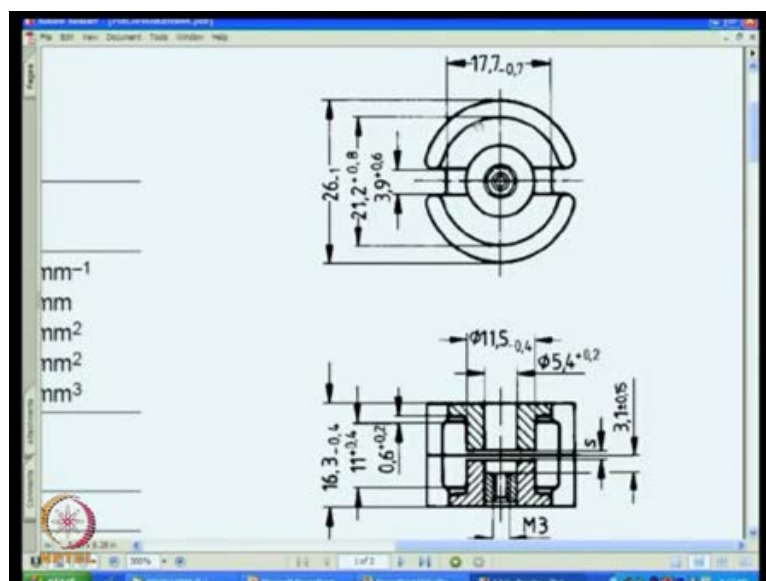
known as low profile course. And if we go back again here there are other types of low profile course which also has a smaller height, but a little bigger than the previous one.

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If you see the size of this, this course are called E F D type of course, where the coil is ranged at this point and there are two E cores one above the other. If you can put you will be able to get the required magnetic circuit, but the magnetic circuit is now ranged in the circuit with height in this direction so that the overall height is only 9.1 millimeter.

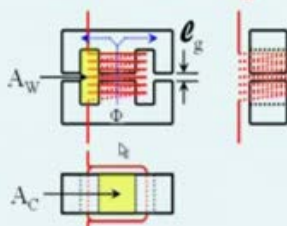
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So, this is also a low profile type of core there are other core which are known as part core this is in the shape of a part will see this here on the shape of part two cylindrical hollow cylindrical things put one above the other. These core are very advantageous because it has magnetic material all round so the leakage flux into the space outside will become very small. So, such part cores without any air gap will let out very little of energy and as radiated energy. So, such core is very advantageous where you wish to have very little radiated energy out of the inductor.

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Switched Mode Power Conversion
Back to the Design



$L = 20 \mu\text{H}$; DC Current of 5 A;
 13 Turns of 16 SWG
 Core Type E25.4/10/7
 Airgap 0.205 mm x 2

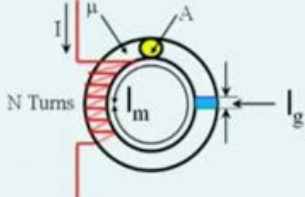
MPTEL

There were many other geometries available from many magnet many manufacturers so on. It come back again to our design so this particular design requires inductance of 20 off to design an inductance of 20 micro Henry carrying of current of 5 amperes. We need to put 13 turns of 16 gage wire on a core type, which is E 25.4 by 10 by 7 with an air gap of 0.205 mm everywhere here, here, here.


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Switched Mode Power Conversion

Practical Design of an Inductor


$$L = \frac{N\Phi}{I} = \frac{N^2}{\left(\frac{l_m}{A\mu_o\mu_m} + \frac{l_g}{A\mu_o}\right)} \cong \frac{N^2 A \mu_o}{l_g}$$

Inductance is Independent of Core Material
Inductance is Independent of Core Shape

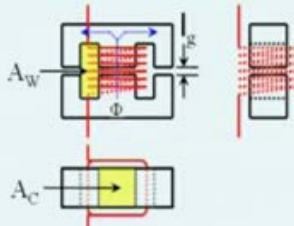


So, such an inductor will give you the inductions that you want ideally, but in practice the practical design has a few non idealities. For example, we had seen that the design consists of an approximation where we assume that the reluctance of the magnetic path is negligible compared to the reluctance of the gap. This will be true if the reluctance of the gap is really large compared to this the inductions is independent of the core material. We had said mainly because this is negligible, but if this is not negligible in comparison with this term there will be an error in our calculation and your calculated value of inductance will be different from the measured value of inductors.


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Switched Mode Power Conversion

Back to the Design



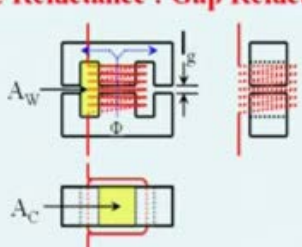
Assumptions:
Core Reluctance \ll Gap Reluctance
Fringing Field at Gap is Negligible



The assumption was that core reluctance was much less compared to gap reluctance. Another assumption that we made was the flux was only in the gap and there was nothing leaking on either side of the gap that is known as fringing effect in re assume that there is no fringing field at the gaps.


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Switched Mode Power Conversion
Core Reluctance : Gap Reluctance



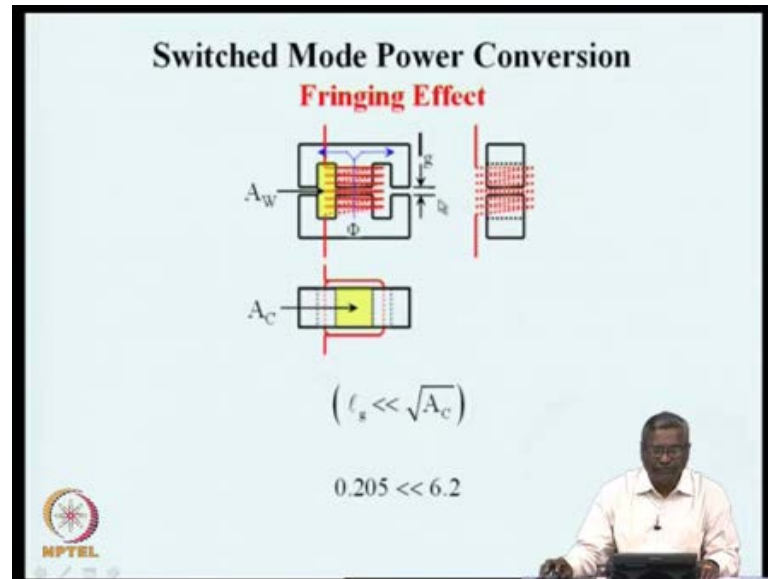
$$\left(\frac{l_m}{\mu_r} \ll l_g \right)$$

$$\frac{47.2}{1510} = 0.031 \ll 0.205$$



Now, this two can be ensured by looking at certain in equalities for example, we had said in the reluctance term the first term L_m by μ_r must be very much less than l_g . In the core that we have selected the magnetic length is 47 mm reluctance is 1510 millimeter. So, this ratio is 0.031 and in comparison with the gap length this is a much small is about 1/8 and so we might say that a the designs or dissatisfactory. If you use alternate material, which has about 4000 has μ_r then this becomes really negligible compare to the gap which is 0.205. So, the core reluctance has to be very much less than the gap reluctance the way to cross check that is whether l_m by μ_r the relative permeability of the material is very much less than the gap that we have calculated.

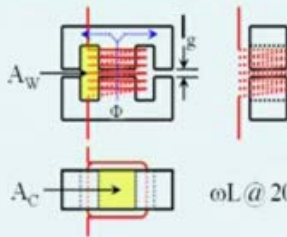
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The next one is the fringing effect where there are no fields on the sides of the gap. Now, this will be ensured if the gap length is very, very small compared to the width of the gap. The width of the gap is approximately square root of the cross section of the gap cross section of the magnetic path and the gap in our case is 0.205 and the cross section was 38 millimeter square. If you square take a root of that it is 6.2 mm so we see that definitely 6.2 is almost 30 times less than the dimensions along the normal to the gap. So, this inequality is certainly satisfied and fringing will definitely be negligible in this design. So, this is an inequality which one has to test as soon as the design is complete.

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Switched Mode Power Conversion
Parasitic Resistance of Inductor



$\omega L @ 20\text{kHz is } 2.51\Omega$

$$R_l = \frac{\rho L_w}{a_w}$$
$$R_l = \frac{1.76 * 10^{-6} * 13 * 40 * 10^{-1}}{1.67 * 10^{-2}} \Omega = 5.5\text{m}\Omega$$


Now, in this particular case or as in any other inductance design what we design is and inductance of 20 micro Henry, but because we have put a conductor this also will have a resistance. This resistance is known as the parasitic resistance of the inductor and that value can be calculated if we know the mean length of the turn number of turns and the resistivity of the wire for example, in this case we use copper. Copper has 1.76 micro Ohm centimeters the resistivity 13 number of turns and each turn is 40 millimeter in length we converted into centimeter.

The cross section of the wire is 1.67 millimeter square converted into centimeter square that gives a resistance of the wire at 5.5 mill Ohm. This has to be compared with the inductive reactance at the frequency that we are interested for example, L of 20 micro Henry at 20 kilo hertz excitation will have an impedance of 2.51 Ohm ωL and in relation to that this 5.5 is negligible. So, we can say that this is almost an ideal induction at the frequency that we are considering or the parasitic resistance is negligible because it is less by a factor of almost 500.

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Switched Mode Power Conversion

Losses in the Inductor

0.36 W per set
If the DC Output Voltage is 7.5 V,
This loss is 0.1%.

The next theme on has to look is how much power is lost in the inductor this is given from the data sheet. The data sheet also will give you for the core that we have selected what will be the losses at the given operating frequency per set for each of the cores set. In this case 0.36 watt is the loss in that and if we have use this in a power supply in a power conversion were we are interested we are concerned about say 7.5 volts power converter current being 5 ampere. So, 5 into 7.5 are about 40 watts in comparison with that 40 watts this 0.36 watts is about 0.1 percent. So, definitely this is a very, very small quantity off a loss and this could be a could be allowed.

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Switched Mode Power Conversion

Measurement of L

$$V = L \frac{dI}{dt}$$

Pulsed Voltage and Current Rise

Whenever we make this inductor it is necessary also to measure them how do we measure the inductor. In this particular case it is possible to make measurement after having built the inductor one could measure it by exploiting the same relationship that we know. The inductor is defined by the relationship between voltage and the current through V is equal to L multiplied by dI/dt . So, if we can set up a circuit to apply a certain voltage a square wave type of voltage to this inductor. Whenever a positive voltage is apply to the inductor the current through the inductor will increase because in this equation a constant voltage will result in a constant dI/dt for a given inductance and that rate of rise of current is V/L .

So, during the positive voltage region the inductor and the current will increase linearly with the slope of V/L . Then following that if we apply a negative voltage during that region the current will now fall with the same rate of rise, which is minus V/L . So, if we taken inductor which is to be measured it is possible to apply voltage, which is of square wave in nature just one pulse measure the current and find out the slope. And from this slow and the applied voltage it is possible to evaluate what is the value of inductance.

This is an important a test because after having design the inductor we would find that we have the core reluctance is a negligible compared to the gap reluctance. We will find that the fringing is negligible and so on and practically the calculated value and the a actual value have to be very close to each other. This test will clearly tell us how close is the realized the inductance in comparison with the design inductance. This is called a pulsed voltage test are a pulse voltage and the measurement of the rate of rise of current.

This is one very reliable way of measuring inductance and this is also very inexpensive way of measuring inductance unlike many a equipment, which are needed to measure. This is a very inexpensive way in the lab you can set up a small circuit in a in order to inject a positive and negative voltage to the inductor put a current probe and measure it using an oscilloscope. How this current is rising and falling measure the rate of rise of this current related to this voltage and evaluates the value of inductance as very very simple test and very accurate test. If a you have a very good current pro and if you have a good measurement for this voltage there is nothing can that go wrong and the sole circuit can be put up very inexpensively in the laboratory.

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The slide content is as follows:

Switched Mode Power Conversion
Measurement of L with LCR Meter

$$\vec{I} = \frac{\vec{V}}{j\omega L}$$

$$|\vec{V}| = \omega L |\vec{I}|$$

Sinusoidal Voltage and Current

NPTEL

There is another way of measuring the inductance using an LCR meter. This is a more expensive way; you have to have an LCR meter, a meter which can measure inductance, capacitance, and resistance. How these meters measure the value is to follow the steady-state performance for a sinusoidal excitation of the inductance. The same equation $V = L \frac{dI}{dt}$ for discrete or a transient current can be returned for sinusoidal excitation as the current. Sinusoid is related to the voltage sinusoid through the complex impedance of the inductor. The impedance of the inductor has a magnitude of ωL and a phase relationship of 90 degrees.

So, in this particular case if I apply a voltage across this inductor, which is the reference phase, then the current in the inductor will be lagging this reference phase by 90 degrees on account of the nature of the impedance. The magnitude of this current and the magnitude of this voltage are related by the impedance of this inductor, which is ωL . Impedance is a function of ω ; it is also related to L , and you find that the ratio of V and I is ωL . And in an LCR meter, sinusoidal voltage of a given frequency is supplied to the coil, and the current that is flowing through it is measured internally, and in between, these two values are related directly.

You will get your display of what is the inductance value in Henry. Many times this test can be carried out at any frequency that you want at low frequency like 10 hertz or 100 hertz, or at high frequency such as 100 kilohertz or 1 megahertz. Many LCR meters will

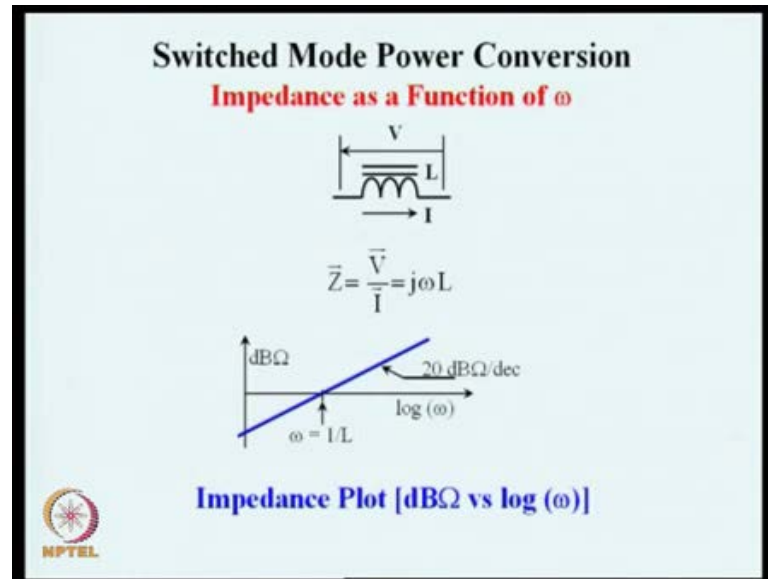
give you a fairly wide range of frequencies at which you can make this measurement and on account of that it is fairly expensive also. Another important limitation of measuring inductance with LCR meter is that the current with, which the coil is excited are the voltage with which the coil is excited is normally very small.

(Refer Slide Time: 50:24)

The slide is titled "Switched Mode Power Conversion" and "Measurement of L". It features a circuit diagram of an inductor with voltage V and current I . Below the diagram is the equation $V = L \frac{dI}{dt}$. A graph shows a pulsed voltage $V(t)$ (red line) and the resulting current $I(t)$ (blue line) over time t . The current rises linearly during the voltage pulse. The NPTEL logo is visible in the bottom left corner.

So, the inductor will not be tested at its rated current, but it will be tested at a very low current on account of that the inductance measured with an LCR meter is not as reliable as the inductance measured by this direct V and dI by dt method. So, far per electronic applications normally the first method is preferred you might use use an LCR meter just to cross check the number so they are ok, but the result obtained by the pulsed voltage and current rise test is a more reliable test.

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There is another way of looking at inductor also as impedance, which is a function of the frequency for example, the relationship V equal to $L \frac{dI}{dt}$ is the transient voltage and current equation. The current vector is V divided by $j\omega L$ is the steady state voltage current relationship for sinusoidal excitation for any inductor. We might decide we might define the impedance of this inductor as the ratio of the voltage vector and the current vector and it is a complex quantity, which is $j\omega L$. If inductance value is L and if we are considering a frequency of interest, which is ω then this impedance is $j\omega L$ were these ωL is the operating part of the inductance, which will give the ratio of the magnetic magnitude of the voltage to the magnitude of the current and this j is related to the phase displacement.

So, if you apply voltage at certain phase relationship the resultant current will be behind the voltage by 90 degrees. That is what is j operator is and if we only consider the magnitudes of the voltage and magnitude of the current we can say that the impedance as a magnitude has the magnitude of this or a magnitude of $j\omega L$, which is nothing but ωL . Now, it is a one way of representing this as here dB ohm impedance in the units of dB ohm because impedance has a unit of Ohm and normally dB is defined for ratio as we might define a unit called a dB ohm, which is 20 times log to the base of the magnitude of Z .

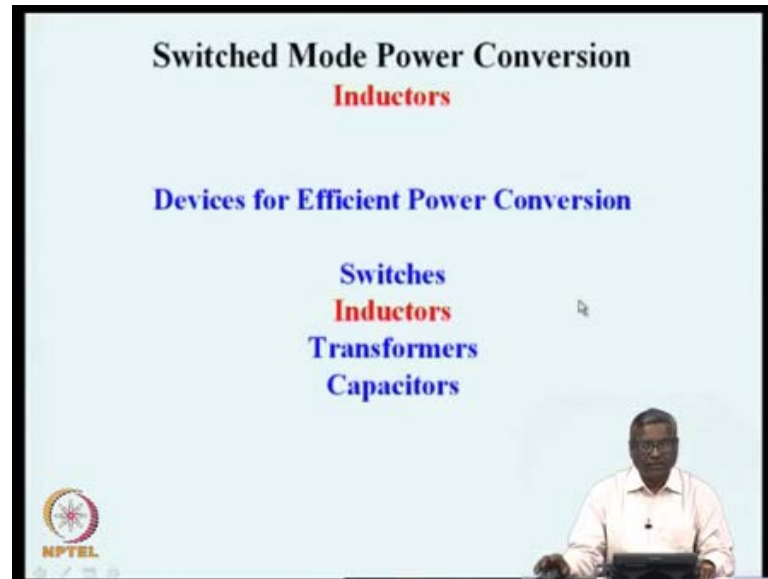
Now, we will see that that is nothing but 20 times log of ωL and if we take it in the decibel scale you will find that as ω is increasing in magnitude. In this particular plot the ω is taken in the x axis to a logarithmic scale. A ω increases by a factor of 10 say from 10 hertz to 100 hertz the dB value will increase by a factor of $20 \log_{10} 10$, which is 20 dB. So, if you plotted in this scale dB Ohm and $\log \omega$ the impedance of an inductor will have a positive slope of 20 dB Ohm per decade of frequency it will be a straight line whose slope is defined by that and it will cross over the 0 dB line 0 dB is corresponding to 1 Ohm.

So, 1 Ohm is ωL product is 1, which means ω will be equal to $1/L$ so that is what you see the 0 dB value of this ωL value will be at a frequency of $1/L$. So, any inductance you can recognize in the impedance plot as a line straight line with a positive slope of 20 dB crossing at 0 dB line at $1/L$. This is a very, very simple test if you have a network analyzer and connect the impedance to the inductance to network analyzer and sweep the frequency from minimum to maximum. You will find that the impedance will follow a straight line and where it crosses 0 dB that particular frequency is $1/L$.

This is one important way of measuring an inductance later on we will see that when we try to compare several types of impedances R L C and then compound impedance is like RL, RC, RLC and so on. These impedance graph is a very, very simple and effective way of identifying how a particular circuit element works at different frequency ranges. So, this is also another important way of characterizing an inductor we have seen that you can see the inductor as the transient relationship V is equal to $L \frac{dI}{dt}$. Whenever voltage is applied the current follows V by L slope and when the voltage applied is negative this current is having a negative slope.

Another relationship is sinusoidal excitation and the current, which is lagging the voltage in the magnitude will relate. What this ωL is? And if you know the frequency of excitation it is possible to find out the value of L . Another way of doing that is two from a network analyzer evaluate the impedance find out at what point you have 0 dB value and that is an inverse inverse of inductance.

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What we have seen in the past one hour or about inductors how they are seen as a circuit element. How they are defined how they are specified value of inductance and the current through the inductor and how the electrical quantities of voltage across the inductor, current through the inductor and the inductance value or related to each other. Through the transient relationship $V = L \frac{dI}{dt}$ and then how inductors store energy and this stored energy is in the magnetic field, which is in the magnetic circuit of the inductor. The magnetic circuit itself has a magnetic material with certain permeability, which has certain magnetic length and certain area of cross section across, which the magnetic field is established.

We saw the concepts of reluctance flux linkage, the definition of the inductance as flux linkage per ampere and so on. We saw the practical limit such as saturation limits thermal limit and so on and how to put all these things together in the form of in the way of designing an inductor and we saw a sample inductor design. We had also seen the nonlinearities, which are the core reluctance in comparison with the gap reluctance the fringing effect how to take care of this effect. So, that your design will be close to what you want we have seen how the inductors are measured and the different ways of measuring the inductors and so on. In the next lecture probably we will see the other components which are transformers capacitors and so on, and once all the devices are covered when we will go on to power conversion circuits and so on.