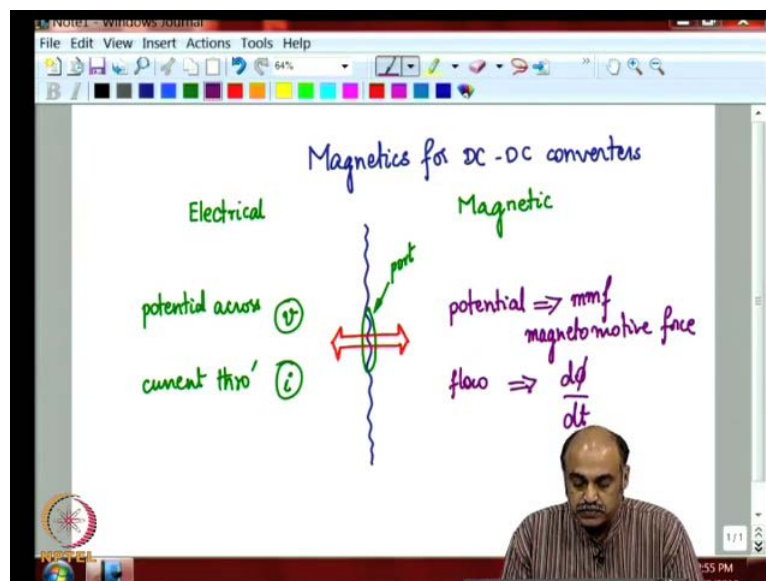


Switched Mode Power Conversion
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Lecture - 39
Magnetic Design

Good day to all of you. Today, we shall discuss on one more important aspect of DC-DC converter, which is the magnetic components. Recall in the last class that we mentioned that to design DC-DC converter, three main components and the selection are important. One being the power semiconductor switch, the other one is the capacitor and third is the magnetic component, both the inductor and the transformer incase of isolated DC-DC converters. So, today the objective is that we try to understand something about the magnetic components, how we would go about designing this inductors and transformers such that it can be included in a practical DC-DC converter.

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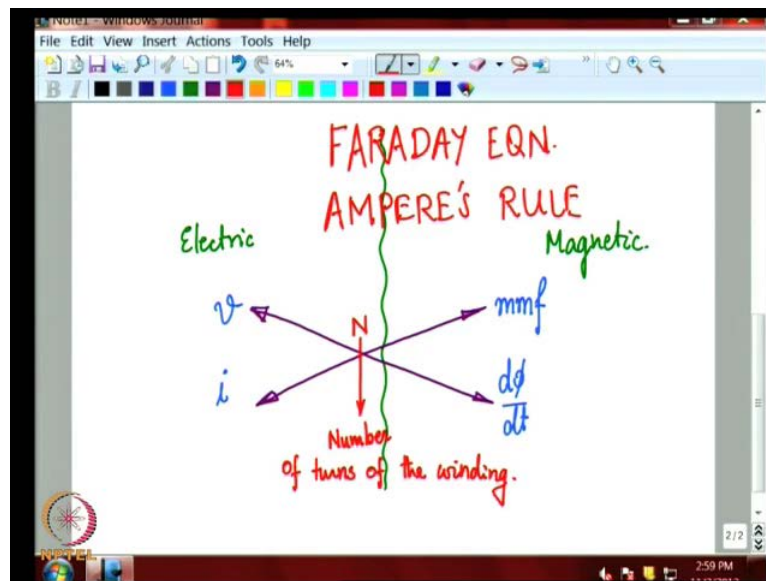
So, the magnetics have always been a domain, which is, which has been a pretty much difficult to understand and implement. However, once a design is performed with magnetic components, the magnetic component being passive, they are very, very robust. Nothing happens to them unlike the power semiconductor switches or the capacitors. Both of them blow once the rating is exceeded. In a magnetic component, the only rating

that can be exceeded is the currents and the saturation levels which will cause other components to get damaged, not it.

So, let us see the magnetic components from two viewpoints. One view point is the electrical and the other is the magnetic domain. So, these two are tightly interfaced and they interact through a window through which there is energy transaction. So, let say we call this as the energy port or the power port. So, this is the port. It is like all ports, electrical ports.

It has 2 conductor points, 2 wires and 2 variables. So, on the electrical side, we have v the voltage and i the current variables. Now, v is the potential across the port and i is the current through the leads of the port wire. Among the other side, you have a potential variable and a flow variable. The potential variable is called mmf, the magneto motive force. The flow variable is called $d\phi$ by $d t$.

(Refer Slide Time: 05:34)



Now, these domains are linked together by 2 very powerful and fundamental laws, the Faraday's equation. How so? How? There is also the ampere's rule or equation. Now, these two important rules and principles govern the entire interaction between the electric domain parameters and the magnetic domain parameters. So, let us say we have the potential variable v voltage across the port and i is the current to the port. You have mmf magnetic motive force, the potential variable on the magnetic domain and rate of

change of flux, not flux remember, rate of change of flux. In fact, mmf is flux by reluctance.

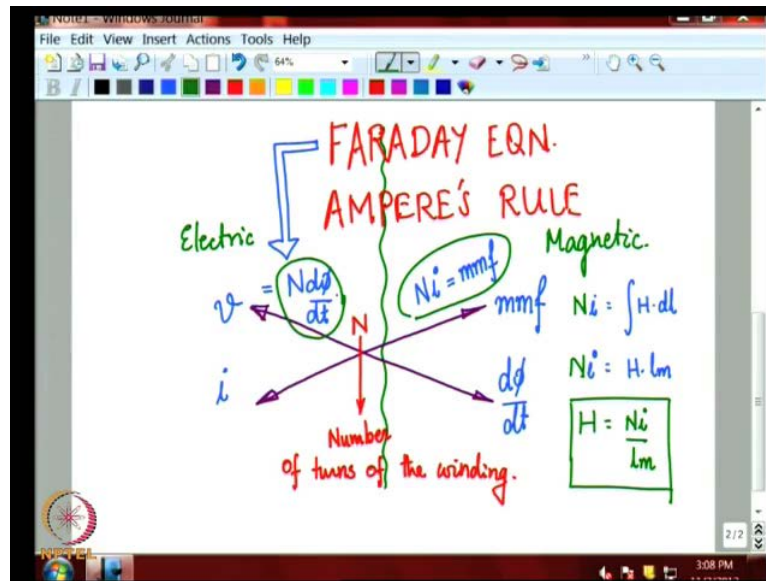
Mmf is flux into reluctance and the flow is the rate of change of flux. Now, how are the variables related? The relationship, the linking is between the cross variables. The potential variable on the electrical side is linked to the flow or the kinetic variable on the magnetic side. The kinetic variable on the electrical side is linked with the potential variable on the magnetic side. Both these linking happen, take place with the parameter N . What is this N ? N is nothing but the number of turns of the winding. So, to make sense out of what we have discussed till now, let us see how practical magnetic components look like and try to understand what it is. Now, here I have a piece of magnetic component. Observe this.

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This is the core. This is a material. Here, in between, you see the bobbin. On the bobbin, you see the windings. These are enameled copper windings. You can remove the core. The core is now split into 2 half, the bobbin along with the copper windings and the removed copper winding is having 2 terminals here. Now, we have 2 half of the core. Now, this copper current carrying copper conductor which has N turns and this N turns is wound on to, is wound or around the magnetic core material. So, let us get back to this later on, but from now assume that we are able to wind the copper coils as shown. That is the N number of turns, N that we have been talking about.

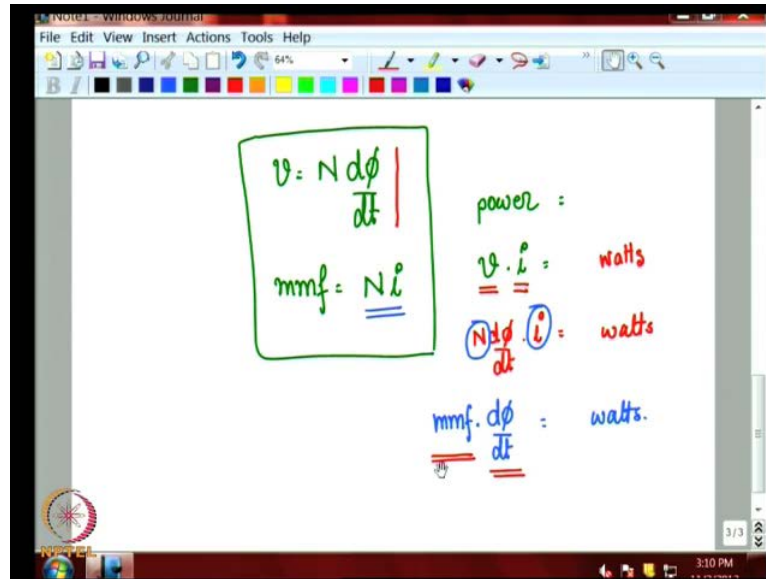
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So, now, coming back to the white board; so N is number of turns, copper turns which are wound on the core, magnetic core. How are these related? So, v is equal to N times $d\phi$ by dt . This is called faraday's law. The voltage across the winding is equal to N times the rate of change of flux within the core. That is the first. Now, to link the mmf and the current, we have the ampere's rule which states Ni . It is the ampere turns are equal to mmf. So, this will come as the result of the ampere's rule. So, the ampere law, by ampere's law, we have i , which is integral of $H \cdot dl$, where l is the magnetic line integrating over the whole line. You would have i , which is $H \cdot lm$.

Now, if there are many turns, the force within the core is accumulating effect of all the turns. Therefore, N comes into the picture. So, it is Ni cumulative effect of all the turns, which is the integral $H \cdot dl$. Therefore, Ni which is equal to $H \cdot lm$ and the force field within the magnetic field within the magnetic core is given by Ni by lm . So, this is as per the ampere's rule. From the ampere's rule, we have these. From faraday's law, we have these. Now, these 2 laws are the most critical principles on which the entire magnetic field in the electrical domain interaction rests.

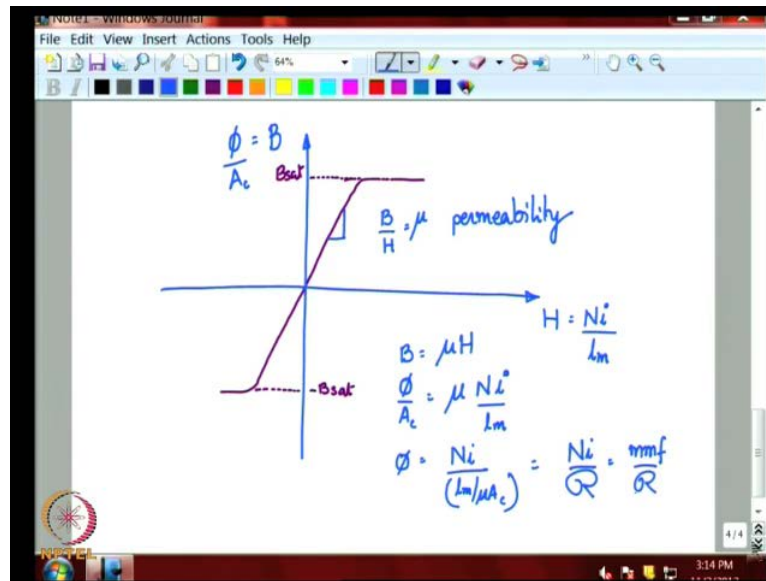
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So, now, v is equal to $N \frac{d\phi}{dt}$. mmf is equal to Ni . These 2 rules are the foundations of which all other magnetic based equations that are used in design. Now, take the example. The power power is v into i and in the magnetic domain. What is it in the electrical domain? You have a potential variable. Multiply the kinetic variable, which is a power watt. Now, let us take faraday's equation. Substitute it for the potential variable v . So, you have $N \frac{d\phi}{dt}$ into i is the power in watts.

Now, from the second equation, mmf is equal to Ni . Let us mind these. You will get $\frac{d\phi}{dt}$ into mmf is again power in watts. You see the power in watts in magnetic domain is also composed of the 2 variable, the potential variable mmf and the kinetic variable of the flow variable $\frac{d\phi}{dt}$ product of which is watts power.

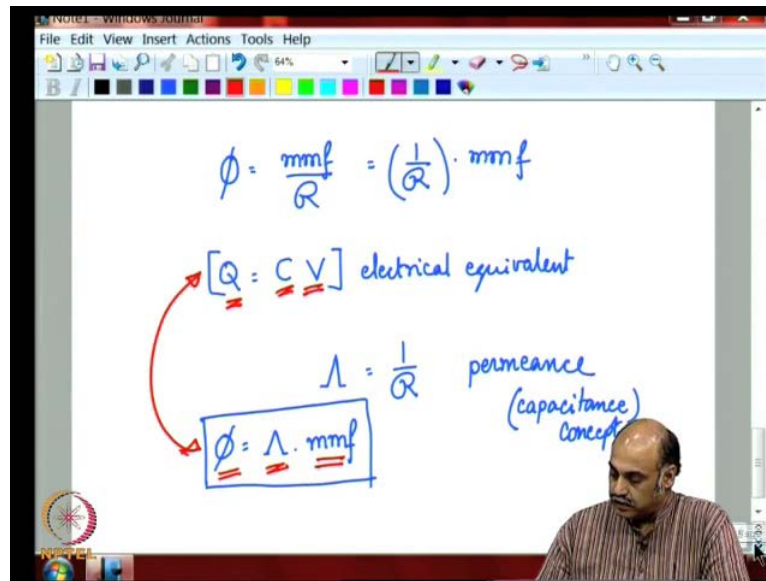
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Now, in the literature, you would see a very important graph B versus H. B is called flux density, which is flux to the core by core cross section area. H, we saw coming from ampere's rule $N i$ by $l m$. Now, this would normally be a straight line like that. But, it is not straight through out after some stage. The flux saturates. There can be no $d \phi$ by $d t$. Not only does it saturate the core, magnetic core has memory; it also demonstrates a clear cut hysteresis curve in this manner. How? What to understand? We shall consider the virgin specimen without the hysteresis, so that you get a clear understanding of what is happening within the core.

So, let us take out the hysteresis portion. Now, this slope B by H is called μ , the permeability, the B which is equal to μH B is nothing but flux by the area cross section of the core $\mu N i$ by $l m$. So, flux which is equal to $N i$ by $l m$ by μA is nothing but $N i$ by reluctance or mmf by reluctance. This is one very important relationship.

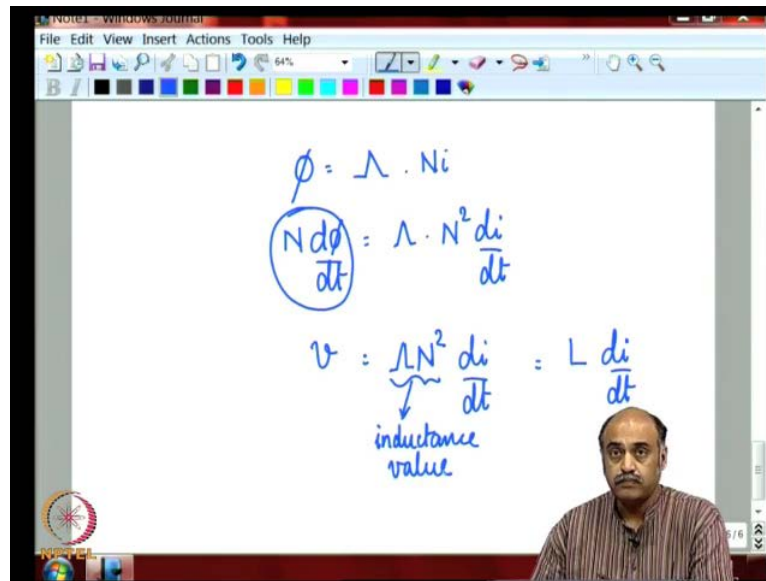
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Flux is equal to mmf by reluctance. In the literature, many a times, it is called ohm's law of the magnetic circuit. However, remember that in the ohm's law, this R is not the R, the dissipative resistance. It is more related to Q is equal to C V. So, this is more related to Q is equal to C V formula rather than I is equal to V by R. Now, this is the electrical equivalent.

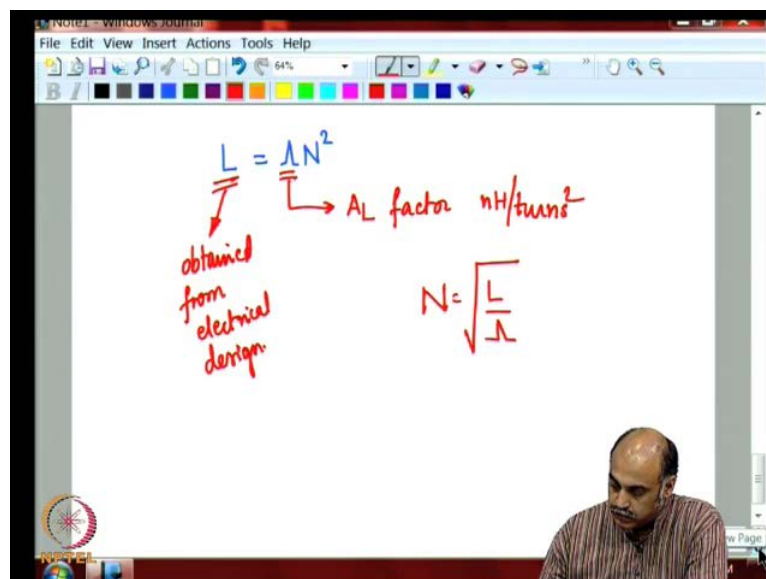
You can put it in that form 1 by reluctance into mmf. Now, what is one by reluctance? I am going to introduce one more term called permeance, which is 1 by reluctance. So, permeance is closer to your equivalent capacitance concept. So, it is phi which is equal to permeance into mmf. Now, compare these 2. Flux is like the charge mmf. The potential permeance is like a capacitance. So, the reluctance is 1 by capacitance; 1 by C V equivalently.

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So, now, let us go further. So, flux is permeance into mmf, which is $N i$. So, $N d \phi$ by $d t$ is multiply both sides by N N square $d i$ by $d t$; and differentiating with respect to time. So, what do you see? You see that this from faraday's law is voltage v and this is equal to something $d i$ by $d t$, which is equal to inductance L into $d i$ by $d t$. So, we define permeance into N square as inductance value.

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So, inductance L is permeance into N square. This is the important relationship because in many of the core catalogs, this permeance value is given as what is called as A_L

factor. It has the unit of nano Henry per turns square. So, this is given in the datasheets in many of the gapped cores or the inductor type cores. This permeance is used to design a number of turns.

Now, L value is obtained from your dc dc converter electrical design based on the amount of current ripple that you want to have the voltage across the inductor in all those things. The L value is obtained from electrical design. Then, N, the number of turns you need to put for the particular core would be L by λ permeance square, square root because this is nano Henry per turns square. Root of that will give you the number of turns of wind that you need to get the value of N. So, this is a very important relationship. We can go one step forward.

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$$L = \lambda N^2$$

$$= \frac{1}{Q} \cdot N^2 = \left(\frac{\mu A_c}{l_m} \right) \cdot N^2$$

$$L = \frac{\mu_r \mu_0 A_c N^2}{l_m}$$

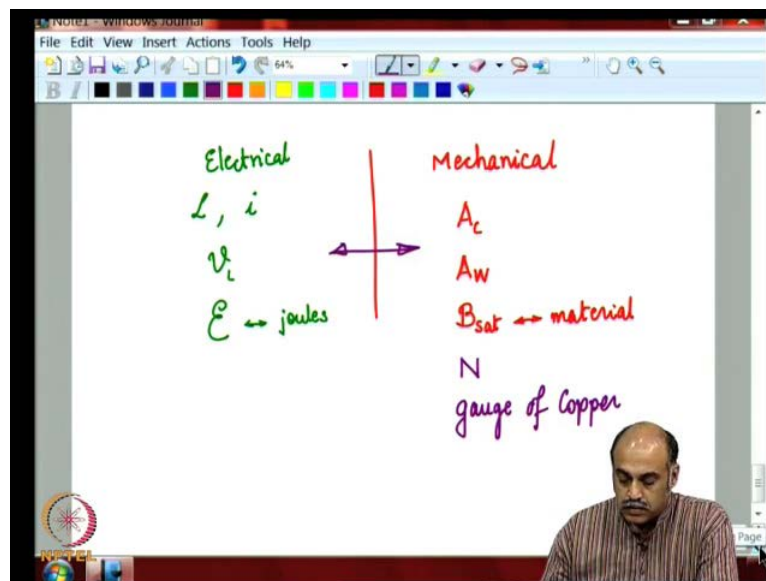
$4\pi \times 10^{-7} \text{ H/m}$

L is equal to permeance into N square, which can be written as 1 by reluctance into N square. 1 by reluctance is $\mu A c$ by $l m$ mean magnetic length into N square. μ is nothing but μ naught and relative permeability, core cross section area, N square by the magnetic length. This is another mutual relationship; in case you are unable to find the value of permeance from the core catalog or in case you are using a transformer core for which do not know the permeance. You are using a transformer core to build an inductance in which you need an air gap; in such cases, this relationship is useful.

This is basically derived from the same relationship, only that you know the play of variables within the play of core cross section area and magnetic length core, which is

given. Relative permeability for core material is known. This is the permeability for free space, which is $4\pi \times 10^{-7}$ Henry per meter. So, in this way also inductance L can be found out, but as the inductance L is obtained from electrical side, L is known, core cross section is known from the datasheets of the core material, μ_r permeability, relative permeability is known from the core material data. I mean magnetic length is known. From that, you can find out N . So, let us see.

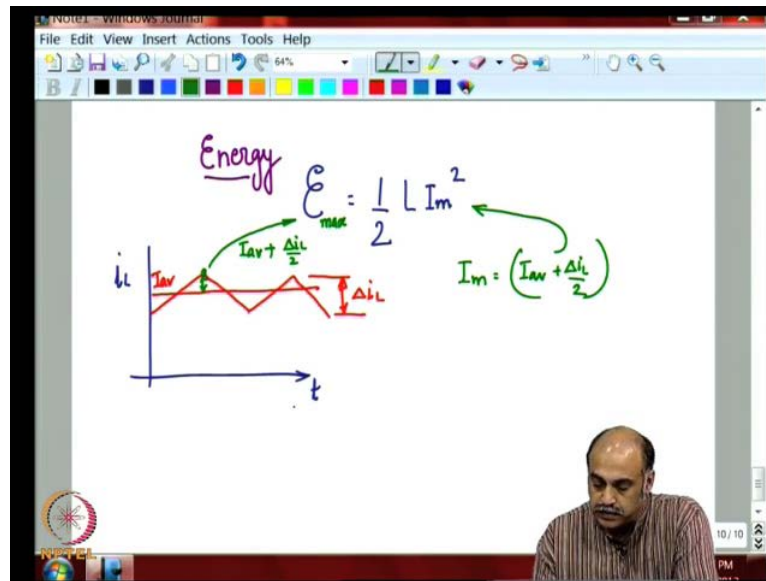
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So, let us see how we now go about finding the inductor relates to the physical parameter in a logical way. So, on the mechanical side, we have what is known as core cross section area. Then, window area you have B_{sat} again coming from the material. On the electrical side, we know how to get the value of the inductance. We know what the current going through the inductance is. We know what the voltage is across the inductance. From here we know what the energy in joules is; which the inductor will have to store.

Now, from the electrical parameters, we need to arrive at the physical parameters such that you can select a physical core for winding the material. Apart from this, we also need know what the number of turns is, what is the gauge of a copper wire? That is the thickness of the wire and the diameter of the wire, which we need to wind. Now, these physical parameters, we have to relate to the electrical parameter.

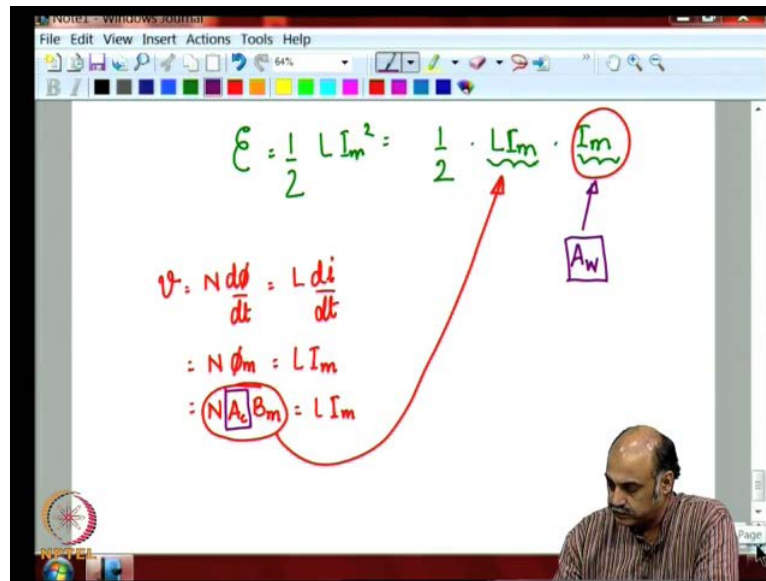
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So, first let us say from the electrical side, we have the energy, which can be easily found out. The energy, the energy E is half $L I_m$ square. This is the maximum energy that the inductor will have to store. What is this I_m ? Typically, in inductor, dc dc converter, if we should plot the inductor current, it is of this form. So, the current rises during $d t$ and falls during $1 - d t$. This is Δi_L .

Let us say that this is most of the times i naught if it is a work converter. If it is a boost converter, it is i_n . So, let us say that this is the average value. So, where is the maximum energy? When the current is maximum, you will see that maximum energy is stored. So, this value is nothing but $I_{average} + \Delta i_L$ by 2 plus this. So, I_m is $I_{average} + \Delta i_L$ by 2 and use that here to calculate the energy. Now, how this is calculated in the electrical side? Let us try to relate this to the physical parameters.

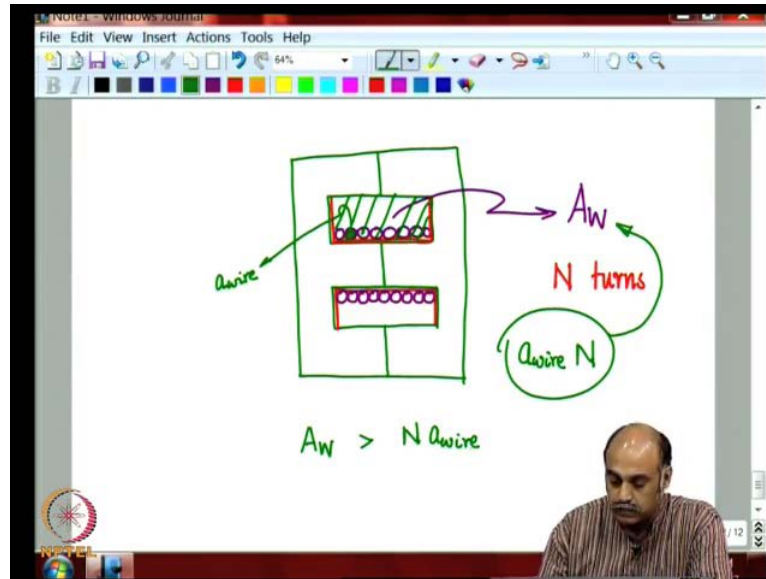
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Let us make a split, \mathcal{E} is equal to half $L I_m^2$ is half $L I_m$ into I_m . Let us make a split. Let us take this from 2 different, from the 2 different rules that is 1 is the flow based and another is the potential based. If you take the Faraday's equation, where we know that v is equal to $N d\phi$ divided by dt , which is equal to $L di$ divided by dt .

As the current is linear, you can as well write it as $N \phi_m$ is equal to $L I_m$. But, in the same time, a given time, if the current has reached I_m , the flux would have reached I_m . $N \phi_m$ is written as $N A_c B_m$. B_m is the flux density that is equal to $L I_m$. So, this we can incorporate here. Let us come to the other aspect. What to do for this? Let us take another physical parameter. Notice, here we are trying to relate to this physical parameter that is core cross sectional area. Let us try to relate to this the other one, the window area. Now, let us see what the window area is.

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If you see, we have this core. Let us take simple ee core which is like this. So, this is ee core. You have 2 ee core joined together. The former comes in here. The red line indicates the former. The red line indicates the former. Also, this is, on the former is wound the windings. So, you will see windings wound in this fashion. You will see the other end of the windings also here.

So, this is how it would appear here. To picture this better, let us go back again and look at the inductor core. So, you look at this core. Let me take out the bobbin. Keep it aside. Then, we have this core. Now, this is what I have been drawing bobbin just now; 2 halves which are joined together. You see here this is one opening and this is another opening.

The wire passes through goes over and comes out through the bottom. How is the former introduced? You take this former and you are introducing it like this. So, you see here a light portion is the former portion and on the former is wound the copper wire. So, if I take a section cut, if I take a section cut, you would see exactly the figure I have drawn there with many windings.

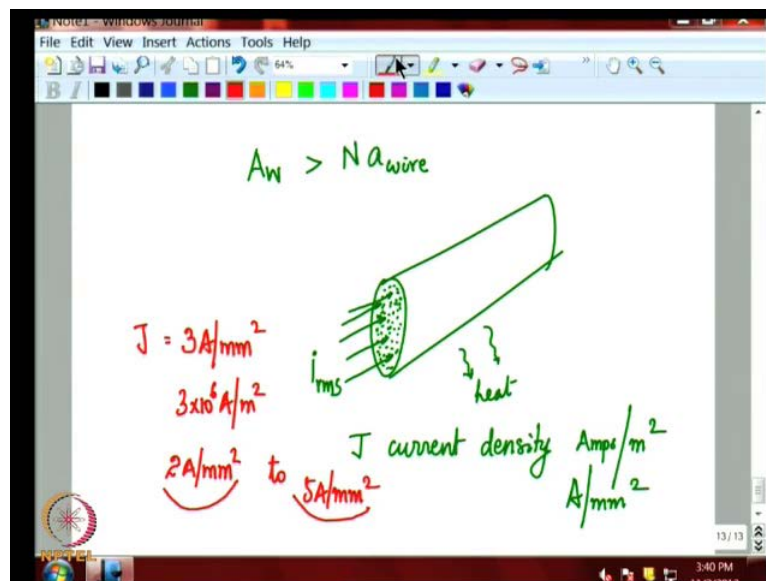
Now, this winding has to fit into this area. That is work. Let me put this together. The entire wire is fit into this area and that is called the window area. This is called the window area because other portion open here, other portion opening here is nothing but carrying other half of the turn. Therefore, in 1 complete turn, half of the turns is sitting in

this opening and other half of the turns is sitting in this opening. So, you consider only you 1 opening as something available to you. So, this window area is called A_w .

Now, we are coming back to the white board. So, this window area is called A_w . Now, within this A_w window area, what should fit? All this N turns should fit. Now, each of the entire N terms has a cross section area. Now, let us say, it has across section area a wire. a wire is decided based on this thickness of the wire. The thickness of the wire is decided based on amount of current that a wire has to carry.

The amount of current on the core is obtained from the electrical circuit side by electrical analysis. So, a wire is the thickness of the; is the cross section area of the wire. N of them should fit into this window area. So, N of them should fit into this area. So, A_w should be greater N into a wire. Now, let us some simple math.

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A_w should be greater than N into a wire. Normally, the amount of current that is passed in in through the copper conductor is based on experience. So, let us say these are current lines through. So, the current i which flows through there is rms component of current. It causes heat to be dissipated in the copper conductor. If the current is varied, then meaning for given area, cross section area, the rms current is very high, then conductor can be very hot and lead to lot of i square lot of losses.

Therefore, there is appropriate amount of current density that is has to be allowed. So, let us call J as the current density. It is usually expressed terms of amps per meter square or amp per mm square. So, normally, an amount of 3 amp per mm square or 3 into 10 amp per meter square is the amount of current density that is consider as nominal.

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$$J = \frac{I_{rms}}{A_{wire}} = \frac{I_m / K_c}{A_{wire}}$$

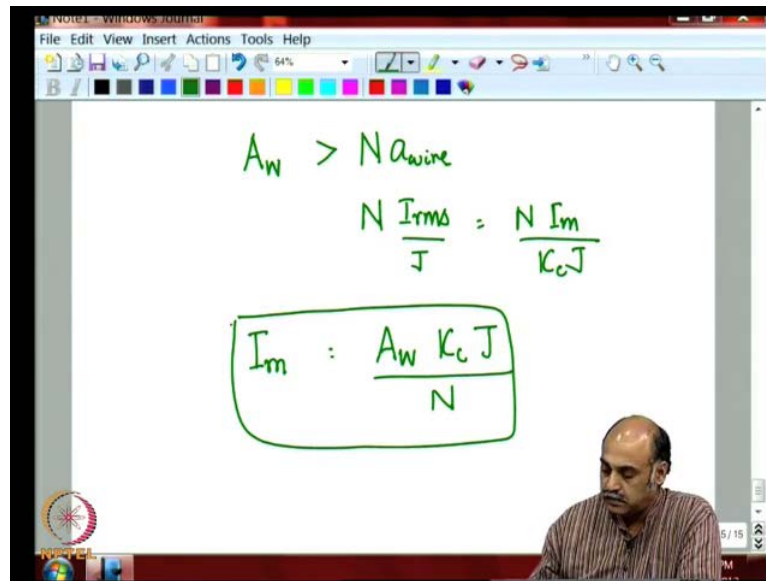
$$A_{wire} = \frac{I_{rms}}{J}$$

$$I_m = K_c J A_{wire}$$

So, it means that when you say 3 amps per mm square for every 1 mm square of this cross section area, 3 amps is allowed to flow through that cross section area to keep that wire reasonably good. However, you will find designers taking a conservative value, 2 amps per mm square very high powers to up to 5 amps per mm square for very low powers. This will lead to copper being cooler. This will lead to copper becoming hotter. You have more I square losses with higher current density.

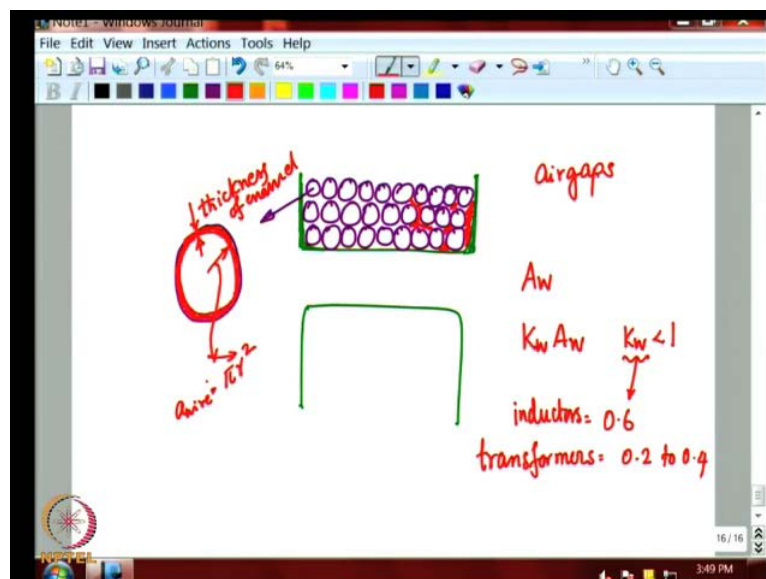
So, J the current density is I rms by the area of cross section of the wire a I rms can also be written in terms of a crust factor I rms is nothing but I m equally by a crust factor by a wire, just put everything in terms of equality. Therefore, the wire cross section area can be calculated using, using this above equation I rms by J. Now, we are coming back to this issue of J and this, let us put in this fashion. I m, there is under feeding to this equation equals J K c a wire. Now, this is this is another interesting relation that may be used. Now, going back to this, we wanted this inequality to be satisfied. What is that inequality?

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Now, you saw window area should be greater than the number of turns times a wire I m. From this discussion, we just concluded, we could say A_w is $N a_{\text{wire}}$, is nothing but I_{rms} by J . It can be written as $N I_m$ by crust factor J . So, let us find I_m as something that is should be equal to $A_w K_c J$ by N . Let us here, there is one more term that we need to add. Keep it in back of the mind one more a practical term.

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We saw that can I take a wire, the bobbin when you start winding, you have a wires cross section, which appears like this. I will consider just only the conclusion. Other portion is

duplicate, other half of the winding. So, let us say, put the layers, layout of the winding of this fashion.

There are many gaps here because the windings are circular cross section. Then, you can see a lot of air gaps here. Also, the thickness of the bobbin it is self, which is eating into the window area space. The thickness of the bobbin, which is to the window area space, the air gaps which into the window area space and this wire itself ;if you take this wire, if flow this cross section is having 2 parts.

One part is out of shield, which is the enamel because the windings are sitting. Next, you do not need, you do not need to short circuits happening between the windings. Therefore, the windings are, the wires are covered with coating with the material, which provides a natural insulation. Therefore, the wire cross section, what we say as a wire is πr^2 , where this r is actually less than the total wire etch radius. The cause of enamel is away, some area.

This is the thickness of enamel, the air gaps, the bobbin itself. The enamel of the wires all eats up into the window area plus there is an additional area issue sitting, which is skill of the winding. That is also eaten into the window area. Now, what is available is not A_w ? What is available is just $K_w A_w$, where K_w is a factor which is less than 1. So, this factor has arrived at typically this around 0.6 for inductors, copper wound inductors and for transformers. It would be anywhere, somewhere. 0.2 to 0.4 would be lesser because it talks for transformer. It is multiple windings. So, there will be a layer of insulation, which is also in each of its same window area. So, what is actually available is $K_w A_w$.

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$$K_w A_w > N A_{wire}$$

$$K_w A_w = \frac{N I_m}{K_c J}$$

$$I_m = \frac{K_w K_c J A_w}{N}$$

If I lead, $K_w A_w$ should be greater than N into a wire, which is copper cross section wire cross section. So, it is basically by using equality $N I_m$ by crust factor J . I_m can be written as $K_w K_c J A_w$ by N . So, here we see that we are relating it could be other window, other area the window area.

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$$E = \frac{1}{2} \cdot L I_m \cdot I_m$$

$$= \frac{1}{2} \cdot N A_c B_m \cdot \frac{K_c K_w A_w J}{N}$$

$$A_w A_c = A_p = \frac{2E}{K_c K_w J B_m}$$

Now, coming back to our energy equations half $L I_m$ into I_m , which is half. Now, this was $N A_c B_m$ into $K_c K_w A_w$ and J by N . So retaining these two physical parameters on one side, you will get A_w into A_c , which is called area product as 2 times energy

value divided by $K_c K_w J B_m$. This is an important equation, an energy obtained from the electrical circuit parameters B_m values from the core material, which is suffer that is 0.3 has saturation. And the operating point may be around 0.25 J 3 ampere meters square; K_w around 0.6, K_c is 1 in this case, and you can calculate the area product. Once you have the area product from the catalog of the cores - core catalog you around through the area product value and pick that area product value, which is just more than calculated. That would be a core that will be used of the inductance. So, at this point, we know how to select the core for the inductance. In the next class, we will look at how we introduce air gaps if necessary and choice of the wire gauges and then see how to modify the design for a transformer.

Thank you.