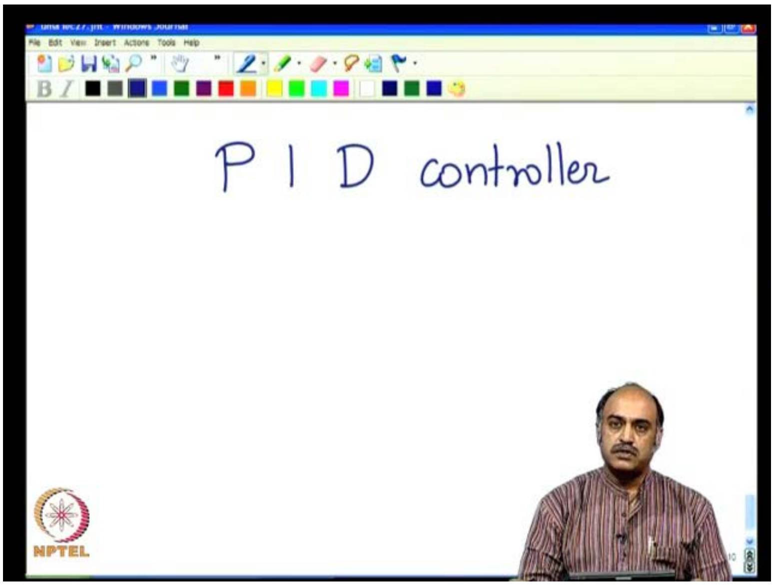
Switched Mode Power Conversion
Prof. L. Umanand
Department of Electronics Systems Engineering
Indian Institute of Science, Bangalore

Lecture - 28
PID Controller-II

Good day to all of you, in the last class we had been discussing about the P I D controller.

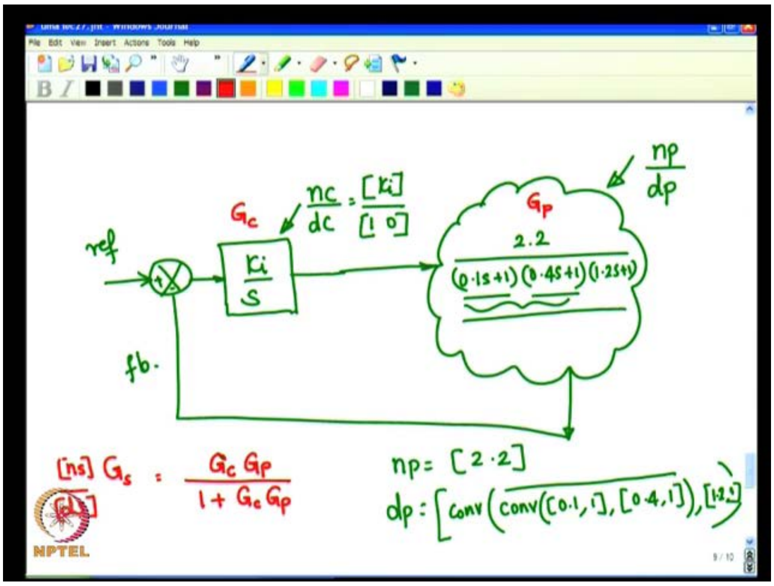
(Refer Slide Time: 00:24)



We had begun our discussion and started with the integrator as a controller and including it into our scheme of the system. We saw in the last class how the integrator behaves and or way it gives infinite gain near the DC or the steady state region. It gives the infinite gain near the DC of the steady state portion of your response the error is 0 at the portion of response near the transients the integrator varies at the higher frequency. The integrator gain is much lower significantly lower and therefore, the error is not 0 in those portions. So, whenever you have a disturbance, whenever there is a change in the input, we saw change in the stepped input and near around the time zone of the step change you will definitely see an error in the output.

As time progresses and system reaches steady state or the DC zone, you will see that the error reduce us to 0 because the gain at DC per the integrator is infinite. So, this fact we try to understand better by trying to implement it in mat lab simulation environment. We will of course use that environment in the discussion today also.

(Refer Slide Time: 02:46)

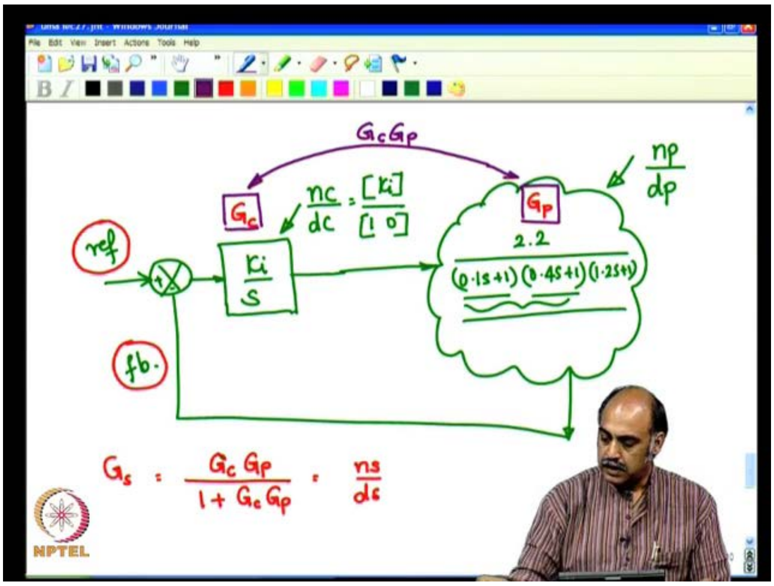


If you look at the example that we have taken in the last class we had an example system, an example system I have shown here this is third order system you have one pole at s is equal to 1 by 0.1 or s is equal to 10 where is another pole at s is equal to 2.5. Another pole at s is equal to 0.8, now this third order system is being controlled by a control input here. This is coming at the output as the output of the controller and the controller that we had been using is just a pure integrator an integrator scaled by a gain k i. So, this is what we try to include into the mat lab environment and in the mat lab environment we included we controller as a numerator polynomial and the denominator polynomial.

So, also the plant as a numerator polynomial and the denominator polynomial we use the convolution function to do the multiplication of these polynomials. We saw the time response of the output where is the one which is fed back the signal is fed back compared with the reference input which was a step input. We also saw the frequency response of this integrator and try to compare the behavior understand the behavior of this entire system.

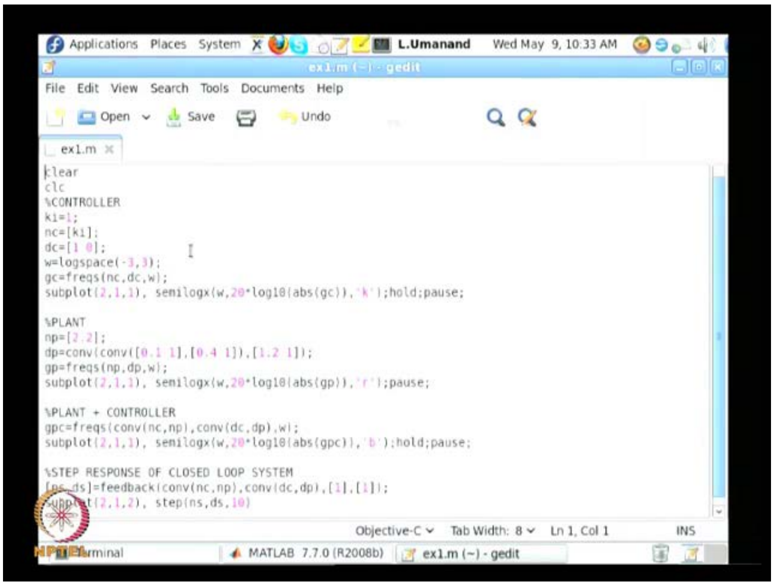
So, we shall continue to proceed in this fashion, we shall call the controller from now on as the G c the controller transfer function as G c and we shall call the plant or the convertor as G p the plant transfer function. So, the close loop transfer function would of course be G c G p by 1 plus G c G p and this we will call it as the system close loop system transfer function G s and this has numerator polynomial and the denominator polynomial n s and d s. So, let us unclutter the screen and just keep these things here let me call this as a numerator polynomial and the denominator polynomial.

(Refer Slide Time: 06:21)



We are going to look at this input step input, we are going to look at the controlled output, and we are also going to look at the frequency response of this. We shall look at the frequency response of the plant alone and we shall look at the combined frequency response of the plant and the controller G c G p together to get an understanding of what is happening. So, this system let us incorporate it into mat lab and see how it behaves, now let me shift the computer screen to the mat lab environment.

(Refer Slide Time: 07:24)



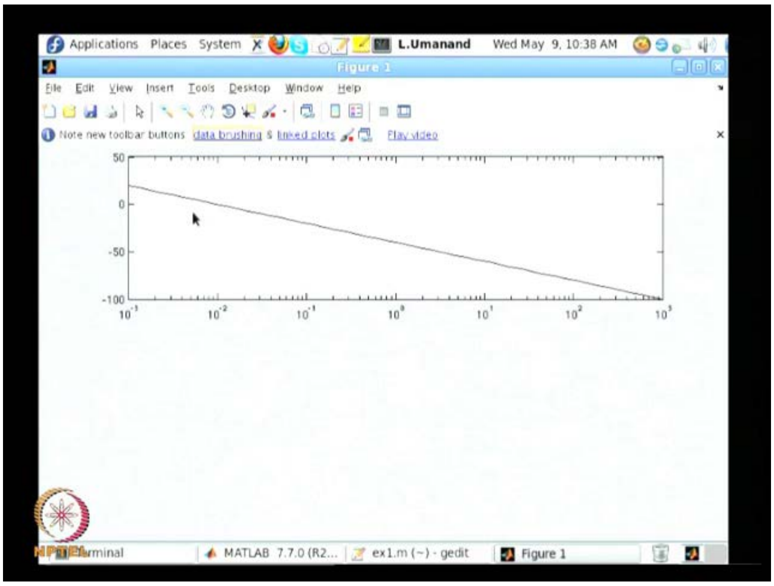
Now, we are on the mat lab environment as you can see here let us take a text editor and write down these things this is small script exactly what we did in the last class, let me mark this and bring your attention. Now, you see here the controller is defined by a numerator polynomial and denominator polynomial the controller. Here is an integrator we also defining the frequency from 10 to the power of minus 3 to 10 to the power of 3.

We are getting the controllers frequency response by giving the parameters the numerator and the denominator polynomials the controller then as a sub plot plotting as semi log plot of the gain verses the frequency. So, this is what we have read we the actually perform in the last class the same thing has now been put in to the script file. Now, we have here; let me bring your attention to this part of the script. It is the script for the plant has the numerator polynomial and the denominator polynomial exactly same as what we did in the last class the plant frequency response. Then, sub plotting it on the same graph see that we are plotting it on the same sub plot and the semi log acts of that one in a different color.

We use a red color here and we are use a black color for the earlier, then we shall combine the plant and the controller transfer function together by using convolution convolving the numerator and the denominator polynomials of the plant. Then, obtaining the frequency response of the plant and controller transfer function together and again plotting it on the same sub plot and we shall give a blue color to that one. So, in sequence we shall plot these frequency responses and then finally, we shall try to extract the close loop response. We are doing close loop feedback of the controller and the plant numerator polynomial controller and the plant denominator polynomial unity feedback numerator and denominator which will give G c G p by 1 plus G c G p.

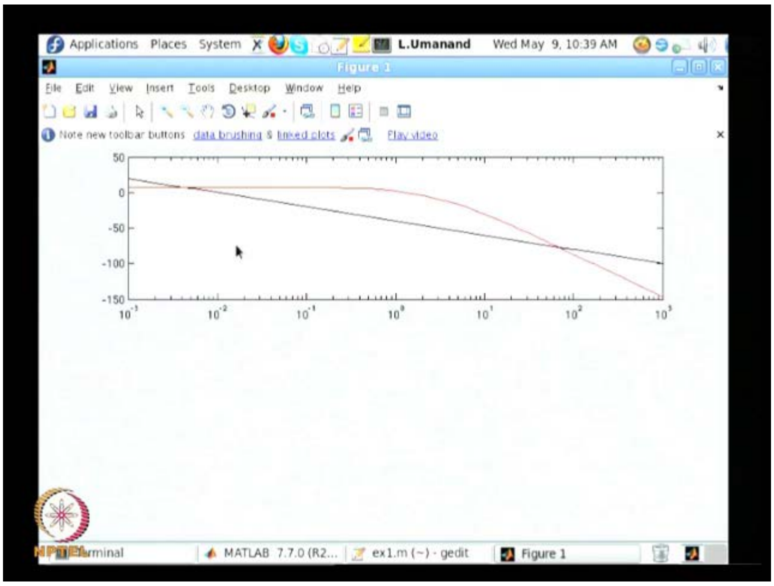
This is the closed loop numerator and denominator polynomials then let us do a sub plot because this is a time plot unlike the frequency plots of the previous ones, this is a time plot, this is verses time and for a step response. Yesterday, we just saw how it was how the integrator just 1 by s was behaving, today let us start with a small value of gain 0.01 and this is small value of integrator gain which we are having.

(Refer Slide Time: 12:10)



We shall run this script as such here in the mat lab work space remembered that we have given the name here as example one e x 1 dot n, so we shall just type e x 1. Let me expand this, so you get a better this is the integrator minus 20 dB per decade, remember that this is not DC, this y axis here is not DC it is not 0 frequency, it is 10 to the power of minus 3. So, as it starts going to the 10 to power of minus 6, 10 to the power of minus 9, so on this value will start hitting a very large value and that in fact is the plus point of the integrator where at DC we want an infinite value to make the error 0 at steady state.

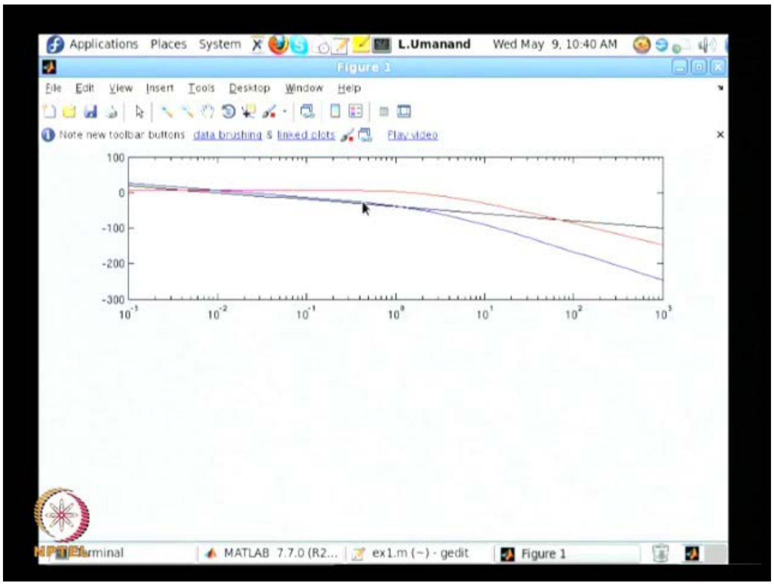
(Refer Slide Time: 13:36)



Now, we shall see the frequency response of the plant in the red color, so this is the frequency response of the plant you see that it is flat of the sub point and then starts behaving like low pass filter and being a third order. Ultimately, you will see that it will start reducing at rapid rate of minus 60 dB per decade, so this is how the plant frequency response looks like.

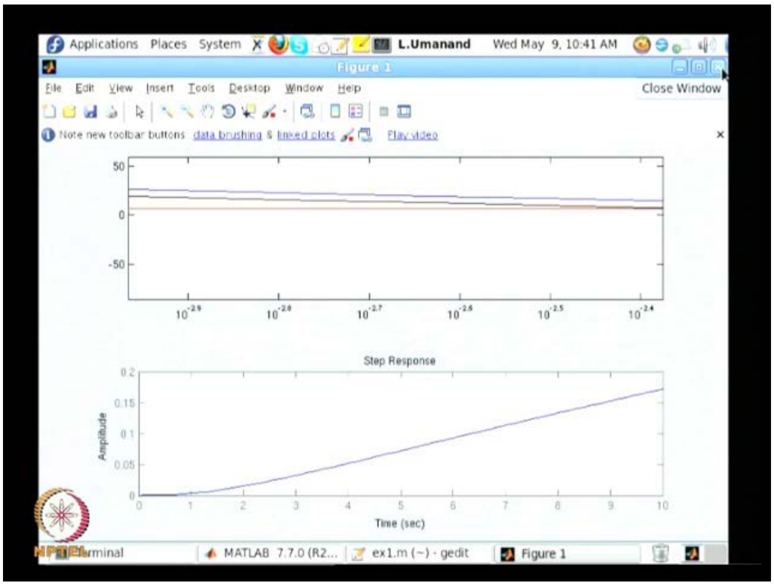
Now, let us see the plant and the controller together, so what would happen is that in the log scale multiplication of G c and G p is nothing but addition of the log dB amplitudes. So, you just have to add the integrator dB gain with the plant dB gain, so you will see that at near around the DC of the steady state the plant gain is pulled up. Now, you see this is zero line and here below zero which means attenuation the plant gain is pulled down, so that is what the modification would happen.

(Refer Slide Time: 15:04)



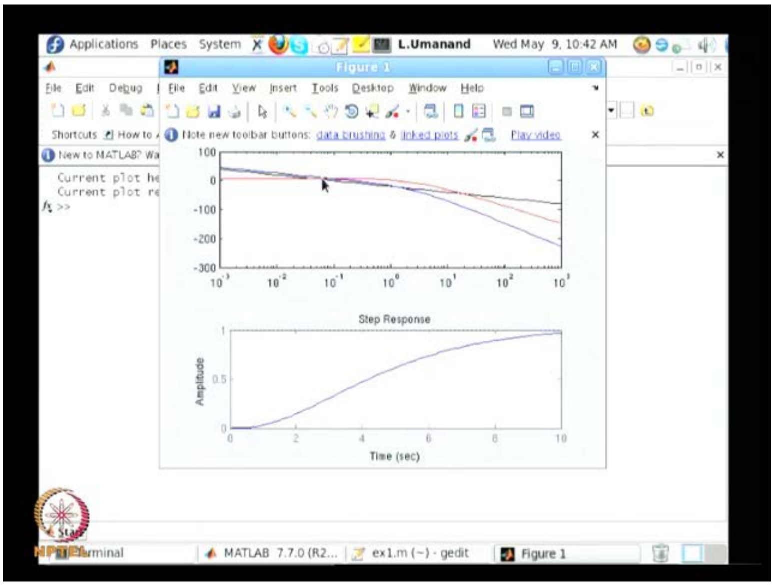
You see the blue line, so the blue line is actually getting pulled down up to this point compared to the red line, the plant line and the blue line goes up the gain is gain as actually increased near the DC.

(Refer Slide Time: 15:28)



So, if you allow me to expand here, you will see that the blue line is actually increased has been pulled up. Then, if we look at the time response you see the time response we have given a very low gain is actually very slow, it is now even reached steady state in time 10 seconds. So, let us try by increasing the gain k i, let us increase the gain k i by a factor of 10 from 0.01 to 0.1 and see what happens.

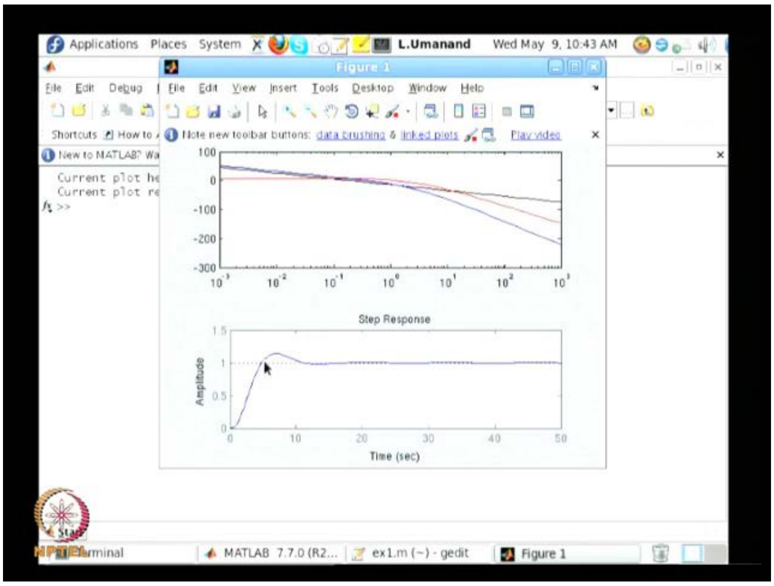
(Refer Slide Time: 16:30)



You will see the time response the integrator response; this is the plant response, frequency response and the response of the system. You see that it is cutting the zero line much further which means the band width has improved. That should get reflected in the time response, you see the time in the time response it is already trying to reach unity at 10 seconds.

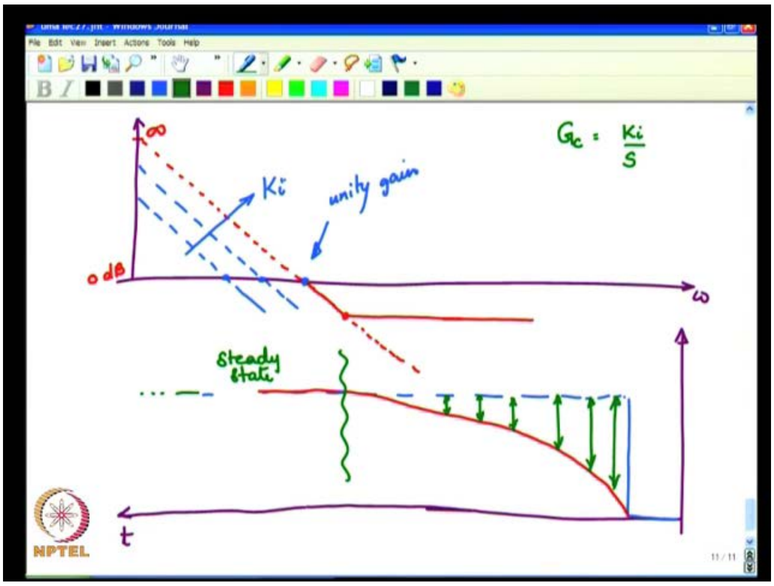
You could further increase the gain, so let us say we double it 0.2 and you will quickly see that this is the integrator, let me expand that the system response this is the system and the controller response together blue line. The time response we have over shot and then it is ultimately trying to stabilize some point later on. So, if you want to view more of the time response you can go and change the end time or the final time, here you can probably say 50 seconds up to 50 seconds.

(Refer Slide Time: 18:12)



You can see the response the time response of the system, so like that one can fine tune the gains of the integrator to achieve better closed loop response. So, what is said that we have done here.

(Refer Slide Time: 18:50)

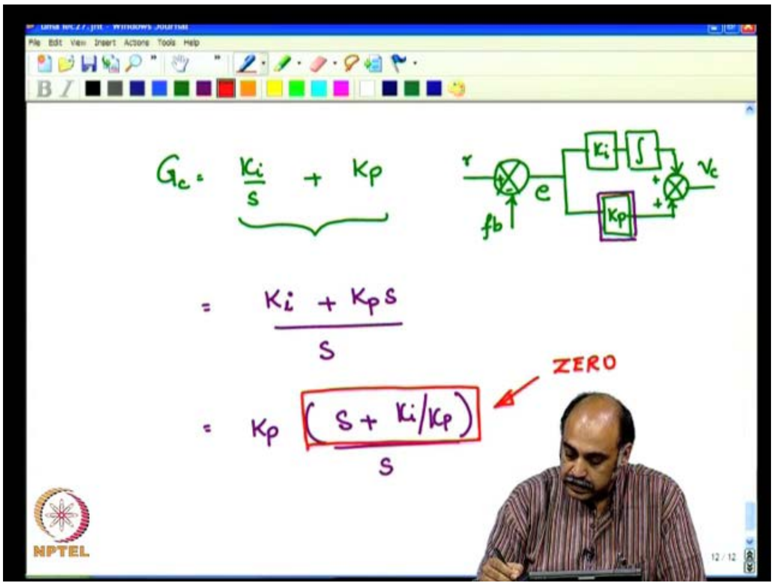


We have the frequency plot and the time plot, the integrator starting from a very large value infinite value going down at 20 dB per decade. So, if this were 0 dB this implies unity gain if it is 0 dB that implies unity gain and we saw that we could chose any of the parallels by choosing appropriately the value of k i. This scaling for the integrator which actually cuts the 0 dB line or the unity gain band width at different points.

So, as k i is decreased you will see the unity gain band width reducing if k i is increased you will see the unity gain band width increasing. So, if I increase k i, you will be choosing you will be choosing higher and higher band width. Now, let us let us say there are some point of time we have these step response, let us say this is the step change that i you and the response with the integrator was something like that. The intention now is to reduce this error ideally, I would like to have 0 error at every instant of time however that is not possible you can have infinite gain, now only at that steady state.

So, the steady state error is taken care of how we reduce the error near around the transients, so let us say we shape the curve in the following fashion. So, let us say at around this point the integrator gain with frequency was coming down in this fashion. Now, instead of coming down let us let us make a change at this point instead of coming down it starts going like this how do we bring this about this is possible by introducing that introduce some component. Let us introduce a proportional component or let us say in the transfer function write now the controller transfer function G c was k i by s, let me go to the next page.

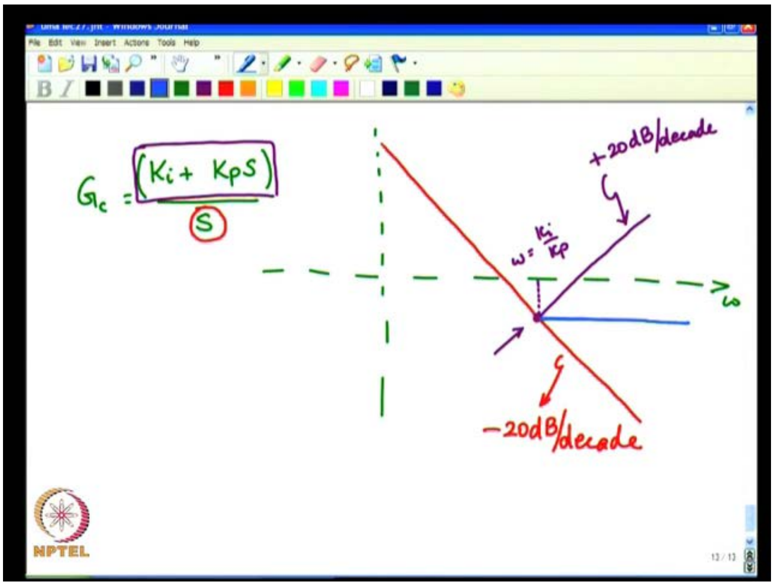
(Refer Slide Time: 23:58)



Now, let us make G c as K by i s plus another component just a proportional gain, so this is equivalent to seeing you have a comparator plus minus you have the reference you have the feedback the error e it use to go through just an integrator. Now, you are also making it go through a proportional gain and you add it up plus and plus and use that has your controller output.

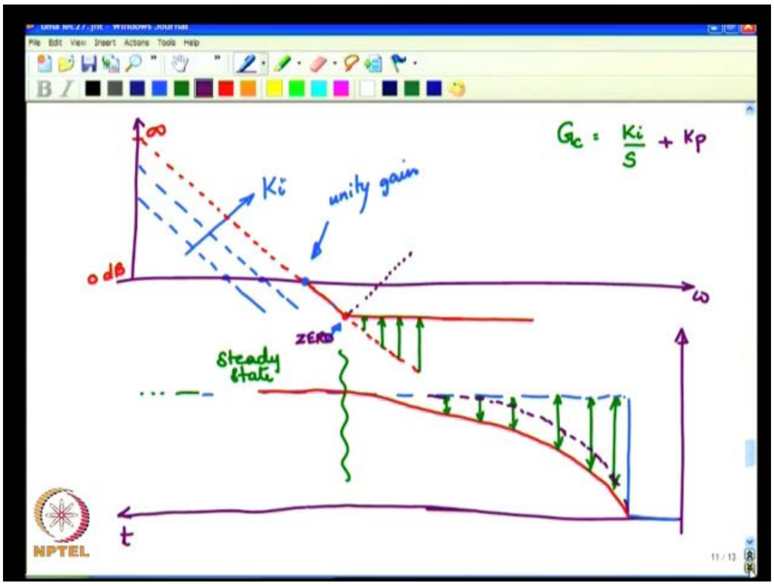
Now, look at this, if you combine these two you have k i plus k p s by s or if you want to simplify it in some form you could say k p s plus k i by k p by s. So, just by adding this new component we have introduced in the numerator as 0 in the numerator polynomial you will see as 0 coming in to the picture, so what is the character of the 0.

(Refer Slide Time: 26:31)



So, you have you have such a transfer function, now G c if I take the frequency some point of time, sorry some point in the frequency plot space the integrator we saw it just keeps going down at minus 20 dB per decade. Now, if you look at the 0 like a derivative wherever it comes in to effect, now let us say it comes in to affect this point. This point corresponds to corresponds to this frequency of omega is equal to k i by k p the 0 will start increasing at plus 20 dB per decade. So, taken together this minus 20 dB per decade and the plus 20 dB per decade of the 0 will cancel and you will get a gain which is kind of independent of frequency.

(Refer Slide Time: 28:37)



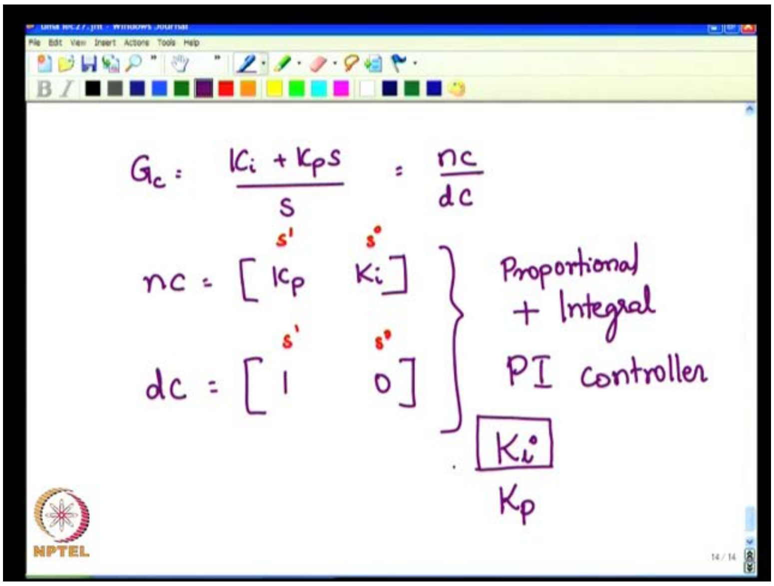
So, that is what is happening at this point here when we say we just want to make it flat like this Independent of frequency. It implies that at this point we have inter post or we have introduced as 0 we have introduced as 0 to become active at that point at that of frequency. So, which would mean that 0 would start going like that the pole the integrator pole contributing minus 20 dB per decade. So, from then on you will see that they both will cancel, and then you will have a flat portion like this. Now, what have we gained, we have gained in gain remember that if we had not introduce zero at this point the integrator, the controller gain would have just dropped like that.

But now the controller gain is more than what it would have been if we had not added the zero understand that if we had not added the zero the controller gain would have just dropped off along this line minus 20 dB per decade. Now, because we have added the 0, the gain controller gain has now become flat here as you see compare to the earlier case as frequency is increasing we see this green arrows are the amount of gain, so this gain in gain has a tendency to help the transitions.

You see that there is no change in the DC portion the DC portion of the controller gain remains the same which means this portion of the time is formed is not going to get expected. Only the high frequency portion of the gain has been increased compare to what it was before. Once, the high frequency portion of the gain is increased, you will see as faster as pans in the transition portion of the time response.

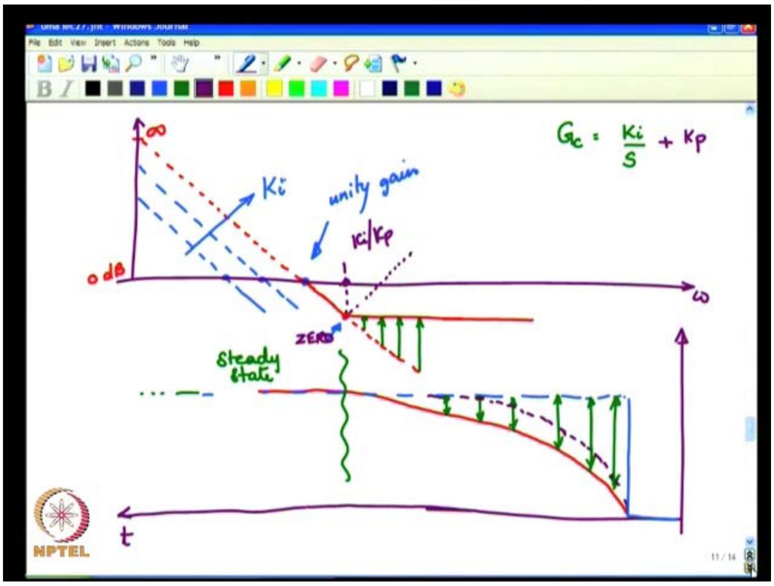
So, we expect that the time response should be faster like that, so this effect is brought about by the introduction of 0. We should say the introduction of the proportional part the P part of the P I D controller. Now, let us see if we are able to introduce the proportional part into the controller in the same controller that we had been working with in mat lab, so what will become the numerator and the denominator portion of the polynomial.

(Refer Slide Time: 32:46)



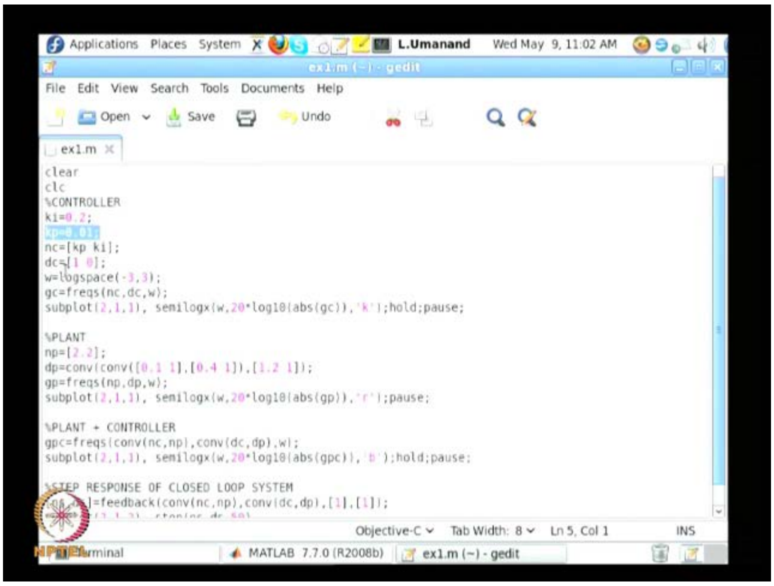
So, we have G c as k i plus k p s by s which is the numerator polynomial and the denominator pole for in mat lab. You can say that numerator polynomial is nothing but k i where this is the s to the power of 0 term this is the s to the power of 1 term and the denominator polynomial is 1 and 0 s to the power of 0 term and s to the power of 1 term. So, this becomes our new controller representation in mat lab this is the proportional plus integral part of the controller or the P I controller. You have two things to design, choose here you have to choose k i and k p you have already chosen and fixed k i.

(Refer Slide Time: 34:27)



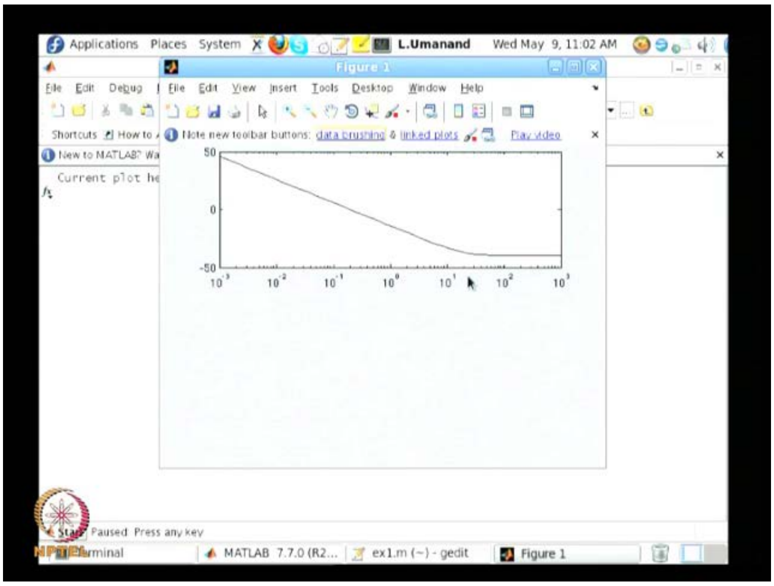
So, you need not touch that, now you need to choose only k p to decide the point of introduction point of introduction of the 0 and the 0 is actually decided by k i by k p ratio. Now, in the mat lab environment let me close all the previous things now let us open that script file, so in the script file we still using the same plant, so we need not disturb this portion. These are all the terms of variable controller in plant variables, so this portion also will not get disturbed the only place, where we are going to introduce change in the controller numerator and denominator polynomials.

(Refer Slide Time: 35:23)



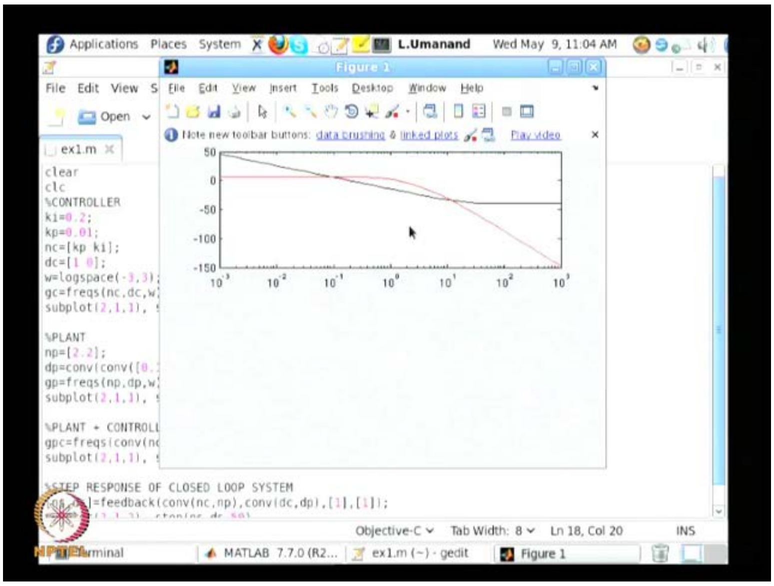
We also introducing one new degree of freedom which is k p, so let us say k p equals let us again start k p with a small value 0.01 and the numerator polynomial becomes k p k i is not this what we just now saw. The denominator polynomial is by s, so there is no change in the denominator polynomial it remains as 1 and 0, coefficient of s. Now, this controller becomes proportional plus integral controller all l's remains same, so let us see what we can achieve we can probably try to tune this k p increase and decrease and see what the effect is.

(Refer Slide Time: 37:42)



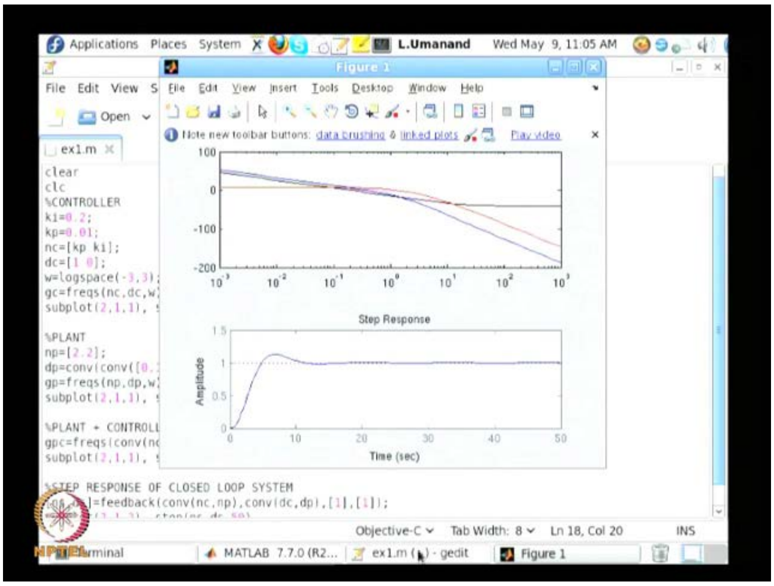
So, let us run this again x one notice that the change we have this, this is the integral portion and somewhere at this point we have introduced a 0 at what point at the point where you have s is equal omega is equal to k i by k p. We know that k i is point two k p is point zero one k i by k p is nothing but 2 and you see here nothing but 20 and you see at around 20 here there is a change in the slope and start going to flat gain at this point of time. So, this is precisely what we wanted to achieve that is change in the slope of the controller curve.

(Refer Slide Time: 38:18)



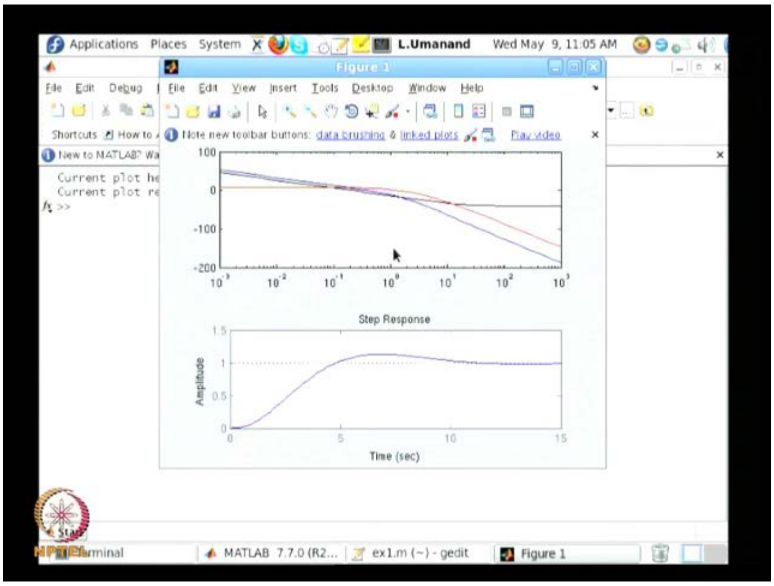
Now, super impose the plant curve there is no change in the plant curve is a same plant and together with the plant and the controller. So, you see that the DC portion the study state portion will remain more as same, so you will see the consolidated curve taking higher value here. Then, start getting attenuated from here onwards that attenuation here will be not as much as it had been in the case of the just integrator. Therefore, you have now a higher gain at this in this region that is in the higher frequency regions and in the time response it gets reflected as better transitions are better dynamics.

(Refer Slide Time: 39:06)



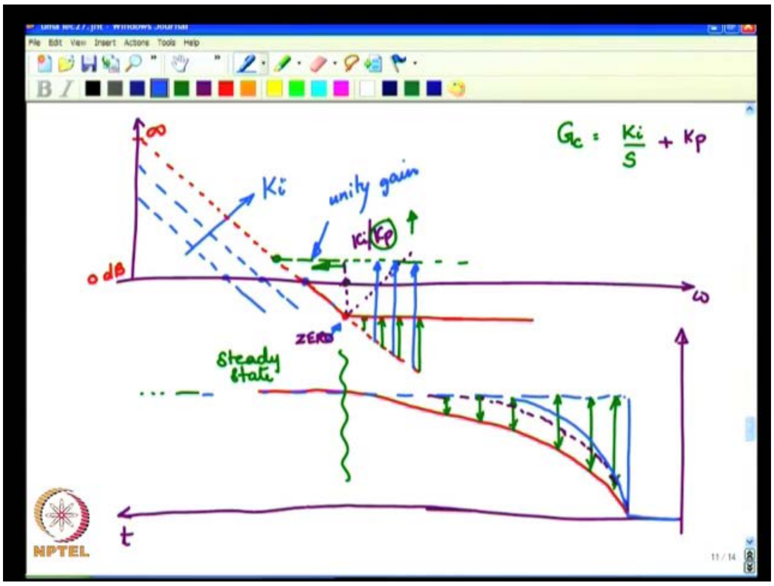
So, this is the consolidated curve the blue curve and let us say this is the time response the time response there is not much change in the study state you probably should absorb may be in this regions. Now, let us say we look the time curve at up to let us say 15 seconds here, so that we assume in we shall we shall change the time response portion to give as a response up to 15 seconds, now look at this rise time around 5 seconds.

(Refer Slide Time: 40:21)



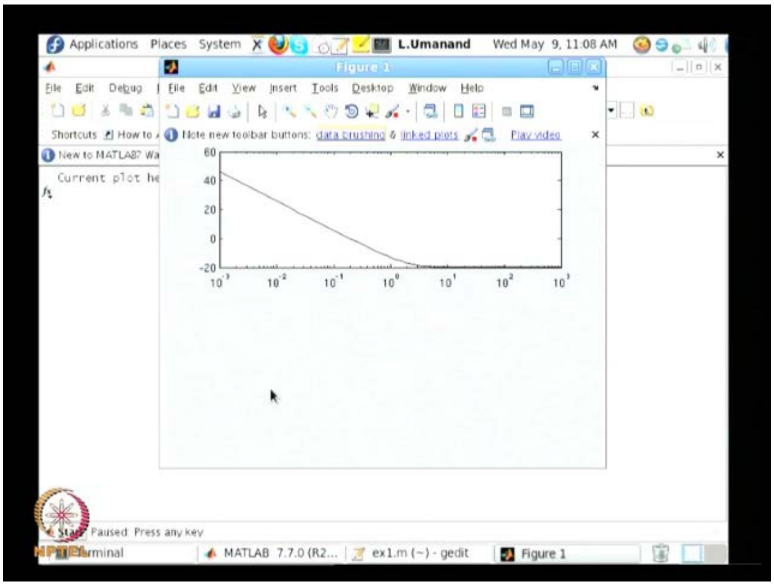
Remember that we shall re execute it by adjusting the value of k p, what happens if we change the value of k p.

(Refer Slide Time: 40:45)



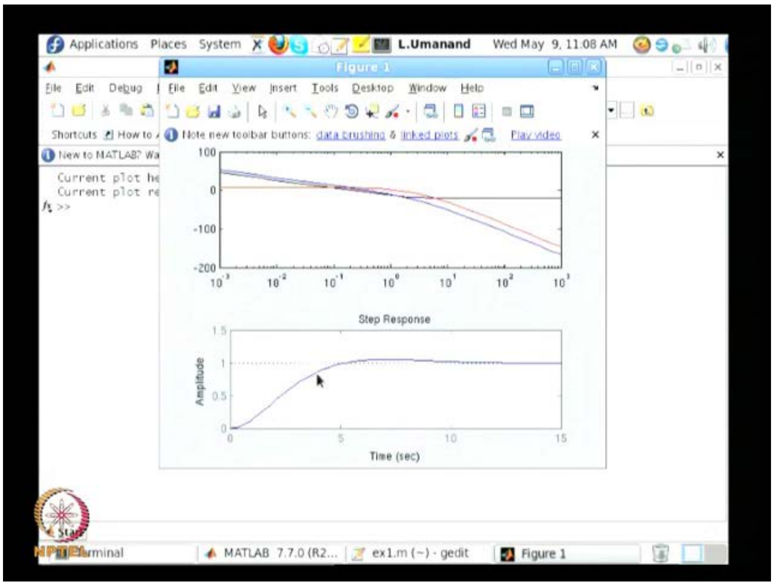
Now, let us go to the white board at this point and see let us see here what happens if we change the value of k p we are not touching k i. You are going to increases the value of k p what does it mean this point is shifted lower to lower frequency point which means at a lower frequency point itself. The flattening will start walker which means the gain is actually higher the gain in gain is more. So, one would have gained much higher the gain by increasing the value of k p a much higher gain should actually reflect in a better or faster response here so this is the effect that we want to actually see.

(Refer Slide Time: 42:09)



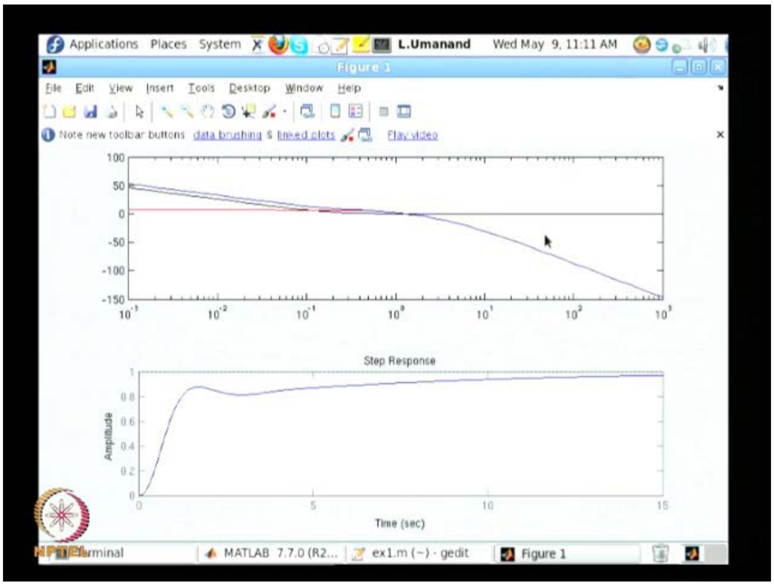
Let us again shift to the mat lab environment, let us increase this k p by a factor of ten we make it 0.1 and the re execute the script file. So, you see you see the turn point at which it takes a turn is at 2 here and the dB gain is minus 20 dB and in the earlier case it was still further down.

(Refer Slide Time: 42:57)



So, you see that the response is much better with the rise time slightly better than 5, you could further increase and keep trying. So, let us increase to that is 1 k p of 1, so you see that the point it flattens out was minus 20 dB. Now, it is at 0 dB when it is gain and you have the consolidated graph frequency plot blue line.

(Refer Slide Time: 43:51)

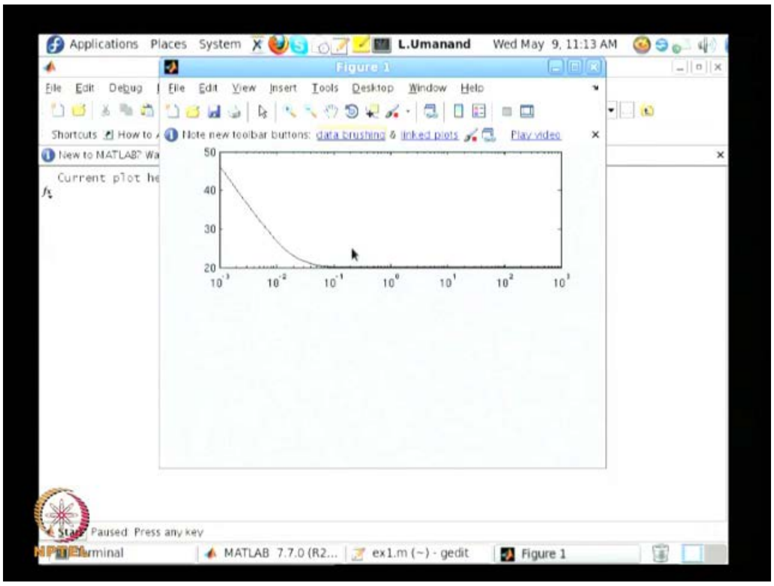


Then, you have, so you see here an nice effect you should notice what has happened there is a fast rise here now this fast rise corresponds to quite high gain that has been flattened because of the flattening of the controller. So, this portion is higher than before so this is rising quite fast here, but we kept on shifting the frequency at which it flattens out to the left there by trying to reduce the band width. So, the gain here there is not much improvement in the gain the proportional part quickly hands over control to the integral part.

So, the proportional part is coming into the zero effect is coming in to effect only for a very small portion of the time here which is corresponding to the high frequency gain. Remember that the system is staying in the high frequency part for a very short while, so if we expand this the moment there is a disturbance the moment there is a disturbance accept change the system is immediately shifted to the high frequency state. The system starts gradually coming down to the DC part the study state portion in the time response this is the disturbance of the transitions state gradually going towards the study state.

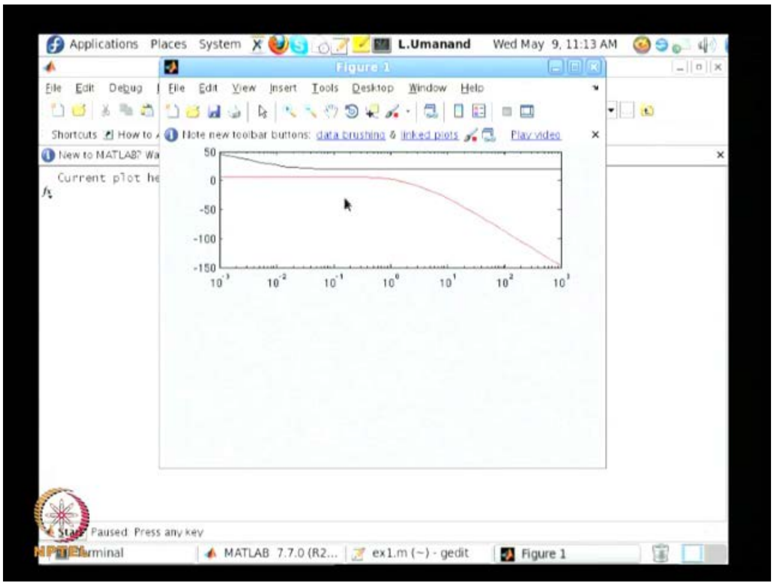
Now, as the system is going from a high frequency as it ad state towards the study state it is passing the proportional part of the proportional effect. Once it has passed into the proportional effect and goes into the integral gain effect, you see the integral action coming taking over at this point. So, this is the proportional part proportional action proportional action loses control here gives hands the battle over to the integral part the integral part is taking over from here taking it to a 0 study state error. This is the effect that you would you seeing by modifying these parameters, you could probably experiment with the many more things. You will see that probably you will get a very over shout because of this high gain.

(Refer Slide Time: 47:19)



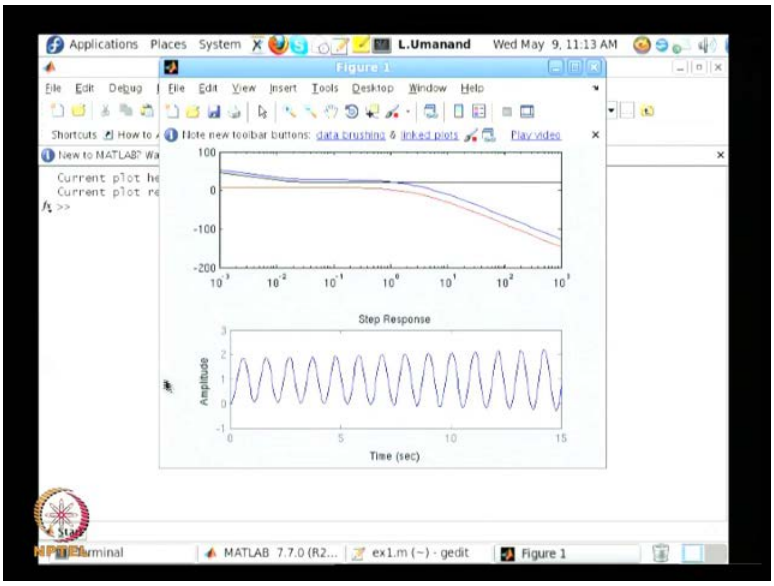
Then, quickly it will hand over the control to the integral part which will take its own time to again study state.

(Refer Slide Time: 47:22)



Now, see the consolidated, the consolidated gain integral.

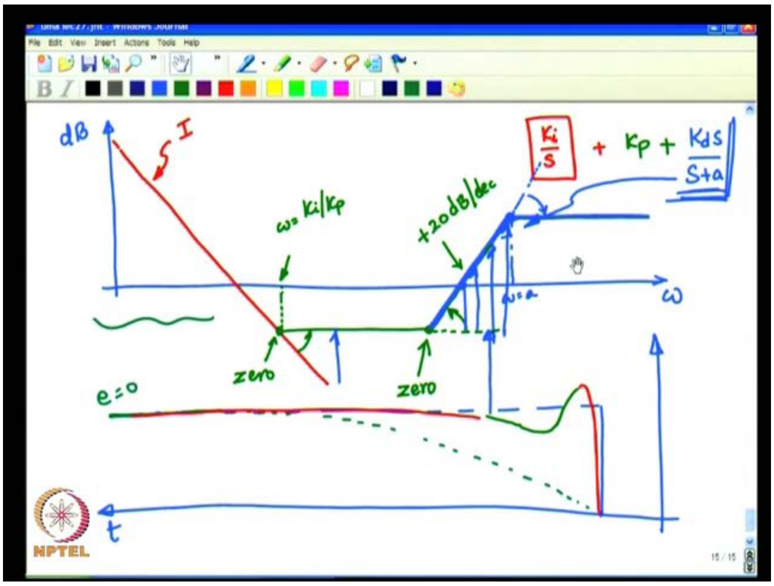
(Refer Slide Time: 47:27)



Then, you have study portion and then the following portion here, this is basically because we have increased the proportional gain much and then it is taking quite a long time to stabilize. You will see that it will take it is oscillating around one and it is growing and probably becoming stable. This is where you will need to find tune these gains to be within the bounce of stability, so you normally will start from low value and start increasing it till you are achieve satisfactory step time response.

Now, we will of course, come to come later on discussed the sequence in which we will be increasing the various parameter gains before we stop at the best frequency response or the best parameter value to get the best frequency response. That sequence we will of course cover later we still have some more topic to cover in terms of the next component which is the a derivative component how do we include the derivative part.

(Refer Slide Time: 49:29)



So, if we look at the controller we have the proportional integral the proportional integral, now we want to include one more part. So, let us look at one more aspect of our P I D controller and that is the derivative part may be we shall see the effect on the white board first you saw that we have the integrator. This is the integrator which is going at minus 20 dB per decade and then let us says we have the proportional part. Let us say somewhere here we introduce a 0 where is we started with k i by s and do this let us add k p and addition of k p introduced as 0 at this point of value k i k p k i by k p.

Not only that, it change the shape of the gain curve what was going down at minus 20 dB per decade it became flat and then somewhere at this point, let us let us do one more thing, let us have a 0 at this point. Now, a 0 means one more plus 20 dB per decade change, so from minus 20 dB we added a 0 it became 0 dB because we added plus 20 dB every decade.

Now, at this point if you introduce one more zero you will shift the curve, again the different loop, now this will be going at plus 20 dB per decade keeps going like that, but it cannot keep going like that as omega tends to infinity. We cannot have just plain k d into s there will be an s plus you need to have a pole k d into s this not realizable k d into s by s plus a is realizable. So, what would this goes on at plus 20 dB per decade at somewhere at this point this pole comes into effect.

Let us say we design such that this pole comes into effect at around this frequency, so what will happen what was going at plus 20 dB per decade will take a turn back minus take a turn back by minus 20 dB per decade and go in a manner which is flat again. So, this is coming into effect at omega equal to a, this portion this blue portion highlighted dark and portion is a derivative component.

We saw that we had we started with three components, one was the integrator component, then proportional component and the third component the derivative component which has a 20 dB per decade rise. Then, after particular point and frequency pole comes into the picture and then behave behaves in a way the gain flattens out. So, this is the derivative component and this is achieved by adding this k d s by s plus a, now what have we achieved by doing this look at the response.

So, let us say we have a step input at this point by the integral action at around DC I know the integrator is giving a very high gain and therefore, e is equal to 0 steady state here, but integrator alone would have given a response something like that. Then, as the system the moment there is a dynamics as the system is changing from as agitated state towards steady state or DC the gains here and this gains coming to effect. So, the moment there is agitation moment there is a transient the gain in the high frequency region comes in to the picture.

They are the once which are trying to pull the response up, so the derivative action which is here comes first into play probably pulls the output quickly up. Then, as a system is traversing from high frequency to the low frequencies side the derivative hands over the batten to the proportional part and the proportional part takes over, and by the proportional part gain is much lower. So, it starts to drop and then quickly the batten is handed over to the integrator part and you will see it moving in this fashion.

So, the derivative part quickly pulls it up because the gain in gain is much higher than the proportional see the proportional part that gain in gain was only up to this extend. The gain compared to the proportional part is so much more in the case of the derivative, therefore that would be very fast the action would be very fast however keep in mind all these high frequency portions are noise sensitive. So, this can the gain is here can amplify the noise in this region and if you give a high value of scale scaling k d or k p.

It can de stabilize the system are deteriorate the performance instead of making it better. So, you have to be cautious while including the d portion and the p portion because they are actually gains at the noise sensitive zones of the frequencies pack drum. This tuning has to be done specific to the plants specific to each plant to get the optimal performance best performance. So, we will see how we go about choosing the various values of k i k p k d in the next classes.

Thank you.