Switched Mode Power Conversion
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Lecture - 27
PID Controller - I

Today, we continue from where we left off. In the last class, if you recall. We hadBeen discussing, studying about the different gain shaping components. Recall that the plant a his sensitive to noise. And we saw that the gain, the controller gain that is interposed in between the error and the plant input needs to be shaped with respect to frequency. And we saw three major, three important gain shaping components, one was the integrator. The integrator which falls at the rate of 20 dB per decade, meaning the gain decreases a 20 dB for every decade change in omega frequency. And we saw just the plane gain, the plane gain does not change with frequency.

And the third component was the derivative component, as that pure derivative does not exist. You need to have a derivative n combination with a, a lag or a pole. And we saw that the gain increases, at the rate of 20 dB per decade plus 20 dB per decade. And after the pole, the poles minus 20 dB per decade and the derivative plus 20 dB per decade cancel and it flattens out. So, essentially we had this 3 gain shaping components. And that is what we will be integrating and using to form, what is called as the PID controller or the Proportional Integral Derivative controller. So that is the essentially, what we will be discussing in this class, the PID controller.

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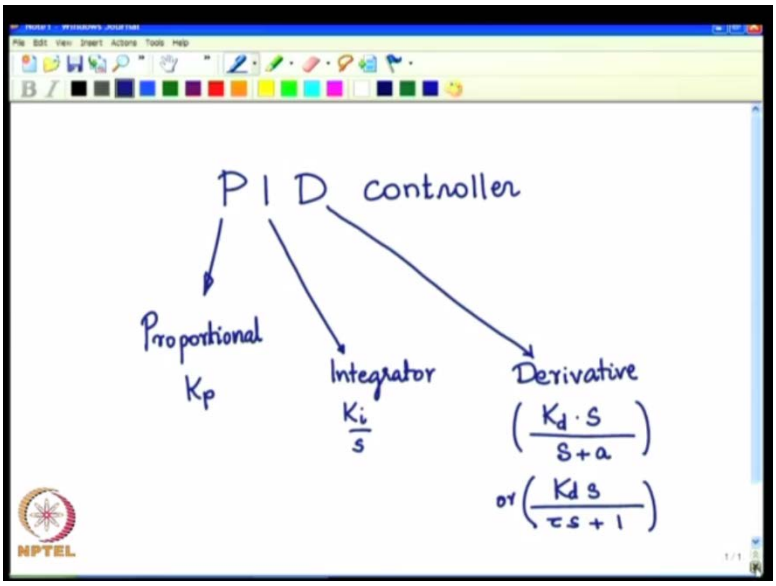

Or, how we go about trying to design this controller, which is consisting of one proportional component, it is a co-proportional gain component, normally called K p, this gain is called K p. There is another component i or the integrator component. And this has a gain K i by s. And here S 1, S suggest the Laplace pole at S is equal to 0 in the S plane. And then we have the derivative component. We cannot make a pure derivative. So, you have a gain, which is of this form. We will call K d into S by S plus a. Or you can also write it as K d into S by tau S plus 1 taking out a common. So, you could express it in this fashion, or this fashion an both the cases, you have a 0 at S equal to 0. And a pole at S equal to minus a or pole at S equal to minus 1 by tau. Depending upon which form you are going to use. Now, this are the 3 component that we studied and these 3 component.

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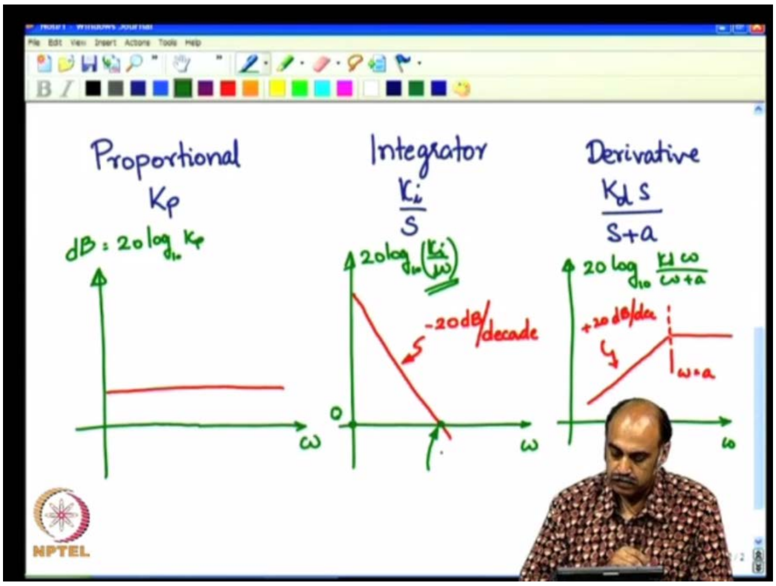

The proportional, proportional are the K p gain the integrator K i by s and the derivative, which is K d S by S plus a. Now, these are the 3 components and, we saw that with respect to omega, these 3 vary in a certain manner. The y axis, the x axis is omega frequency in radians per second or Hertz and the y axis is the gain in dB. And we saw in the last class. In the last the dB gain is given as 20 into log base 10 of output by input. Or log base ten of the transfer function or the gain. So, in, in the case of; in the case of the proportional, this will be 20 log base 10 K p this will be 20 log base 10 K i by omega. And this wouldBe 20 log base 10 K d omega by omega plus a.

So, it will be of this form we take the absolute values and plot the gain. There is also another factor here that will vary with the frequency, which is the phase relationship between the input and output. And that is called the phase plot which is also normally plotted with respect to frequency omega. However for the moment, let us not take too many things confuse ourselves with too many things. We will just look at the gain, gain plot in dB. And we saw that in the case of proportional. It is a constant with respect to frequency. And in the case of the integrator its start fall; its start falling at the rate of 20 dB per decade.

So, it has a negative slope 20 dB per decade for every 10 times change in omega that is the slope. And in the case of the derivative there is the rise at 20 dB per decade, up to the point, where you have a omega equal to a, then its platens out. So, this is plus 20 dB per decade. So, this are something, that you need to having in your mind, while you are designing the controller. And we will use all three of this to manage the overall shape of the gain. Not that, if one is taking of the x axis of the 0 in dB. If this is 0 dB then at this point, run the integrator gain crosses this 0 line, 0 dB line. We can say that the integrator gain is unity, because when will this be 0, this will be the 0. When this portion is 1 unity log of base 10 to 1 will be 0. And that is why this point is called the unity gain point. Now, these 3 will be combined to form the controller.

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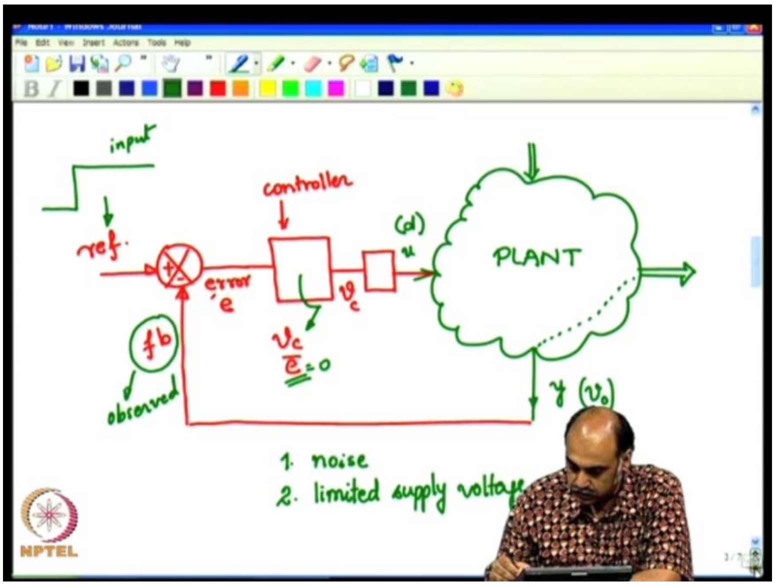

So, we have here the plant. Now, the plant couldBe anything, it couldBe dc dc converter, it couldBe the bug converter, it couldBe the boost converter, couldBe the isolated converter, or it couldBe the any other physical plant. Where in there is a power input and a power output and is the power flow is to be controlled. It has the control input u and an output to be controlled y, which is actually linked as a sensed; as a sensed parameter of the actual power variable. So, in over case in the dc dc converter case, the u is nothing but duty cycle, duty ratio d output many cases is v naught. And this, what we want to a control and this is passed the reference and the feedback are compared reference.

And the feedback are compared to obtain the error e. And this is passed through the controller to get a control voltage v c. And this control voltage v c will appropriately get transform as a compatible input signal u, u according to the type of the plant. Now, this controller we saw has a gain dynamic gain, which is v c by e. Now, this dynamic gain, we saw cannot be infinite, though we desire that it shouldBe infinite, because we want to have error equal to 0. But if we have a infinite gain, we saw in the last class as we discussed.

There are 2 issues; one is the issue of noise, noise which is present in all electronic circuits, which will also get amplify and amplified and it will swamp the plant input. And the second issue is that of limited supply voltage at u. So, the there are limits on u, if it is a duty cycle the limits are between 0 and 1.5 depending on the nature of the converter. There couldBe a limits on v c, itself is apply voltage, what over it powering up the controller circuits, it couldBe 0 to 5 volts, 0 to 3.3 volts, it could Be minus 15 to plus 15 volts or minus 12 to plus 12 volts. Depending upon the nature of the circuits and the power supply is being generated. Now, because of this 2 limitations, we saw that dynamic gain, this dynamic gain should it is not an constant it should vary with frequency and that is the reason.

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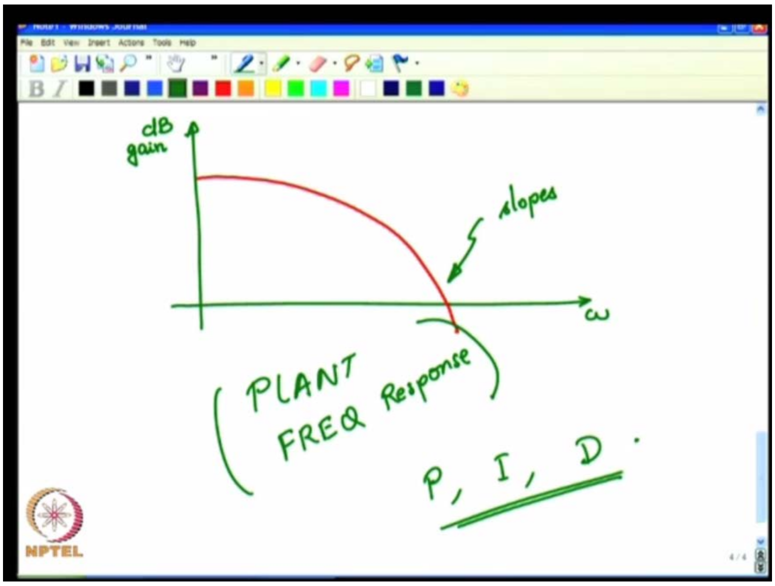

We have been trying to use the various components to get a variable gain K as a variable gain controller. Now, the gain has to vary with respect to frequency, the gain in dB 20 log the actual gain. Now, in most, most physical systems they have a natural gain versus frequency, which is like a low pass filter. The gains automatically start reducing and becoming attenuating after particular frequency.

So, this is something like a low pass character, where the lower frequency gains, lower frequency have a higher gains pass through the plants. And the higher frequency component get attenuated. So, most physical system will have a character like this, what will differ is slopes. So, if it is third order systems, you will have a minus 60 dB per decade slope. If it is first order system as we saw in integrator. It is the minus 20 dB per decade slope, as a second order system minus 40 dB per decade slope so on. So, this slopes indicate the rate at which the attenuation is going to occur at higher frequencies. Now, this plant frequency; this plant frequency response, we would like to modify by introducing the P and I and the D.

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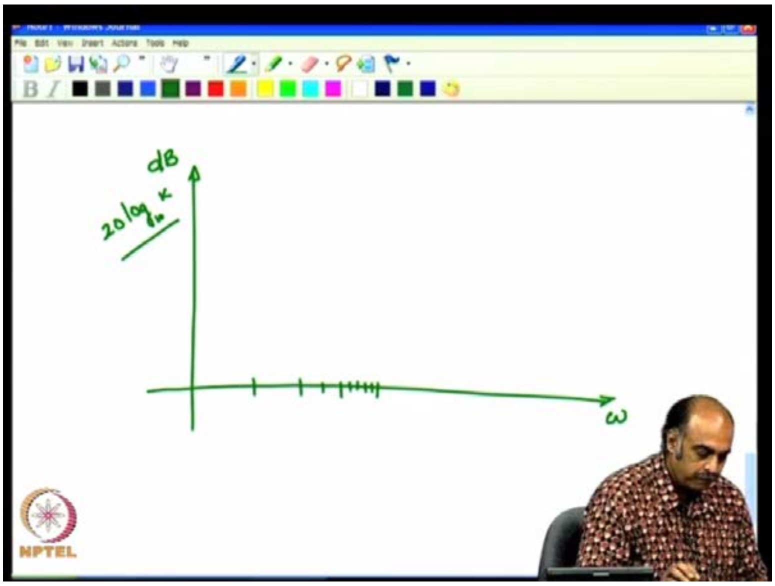

Just to make thing clearer. It is good for you to simulate and get a field for the omega versus dB curve. We saw here that it is 20 log base 10, whatever the gain k. The x axis which is omega frequency, the x axis is not linearly distributed or linearly spaced. The ticks, the tick mark are logarithmically distributed. So, this is the, the points are distributed in a log scale. So, that it compresses the x axis. And you will see, you will; you will see the linear curves as would you normally see in bode plots and such other frequency plots in the literature. Let us switch over to the simulation tool. And get a field for the various frequency plots.

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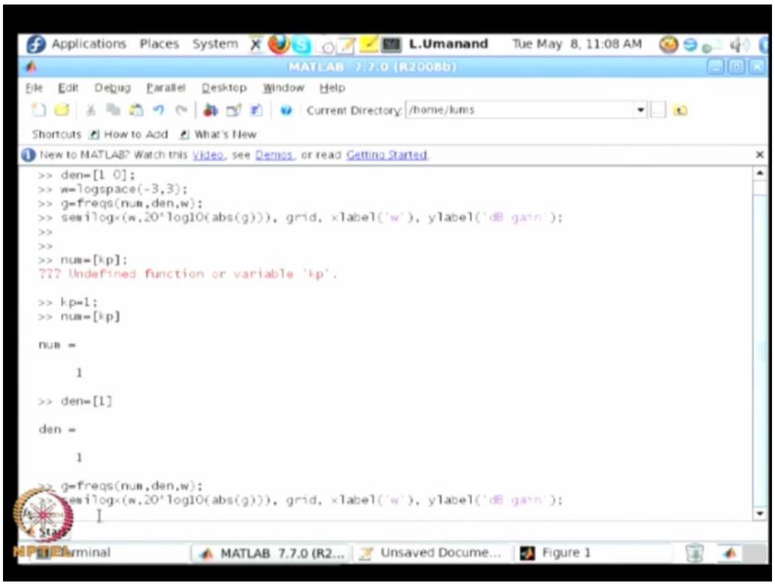

Now here, I have an, the computer matlab. You could also use octave, the open source clone of the commercial matlab. Now, open matlab you will get a command window like this. Now, let us say we first make an integrate. Now, the integrator has an numerator polynomial, and a denominator polynomial. Now, how does that come about? So, if you look here.

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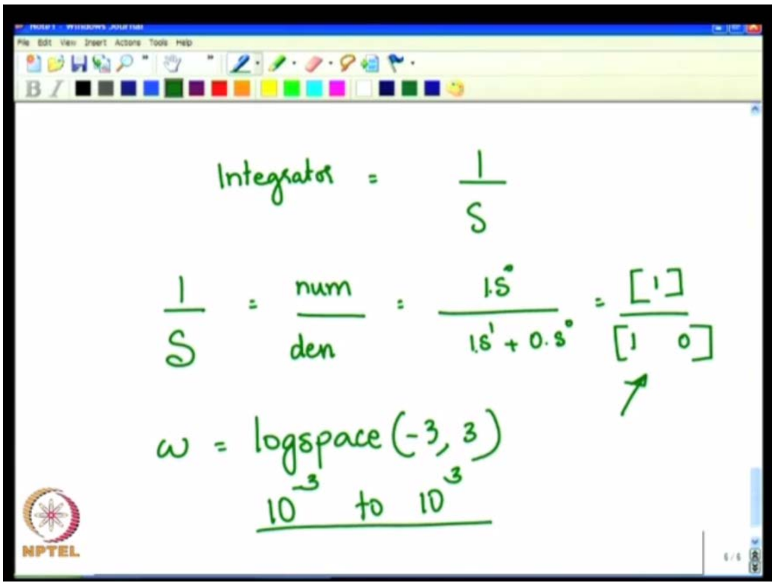

An integrator is returned in the Laplace domain as 1 by S. Now, this 1 by S has this numerator polynomial. And the denominator polynomial, which is actually can also be return as 1 into S to the power of 0, dividedBy 1 into the S to the power of 0 plus 0 into S to the power of 0. So, this can be return as in the matrix form. Let us say only the coefficients of the powers of S wouldBe 1 and here the coefficients of the power of S 0 and 1. So, if you write in matlab vector like this 1 0. It implies that this is the 0th power of S the first power S to the power of 1 and so on.

This is S to the power of 0. So, 1 by S in matlab is written as 1 by this 1 0 vect. This is basically, implies it is a integrator. So, let us say you have this integrator. Let us; let us; let us now define omega. Now, omega log space minus 3 comma 3. Now, this is; this is actually omega given as 10 to the power of minus 3 to the 10 to the power of 3. So, if, if you define omega is equal to log space minus 3 to 3 in matlab, it means that 10 to the power of minus 3 to 10 to the power of 3. The x axis is distributed from this starting value to this ending value.

Now, let us obtained the frequency response g equals frequency response of the numerator polynomial that we have defined denominator polynomial, and the omega that we have defined. Now that would now be calculated input into the variable g. And you can plot using semilog x, the x axis can be logarithmically distributed use semilog instead of plot w, than 20 into log base 10, absolute value of the calculated g. You can give a grid, you can give x lable as omega. You can give y lable as dB gain.

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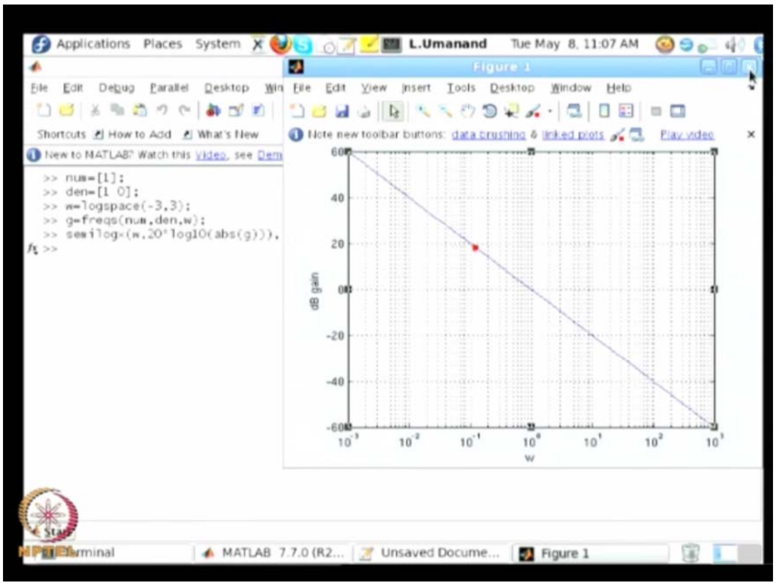

So, you see here, the plotted value. Note that, we have plotted only starting from 10 to the power of minus 3. We cannot go down to omega is equal to 0, because that wouldBe 10 to the power of minus infinite value. And at that value, the value of the integrator as a gain wouldBe infinite. So, we have taken some small value 1000 frequency. And from there an you see there dB is going down at 20 dB per decade. Not this value let us say, you have, let us say we marked this point. This is at 40 and this is at the 10 to the power of minus 2, 1 decade mode 10 to the power of minus 1. If you mark, you will see that it would have traverse 20 dB down in the vertical scale. So, that is what we mean by saying minus 20 dB per decade.

So, this is the frequency plot of the frequency response plot of the integrator which keeps on going on down at minus 20 dB per decade. Let us now see, how derivative would look like. The proportional part is always a constant. And if you want to do a proportional, it is the numerator polynomial wouldBe 1 or just K p, and define K p. K p is equal to 1 numerator polynomial is K p, denominator polynomial wouldBe 1, let us say. So, this wouldBe this numerator and denominator would form your unproportional transfer function of the proportional system. Now, let us look at the derivative, maybe we could go through with this. Get the g for this proportional systems and plot.

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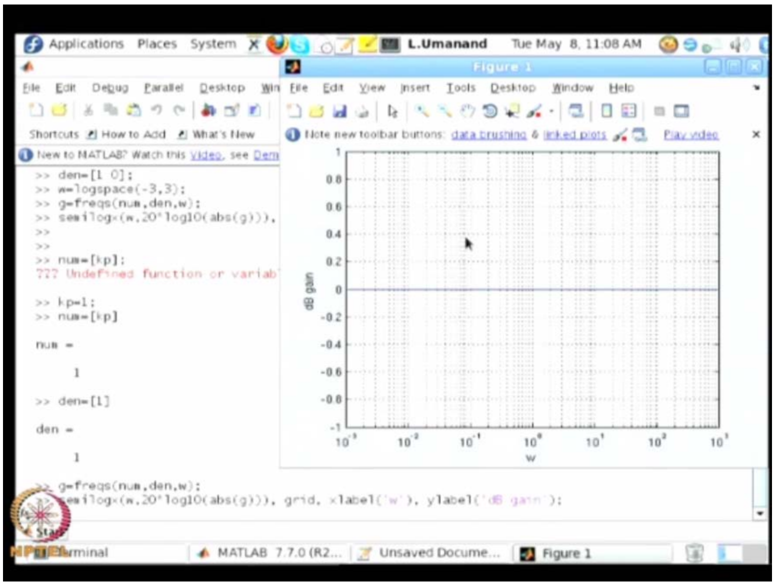

The response, you see this is gain of 1, numerator is 1, denominator is 1, gain is 1 log of the gain dB gain would 0. This is the proportional system, If you keep changing the value of k.

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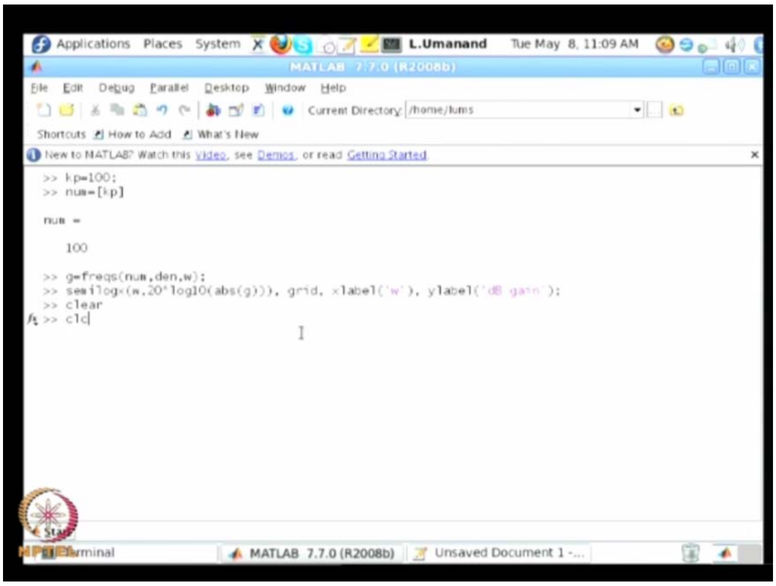

Let say K p is equal to 100. Numerator is set to that value, recalculate the frequency response. And re-plot the response 100, 20 log 100, log 100 is 2, 20 into 2, 40 dB is the, and it is constant does not vary with frequency. So, that is the proportional part for you component. Now, coming to the derivative component.

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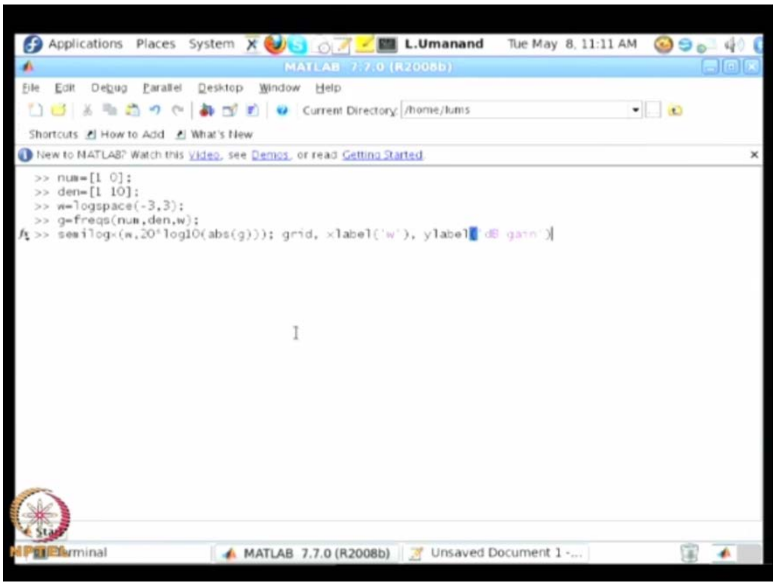

Now, the derivative component has a numerator polynomial. And the numerator polynomial is S. So therefore, you have the S to the power of 1 coefficient is 1, S to the power of 0 coefficient is 0. So, this wouldBe the numerator polynomial. The denominator polynomial is S plus a. So, let us say do you have 1 and 10, a is 10. So, this is the numerator and denominator polynomial omega again log space. Let us plot for the same range and get the frequency response of the derivative system. And plot semilog x 20 log base 10 absolute value of the calculated gain. And your grid x label is omega y label is dB gain.

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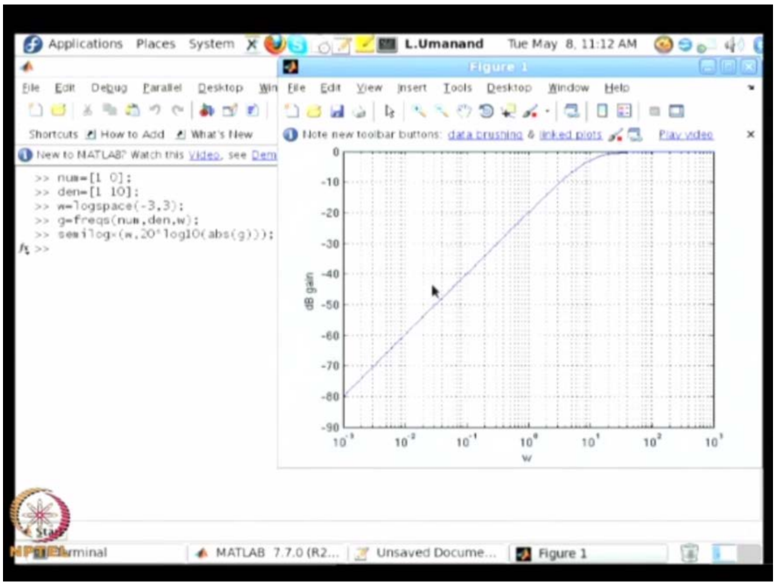

So, you see, it is having a positive gain 20 dB per decade. You can always measure take any dB point. Let say this is 1 point here, where the curser is pointing at 10 to the power of minus 2. This is 10 to the power of minus 2, then corresponding next decade 10 to the power of minus 1. If you see there is a vertical translation of 20 dB minus 60 to 40. So, you have a plus 20 dB per decade at 10, at omega is equal to 10. You see the flattening out. It becomes flat. So, this is actually, the derivative portion S. So, the derivative portion in a pure in derivative, derivative a pure differentiator would keep on continuing at plus 20 dB per decade and so on. But you cannot have a practical or physical system. Therefore, there will be a pole which will occur sometime later, which will flatten it out. You cannot have a ever increasing gain. So, this is the derivative part. So, let us use these 3components in some fashion.

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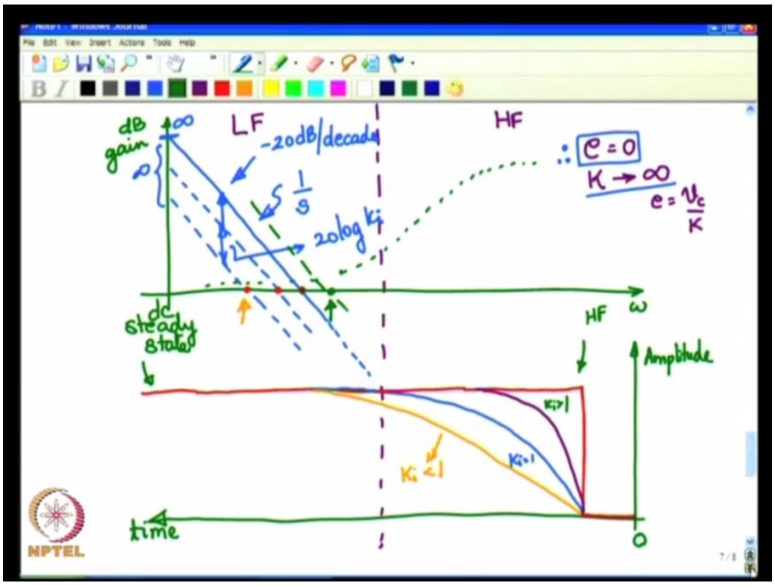

Consider, let us draw 2 graphs here. This is 1 graph, which is with respect to omega and this is the dB gain. Now, another graph I am going to draw. I will put just amplitude as the y axis and the x axis is progressing from right to left. So, imagine that we have x axis progressing right to left. And it is not omega, it is time. So, assume that the time is progressing. You are going to write that time scale progressing from right to left, starting from 0. Now, to the system, that we hadBeen discussing to this system that we have been discussing. Let us give a reference, now this can be a step input. So, we give a step input to the reference here. And observe the output, the measured output FB yeah. So, this is given as input. And this is observed. And we keeping modification to controller. Let us see what happen. Now, you look at this, this is the reason why we have put the access in this fashion.

Consider the frequency plot. In this zone, we are saying omega is close to 0, means dc means steady state. And as you progress in this omega axis, it is higher and higher frequency. And this are higher frequency zones are the transient state. You could have a kind of a disstanding of the dynamics. If you put the time scale going from a right to left. So, if you have an input. Let me draw, that if you have an input, which changes dynamically at this point. And then on steady state, becomes this is unit step.

So, take for example, this case from 0, at this point there is the disturbance as a step change. So, this contains lot of high frequency component. This map directly here to the high frequency component. And gradually there is a transition to more stable state dc. This is where you have the dc. So, you could get kind of an inside into the system on the transition, and the behavior of how the gain also changes dynamically. So, let us visualized like this, this is just a visualization process.

Now, now let us say what is it that we want, we want error e equal to 0. Now, if we want error e equal to 0 the controller gain controller gain K, which is dynamically varying should tend to very high value, would towards infinity. Only then we will have error is equal to 0. Because you, you saw that error is equal to vc by K, the control voltage by K. So quite evidently in the steady state and we saw that if the, we discussed that if K gain K is infinite all frequency bands even the noise get amplified. So, we do not want the gain K to be infinite in the noise predominant zones.

So, if you look at is, let us make as split like that. This is low frequency for the LF zone. This is the high frequency of HF zone. And noise is per dominant in the HF zone. So, in all equipment you will see the noise more prominent in HF zone. So, actually one does not mind having a very high controller gain in the LF zone or more towards dc, towards dc or steady state. So, you see here also, you have the dc or the steady state. And this is where the high frequency happens. So, there is a kind of 1 to 1 mapping and get better inside.

So, let us have high gain at dc or steady state. So, if we have high gain at dc, then at least we are ensure that at steady state error 0. At this state the gain infinite and therefore, the error is 0. So, let us make use of an integrator, the character of the integrator is use of. It has a very high dc gain, infinite dc gain. And then keeps falling keeps on falling at minus 20 dB per decade, at minus 20 dB per decade. So, that is the charter of the integrator. Now, here so, let us say, that this particular integrator that has been chosen has K i value of 1. This is 1 by S. Now for this, let us say for example, you have a time curve which goes like that K i is equal to 1. Now, let me just go to the next page.

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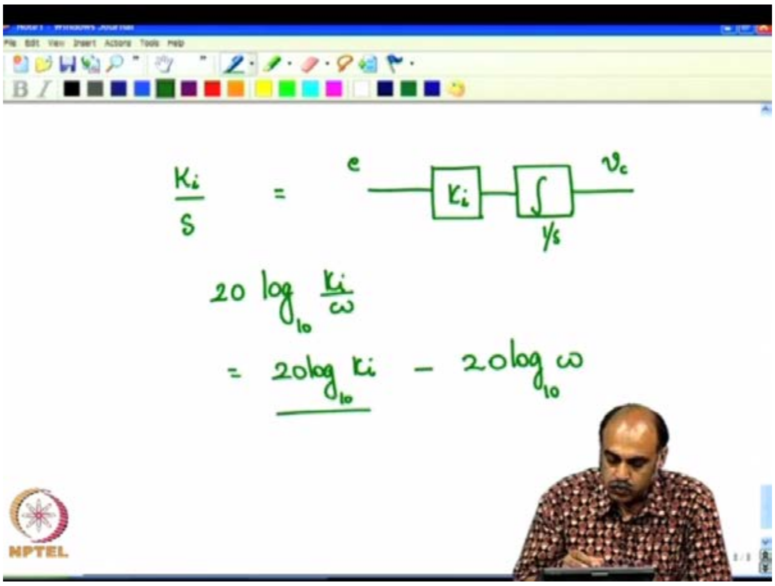

Let us say we have the integrator is something like this. It is in block, we have K i followed by an integrator this is 1 by S this is vc, and this is e. So, dB gain wise what does it mean 20 log base 10 K i by S is nothing but omega, because the real part is 0. So, you have 20 log base 10 K i minus 20 log base 10 omega. Now, if K i are a 1. For K i is equal to 1. This wouldBe 0 and you are just left with 20 log base 10 omega. This is our minus 20 dB per every 10 time change in omega. But for any general, K i you have this component 20 log K i which is constant change with frequency. How does it reflect? Here you have different parallels all coming of family have been infinity. I do not have place here, but just remember that all these are coming from a family of infinite. There all parallels and the distance between them is 20 log Ki, the difference on the gain between the 2. If for the example, this is the normal the K i is equal to 1.

And this is has some value of, then you have 20 log K i is the difference in heightened is parallel. What is the effect that we achieve by this? Look at the bandwidth, where its cuts the omega axis. You see a higher value of K cuts much further in the omega axis. It has the higher bandwidth. This, this one as the lower bandwidth, still further lower bandwidth, which basically means as starts going down K i is reducing. And what is the effect here. You will see that, you will have curves going like that. Or curve going like that. So, here you will see the K i here is less than 1. Less than 1 means as lower bandwidth. So, lower bandwidth and therefore, it is lower. And if you take this here K i is greater than 1, which would mean much beyond removed here, if this is 1. So, you will have a much greater bandwidth and therefore, you will see gate rise faster. So, this effect is basically, what we want when we designed this scale factor K i. So, this effect, infact you can also see, in this simulation. Let us take up for example.

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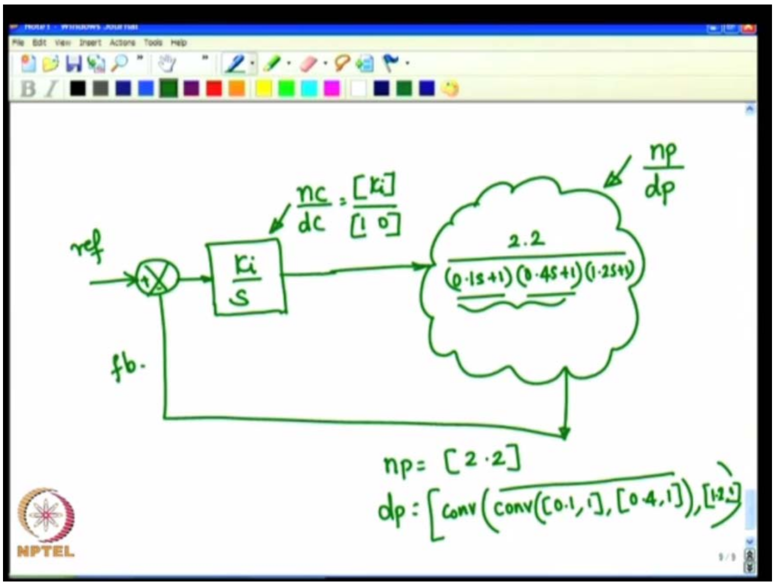

Some systems let me demonstrate to you with a plant. Let us say following plant, which is 0.1 S plus 1, 0.4 S plus 1. This is the third order system 1.2 S plus 1. This is the plant, this is some arbitrary third order system is not a converter just. So, that we get our concepts right K i by S. And reference feedback. So, let us say, we have a system like this. Now, we can call this is the controller which it is let us n c numerator polynomial by dc denominator polynomial. And we will call the plant as n p numerator polynomial of the plant denominator polynomial of the plant, like this have do.

We write this in matlab, it will be K i like that and it will be 1 0. How do we write this, n p will be 2.2, d p will be written as. you can multiply then and put it together. But there is another easier method. You need not multiply, you can multiply polynomials by convolve. Let me convolve, 0.1 and 1. This one polynomial convolved with 0.4 with this. So, these 2 are multiplied to gather by this convolution. Now, the result of this, you can convolve with the other polynomial by using another convolve. So, this would give the denominator polynomial. So, you could do it with any order of polynomial. So, let us try to put this thing together.

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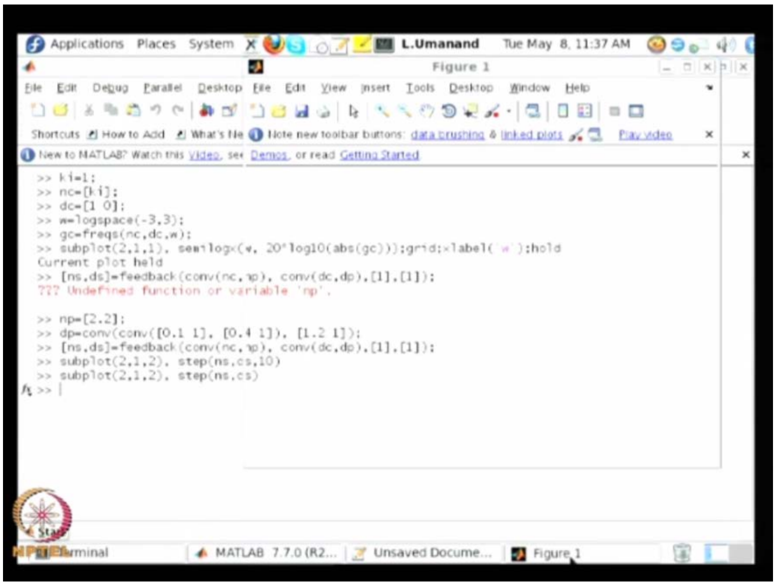

So, let us say numerator polynomial. The numerator polynomial is. Let us define, K i is equal to 1. Numerator polynomial is Ki, denominator polynomial is 1 0. And let us have omega log space. Now, let us see the plot, subplot on the same in the graph window semi log x omega 20 log base 10, absolute value. We shall first putting the frequency response of the numerator polynomial, denominator polynomial and omega as defined above. Then plot semi log x omega 20 into log base 10, absolute value of the calculated gc. And let us hold the graph grid and x label as omega and hold the graph.

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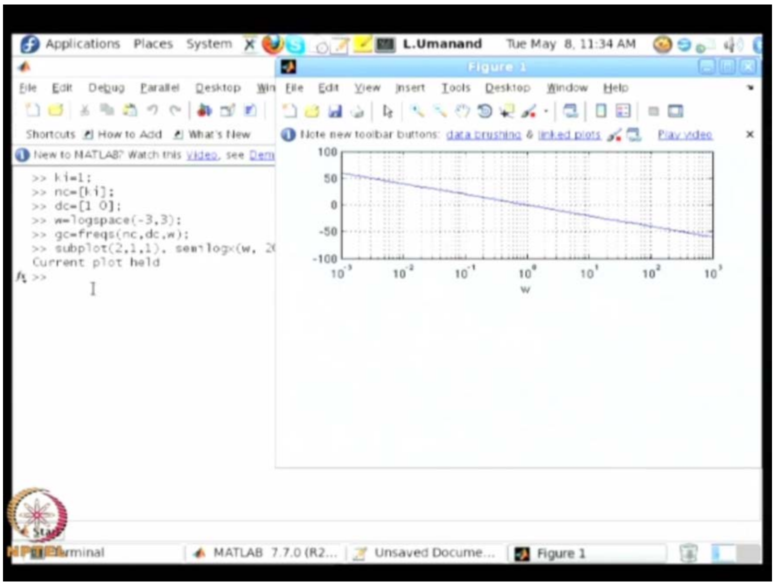

So, this is the integrator with K i is equal to 1. Now, if you want to see the time response. Let us say the fee feedback system is ns. The numerator polynomial of the closed loop systems ns, ds you make. If you to a closed loop feedback of the, you multiply both the controller and. The plant numerator polynomials convolve the denominator polynomials. There is no other component feedback path, just make it unity. And that would give you the, we have to define the plant np on denominator polynomial of the plant. As I just discuss, double convolve 0.1, 0.4. And now close the loop, you have the closed loop. Now, plot that in the other subplot in the space bellow in the graph with the step response.

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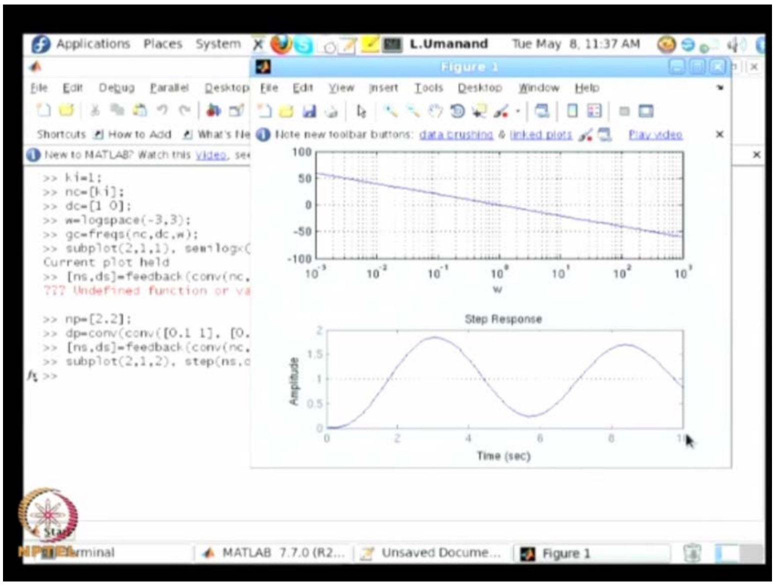

So, this step response will come along with this plot. So, you have, I have plotted here only up to point of 10. You could do that; you could allow it to take its own step.

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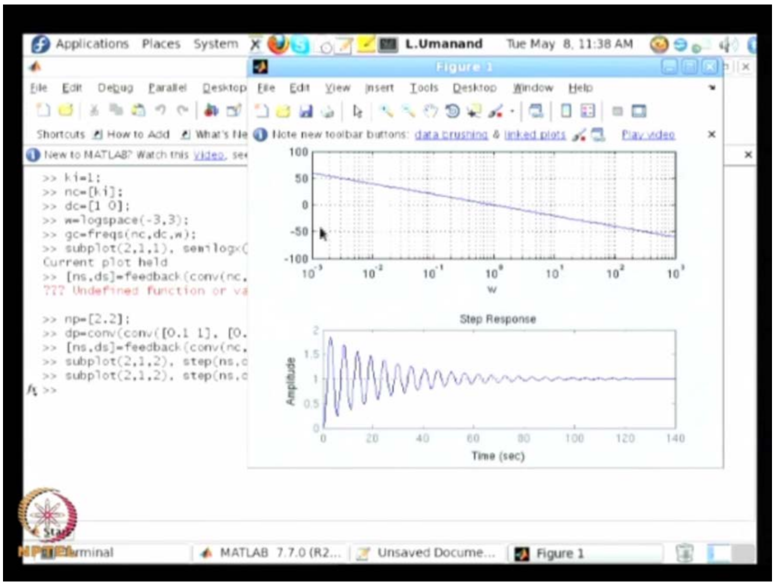

And you will see that the step response wouldBe like this. So, these are the high frequency transition and then ultimately its settles down at 1. There is steady state, there is no error it is exactly 1. That is basically, because steady state here maps to the very high dc gain, which will be there at this region. That is why during explanation, I had inverted the time axis, such that there is one is to one map in between the frequency and the transitions.

So, we of course, will continue from here on and improve the response. You see that the response is not truly satisfactory. It is under damped acceleratory at the beginning during the transient this is when the unit step is given. And then you have 1, the reference. This is the one, which is measured at the feedback point. We have to improve this. So, to improve this how do we shape or modify this gain. So that preciously is what will be doing in the fore coming classes.

Thank you for now.