Switched Mode Power Conversion

Prof. L. Umanand
Department of Electronics Systems Engineering
Indian Institute of Science, Bangalore

Lecture - 24
State Space Model of Boost Convertor

Good day to all of you, in the last class recapping we discussed the concept of state space averaging using boost converter as an example. The circuit averaging method that we discussed we also employed it in trying to obtain the dynamic equations for the boost converter and we stopped at a point just after. We arrived at the average large signal model. Now, today in this class, we are going to continue from there, we are going to develop the study state model, the small signal model and then discuss on the various aspects about the model from there on. So, a quick recap, we discussed on the boost converter state space method of obtaining the circuit averaged model.

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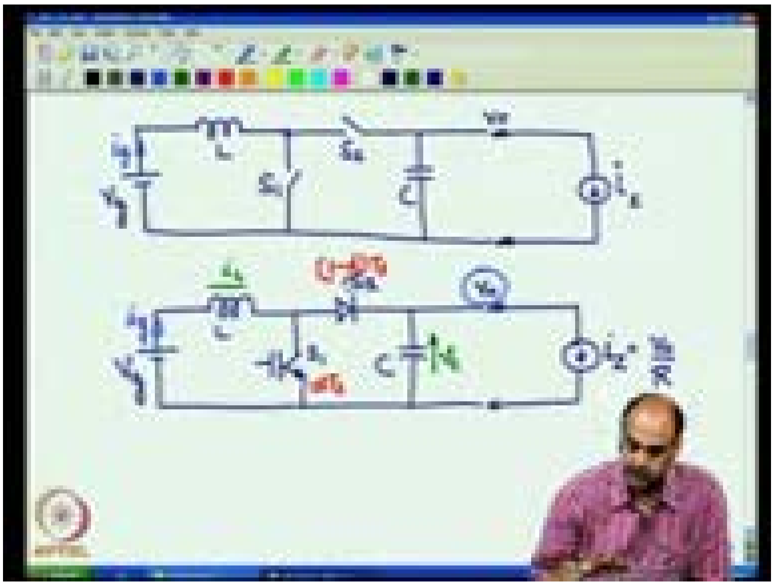

This is the boost converter which we discussed, it contains the input source v g input current i g l two switches s 1 and s 2 c and load current I z. This is also representable in terms of switches here it can have a IGBT here, or diode as these two switches IGBT switches during v t s time and the diodes comes into action during 1 minus d T s time.

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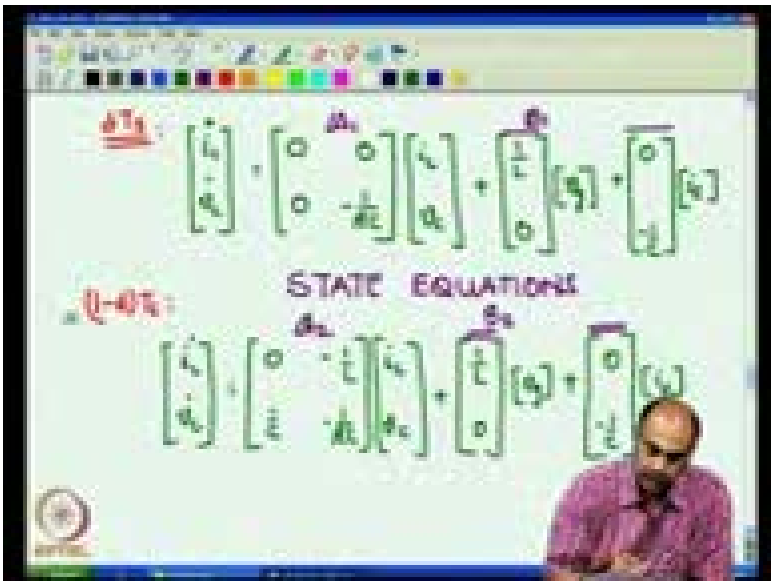

And from here we started developing the state equations during d t s time this was the state equation that we developed and during 1 minus d t s time this was the state equation that was developed.

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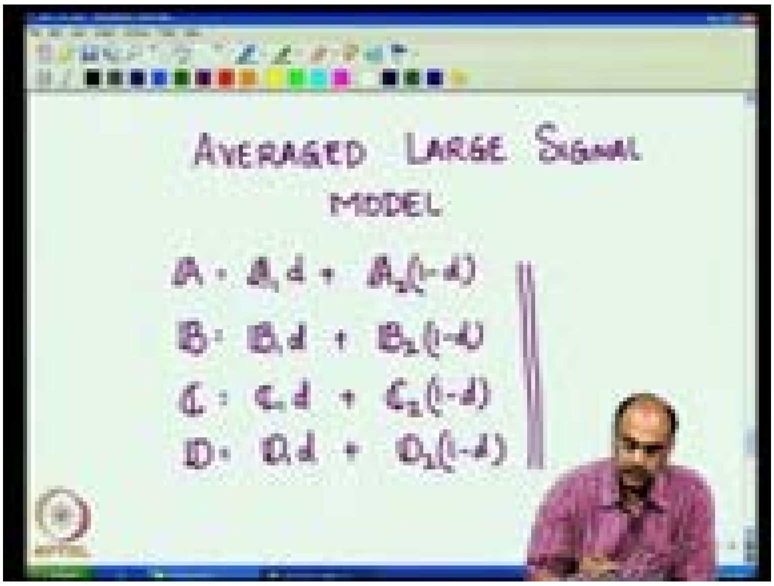

Then we started combining the equations to obtain the average large signal model. So the average large signal model is obtained by combining the state equations and the matrices obtained for the d T s period and that obtained for the 1 minus d T s period, and they are time averaged a 1 into d plus a 2, 1 d minus so on so forth for all the matrices.

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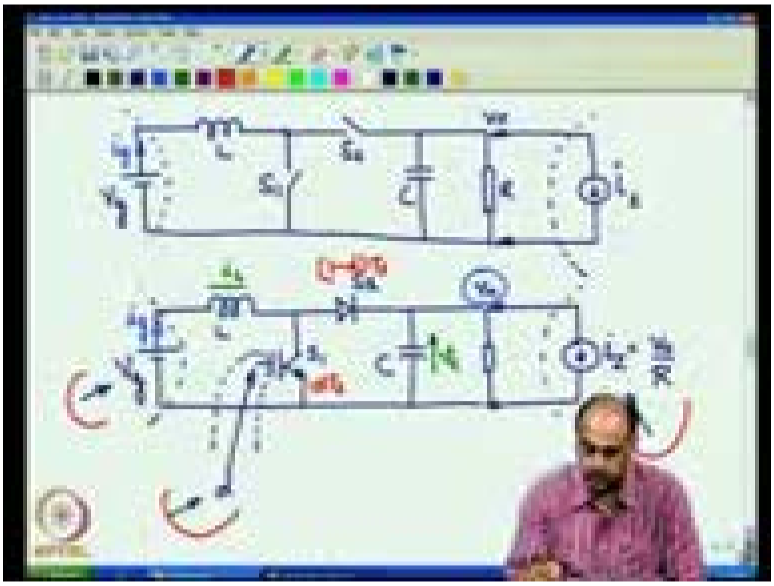
Then based on that we obtained the averaged large signal model, which is like this. (Refer Slide Time: 03:43)



So here note that we have used the term r. So, the term r there is basically to have a concept of a bleeder resistance here. So, you have a resistance r here and to that r in parallel you have current source attach and the current source basically, gives you the idea of the varying load. The load is the external input to the system and therefore, that can be consider as one of the inputs the v g is the external input to the system, so that also can be considered as one of the inputs and another input that you can consider is the control input. Here the d so to this you are giving the duty ratio as one of the information signal this is an energy signal this is an energy signal this is an information signal and this is also an energy signal. So, you basically have three inputs to the systems, which you need to observe and understand.

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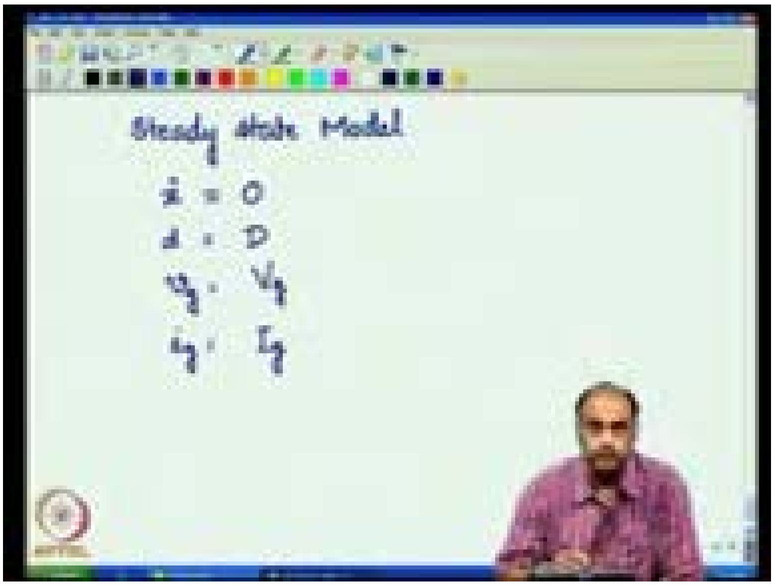

Now, that we have recapitulated on what we were doing we shall now proceed from here. Now, in the averaged large signal model, we shall obtain the steady state model; steady state model can be obtained in a simple way, where in all the derivative term all the x dot terms will become 0, which means steady state in the equilibrium state, you do not have the derivates and all the variables will take the upper case forms, where will d settle at and upper case D this is the equilibrium condition the d. And likewise, v g will become V g, i g becomes I g and so on.

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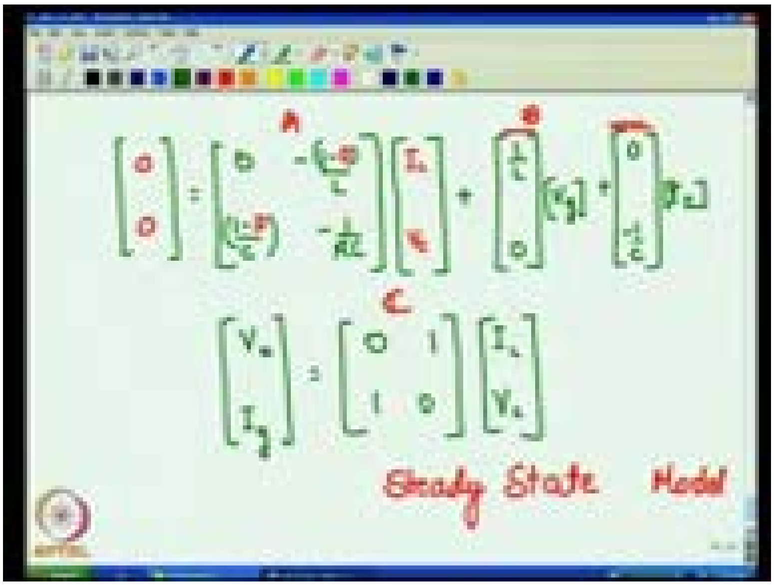

So, from the averaged large signal model we shall mark of this copy that and we shall paste it here and edit this to form this steady state model. So, the steady state model is obtained by replacing thing the derivative terms with 0 and replace the lower case d's with upper case D like this and these terms become upper case IL and V c vg I z V naught and I g or the steady state outputs that we want to get out from the state variables now this becomes the steady state models for our converter. All that we have done right now is replaced all the variables with upper case values and we have removed the derivative terms made it into 0.

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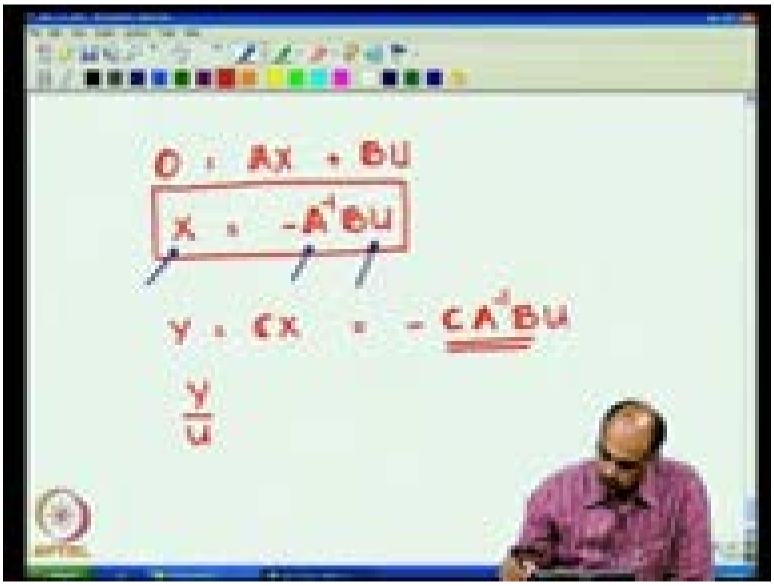

Now, this is of the form 0 is equal to A X plus B U. Now, here X and U have become upper case. So, you have steady state state variable and a steady state input variable X and U X. Steady state state variable X is obtained from by simple manipulation of these minus B U and then pre multiply by a inverse this would be the steady state relationship between the inputs. The inputs scaling the system parameters and the state variables likewise if you want to include the output equation into into this you have let us Y which is equal to C X which is minus C A inverse B U. So, if you want to get the transfer relationship between any 2 variables you have Y by U, if it were all single variables Y by U would give you all the transfer ship and that is equal to C A B however in the matrix form we do not write it in that fashion.

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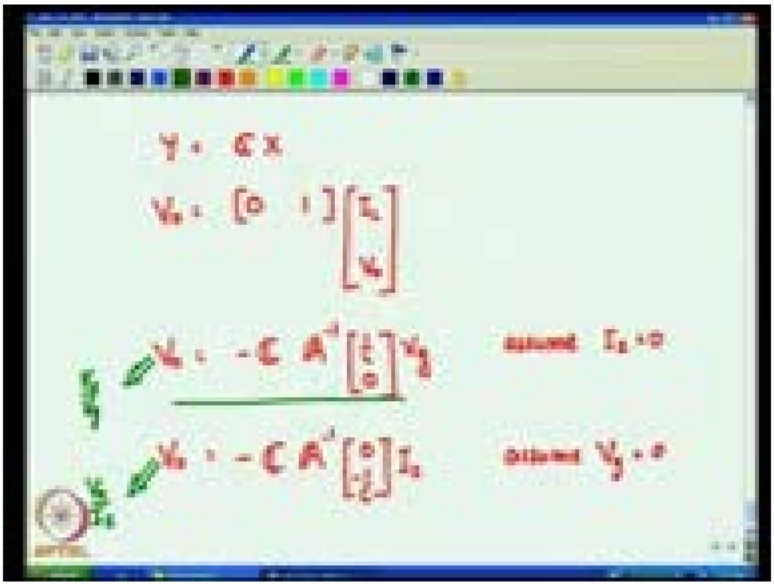

We apply it to specific variables here now here Y is equal to C X example would be V naught. Let us say V naught is equal to X in our case was I L and V naught. So, this what would be C X and V not is equal to C and X is written as A inverse and you have to take that matrix which, for which you want the transfer relationship, so let us say we want the transfer V G then multiply it with the B matrix corresponding to VG into VG assuming I Z is equal to 0.

When you want to get the transfer relationship, you are getting the transfer relationship of an output variable to and input variable assuming all other inputs are 0. Likewise you would also get the transfer relationship V naught with respect to I Z, sorry this is minus here C A inverse. Now, the B matrix corresponding to I z this will give you and here you assume V g equal to 0. So this gives you V naught by V g and this gives you v naught by I z steady state output inference.

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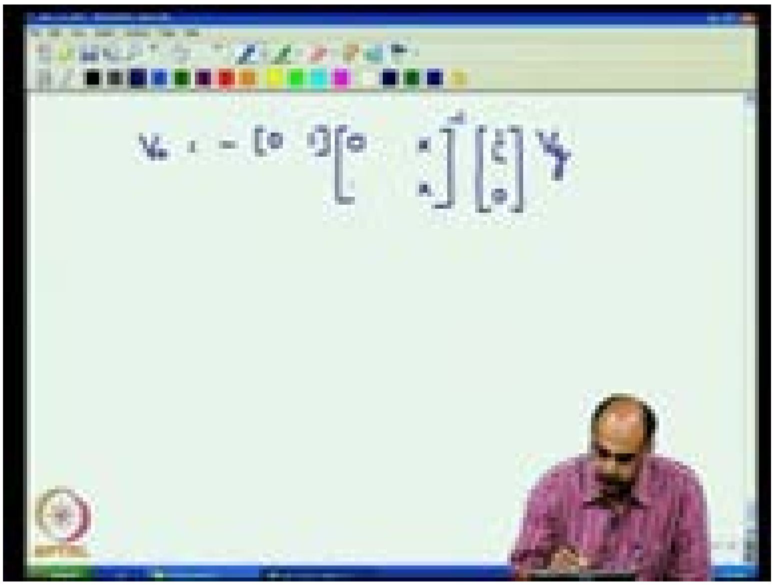

Now, take for example, this V naught is equal to minus C and A inverse b 1 by l and 0 Vg. We need not calculate of the all the elements because this is 0 these 2 are do not care there are values, but we are not bothered in obtaining the values we shall obtain values only for this portion.

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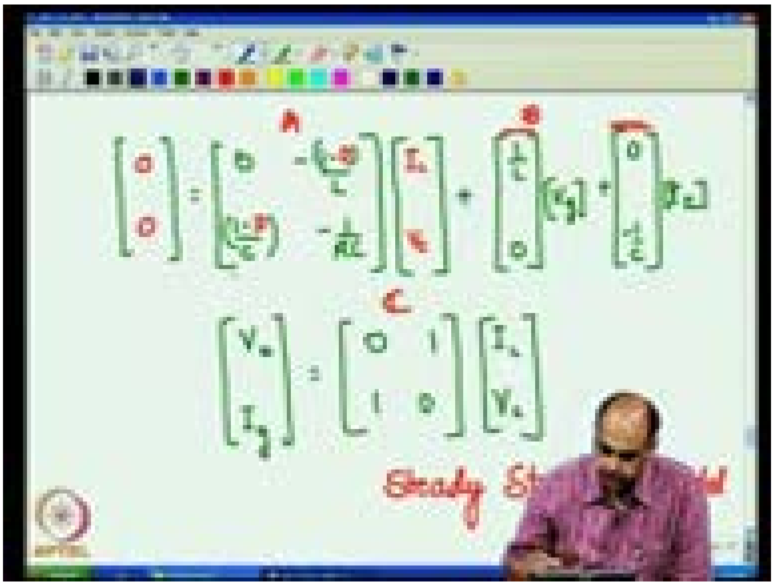

So, the determinant in this case could be find the inverse the determinant of this matrix is 1 minus (( )).

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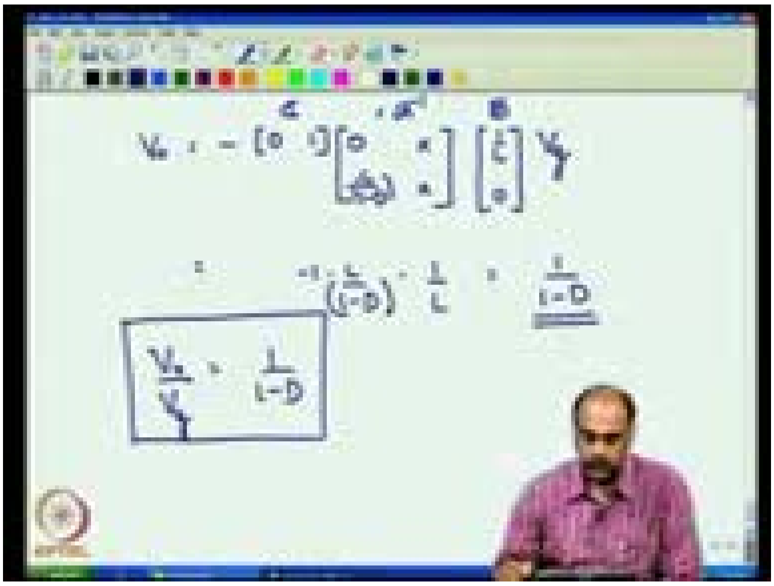

Multiplied by multiplied by the co factor and the cofactor is 1 minus d l by 1 minus d you can work it out by hand on paper on this on multiplication gives you 1 into l by 1 minus d into 1 by l which is equal to, sorry this is actually A inverse this is equal to A inverse I do not have to put there, so you have here c A inverse this is B corresponding to the Vg input. So, you see the v naught by Vg which is equal to 1 minus d and this is the relationship steady state relationship that you had discuss very early in the course likewise steady state relationship of the any output variable to the input variable.

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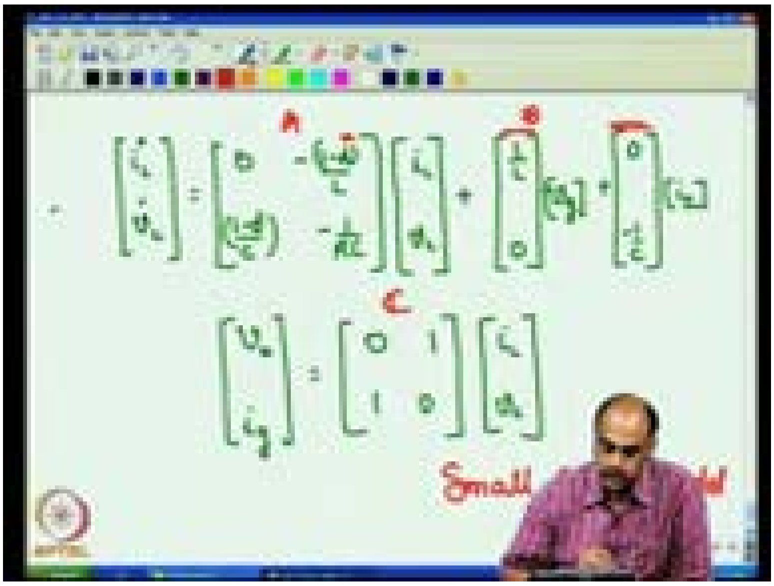

So, preceding from here we can.

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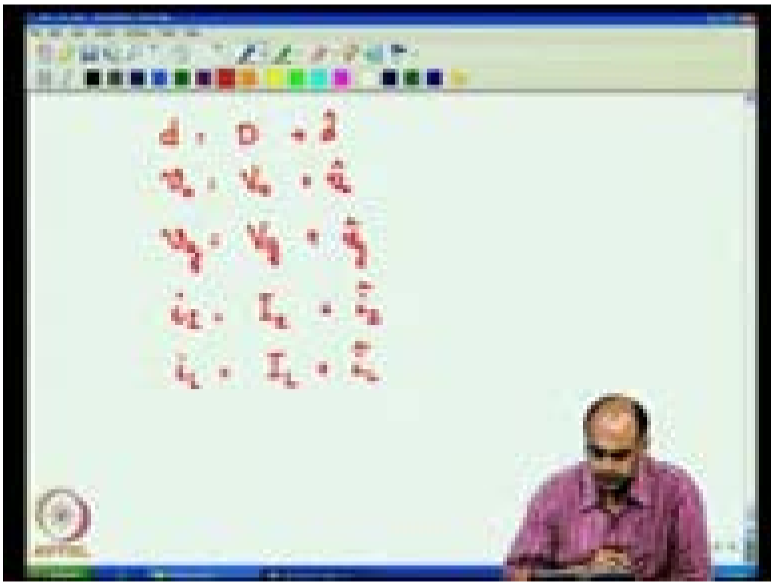

We can now try to get these small signal models so these small signal model is obtained by substituting in the average signal model for every variable a steady state part and A small signal variation about the steady state or equilibrium point v naught is equal to v naught plus v naught hat, so the variation about the.

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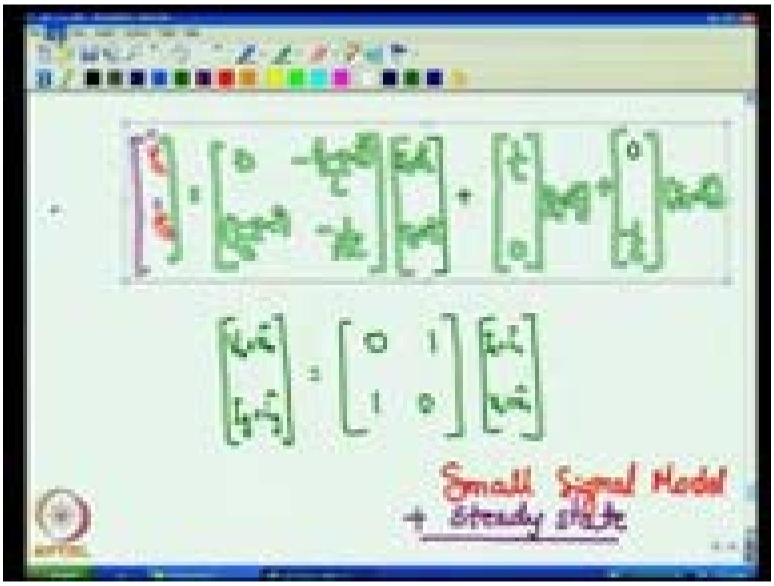

Equilibrium point were indicating by hat Vg is equal to Vg plus Vg hat Iz is equal to steady state part plus and you have I l which is having a steady part plus I l hat and so on. And we tried to make this substation everywhere, so when we tried to make this substation there will be new terms which will be coming in. So, this will get substituted as I l dot plus I l hat we see dot v c hat. Now, this portion becomes minus d minus d hat v plus d likewise minus d minus d hat and these variables will become also I l.

Plus I l hat v c plus v c hat I will vg plus vg hat and I z plus I z hat and likewise the output equation. So, the output equation you could take v naught as the output v naught plus v naught hat or you could take I g has output I l plus I hat plus v c v hat. So, still the small signal model is not complete this is actually having small signal plus steady state plus steady state included.

Now, we have to split this remove the steady state part we have to split this and remove the steady state part and then you get the small signal part, so we know that the I l dot v c dot I r the derivatives of the steady state, so they are apparently 0 therefore, we cannot remove we cannot remove that part. We have basically the small signal dynamics and here we shall perform new operations take this portion into a new page of this fact.

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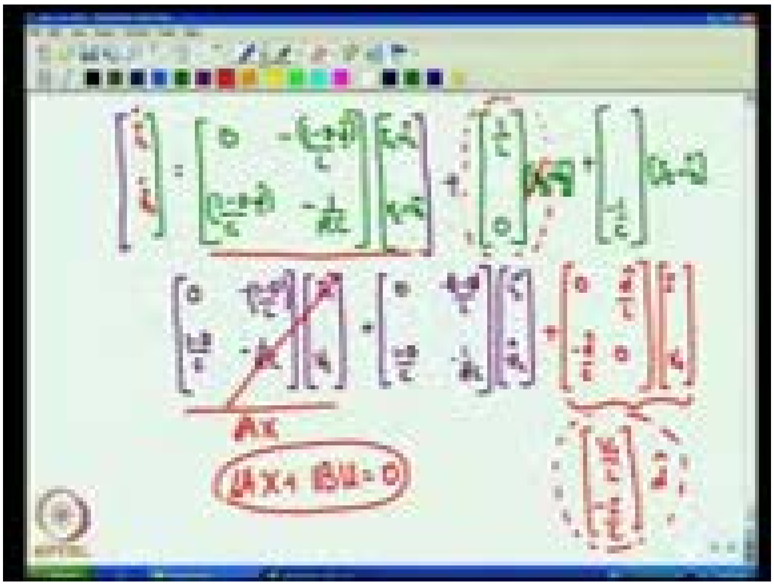

Yeah, now the operations that we in the form is this very simple we split this matrices. So, 0 minus 1 minus d by l 1 minus d by c 1 by r c and I l v c plus 0 1 minus d by l 1 minus d by c 1 by r c and I l hat v c hat. So, this matrix is split into these 2 parts of course, we consider these 2 hats, so that will come as 0 d hat by l minus d hat by c 0. I and v naught. So, what we shall now do from here is to use d hat we will take out d hat and push this steady state components inside, so this actually would become I by l v naught by l.

That is trying to link 2 and this 2 minus I by c d hat and that can be written in this form here this is one of the steady state part that is the steady state part the ax part steady state part the ax part and then you have as steady state b steady state part here b u part. So, that part and this part is that that equal to 0 the steady state part so equal to 0 because A x plus B u equal to 0, so this part, so Vg into one by l become can be eliminated.

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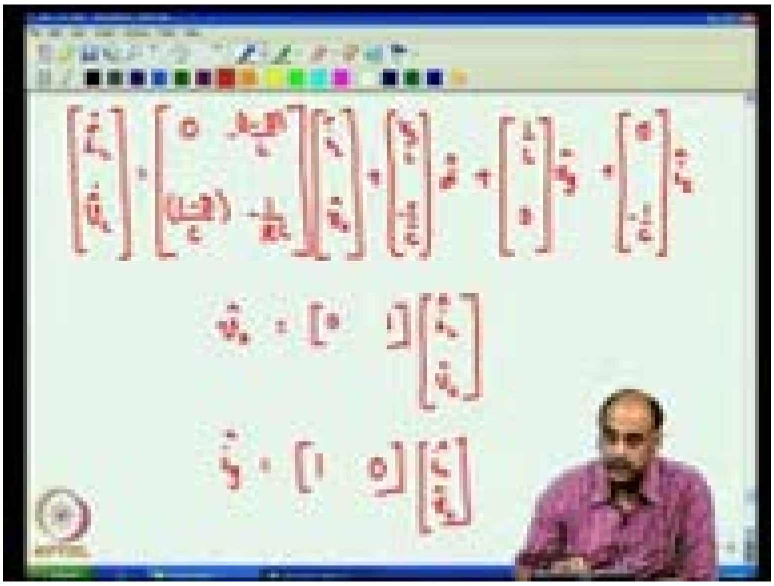

So, from here you simplify you land up with the following, I l hat dot, v c hat dot. So, these are becomes 0 1 minus d by l 1 minus d by c minus 1 by r c. And you have only the small signal components this is corresponding to this corresponds to this portion plus v c by l minus I by c into d hat that corresponds to this portion plus 1 by l and 0 Vg hat. That corresponds to this portion and finally, this portion we incorporate it in here 0 minus 1 by c I z hat. So, this becomes the state equation of the small signal model plus you can have the equation of the output models output equation would be of this form 0 1 I l hat v c hat, so this could be for one equation or you could have output as I g the input current itself.

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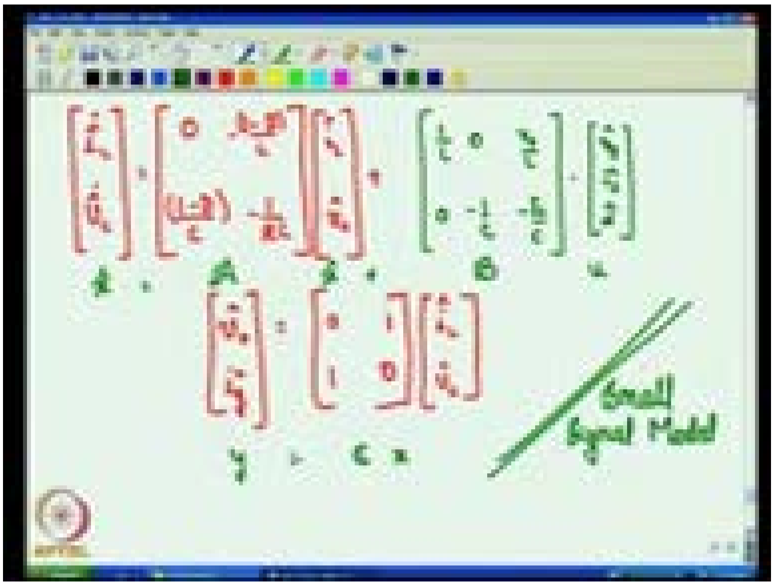

Which is of this form so either you could have this or this. The output equation also could be combined together in the following manner you could have I g hat like hat 1 0 so this also results in the same output equation y is equal to c x x dot hat equals a x hat plus another matrix. Let us say m d hat plus or we will put it b 1 d hat plus b 2 V g hat plus b 3 I z hat you could also combine the three inputs in the following manner. So what we shall do these three inputs.

We shall combine in this fashion to make it into a common input where in you are having three inputs Vg hat I z hat d hat and Vg hat 1 by l, then you have 0 minus 1 by c then v naught by l minus I l by c. So, this forms this would form that b matrix and the u vector. So, this model here would represent the small signal model of the boost convertor.

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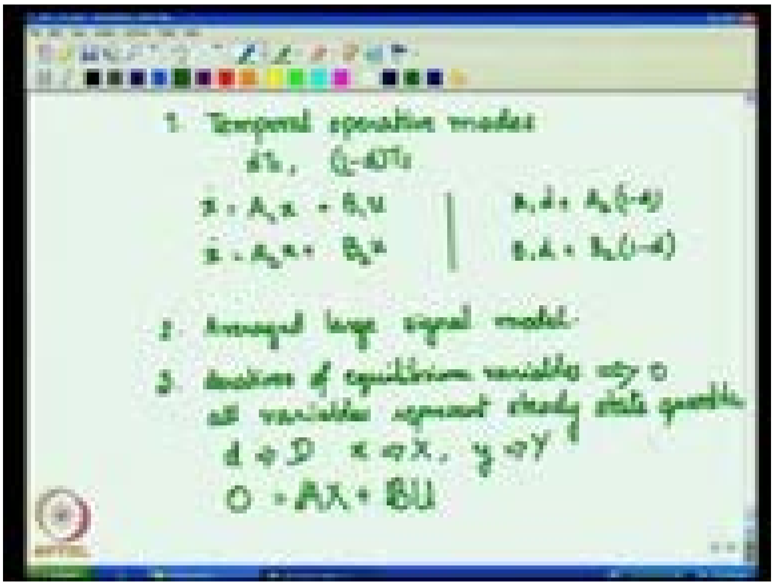

So, summarizing the process we first from the circuit depending upon the temporal modes operative modes that is d t s 1 minus d t s timed duration split the circuit and obtain the each models for each of the temporal modes. That is you have a 1 x plus b 1 u and that would be extra and then you would have a 2 x and b 2 u then you can combine them by making a averaging in this fashion.

Likewise for the output equation also once you have combine them you have what is known as the averaged large signal model. Then from the average large signal model the derivatives of equilibrium variables or states are set equal to 0 and all other variables represent steady state quantities that is d becomes d x becomes x y becomes upper case y they represent the steady state quantities. So, you will get a x plus b u subsisting you will get the steady state model after you have got the steady state model

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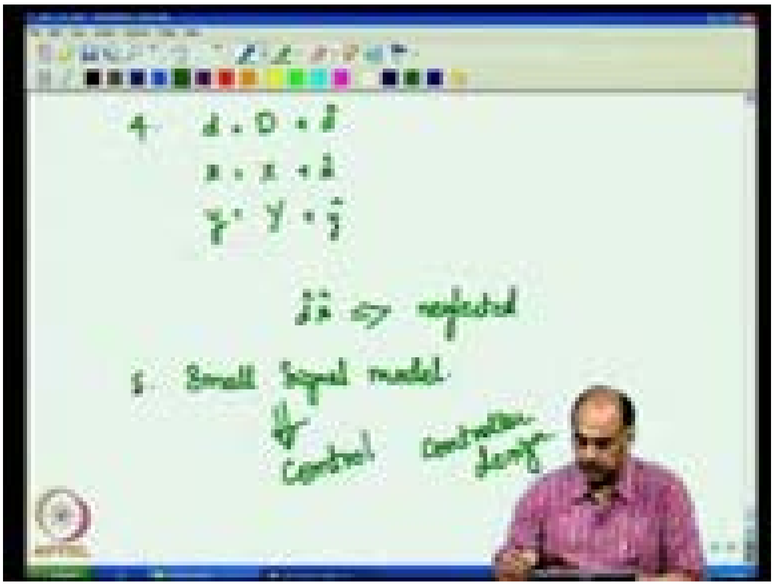

The average large signal model is split every variable has a steady state term and and small signal terms a small signal variation the neighborhood of the operating point. So, this average large signal model is split up with these remove the steady state portion and to the remaining terms apply the constrain the product of small signal terms like example this is in significant and this is very small and can be neglected take only the first order small signal products

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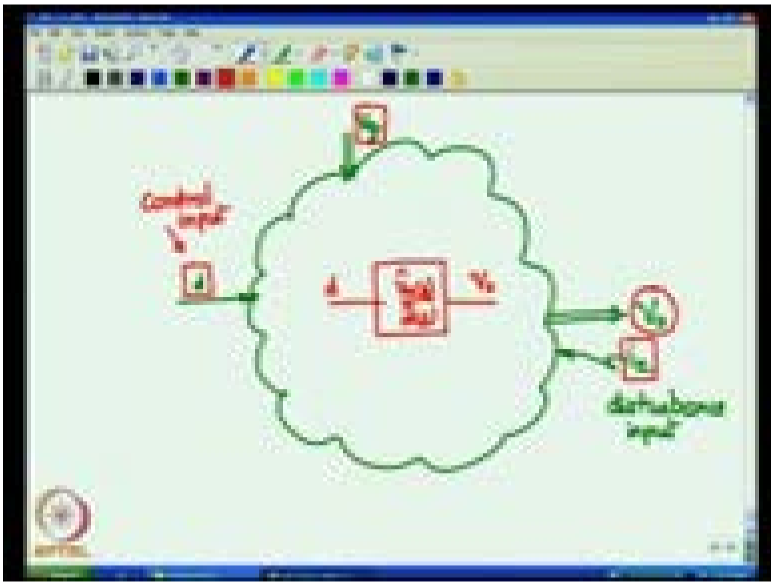

You will land up with the small signal model and this small signal model to be used for your control or controller design controller design. So, if you look at our plant for example, in this case is the boost converter and let me say I will controlled convertor is something like this we have a power input which is called vg and we have and we have an output which we are calling it at v naught.

We have a controlled input d you also have controlling the load I z which is coming as an external input and this is an disturbance input, this is not a controlled input the load is not under your control it is an external disturbance the load change depending upon the application. Therefore, it is consider as a disturbance input. V g itself itself is an input because the variation in vg is to e b taken into account by varying d and d indirectly varies g to the system.

So, you have let us say an output and three inputs and half which is d is the control input this is called d control input. So, when you want to design you need to have a transfer function of the system v naught with respect to d, so if you take the transfer function which is the (( )) transform of output by input you are required to obtain v naught s by d s or if it is the small signal model v not hat s d hat s. So, this is the controlled t funcanfertion model that you will be using.

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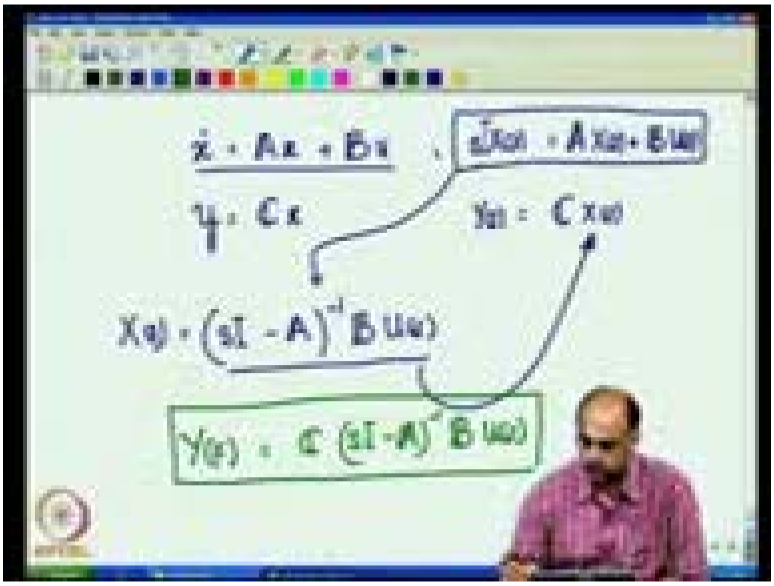

So let us see now how we go about getting the transfer function from the state spares model you know that the state spares model is given by A x plus B u and y is equal to c x now, if you take this the derivative term take the (( )) transform. So, you have s x s a x s plus b u s y is c x s. So from here you have s I minus a inverse b u s is x. So, you have actually have an eye here s I minus a push it to the other side s I minus a inverse b u s. Now, this you substitute here, so what do you get you get y of s, which is c s I minus A inverse B u s.

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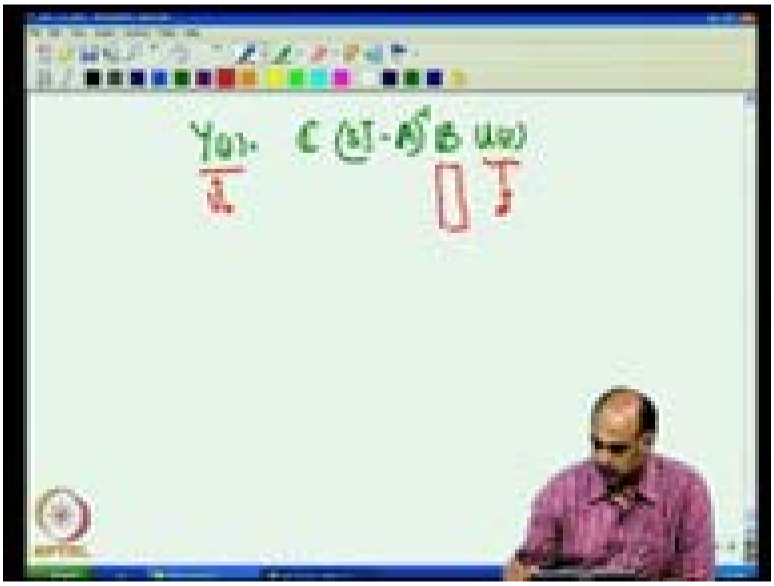

This is how you would obtain the transfer function, so looking at that c s I minus A inverse B u s, if you want to obtain v naught as your output variable and input variable is d hat.

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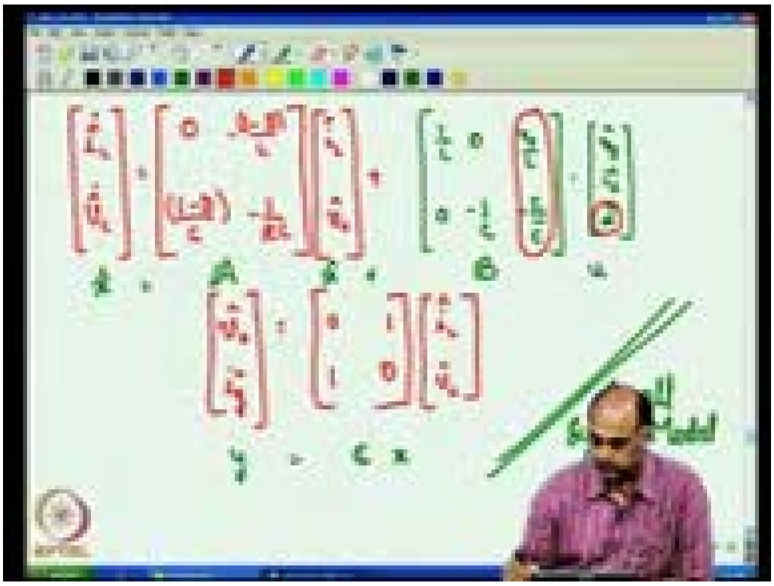

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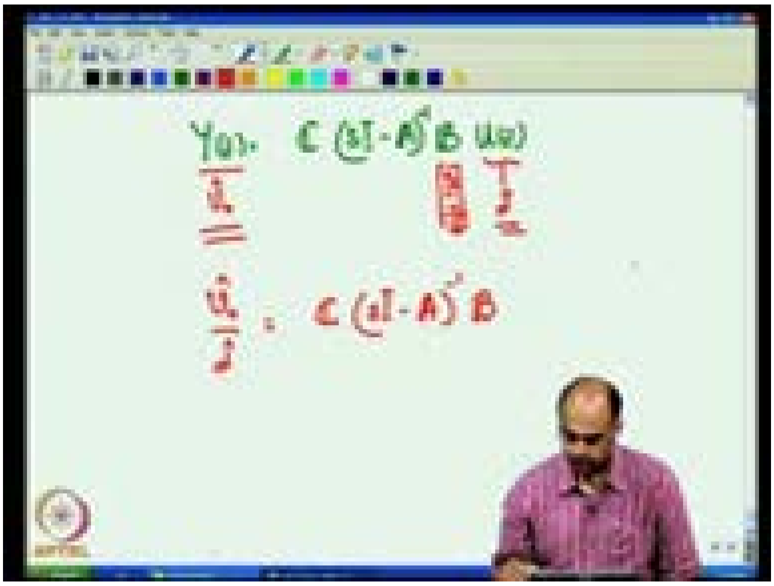

The B will correspond to only that portion corresponding to d hat. So, if you take the small signal model here corresponding to d hat you have this column of the b matrix. So, only that column need to be consider because transfer functions are for single input single output only, so that is v c by l I l minus I l by c and s I minus a inverse is the A matrix which is same for all input and that can be operated upon to obtain the transfer relationship between this or this or v naught hat by d hat is equal to c s I minus A inverse.

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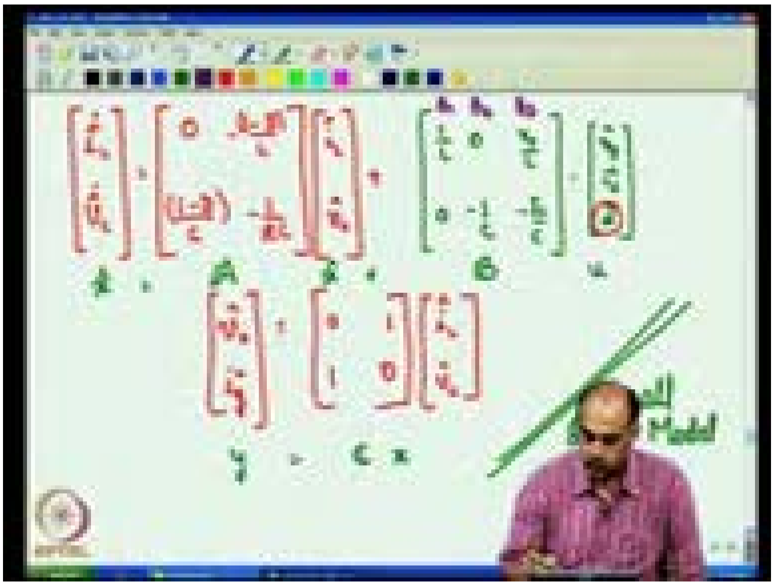

If we consider the the b matrices columns, so let us say b column 1 b column 2 b column 3, so b column 1 we used for Vg b column 2 is used for I z b column 3 is used for d in trying to obtain for the heat transfer functions.

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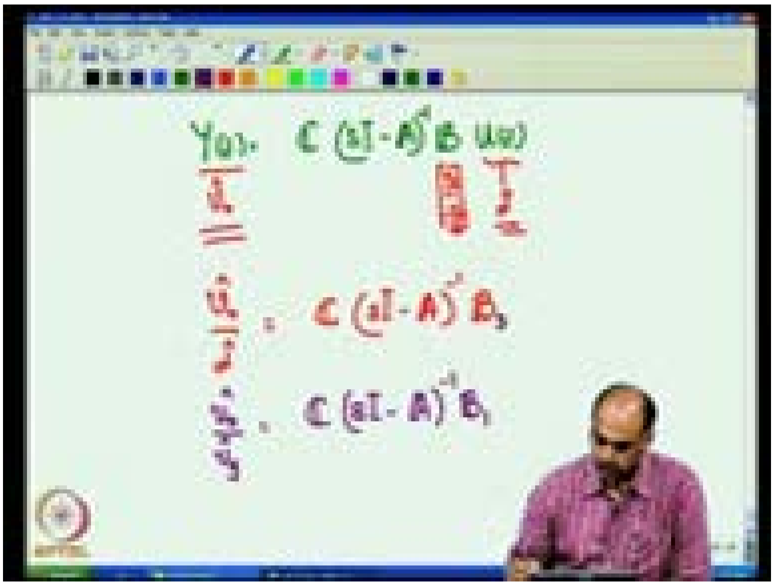

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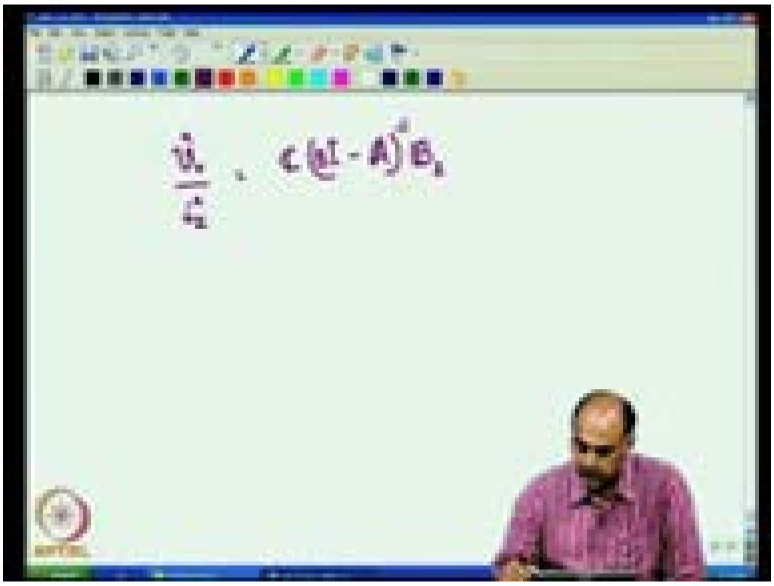

So, you use B column 3 in this specific or if you need to have v naught hat by v g hat you would s has I minus a inverse b column 1 or if you would like to have v naught hat by I z load with load transfer function load disturbance, you will have c s minus a inverse b column 2 and so on like that.

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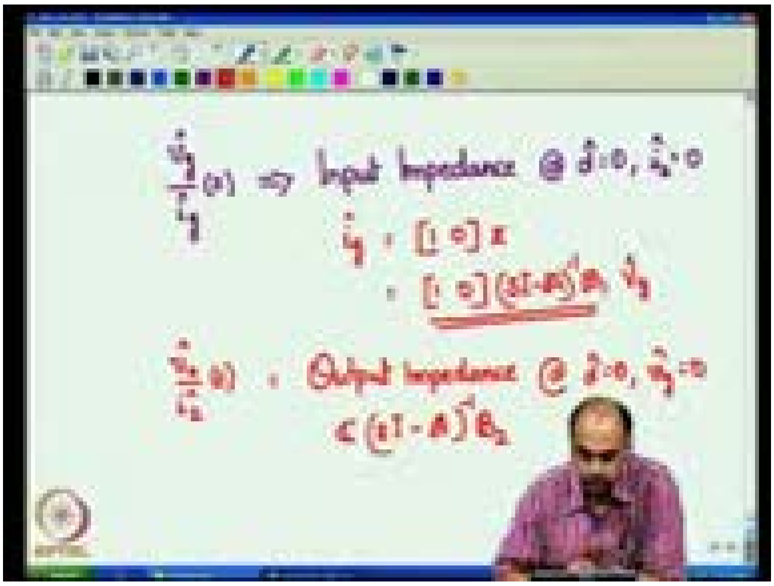

So, there are certain names which have been given and you will find them in the literature this are common names, so if you get the transfer function V g hat by let us say I g hat lap less transform which is obtained by c Si minus A inverse, no these are general terms wipe it out. This is called the input impedance transfer function at d hat equal to 0 I z hat is equal to 0, how do we obtain this? We know that I g has an output is given by a c which is 1 0 by like this by x.

Therefore, we can obtain it in this fashion s I minus A inverse and the input b 1 for Vg Vg hat so I g by V g hat is obtainable by this, inverse of that we will give you the impedance give the input impedance then another common transfer function is v naught hat by I z hat (( )) and this is called the output impedance. Now, hat that is the hat is equal to 0 V g hat the 0 I g hat is the disturbance of the input, so I z is the input v naught is an output, so this directly obtainable using s I minus a inverse b 2.

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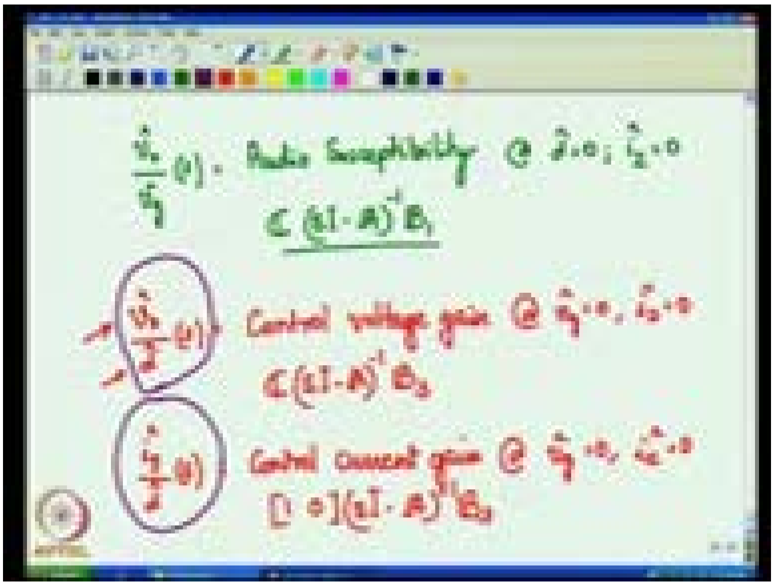

Another transfer function is v naught hat by Vg hat, this is the input output voltage relationship this is also called audio susceptibility. See, d or d hat is the control input that control input is actually changing Vg and Vg hat is also one of the inputs and variations in Vg can also get reflected and changes in the output and these output are low frequency low signal variations. They are in the audio range are the low frequency audio and the regulation susceptibility low frequency disturbances is called the audio susceptibility such a name is given and this at the d input being 0 and the I z input being 0. So, this a is also directly s I minus A inverse this is the b 1 matrix.

You have another transfer function this is the control voltage gain at Vg hat is equal to 0 I z hat equal to 0 and this is obtained see d is an input this is the output c s I minus a inverse b 3 column, likewise you have you saw that you could have another output vg the input current. So, if input current is an output and it is getting controlled by the control input d the it is called control current gain and this again I z hat will be equal to 0 and this will be calculated as c (( )) a inverse b 3 column. So, this is how you get these various transfer functions, but for control remember that you are doing the control with duty cycle.

If you are doing output voltage control, you will be using this transfer function. If you are doing the input current control you will be using this transfer function. Now, in the till now we have done some derivatives on these converters through the process of circuit averaging and what we have done is on paper consider the switches as an ideal. And then find out the average large signal model the steady state model and the small signal model. And from the small signal model you can derive the various transfer functions, so you have a state model and the various transfer function that can be derived and used for analysis purpose and for controlled purpose.

Especially the transfer function with respect to the control input that is the duty ratio will be used for you can do either voltage control or current control. More or less this is the approach for all the converters and I suggest that you practice with different isolated and non isolated converters in this same manner. In the next class we will try to use mat lab and probably also spies to do some simulation and see how, how these plant performs so that it will become more intuitive and more easy for us to understand when we take up the controls.

Thank you.