Switched Mode Power Conversion
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Lecture - 23
Circuit Averaging - 11

Good day to all of you in the last class we saw how we can go about developing the state space representation of a DC DC convertor, specifically we did work using the buck convertor as an example. Now, in this class we shall continue on the same topic of circuit averaging and we shall consolidate at what consolidate what we learnt in the last class with few more examples. We will see what we want to do with the state space representation that we have obtained in such a manner.

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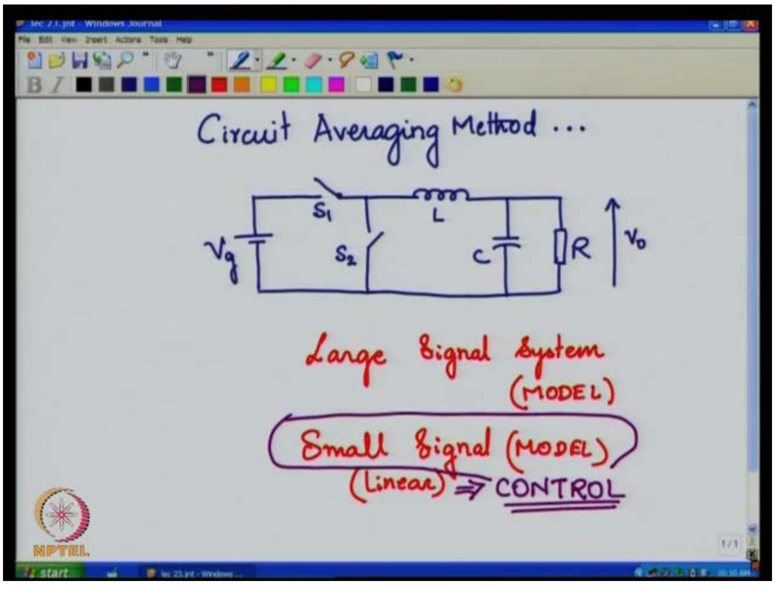

So this is circuit averaging method continued circuit averaging method we shall continue it. So, in the last class we had developed the model for the buck convertor which is represented by these two single pole single throw switches and an inductor on the capacitance. By now, you are familiar with the buck convertor which is attach to a load r s 1 s 2 V g and this voltage across the resistance, supposed to be v naught.

Now, here this buck convertor is a system where all the variables within it are actually swinging from 0 to some nominal operating time. This is a large signal large signal system having a large signal model large signal system on this has a large signal model.

It has time warring tricks due to the switching character two switches s 1 and s 2 and the switches circuit 1 the switches two is on we have another circuit and these two operating more smarting picture points of time during d t and 1 minus d t.

So, from the large signal model we develop the averaged large signal model and from the average large signal model we removed the study state model and having the removed state portion for model is left out is this small model small signal model. Now, this small signal model is the one which is linear and this is what will be used for control process for all around your control sign. You will be using the small single model, now let us revisit something that we did learnt in an earlier class.

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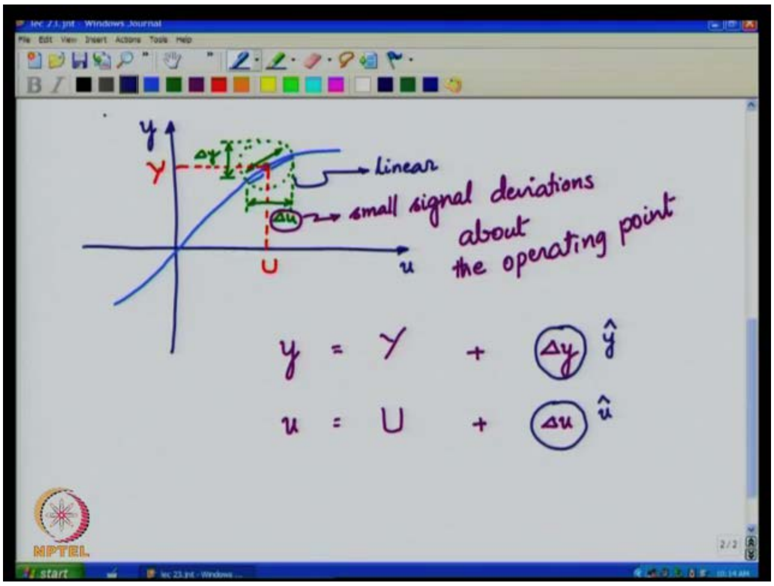

If you remember we had that transfer curve and this transfer curve between input and output could have let us say some kind of characteristic like this. We have an operating point and about the operating point, about this operating point in the neighborhood of the operating point. Let us call this as the neighborhood we say the system behavior system behavior in this neighborhood is linear in this neighborhood is linear. So, this non mineral study state operating point for an input U, you have a corresponding input Y and for a variation in the input delta u you have a variation in the output delta y.

Now, this these two are called the small signal deviation deviations about the operating point about the operating point which is the state linear or the equilibrium condition. So, we see that the large signal model y is composed of two dusting parts the study state part plus a small signal part delta y. In the Same way this large signal input u is composed of a study state part plus delta u, now this delta y delta u are the derivations in the neighborhood the operating point.

We have actually termed them with a different symbol in when we were developing the state space representation we used to call them if you recall y hat and the delta u used to be called u hat. Therefore, we can say y the large signal part is nothing but, a study state part represented by upper case y plus a small signal part representation the lower case letter with a hat on top.

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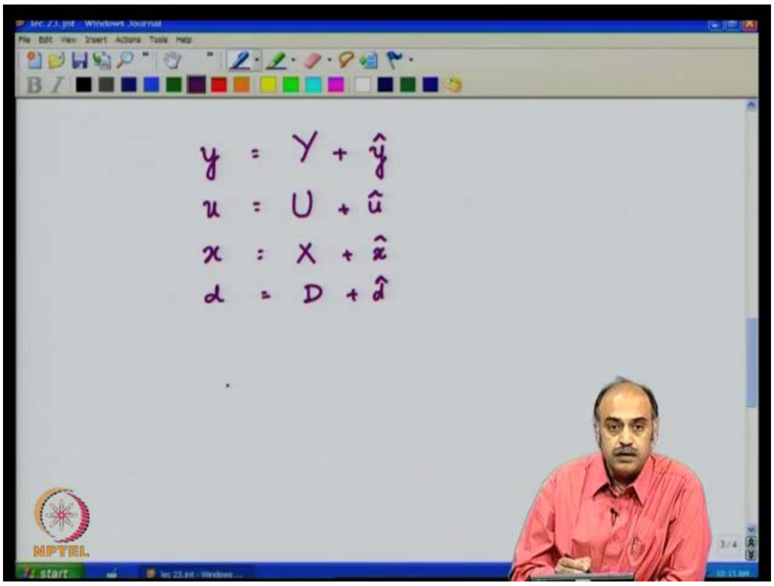

So, you have used which is having a study state part u hat x which has study state part x hat so on d which has a study state part plus d hat so on, all this deviations are around here, now after having done the small signal model.

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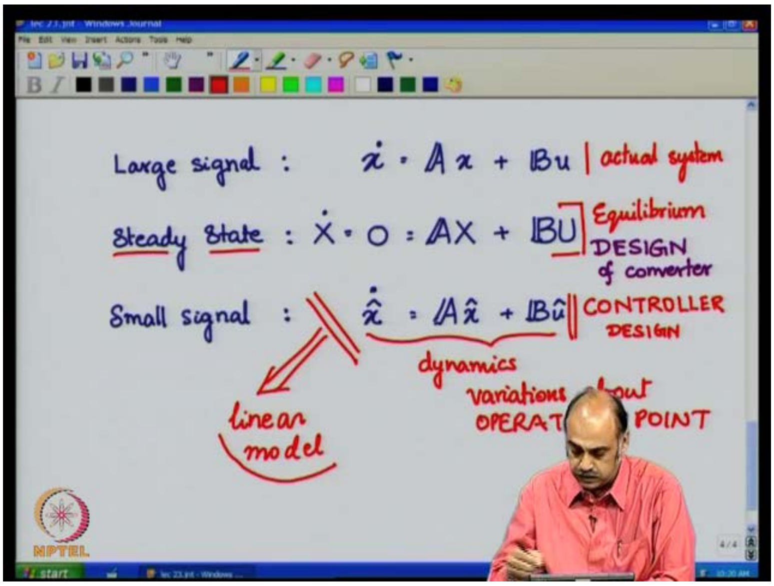

What do we do with it, we have three models here large signal model which is given by x dote which is equal to a x plus B U, you have the study state model which is 0 because X dot is equal to 0 x dote equals 0 equals A x plus B U. This is the steady state model and then you have a small signal, so in the small signal model we have x hat dot variations A x hat plus B U hat. So, I have written only the state equations likewise the output equations also you will have these three models. So, this large signal model is actually the one closes through the actual system it represents the actual system more closely the steady state model represents the actual system during equilibrium condition.

As such, most of the time the system is going to be in a steady state for a large portion of a time and therefore, you are design the design of the plant is done using these equations. So, this is very important, so this steady state model especially for designing the plant itself. So, you would have recalled that design of plant or converter designing the value of inverter designing the value the c so on and so forth, probably in much later we will see how you use the steady state model to design the converter.

Then, the third part the one which we have struggled so long to get the small signal model is used for control purposes control design control design and also to design the controller. So, for both these purposes we will be using the small signal because to note that it is the small signal model it gives you which gives you the idea of the dynamics. It gives you the idea of the dynamics implying that variation about variation or deviation about operating point and therefore, the control charge these deviations are made 0.

So, one of the constraints is that controller design is based on all methods where it assumes the plant is linear and because of that we need to have this model especially. As a linear model, which is what it is we do the linearization by neglecting the second order terms of the deviations basically what we have done is obtained linear model that is why the value of B do not have time.

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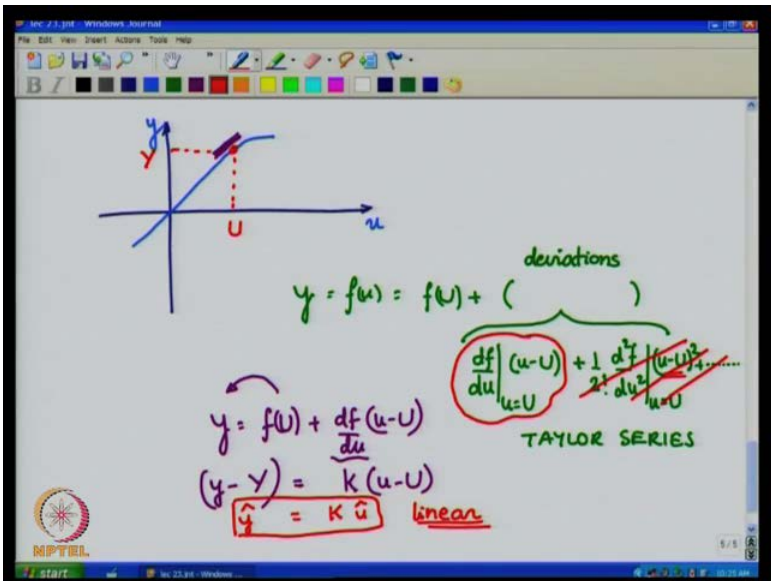

If you recall more generations, we had we had mentioned that earlier and discussed that earlier let me just recap that because it is very crucial. We should understand and that you have the steady state part you have the deviations in the neighborhood of the operating point. If you take if you take y which is the form function of u which is nothing but function of steady state normal operating point terms plus which contain deviations due to the input deviations of the operating point.

Now, if you this recall if you recall we had used the tailor series to b f by d u deviation of f with respect to u evaluated u equal to nominal rating point into deviation U minus U plus 1 by 2 factorial d square f by d u square evaluated at the nominal operating point minus u whole u minus u whole square plus, so on keep going. So, this is nothing but tailor series expansion and out of you know that these deviations are very small quantities with respect to operating point, so they are much lesser than one and the square term. Will the normalized be still further lesser than 1 and therefore, the highest order terms above square, this itself be smaller further smaller.

Therefore, all this higher order square terms will be insignificant we can neglected which means out of the sailor series terms there only taking order terms. That makes it why which is equal to f of u plus d f y d u evaluated at U minus, then do at in this neighborhood of the operating point. We see that it is linear and therefore this is nothing but a constant k.

Then, when you the f of u onto this side it becomes y in that steady state which is equal to k u minus u so this is nothing but y hat the deviations about the operating point k which is the constant it is the d hat. So, this is actually the small signal and you see that the small signal model is linear and this is exactly what we have done in obtaining the small signal the state representation model of the DC DC converter.

Here, we obtained the average signal model large signal model we removed the steady state model and when we were doing the large signal model multiplying the various variable which contain the upper case and lower case atom. We neglected the small signal multiplication terms assuming they are insignificant, therefore we had only the first order terms which made the automatically the whole system linear.

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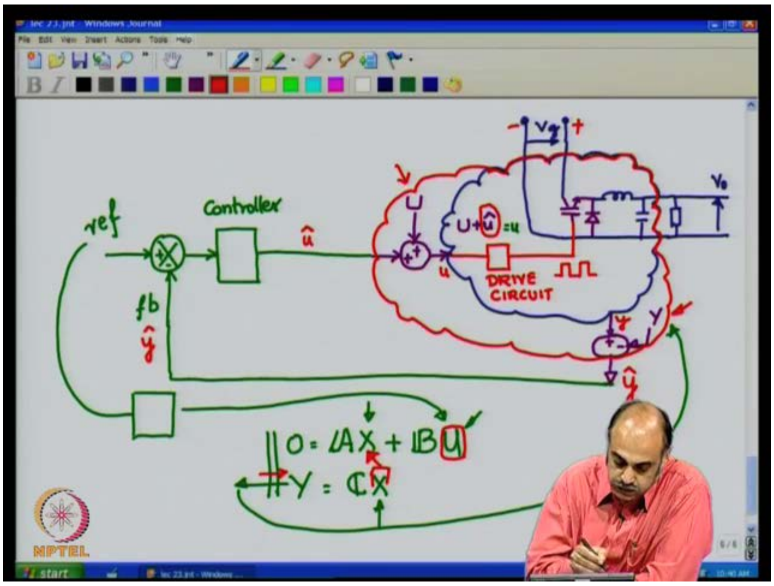

So, this what you would arrival therefore, if you now go back to the plant, now this is the plant which is having let us say which is having let me put it like this. So, from here you have a switch another switch inductor capacitance and this goes out to the lower and this is our conductor. Here, you have you were load resistance that may raise it here and so that you are able to c properly this will be the lower resistance.

We are going to put the input here this is v g this is v not which come out like this, now this is our plant this symbolic enclosure which indicates that whole thing is the plant. Now, to this plant we have a control input u and we have an output to be controlled which is Y and somewhere here. Let us have a comparator plus minus where you place the reference this is the feedback portion and the output of the comparator goes to the controller. The output of the control is the input which is this the structure control structure of the plant now what is this control input going to control where is going to go.

So, this control signal actual gets simplify we call it is the dry circuit, so circuit and output of the dry circuit is going to control this switch and the other switch is controlled it is automatic by virtual of fact that this can be divert. So, let us replace that is will become at diode and we will have a control switch so here let us have a diode. Here, let us have a controlled switch like the bit, so this dry circuit is going to control this.

So, you have the dry circuit output giving something like that pulses so whenever the pulse is high the switch is on the moment the switch is on positive is going to come to this point the diode point and the diode get reversed biased off. Therefore, it is off, so this would be like mode of operation where the supply we going to B and C and that the second mode there is a polar reversal of the inductor voltage is within it will free. So, automatically lets conducting and also this switch one minus d period, so this is the kind of situation you have. Now, let us see what the controller is suppose to do and here if you look at then this how much large signal this is a large signal or plant.

The plant always is large signal is existing is large signal model, now out of this large signal model there are two parts we have split here this is equal to a steady state part plus the small signal part and the u here that you have. Suppose, it containing a steady state part plus a small signal part, therefore here within everything is large signal, but the controller is design to handle only the small signal or the parts, so the controller is both were and going to handle only this.

Therefore, the controller is designed to address this which means what we are feeling back should y hat and what you are getting out of the controller is u hat. So, it is no longer U, but u hat, U is within, this is U, it is within which implies that there are two small blocks changes that will happen and that is one is here and another is here.

So, what are those changes, so let us put harder or subtract that you may call it that point and what one harder or subtracted of this point and that is what is now considered as the new output and that is called the new input to the system. So, we will say this is u and this is Y, so as usual this two are retain same large signal value, but what you are actually giving to the controller is here, so the controller is looking at this kind of a model.

So, it will change and go like that, so this red is the plant that the controller is going to look at. So, what do you have here the output here is nothing but y hat you need to have y hat because that is what you can feed back to the controller because the controller can handle only this small signal. So, controller is also seeing only the small signal model of the system which means that here you are subtracting this is plus minus can you make out your subtracting.

Let me make it more a clear inside here the y output large signal output is subtracted this is plus this is minus from Y. It is the steady state portion so large signal minus the steady state is giving you the small signal, likewise you need to add where you need thing plus and plus you need to have the steady state. Now, the steady state quantities are obtained from the obtained from the study state model you know that 0 which is equal to a x plus B U and you have y equals c x plus B U. Now, the output does not have the direct as you saw in the case of the buck convertor this is not that this portion is what we have.

Taking these two equations, you can always get y let us say if you know the input u you can get the x because from this equation the only unknown will be X. Once you know the x from this equation you substitute here and get out Y, so that is the y you will use here and how do you get the u from the reference you have the reference and from the reference you have the study state relationship to get U.

So, this is an inverse relationship I will take another color so how does it go the reference is nothing but what you want the output to be v not or what you want y to be. So, y is given and from y you can get x, so from x you substitute here, then you can get U, the reference for the study operating point. So, this from the inversing equation you can get this and from the other equation you can get Y, so this way you can use the study state model to subtract and add and get this model values. So, let us now complete this, now here the reference are the reference is split into two parts, so you will have two parts for the reference which we will be splitting.

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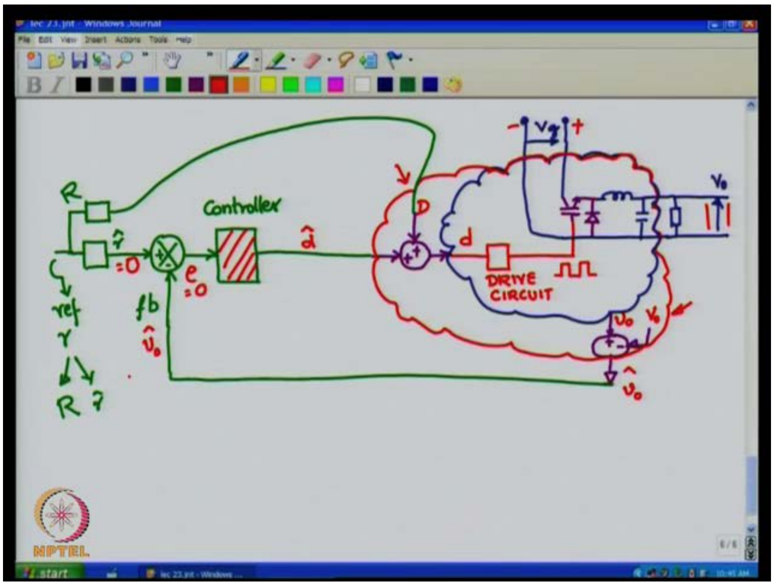

So, let us say this is reference are now r itself is having the study states part and R and hat part r hat should be the reference here and R is what should go to directly to the study state part. So, there should be something here and there should be something here which direct which directly takes the r part here. So, the study state part is directly given to controller the controller is by pass for giving the state giving state part of the reference this will be the r part of the reference. Now, what should be the r part of the reference I should not have the d v and therefore it should be 0.

So, this should be equal to 0, there should not be there should not be deviation for the reference so y hat should eventually be such that controller should take care that y hat eventually reach 0 by making this 0. Then, the whole system will be equilibrium provided by represent y the study state, so this will be these structure of the controller before any controller that matter, so we have specifically taken this DC DC converter mode.

So, if you look at the issue if you look at the issue of the controller design it boils down to designing this controller as for small signal model as shown by the red. Now, in this case for the buck converter the input u what is the input u the input u is nothing but, the deviations of the duty so here you are going to trigger to this fridge and what to the switch it is the duty cycle. So, if you keep on raising is things and c replace it by appropriate so for the appropriate symbols.

So, for the converter here this will be the input u control input in D and what will be the steady and what will be the small signal this will be small signal D and this will be the upper case D. So, you get d duty cycle which then passed to appropriate ramp and dry circuit will give you the signal for switching on the devices what will be output y hat. So, the outputs here see the output here would be not here upper case V, here it is lower case v naught hat because we want to control this the output voltage. So, coming back here we see that it becomes b naught hat and what are the references, so the references also change in symbol.

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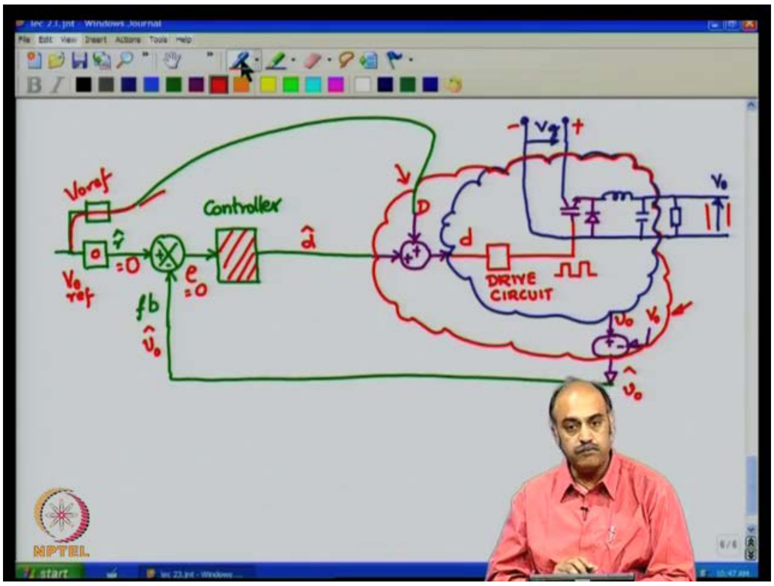

So, this will become v not reference the value is 0 for hat portion and this would become v naught v naught ref portion. So, the study state portion passes through like this and effects directly d by a scalar term and this gets scared multiplied by 0 to provide as a 0 reference equation. So, this will be how the controller if the movement there is deviation separating point that will mean that is the non zero v hat will be 0. That will go as a feedback, it will compare with 0, the other will be non zero control taken action d hat will be provided which will get added to the portion goes into the consolidated the duty cycle value goes into this trigger control and gives the control input.

This will bring the b naught to it not normal value, so this is how the control value equation will happen and for any other convertors this portion will get replace by the declare convertor. So, this is basically the frame work in which we will be find to operating all the DC DC convertors. Now, what we do in the remaining time today is to work out one example completely from scratch that is we start from the basic what we do the DC convertors. We develop the state equation different mode combine them have the average large signal model from the average large signal model subtract the study state model.

From that, you have the neat step which is not a signal model so that would give you and then identify the control inputs up to this point would give you state representation in the complete control. So, we would like we would like to go through step for another convertor the boost convertor for now I have worked it here. I have this sheet with me that it will help in going smoothly I will not do the working here you will have to work it out separately. I will try to give you the in every step the final answer so that we are not derivate in derive and working, but other to CD flow of how we go, so let us start begin by working on this.

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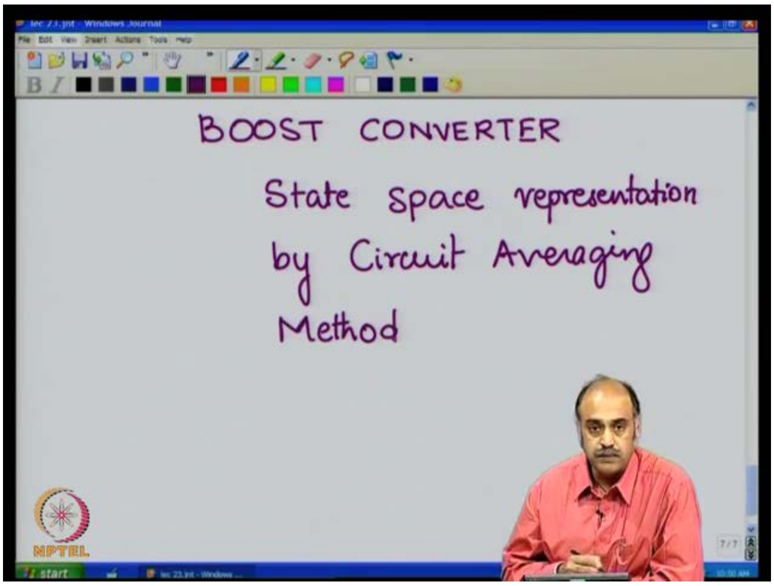

With this boost convertors, we will do it with the step by step and work through this a depend process, so boost convertors we are going to obtain the state space representation by substitute averaging method. So, this is what we want to do, this is our pole and to this end we will do, so first this is simple you know that.

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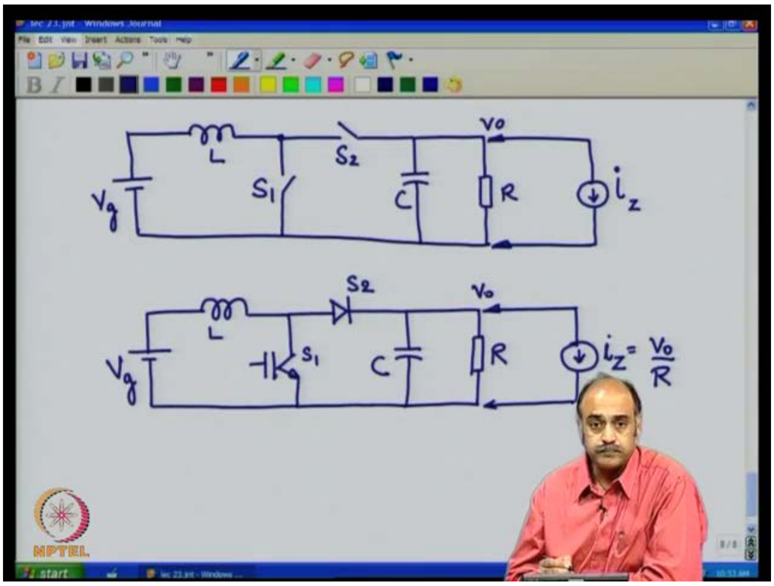

It have been you have the spector and at this load point how one switch how another switch here we have seen and you have the resistance are the resistance. So, this is the called it as switch 1, this is switch 2 RCL V g and v naught, now this can also be also can be written in circuit form where circuit representation of the switch symbol L. This can be a transistor I g b t any of those and switch to s 2 will be a diode you have C and R, this is and we give and we may not here at the output. So, this is the representation in the circuit from now.

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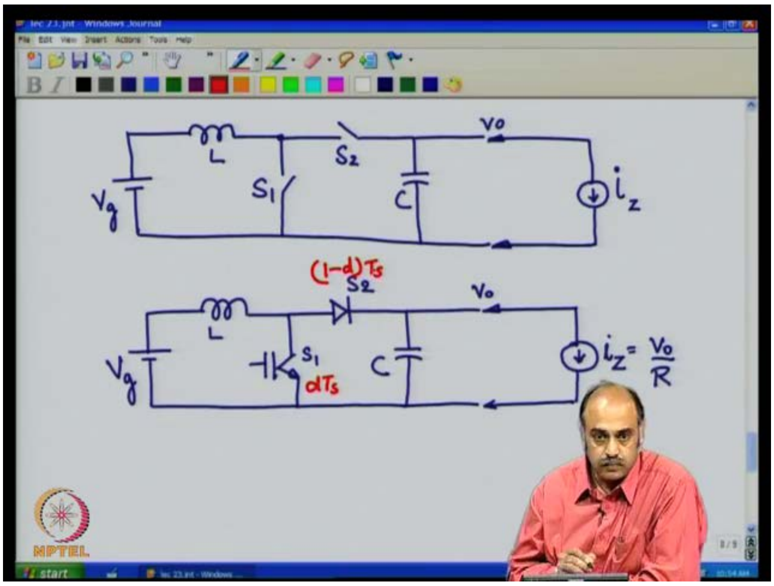

To this, we need to go about equations here we could also make a bit more generalization by at the output. We have v naught and let me give some space here, let us have connected to this at current source because the voltage output v not at this point is constant anything divided by R can effectively be replaced by a constant current source flowing like this. We can label it as I z just for convenience this load can be replaced by a constant current source I z which is equal to V naught by r value.

So, effectively it would look like a slow it is like that this things can also be done such that any value of r you could keep changing in only 3 I z value will change because v naught be controlled. We are trying to maintain the value v not the rate is a constant value now this is also possible thing you will see such also from here we start out and writing down the state space equation this is conducting this is on during the time is on during the time d t s. This is on during the time one minus d t s so during the time d t s, if you look at the equation it is like this.

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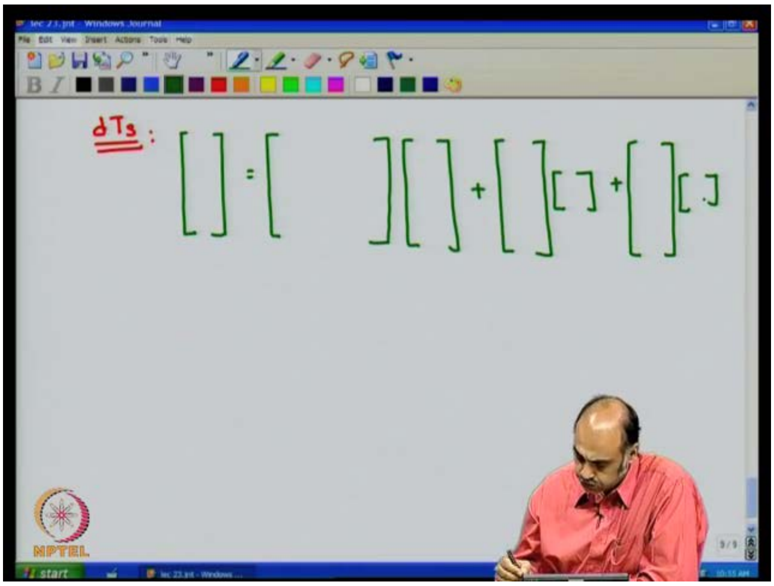

This is the frame work I am going to put in the matrix frame work and then let us fill in the places. So, like that plus I will consider another you will know so let us say for example that we have we have a system like that what are the states of the current I l and the voltage we see these are the two states and v c is nothing but v naught.

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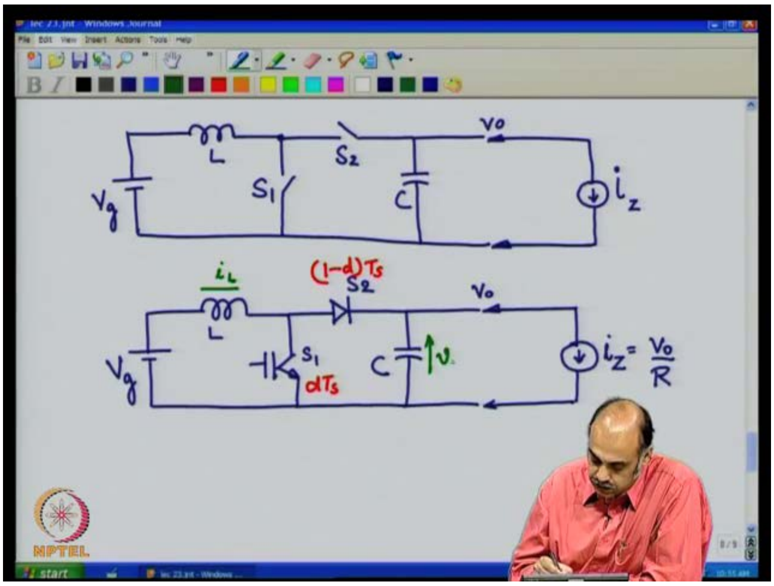

So, you have I l dot v c dot, now I am populating this you can work it out to be something like this.

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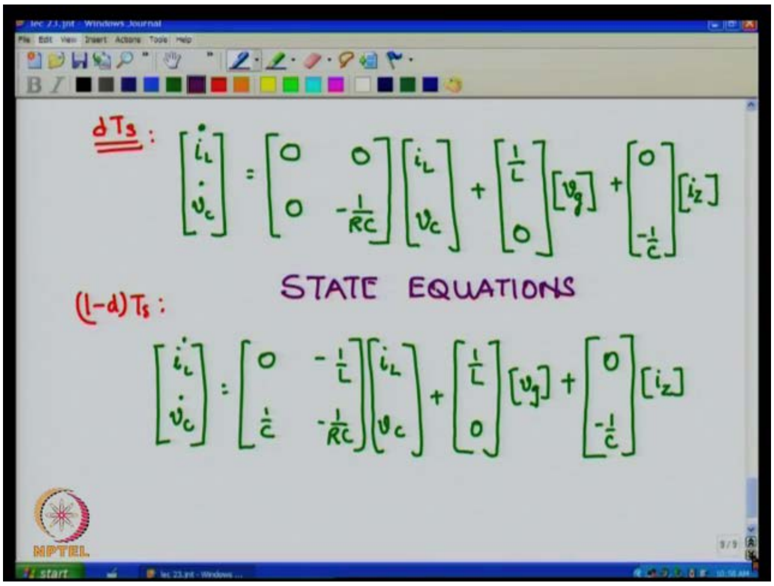

You have i L v c 1 by l 0 you have v g the input, now if you consider the current source input that could also be putting that 0 minus 1 by, so this is the state equation during the time d t s and during the time 1 minus d t s. You would have i L v c dot 0 minus 1 by L 1 by c minus one by r c i L v c dot v c plus 1 by l 0.

You have V g plus 0 by 1 minus c by and I z if that is also one of your control inputs. Now, to both these cases what are the outputs what are the possible outputs that you could have these are the state equations. So, I can put that bold state equations, now let us come to the output equations, now for the output equations, let me go back here one possible.

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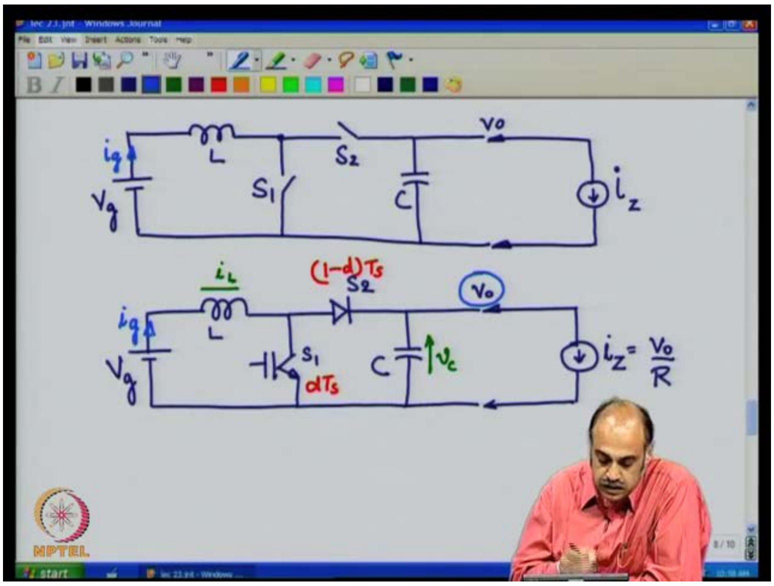

Is this V naught can be an output another possibilities is to control the input current itself I g in a unity power factor which were the boost converters topology I g is normally controlled.

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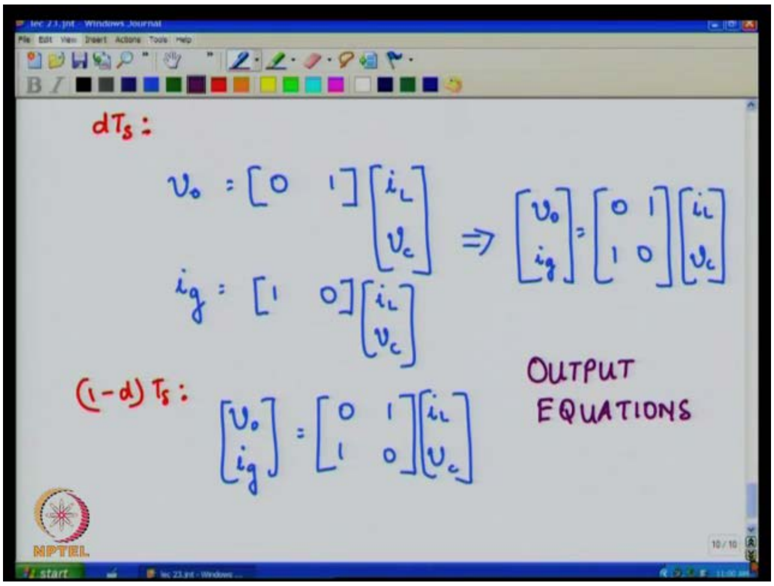

So, you could have that has the output variable, so during this time you can have output is v not one of them 0, one you have I l v c this is one output equation or I could I could also have another I g which is equal to 1 i L v c. You could also put it in the matrix form as v not I g equals 0, 1, 1, 0 like this and i L is c this is and for the case of 1 minus d t is the output equation v naught and I g are the outputs possible outputs and comes like this so these are the output equation output equations. Now, after having output developing equations we need to develop the averaged averaged large signal model.

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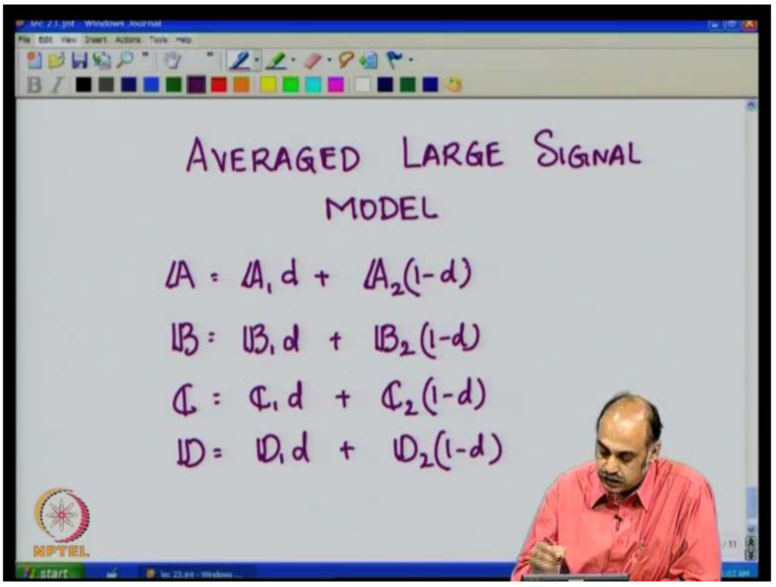

So, in the average large signal model what do we do we are trying to combine the two modes operative modes during d t s and one minus d t s and obtain values. So, if you see here a would be A 1 during d t s A B by r A 2 A 1 minus D that is what we would be likewise you have for b a which is equal to B 1 d plus D 2 into 1 minus B. So, on c 2 1 minus b and these are the generic that we saw how we combine them the average.

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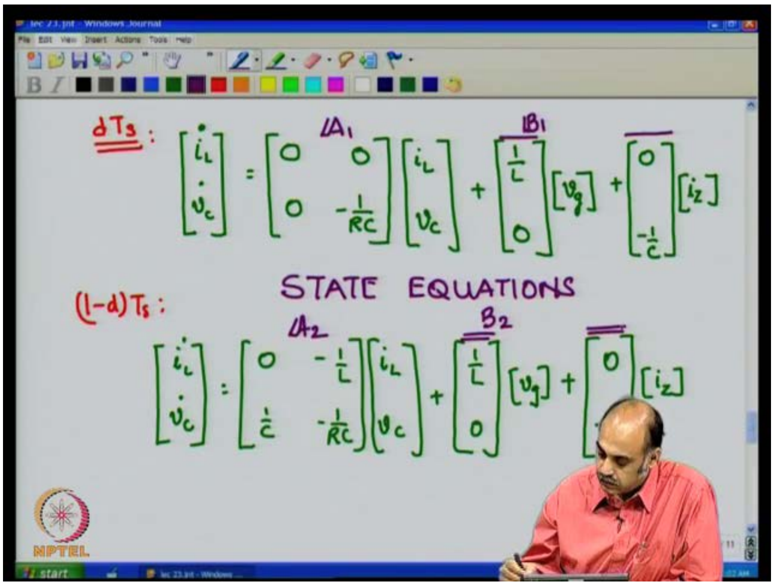

That is A 1 is nothing but here I will write it A 1 and this is D 1 you could apply for these and these together this is A 2 and D 2 this column and this column taken together.

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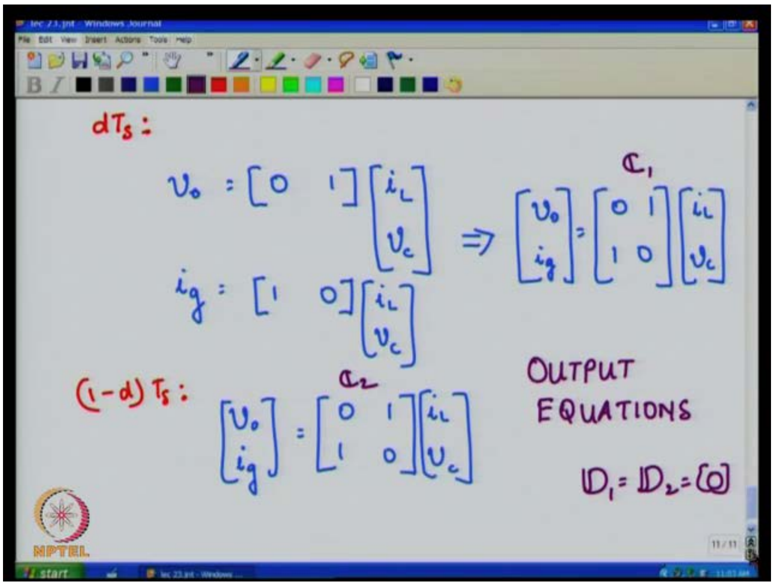

The output equation you could take this one as C 1, C 2 and D 1 equals D 2 equals as zero matrix, so applying those relationship, we will get the average large signal model as follows.

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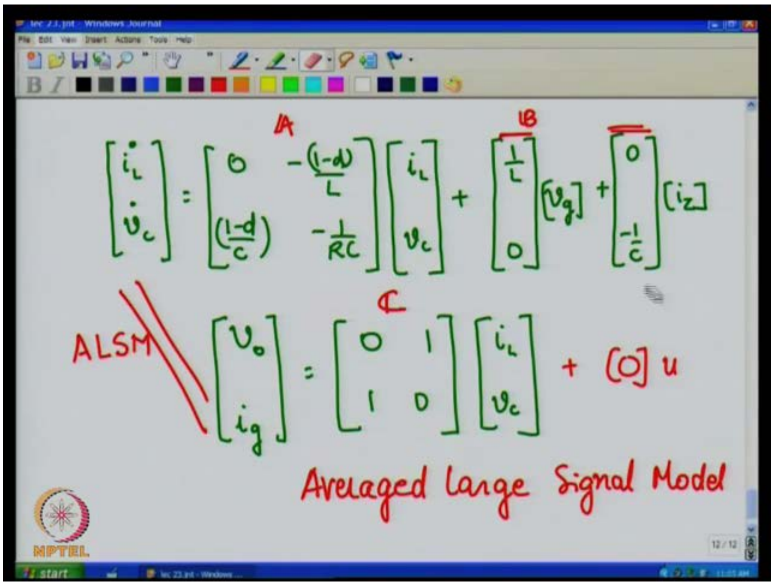

You have i l derivative this the derivative which is equal to 0, 1 minus D by D l 1 minus d by c minus one by r c this multiplied by the state factor x plus you have the two input 1 by and 0 for the v g input plus zero and 1 by c. I set n load has an input y z is nothing but the load, we call it the load and one of the output you could have v not and I g has the other output that is we have consider equals 0, 1, 1, 0 that is the c matrix and i L v c. So, this is your average large signal model averaged large signal model absorber that a matrix D matrix together.

This is one column this is the second column c matrix d matrix is not there because this is 0 plus. As such, we are not using that we will not use it absorb you have a d term time wearing d term d which is a large signal value there which is composed of study state D plus d hat we have a d plus d hat. So, a matrix becomes time wearing and as such you cannot use it directly for controller design application using the linear time wearing approaches. So, the a matrix is time wearing matrix which we need convertor into the time in wearing matrix and that is the job were job of linear were in we remove the state part linear I set and consider e hat has another input because d hat is the controller input.

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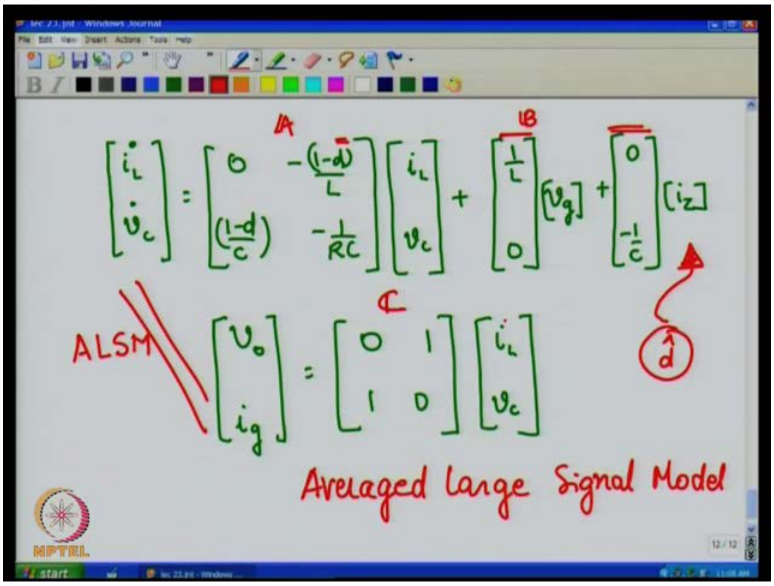

We want it to come here as a controller than the A matrix will become timing wearing matrix state representation becomes for controller implication. So, I will at this point time ask you to practice what should come for the how do you go about doing the linearization by removing the study state part and removing the higher small signal derivation. The linear small signal model try to do that as home work in the next class next section we will continue with this and and see how the obtain the state space representation of the whole system.

Thank you.