Switched Mode Power Conversion
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Lecture - 22
Circuit Averaging - I

Good day to all of you, we continue from where we left of in the last class. We had been discussing about states space representation of DC DC converters. We had split the DC DC converter into two parts. The first part are the first mode were in the one part of the switch is on, the other switch is off. One part of the circuit is operative and when the other portion of the switching time that is 1 minus D T portion of the switching time set same. The second part of the switch is operative, when the first part, the earlier part is disabled; so this is what we had used for formulating the states space equations for the two parts of the circuits. And now, we need to combine them into a single averaged state space model, which takes care of both the situations. So, we continue from there and let us title this course as circuit averaging method.

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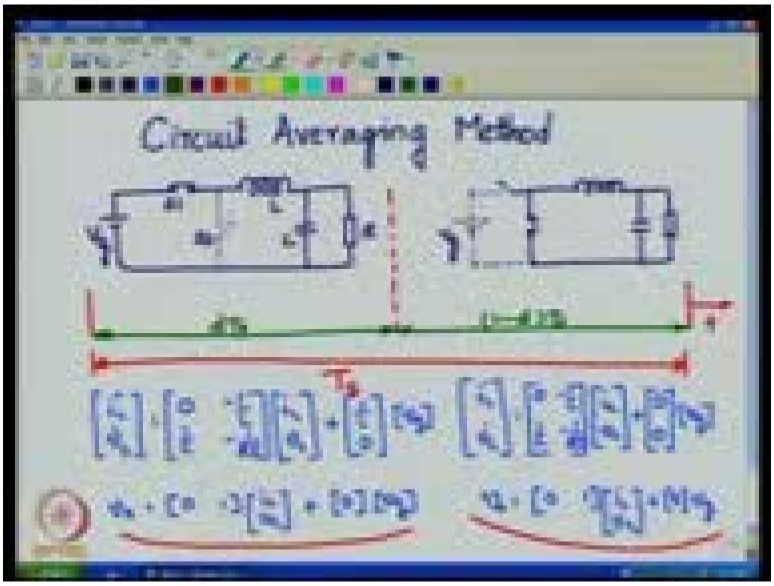

So, this lecture, let us name it as circuit averaging method. Basically what it does is, it tries to average and bring in the, bring in the values of the states has represented in the D T time period and those that are presented in 1 minus D T time period together into a single averaged state space model. Therefore, we had this portion of the circuit, we had taken the example of buck converter were in we had in the first case as switch S 1 and an inductor as shown here with the load R.

So, this is the first part of the circuit were we have not switched on the second switch S 2, only switch S 1 is switched on V g L C and R. On the second part we have only the switch S 2, which is switched on as shown here. V g is effectively removed from the circuit. So, this is the second part, we know that this part and this part together period T or T S. This is one switching period, it is split into two mutually exclusive parts, this is D T S and this is 1 minus D T, so where D is the duty ratio. Now, these two portions circuits are operative at these points.

We had also developed the state equations for these portions as follows, which is i L dot and v c dot. So, this has a matrix like that. i l and V C plus the input matrix where the input is V g. Wherein this case it was 0 minus 1 by L, 1 by C minus 1 by R C, 1 by L and 0. Likewise, for this case i L dot V C dot, which is equal to a matrix, i A matrix multiplied by the state vector plus the B matrix multiplied by the input vector V g. Now, we had the seen that there is no input coming through to the output in this case that is 0 and this is my minus 1 by L, 1 by C, minus 1 by R C, this means the same.

So, these two are the, two state equations for the two modes and the output equation, we said the output is V naught, V naught equals 0, 1, i L and V C, the state vector plus 0, and V g. Likewise, similarly you have 0, 1, the state vector i L, V C plus 0 V g. So, this much we had covered in the last class, we had developed the A individual circuit mode state equations. This portion is the state equation for mode 1, this is output equation for mode one, state equation for mode two, output equation for mode, mode two.

Now, here two things I would like to mention, one is on the variables that will undergo a slight change, I will come to that one and the second point is that we shall combine, we shall combine both this portion and this portion into a single equation. Now, before we do the combination, let us, let us take the case of the variables first. Now, look at the variables here, we are using the upper case D and so also here the upper case D. I am going to erase that and erase this and change it into lower case d, 1 minus d.

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So, it becomes this portion of the period becomes d T S were d is lower case, 1 minus d T S, again d is lower case. Why did we do that change? So, let us re visit some of our earlier concepts to know why we are doing this specific change. If you recall, much earlier we had, we had a transfer curve y versus u and this transfer curve had a linear shape like this. And, now let us say this is the operating point and this operating point is the nominal position of the u variable and the y variable input and the output variable. And, it could have a variation above the operating point in the neighborhood of the equilibrium state. Now, this is our equilibrium state equilibrium state or in other words steady state, steady state.

Now, this steady state portion is having a norman cloture operating point, U upper case U and y, any deviation about the operating point we are calling as u hat. Any deviation about the operating point we call it with small lower case variable and a hat. So, equivalently if you look at any typical inductor current wave form in a buck converter. So, in the buck or the DC DC the converter that we saw, if you look at the time domain evolution of the inductor current i L, it would appear something like, something like this lets say, goes through the transient and comes into the steady state lines.

When you zoom in, when you zoom in to the wave form, it would appear slightly different, because you are looking at the switching, so let me put the dotted to indicate an average effect like that and let us see that during d T it will rise during one minus d it will fall, during d t rises falls rises falls, so on keeps happening in this fashion. So, this would be the wave shape of the inductor current. So, if you look at this portion, we will call that one as the equilibrium or steady state part equilibrium or steady state part capital or upper case symbol.

If you look at the deviations about the steady state that we will have, we will have the symbol, we call it by lower case symbol with a hat. And, the actual large signal instantaneous large signal i L, which is just plane lower case will be the upper case symbol the steady state part plus the small signal part that is the hat part. So, if you look at it, this is the actual large signal value, this upper case signal is the steady state value.

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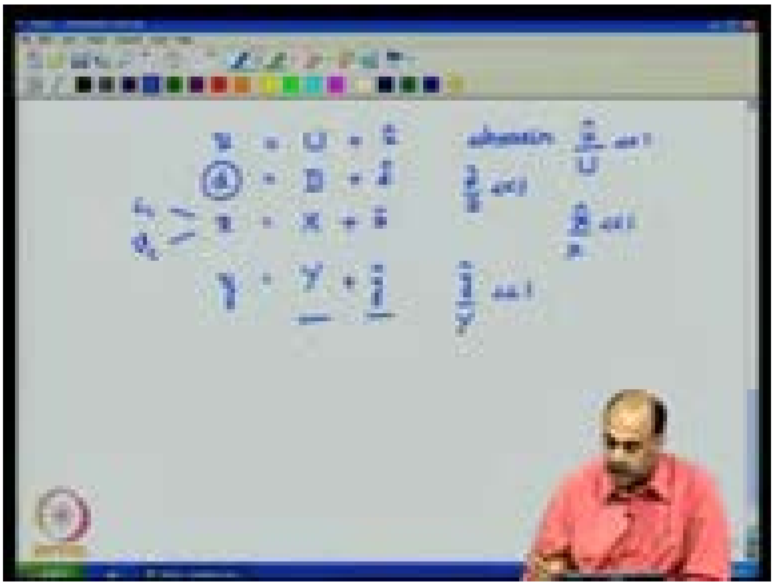

And, this lower case signal with a hat on top is called the small signal value. So, every variable every variable in the physical system has this two parts. So, if you consider the variables like u, d, the state variables, the state variables in our case is two in number, you have i L and v c and the output variable. All these variables have two parts, a steady state part plus a small signal part, wherein the small signal deviation with respect to the steady state part is very much less than 1, very, very small deviation about, in the neighborhood of the steady state or operating nominal operating point.

Likewise the duty cycle large signal value has a steady state part indicated by this upper case D, plus a small signal part lower case d. d hat by d of course is much less than one. Likewise, the states you have the steady state part and the small signal part and the output, which has a steady part plus the small signal part wherein y hat by y is much less than 1. So, likewise every signal has these two parts, this is very, very small compared to the steady state part. So, that is the criteria. Now, that is why I have replaced the symbol d with the lower case d indicating that it is the large signal value, which comprises of two parts steady state part plus the small signal.

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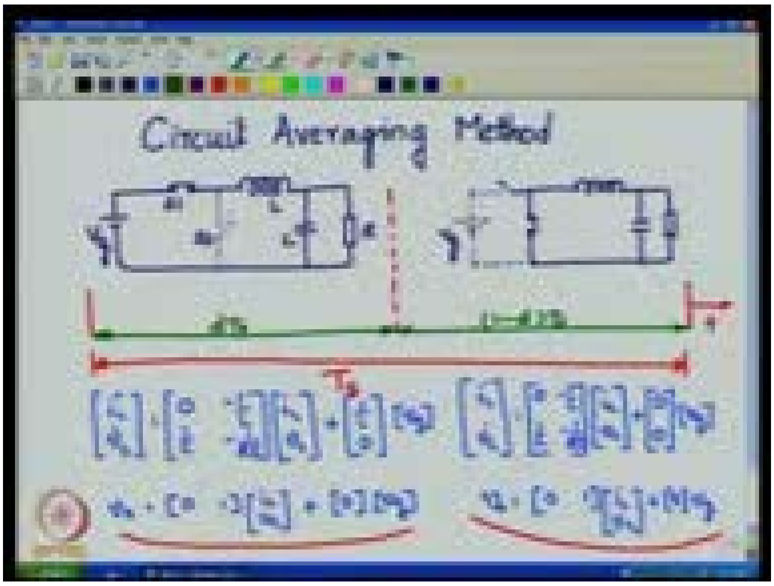

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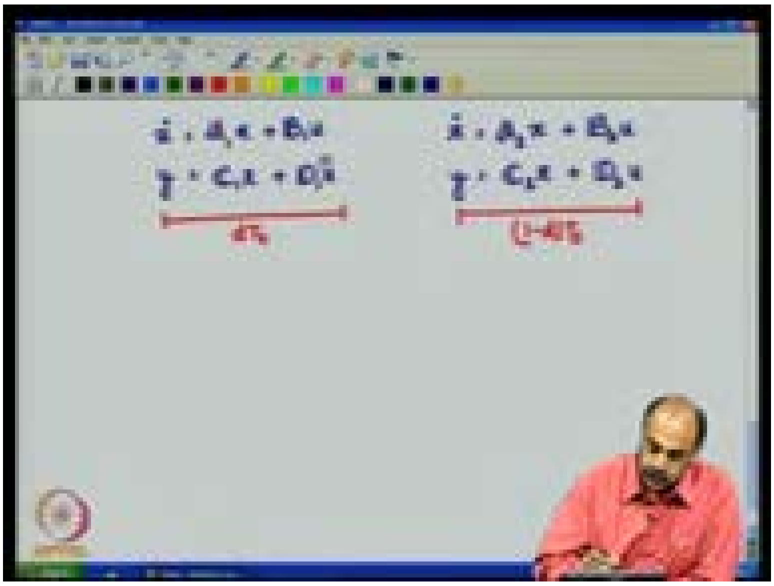

So, to bring in this change in notations I have replaced the symbol for duty ratio without upper case D to lower case d and so also here consciously, so that what we do future, in the future equations to bring in clearly the distinction between the steady state part, the large signal part and the small signal part. Now, coming back to our model, we have the two models, now I will compactly right it as x dot, which is equal to A 1 x plus B 1 u, y output which is equal to C 1 x plus, D 1 u.

This is the model for the first part of the first mode of the circuit, when the switch S 1 is on during the period d T S. During the period 1 minus d T S, we have A 2 x plus B 2 u and y equals C 2 x plus D 2 u. So, this is happening during the time d T S and at this the result of what happened during the time 1 minus d T S. And, remember that this portion of the state equation is nothing but this state equations output equation put together and this state, and output equations during 1 minus d T S has been indicated like this with the subscripts A 2, B 2, C 2, D 2. Now, we need to combine these two together to obtain x start, which is equal to A x plus B u, y equals C x plus D u.

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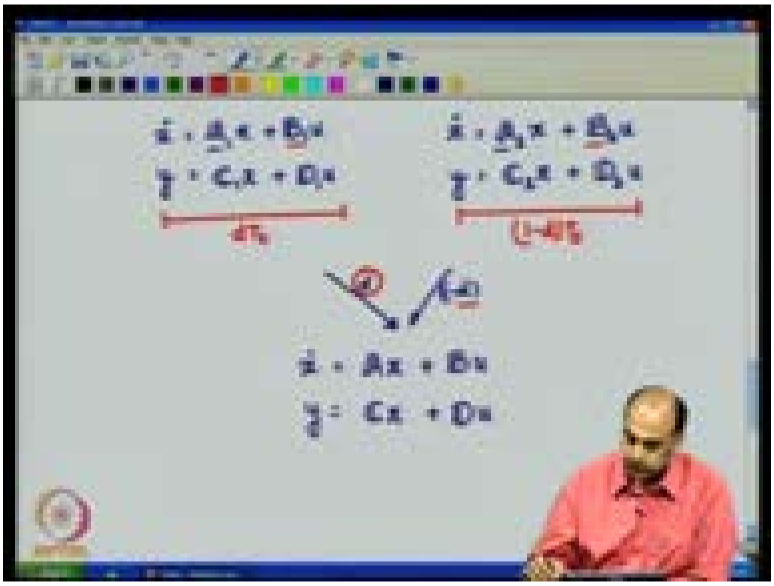

So, wherein this A in is the weighted average of A 1 for a period d, d t s and A 2 for a period 1 minus d T S. Likewise, d is the weighted average of B1 for the period d T S and B 2 for a period 1 minus d T S. Likewise, c is the weighted average of C 1 and C 2, d is weighted average of D 1 and D 2.

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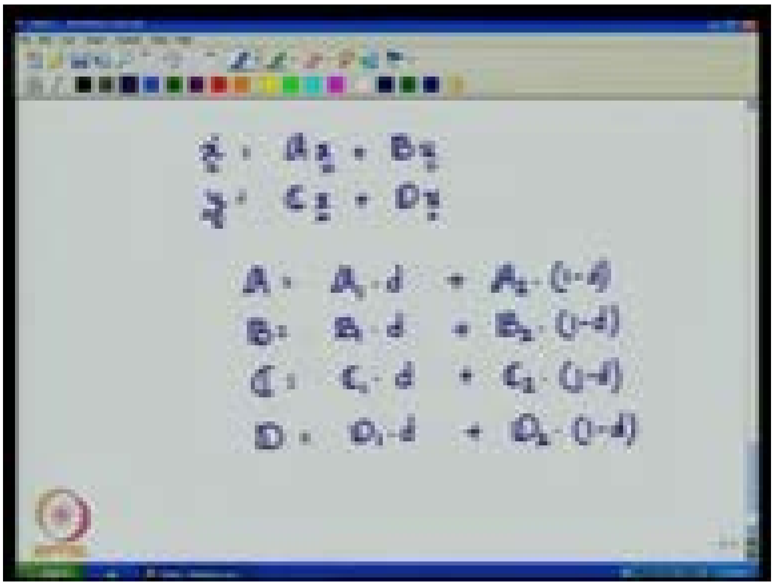

And, if you take the equation x dot, which is equal to A x plus B u and y is equal to C x plus D u. A as we said is nothing but A 1 into d plus A 2 into 1 minus d. So, weighted average of these two, B is B 1 into d plus B 2 into 1 minus d, similarly C is C 1 into d plus C 2 into 1 minus d. D is D1 into d Plus D 2 into 1 minus d.

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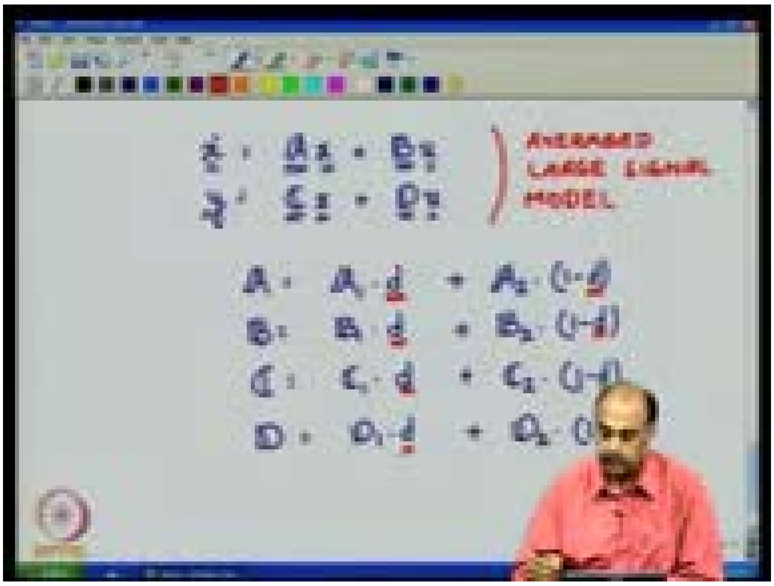

So, this is how the weighted average is taken into account as we saw that we are trying to average the signals x and u with respect to the periods d and 1 minus d as we saw in the last class and we are taking the weightings into the co efficient matrix to, so that the same states as indicated in a period d T and 1 minus d T will be used here. We are not changing the symbols for the states and the input vectors. Only, the weightings has been taken them into the matrices into the A B C D matrices.

Now, you must note that the A B C D matrixes being weighted by d and by virtue of the a fact the d is the duty ratio, which keeps changing with time based on the controllers which I employed to control the output y, The matrices A B C D would become time varying matrices, because d is no longer a constant. This is one issue that we need to address and solve, because the state, the state equation methodology that we will be adopting for a designing the controllers are for linear time variant systems. So, the matrix A and B are suppose to be constants.

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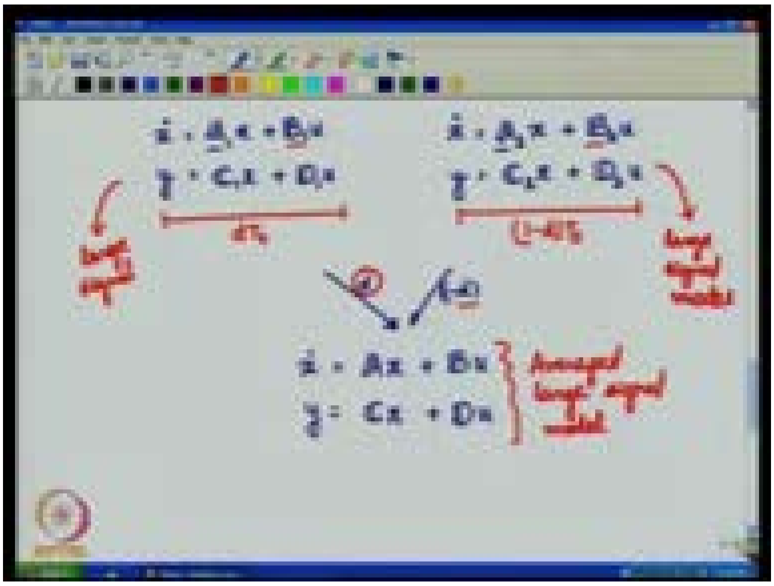

So, coming back to this equation, this weighted are the averaged model together is called where both the A 1 part and A 2 part are brought together is called the average large signal model. So, we began with the large signal model, which is this, this is the large signal model, large signal model, so also this, also the large signal model, these two have been combined together by weighted averaging and this is called the average class signal model. Now, we further reduce it to take care of, there are two issues non idealities, which will creeping, just show see that that is one major issue together with the fact that the A and the B matrices, some no longer constant, but time varying matrices, and these are also to be addressed.

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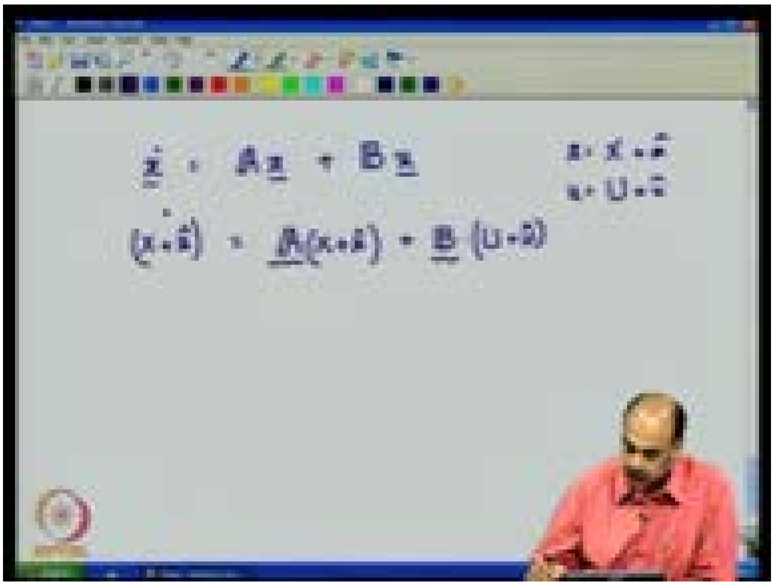

Now, if you look at the equation, x start which is equal to A x plus B u, these are all large signal values, these are all large signals variables. Now, we replace them, because we know that x is nothing but steady state part plus small signal part, u is nothing but steady state part plus small signal part and so on all the variables. So, we will replace it with their split parts dot equals A into the state plus B matrices into the input variable.

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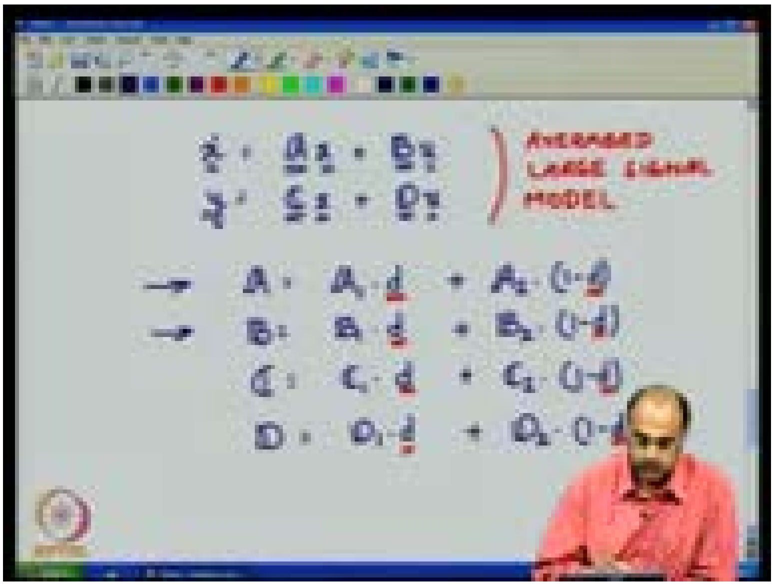

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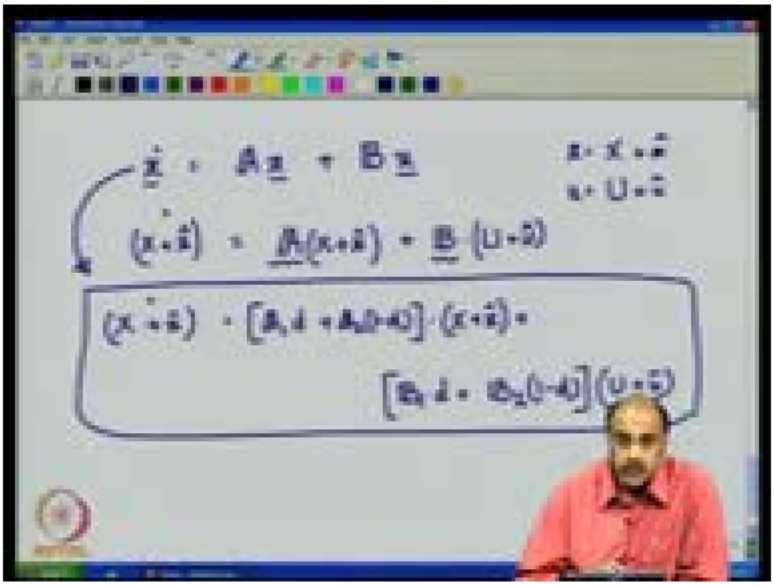

And, this itself A and B itself can be written as these. So, if we rewrite them, why splitting them into the steady state and the plus A 2 1 minus d multiplied by steady state plus B 1 d plus B 2 1 minus d into steady state and small signal part. So, this would be the equation of this average large signal model expanded, expanded. So, a let us, let us now simplify it further and in the process of simplification let us make some small assumptions, valid assumptions.

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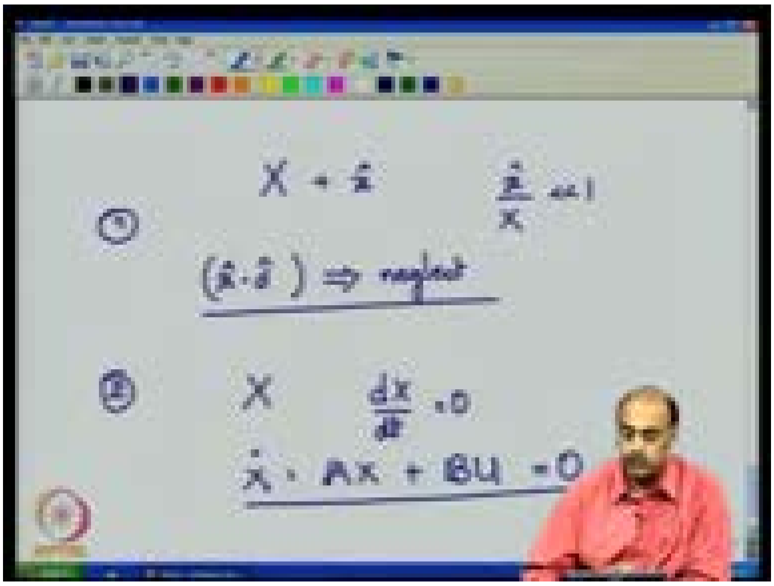

We said that the steady state part is very large compared to the small signal part. Small signal part is very small compared to the steady state part. Therefore, if you have a term which is multiplication of two small signal part then we can neglect this term, because x hat itself is very small compared to x. So, in the normalized form a very small number less than 1 into a very small number less than 1, we are trying to neglect it, that it is not, so to significant.

And, this is the first assumption and in the second the steady state part or the equilibrium part is reasonably steady and therefore, d x by d t is equal to for the steady state part. So, it does not have a slope, it is like a constant d c. Therefore, x dot, which is equal to A x plus B u can be equal to 0. So, this is the second assumption that will come into the picture. Now, using these two you can apply simplification rules for this.

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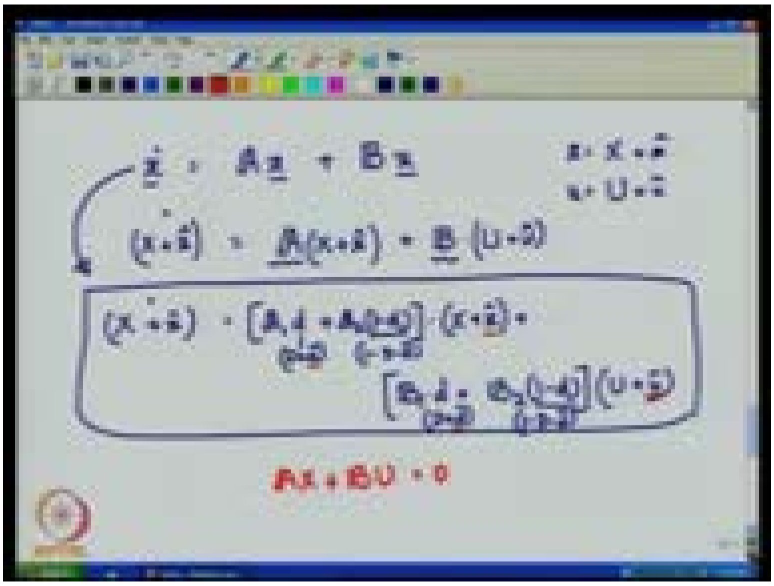

And, observe here that, we have this d, now this d you should further expand, expand it as d plus d hat and this would become 1 minus d minus d hat. Likewise, here you have d plus d hat and this would become 1 minus d minus d hat. So, you have d hat term, product of x hat d hat, product of d hat u hat.

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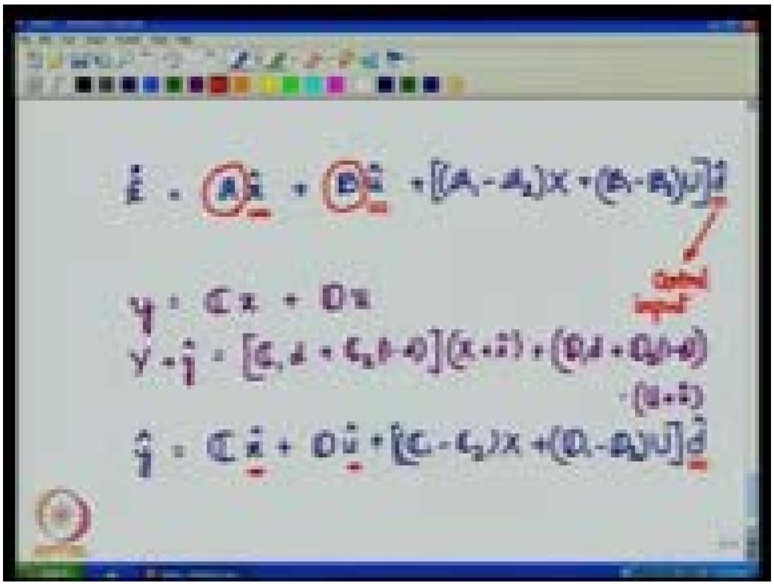

Now, all these product terms you can delay, you can remove neglect and then apply, apply A x plus B u equal to 0, assuming the steady state deviation the state factor is 0 d d x by d T and using that you will obtain x hat dot, because the steady state deviation is 0. You only have x hat dot on the left hand side of the equation, which is equal to A x hat plus B u hat, all are small signal quantities plus A 1 minus A 2 steady state quantity x plus B 1 minus B 2 steady state input quantity u. Now, this whole is multiplied by d hat. So, observe that you have small signal state, small signal input and the small signal duty cycle.

This forms the control input in most cases of our d c d c convector. Likewise, we could in the similar manner take the output equation replace it with Y plus y hat, which is equal to C 1 d plus C 2 1 minus d, x plus x hat plus D 1 into d plus D 2 1 minus d, u plus u hat. So, this can be simplified and on simplification for the small signal you have y hat, which is equal to C x hat plus D u hat plus C 1 minus C 2 x plus D 1 minus D 2 u. This whole thing product d hat. Observe again the same form you have the small signal terms coming in and you look at the dynamics of this state matrix A and B.

They do not contain any small signal varying term utmost they would contain only the steady state of terms which for a given operating point is constant and fixed and therefore they can be used for controller design early in the lineartime in variant mode itself. Now, let us apply our case to the DC DC converter and we have taken up this buck converter, DC DC buck converter and let us try to apply the process.

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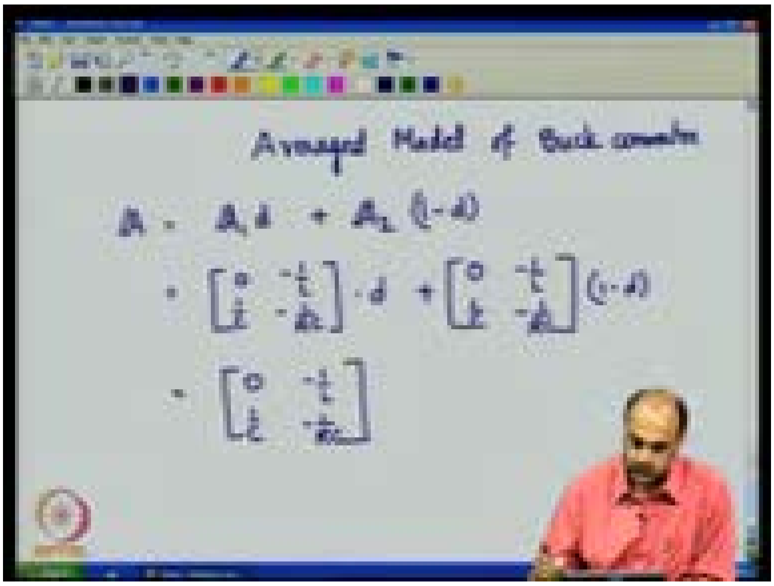

Now, the consolidated averaged model for the d c converter is of this form. I will call that one as the averaged model of buck converter. First let us see A, A is nothing but A 1 d plus, plus A 2 1 minus d, which is nothing but in this case 0 minus 1 by L1 by C minus 1 by R C into d plus A 2 is also same as A 1 In this case in this specific case 1 by D C minus 1 by R c into 1 minus d. so, this will turn out to be as d plus 1 minus d is equal to 1. We will have A as nothing but same as 0 minus 1 by L 1 by C minus 1 by R C in this specific case.

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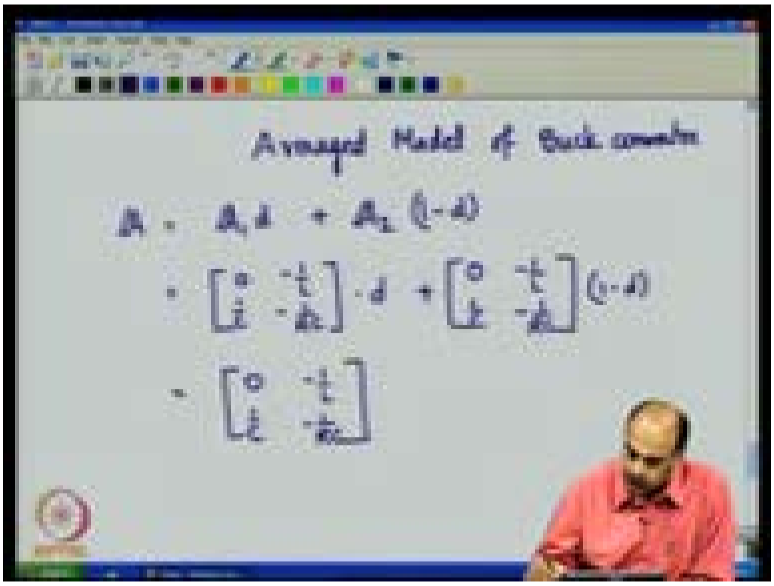

Now, if you take the case of the B matrix, the B matrix equals B 1 into d plus B 2 into 1 minus d.

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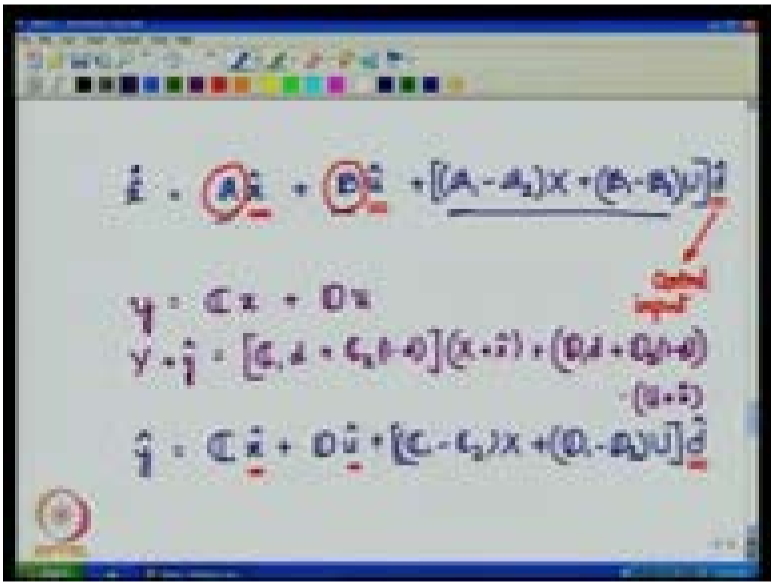

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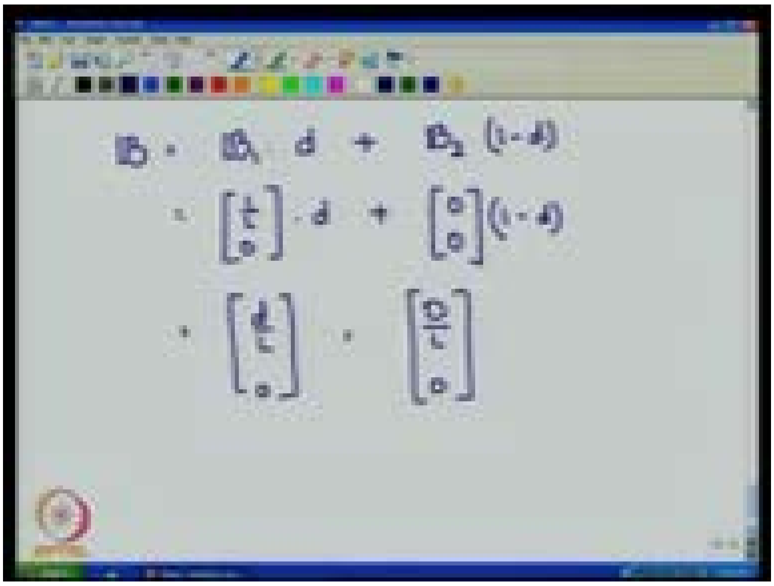

And, we saw that B 1 is, B 1 is by L and 0 in this first case which is d plus B 2, B 2 was 0 and this case. So, this becomes d by L and 0, d by L and 0 and the average small signal model, because this gets split into two parts as we saw and only the steady state parts comes into the portion here, only the steady state portions comes, comes in here. And all these parts it will boil down to only D by L 0 are is if you, if you work out, if you work out this, this equation let me probably give you the, are we saw here that we get this matrices.

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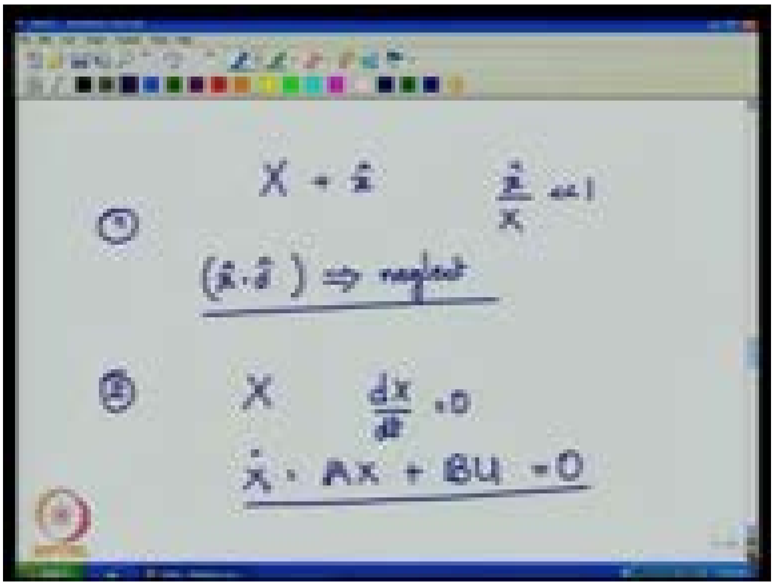

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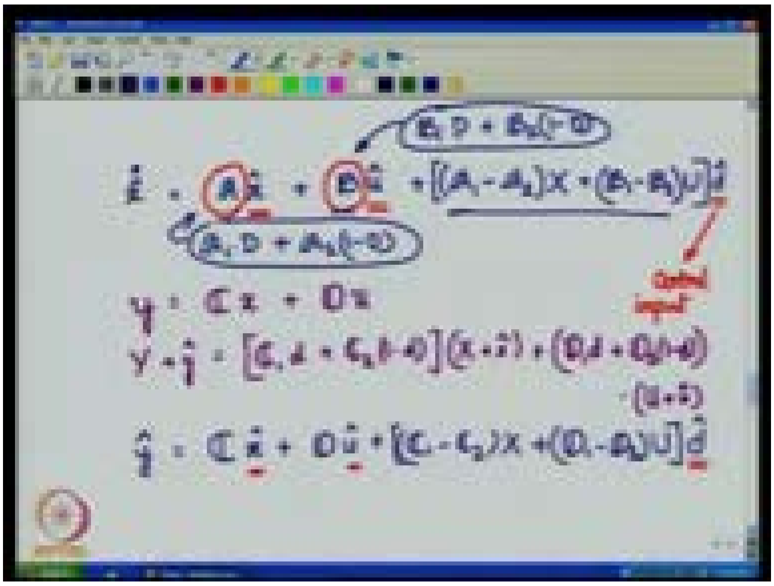

Now, this A matrices is nothing but A 1 into upper case D plus A 2 into 1 minus upper case B. Only the steady state portion comes into the picture here. Likewise, here also you have B into upper case D plus B 2 into 1 minus upper case D. So, that would be the expanded form of this, this having come from the simplification, simplification of this equation.

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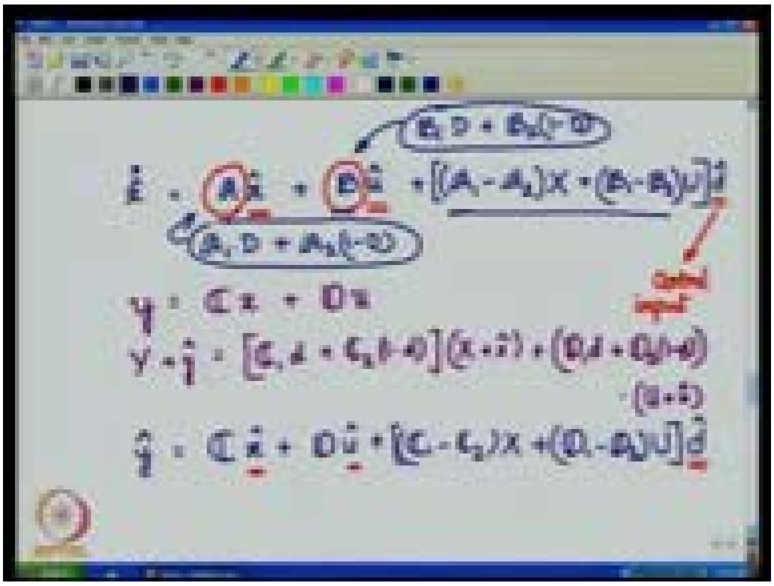

So, if you work out in detail the simplification you should land up with a simplified form for this small signal model like this were A and B for the small signal model is like this.

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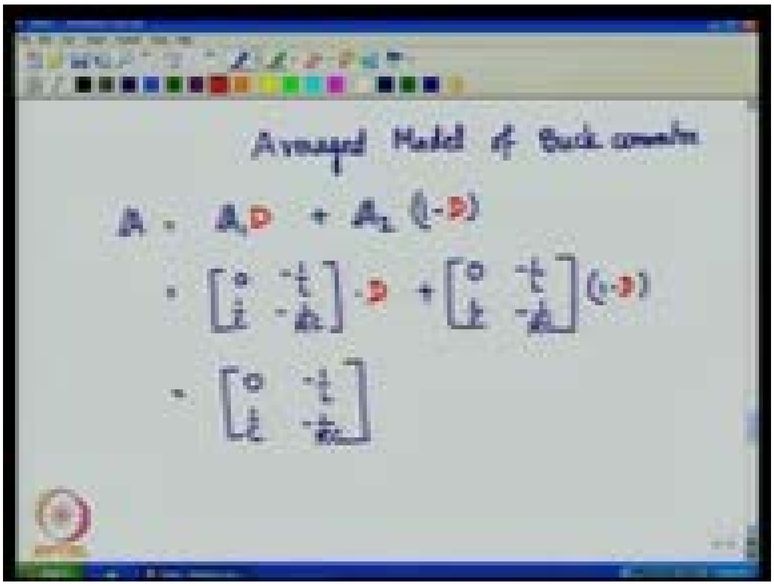

And, that is what we have try to apply here. In fact I should I should probably make this one as upper case directly to the final form here to, this will be the upper case and here also we have only the upper case.

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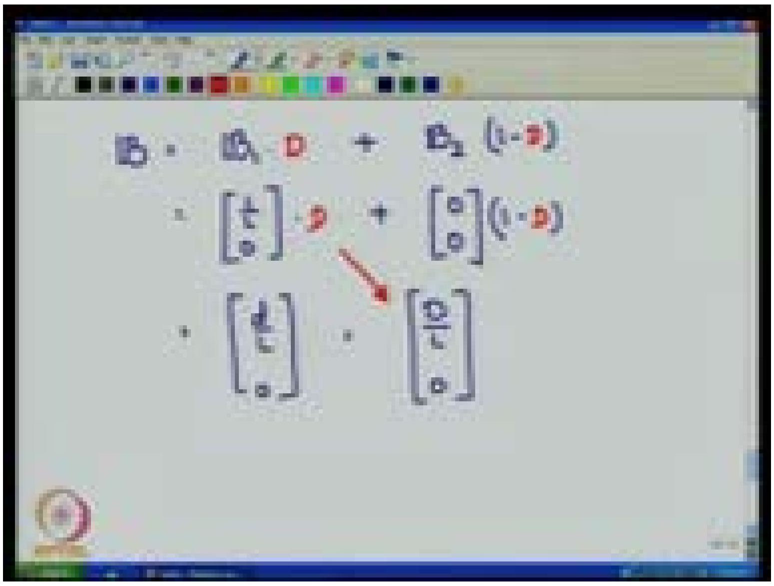

Remember that this is the simplified version, so once we have this from here, it is directly boiled boiling down to this.

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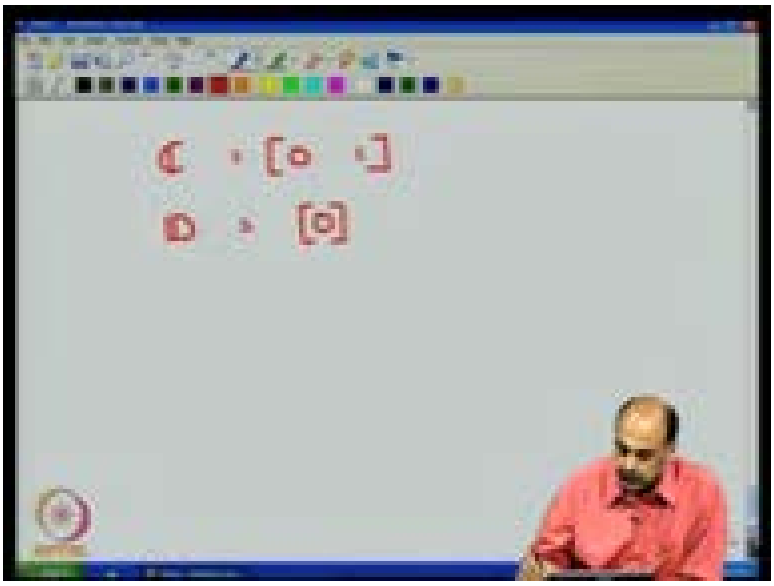

Likewise, the C matrix of course, the C matrix does not show any change, it is same as, same as the C matrix, because this is a same matrix for both. And likewise the D matrix is also 0. It is the same for the both the modes, the mode 1 and mode 2 and there for you will not see change in that one.

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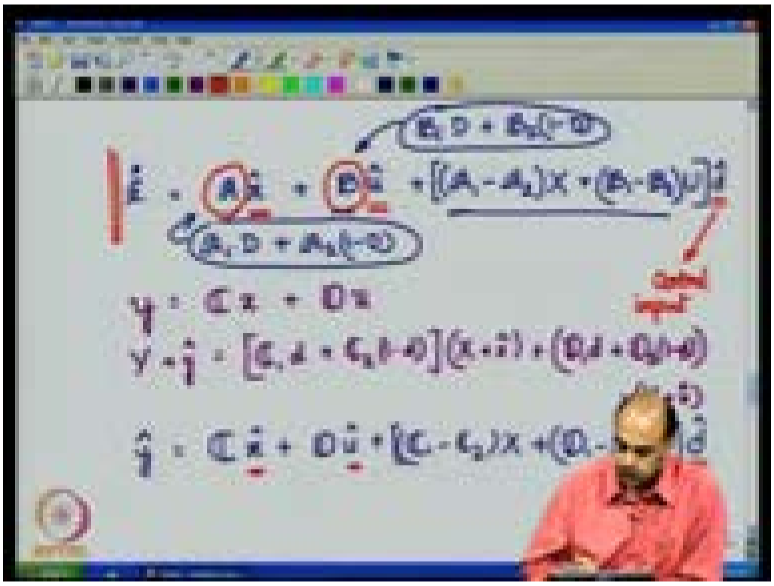

So, there for the average, from the averaged model, we obtained the small signal model by applying these, by applying values to this equations like this.

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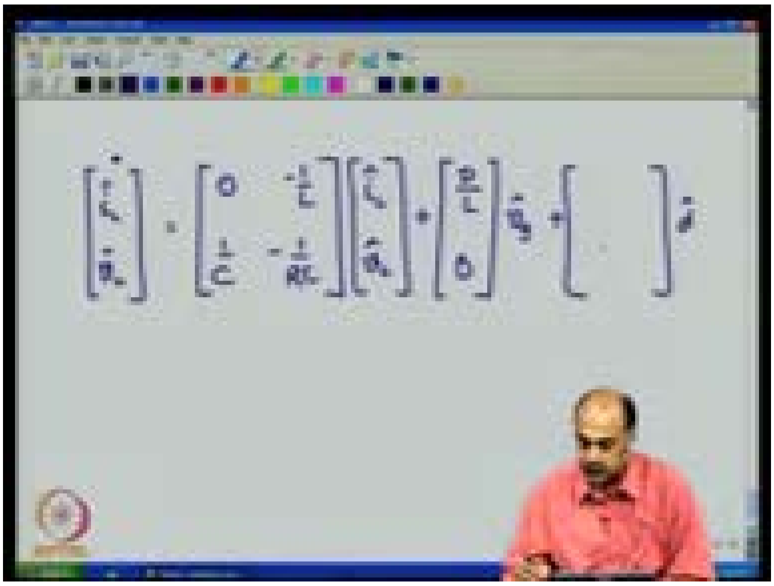

So, small signal model becomes x, in this case specific case, i L hat V C hat derivatives equals 0 minus 1 by L 1 by C 1 by RC minus i L hat VC hat plus and D by L and 0, this would be for V g.

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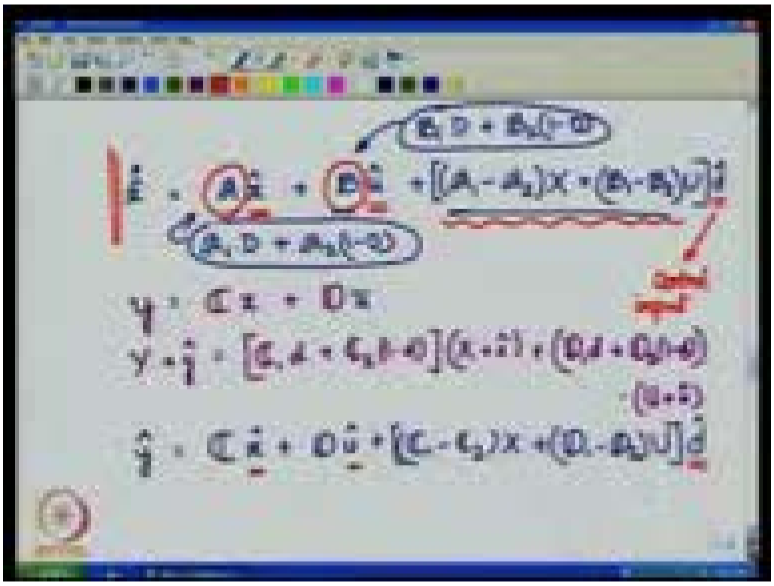

Vg hat plus something d hat. Now, what is this portion? Now, this portion is to be taken as you see, this portion, here this portion of the derivation, so it contains what is, what is upper case u? It is nothing but V g. what y is upper case i L and V naught. So, A 1 minus A naught is 0 in this specific case. D 1 minus B naught is as can be seen.

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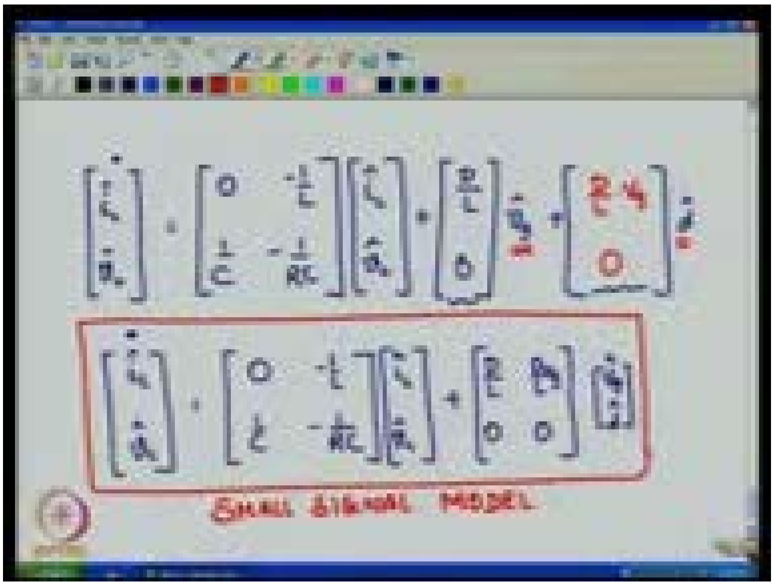

So, in this case you have D by L into Vg. This is 0 here. So, this is d hat. For some cases this and this are considered as in put together as input vector. It can be rewritten in the following manner. i L hat V c hat, i L hat V c hat, this is derivative. Putting common derivative mark for both the variables together plus, now we have B matrix and input. Now, let us combine both this together, we combined both Vg and d as the input variables. So, you take this with rest for the d a Vg variable corresponding to Vg input. This column corresponding to the input.

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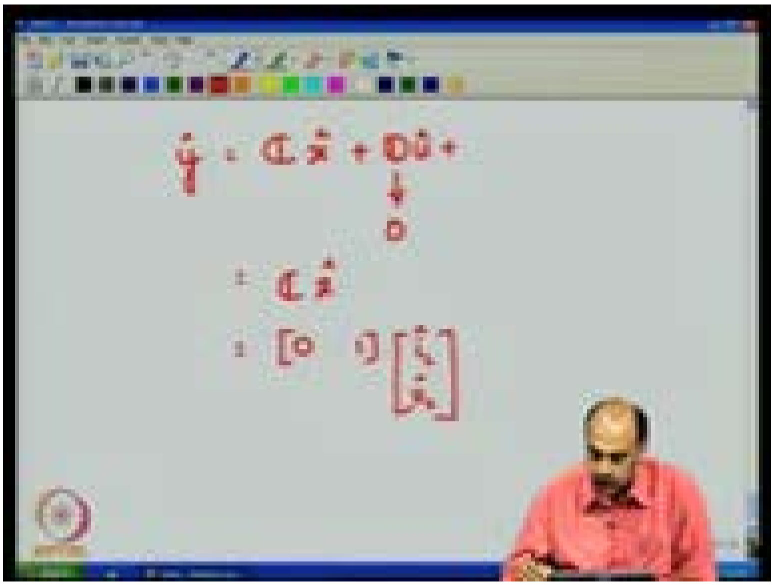

So, this becomes the small signal, this becomes the small signal model which gives deviations about the operating point. All are deviations about the operating point, every variable here use this deviation about the operating point. So, the key here likewise the output variable, which in this case is trivial case where you have y hat, which is equal to C x hat plus D u hat plus du d you have, but in this case d 0.

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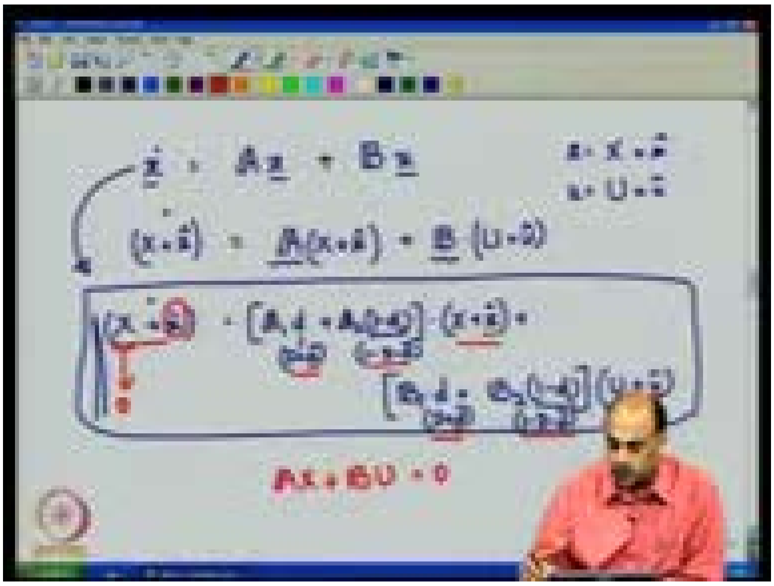

Therefore, it is only C x hat or just 0 1 i L hat V c hat. So, this are the output equation, but you should note that it all starts from the application of, application of this starry state portion, small signal portion to the original equation, large signal, average large signal equation. To this average large signal equation after applying this multiply there is a component by, component you multiply. If there are small signal term multiplications neglect them. A x plus B U terms set them equal to O, upper case X dot this is 0.

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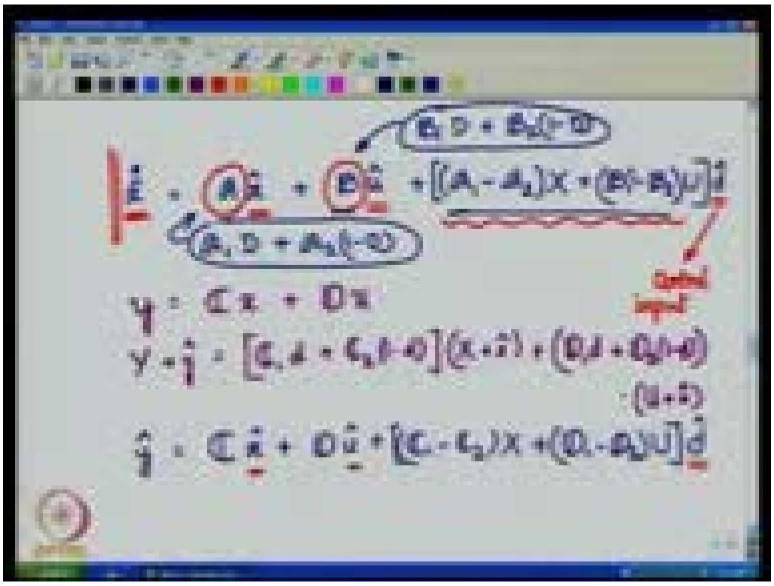

So, which means that this exist the left hand side then on equating them you will land up with and equation, which is like this is just having the small signal deviation, you have x hat where A and B are waited, are waited with respect to the duty ratios which are steady state values. So, A and B contain only constant values for a given operating point. Then A 1 minus A 2 into the steady state value of the state vector in this case A 1 and A 2 are same. Therefore, the cancel become 0. B 1 minus B 2 into this steady state value of input u into d hat. The d hat is again one of the control input and then on simplifying A.

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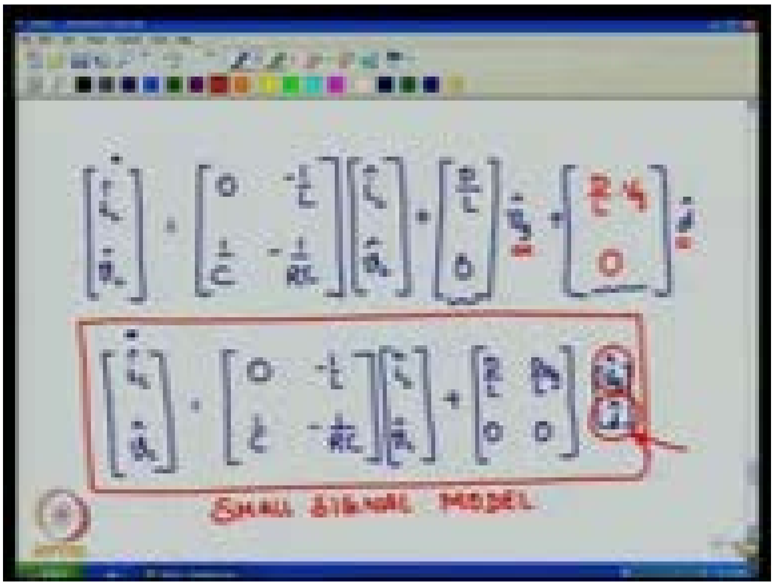

Further you will and applying it to further DC DC converter case, you land up with this small signal, small signal model of the converter. Now, if you look at this small signal model of this converter very, very important future here is that it has not only V g variation about V g as the input, it also haves the variation of the duty cycle has control input, one of its control input in fact this becomes your major control input for controlling the output, the input current and many other, many other parameters, so quite significant parameter for you to control which we will be repeatedly re-visiting and the classes to come.

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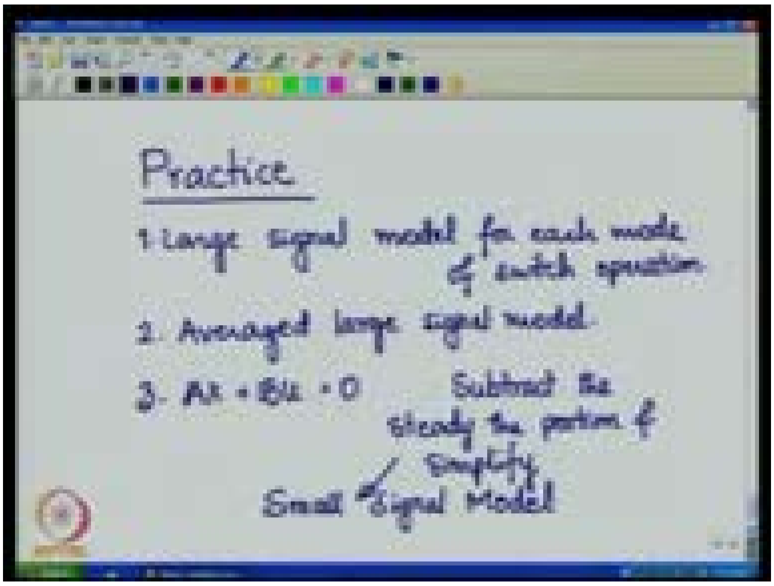

Now, for you to practice I would suggest that you try to apply the concept that we have seen so far. They are actually not complicated, you just go have four steps; first is obtaining the large signal model, practice obtain the large signal model. For large signal model, for each mode of switch operation, average the large signal model to obtain the averaged, obtain the averaged large signal model. Then from the average large signal model as I said the steady state portion of the model is set equal to 0, is set equal to 0, which basically means we are trying to subtract. So, essentially we are trying to subtract the steady state portion and simplify.

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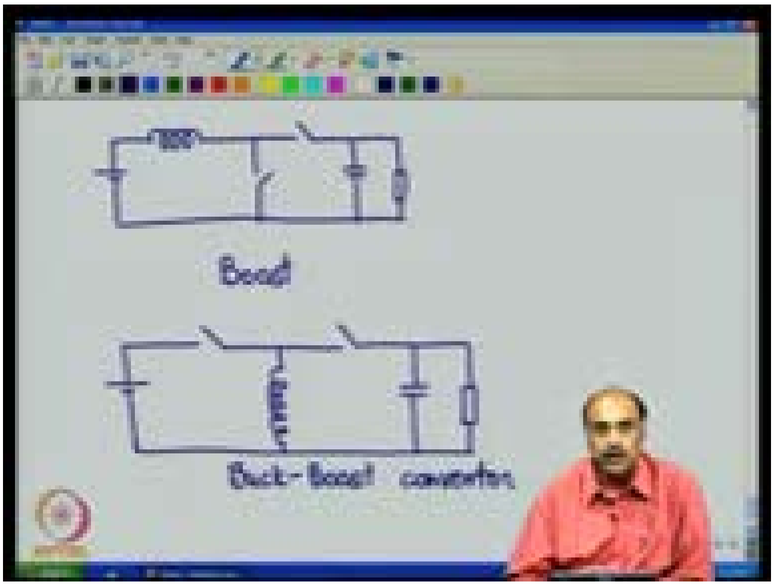

This will give you the small signal model. So, practice this on the following circuits, we will do few more a practice exercises, take the case of the boost converter, which is nothing but the boost converter and also take the case of the buck boost converter which is like this. We have the load; this is the buck boost converter. Apply the methods that we just now went through in this class, this process of circuit averaging, from the last signal to model to the averaged small signal, average large signal model. Remove the steady state model by setting the steady state portion A x plus V u equal to 0. Then obtain the small signal model of the converter. So, up to this point try practicing, we will continue in the next class.

Thank you.