Switched Mode Power Conversion
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Lecture - 21
State space representation – II

Good day to all of you. Today we shall continue with the discussion on modeling of the dc dc converter; till now, we have seen how we can obtain the state space representation of any given circuit. We have seen the first order system in the previous class, we shall continue from there. Take up a second order system and then specifically take up the case of d c d c converter.

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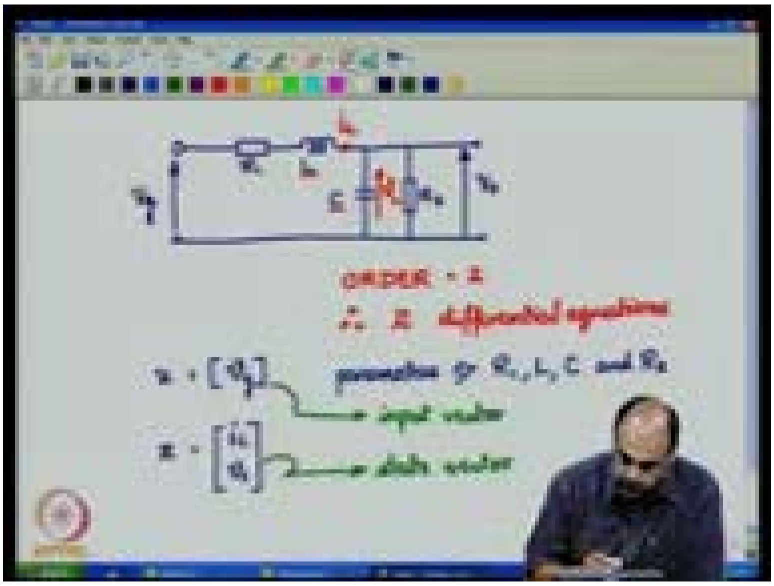

Now, if you see the example of a circuit that we had taken in the last class, the R c circuit, and to this R c circuit we now introduce another element here called the inductor L. So, inductor is one more dynamic element, so if you see the R c circuit, had the R c, and let me call this as R 2. Now, one more element the L Is now added, and at the input let us supply it with a voltage v g, and we require a voltage at the output which is set at v not probably that is what we would like to control. Now, this circuit at the outset looking at the number of energy storage elements, we see that we have the capacitor c as one energy storage element storing energy as half c v square where v is the voltage across the capacitance, and another energy storage element L storing energy within it as half L I square by virtue of the current I flowing the inductor. Now, following the steps that we discussed in the last class, the first step was to identify the energy storage elements, and that was this, and by this we know that this is the circuit of order 2. So, it is a second order system, and therefore, we will have two differential equations, equations that will represent the system.

So, if we have such a case like this, order 2, we need to choose now the variables. I will not now list step one, step two, step three, and will do it in a continues manner, but the process will follow the step as listed in the last class. Now, the variable list, so let us say the input variable u is nothing, but v g the states, where are the states of the system. States are of the system are nothing, but the output the integrator, and in this case we have two, one is with respect to the inductor.

Another with respect to the capacitor. The energy variables with respect to the capacitor being the voltage across the capacitor vc, and the energy variable with respect to the inductor is the current through the inductor I L. now, these two are the state variables. Therefore, we can list them here, so one is I L and the other one is v c, the voltage across the capacitance, and the parameters of the system are R 1, L, C and R 2

So, these are the list of variables that we will be using, and restrict ourselves to in arriving at the state space representation of the above circuit. Note that this matrix is called the input vector, and this array is called the state vector. The self explanatory, because it means that this is the vector or an array which contain state variables, the U vector of the input vector contains an array of input variables.

Now, let us go about writing down the equations, first let us clear the screen by deleting is step listing, the next task after listing down the variables is to start writing down the differential equations one dynamic variable by dynamic variable. So, starting first let us say with a inductor L. Now, we know that L d I L by dt should be equal to the voltage across the inductor. V equal to the voltage across the inductor, and in this case the voltage across the inductor is nothing but this voltage minus this voltage, minus this voltage. Therefore, it is v g minus v R 1 minus v c.

However we have to represent only in variables that are permitted which means state variables and input variables v g minus v R 1 can be written as I R I L into R 1 minus v c which is already a state variable. Therefore, segregating, you have d I L by d t which is equal to minus R 1 into I L which is this divided by L minus 1 by L into v c plus 1 by L into v g, where we have segregated into two distinct parts, this is the part which contains the state variable component and this is the part which contains the input component. So, this is the first part differential equation one.

Now, the second differential equation is the one which is connected with the capacitance C. So, let us clear some space, and write down the differential equation for C. So, you have C d v c by d t equals, now c d v c by d t equals the current through the capacitance.

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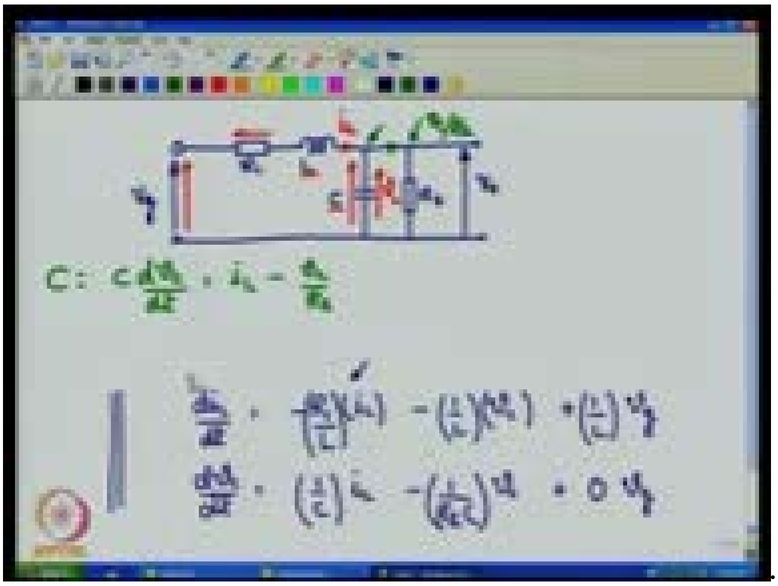

Now, the current through the capacitance at this point is the difference between the I L, and that current goes into the load R 2, and this current which goes into the load R 2 is nothing, but we see state variable by R 2. Therefore, this is nothing, but I L state variable minus v c by R 2. So, this can be written down as follows.

d v c by d t which is equal to 1 by c I L minus 1 by r 2 c into v c plus 0 v g. So, there is no component of the input in this particular equation. Both are state components, the capacitor value c is taken to this part this side of the equation rearranged to obtain these co-efficients.

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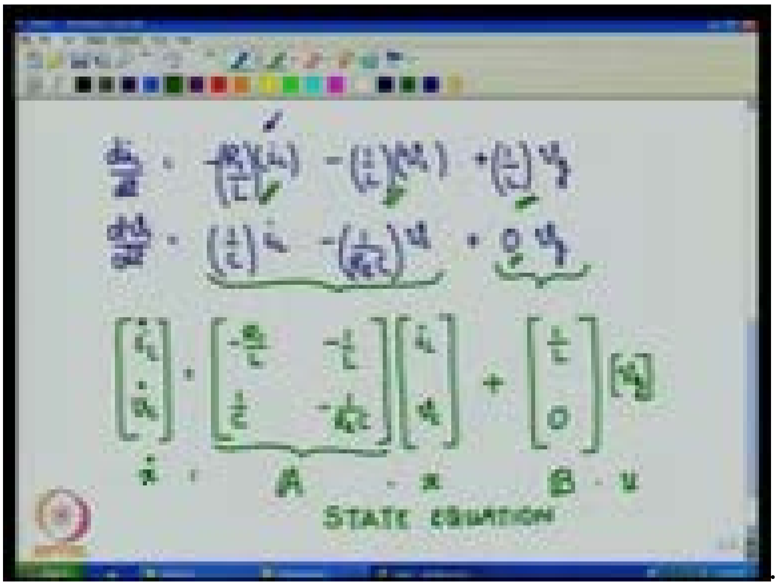

Now, these are the two equations which, which have been developed from the two dynamic components, and these basically form the state equation if they are re-arranged in a proper manner. So, let us do the re-arrangement by copying this equation set into a fresh page. Now, this equation set, we will arrange it in the form of a matrix.

So, I L dot would imply d i by d t, v c dot would imply d v c by d t. now, we represent this portion in the following manner, we have a matrix, we call that a matrix, and that a matrix is multiplied by the state vector. And this is nothing but the state vector derivative. So, what would be the state vector we have already defined the state vector which are the energy variables here I L, and v c. So, correspondingly we can fill in the elements of this matrix by looking at the two set of the differential equations as we have written about. So, the coefficient of I L, we write it down R 1 by L. Now, the co-efficient of v c into its corresponding place, again the coefficient of I L with respect to the second equation, and so on.

So, this matrix, square matrix forms the state parametric or the characteristic matrix which actually defines the system using the parameters of the system, plus still it is not complete. We need to address this part, the input part, so we make up a place holder for this two matrices, one is called the b matrix another is called the u matrix, which is the input. This is nothing, but v g, and then looking at these two values we populate the b matrix as follows.

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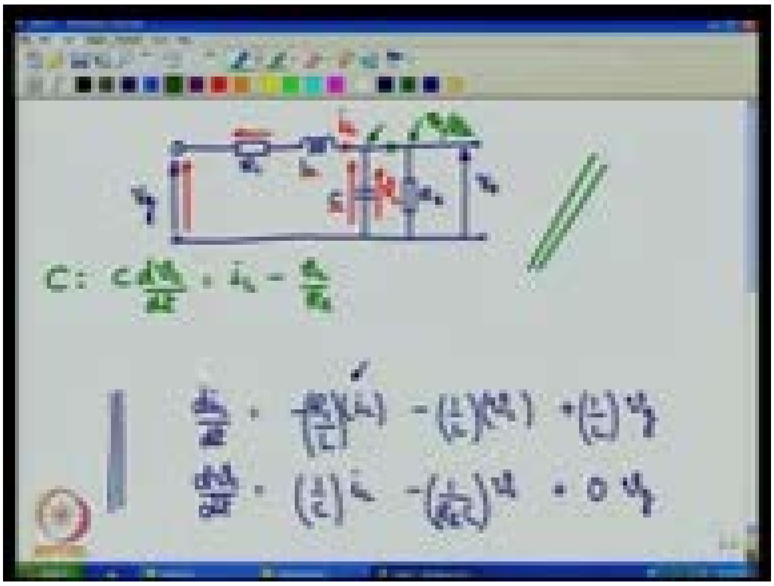

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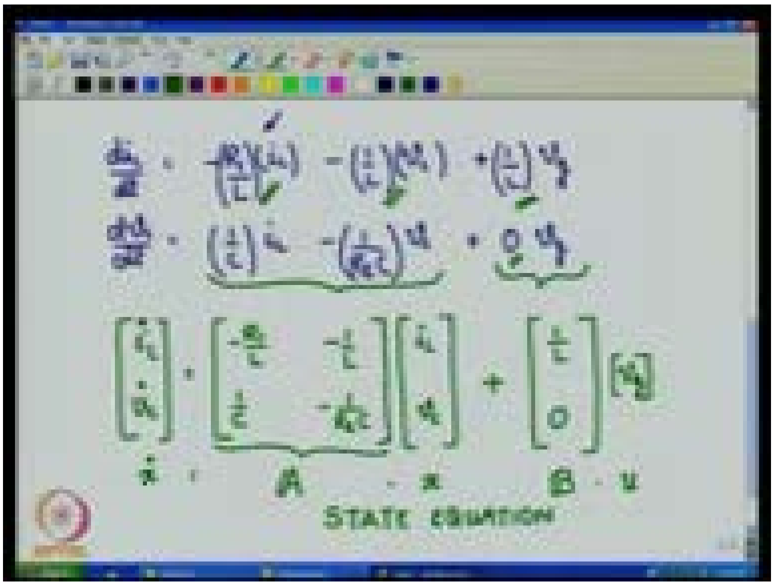

So, notice that this equation is in the form x dot equals a x plus b u, and this is nothing, but the state equation, equation for the second order system or the second order circuit as shown here, or else the circuit. Now, if you observe this equation, this vector, the x dot vector provides the derivative or predicts the slope at that current point of time, and which gives the possible extra pollution for the next, future point of time prediction how these state will evolve in future that was this will give.

This matrix gives the property of the circuit in terms of its components values, and the state vector I L and v c is nothing, but the energy variable already stated, plus the input scaling matrix, and the input itself.

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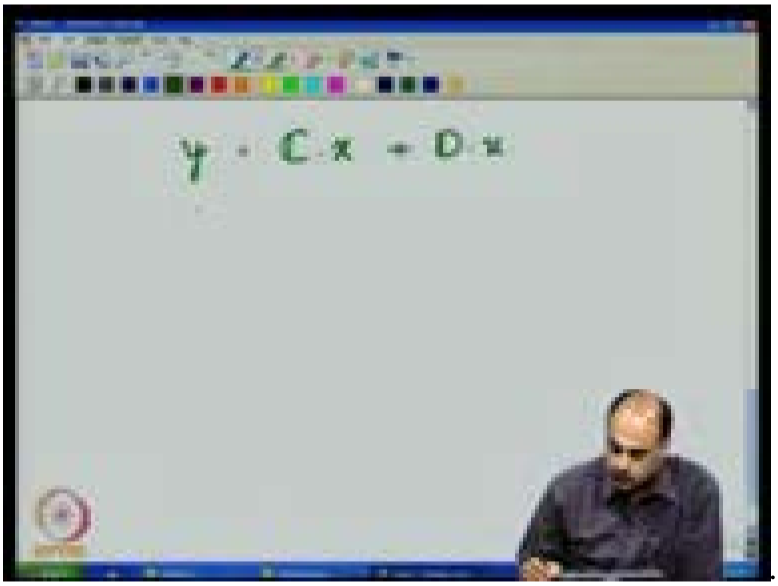

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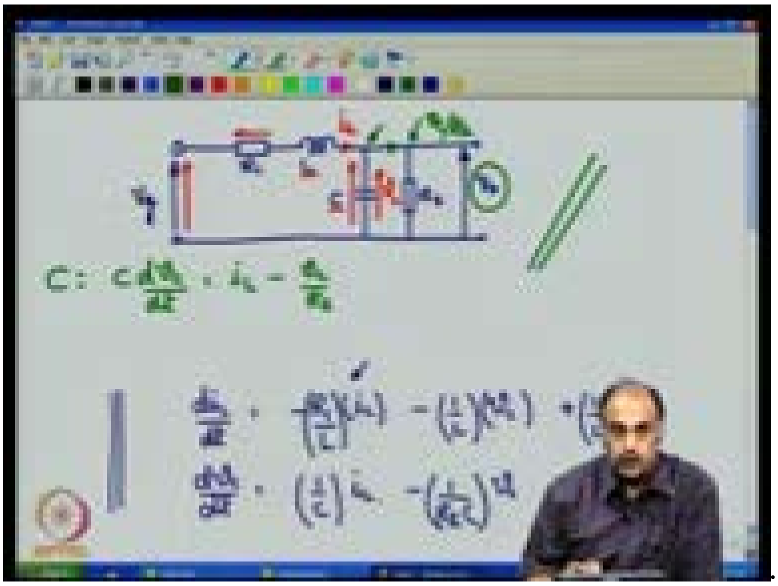

Now, this state equation as a whole would give you the dynamics or the dynamic relation between the various variables signals, and inputs for the system, for the specific system. The next aspect is to obtain the output matrix, output matrix is the form y is equal to C, a scaling matrix, into the state x, state vector, plus D another scaling matrix u which is the input vector. So, what is suppose to the output, the output for this specific circuit is v not. v not is what we want to be the controlled output. Therefore, put that down as the output.

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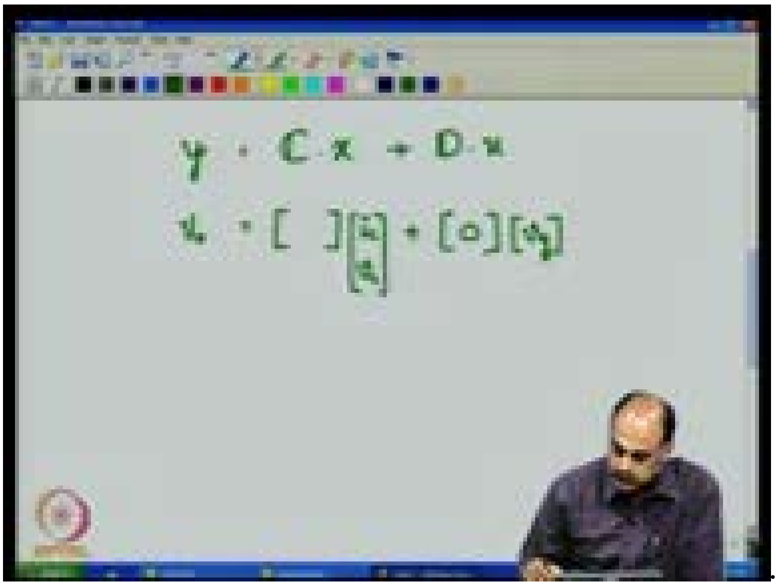

V not equals a matrix which scales the state, and what is the state vector? State vector is nothing, but I L, and v c plus another matrix which scales the input, and input is nothing, but v g. Now, we see that there is no direct feet forward from the input to the output. so D is nothing, but 0, and again looking at the circuit we see that v not is nothing, but the voltage across the capacitance.

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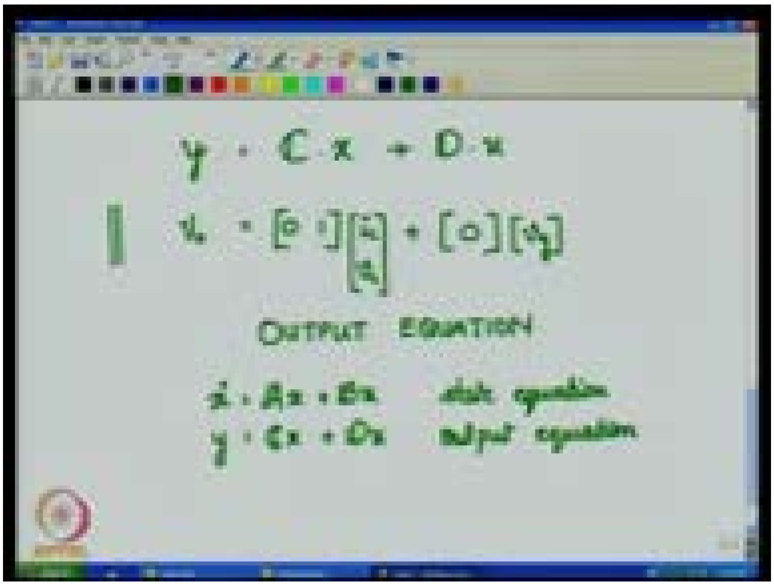

Therefore, we, we scale give the numbers to the c matrix accordingly as 0 and 1. So, directly the capacitor state value, state variable value in this particular case. So, this equation is called the output equation. The state equation, and the output equation together form the state spares representation, and always as I have been telling since the previous class, we represent it in this form.

The state equation, and the output equation, even though it seem to be repeatable it is very fundamental, and important part of developing the model for any d c d c converter. So, this portion has to be understood thoroughly before going on to the next part which is we try to get a state model state representation of a d c d c converter. So, using this as the basis we saw how to get the state equation for first order system, we saw how to get the state equation for a second order system, but both these systems were continuous state, that means the state did not have any jumps between, the circuit was continuous to main circuit, analog power analog circuit.

Now, the d c dc converter case there is a difference there is a major difference where in a switch component is introduced. It is not just an r, L and c. Apart from this three there is one more component which is a non linear component which has two states, either it is on the resistance is 0 or it is of where the resistance towards infinity. Now, these two states change this structure of the circuit, and this is where the complication sets in, and we need to appropriately make assumptions to arrive at a state spares representation.

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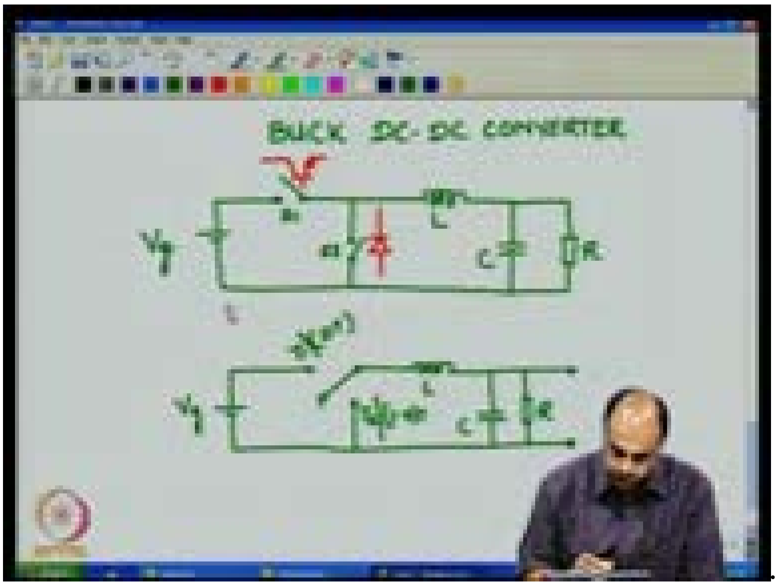

So we shall see how we go about doing that however we will bank upon this basics of how to obtain the state space from equation representation of a circuit of a general circuit, and use that one appropriately for the a c d c convertors case. Now, let us start by considering the case of a very simple d c d c convertor, and that is the buck convertor. To start with, we shall start with deriving the model for a buck convertor d c d c convertor. You are all familiar with the (( )) of the buck convertor when you have discuses its study state characteristic analyzed it, and, so how it behaves operating principles.

So, basically if you look at the buck convertor, it consists of an unregulated source v g v g or v n as whichever symbol is convenient to you, you may use that where is one switch, and another in a passion as shown here followed by an inductor, and c filter l c filter which is the averaging network, and of course, the load R c L. Switch 1 switch 2. Notice that may be earlier you probably may be familiar with this form that is you have two pose in the case of a single pole, double through switch like this. So, this is a representation that one would start with a single through, double through switch where the switch pole is placed at through one for a period of time d t, and placed at through to for a period of time 1 minus d t. This t and this t are different.

Now, this single pole double through switch is represented as two, as two single pole single through switch. This is a s p s t switch, this is a s p d t switch. Both are exactly equalent however while implementing, where implementing normally by two separate switches were in s 1 replaced. By probably as a semiconductor switch like a transistor mass unit or a b t like this, and s 2 is replaced by a device like this, a diode. So, you start with single pole double through switch, and then as you progress you switch over to single pole single through switches, and then replaces the single pole single through switches with the semiconductor switch symbols to go more towards the implementation of circuits.

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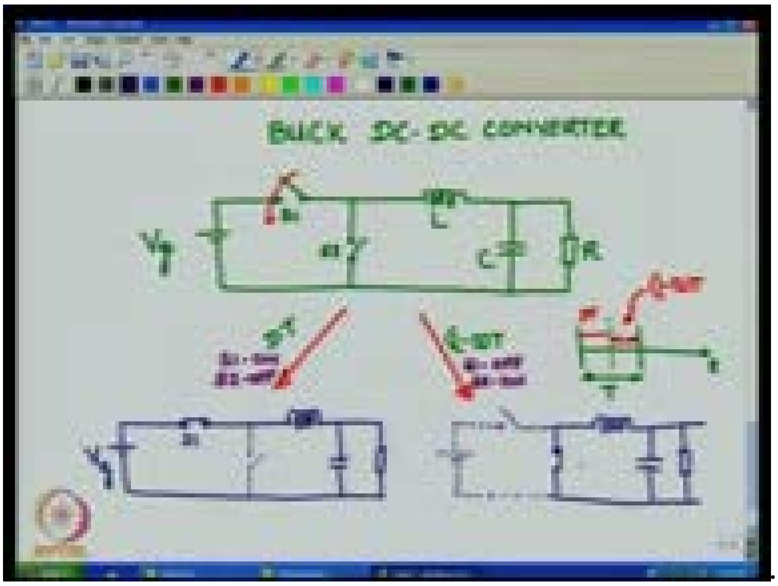

For now, let us raise this portion of the circuit, and see what, in what manner we can model this d c d c convertors circuit. Now, I have shown two arrows hear basically what it means is that there are two distinct time periods within a period d t is a period that if I indicates time axis as d here. And if I indicate here that this is the time t, time period total switching time period t, and this is spilt into two parts, where in this part we call it as d t, d is less than one the duty ratio, d is duty ratio, and this part is 1 minus d t. So, during the d t part, the switch s 1 is on and switch s 2 is off.

So, let me indicate during the period d t, S 1 is on, S 2 is off, and during the period 1 minus d t S 1 is off, S 2 is on. So, these are the two possible modes of the circuit. So, during d t what happen to the circuit? How does it look like? So, during this period you have the supply v g the switch S 1 is on, and drives directly through inductor, capacitor, resistor in this fashion. And, I am going to show the dotted switch S 2 which is not used. So, it is essentially out of the picture, It is not there in the circuit.

During the time 1 minus d t the switch S 2 is on S 1 is off, so which means that the supply which I am going to show doted now is out of picture, and as switch S 1 is removed, is open, it removes that portion after the circuit. And we have S 2 on as shown here, and the inductor, and the capacitor which contain energy by virtue of the voltage across them, and the current through them, and the inductor energy will freewheel through the switch S 2. So, this is how the circuit would look it like when the switch S 2 is on and S 1 off. Now, we have these two circuits, and we need to model then. So, we take the approach that we write down the static equation for, we write down the state equation for this mode of operation of the circuit. Then we write down the higher equation for this mode of operation circuit, and then we combine the two to arrive at some kind of an averaged state space model for that portion. so, we shall do just that.

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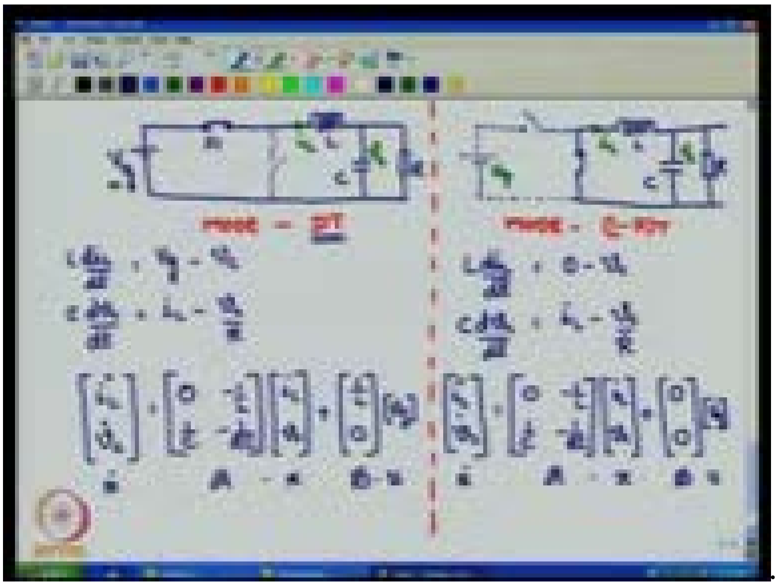

But before that let us copy this portion of the circuit, and take it on to a fresh page, and develop the model for these two. Now, here we will have a vertical split. This is the mode, this is the mode of operation during D T period, and this is the mode of operation during 1 minus D T period.

By inspection of course, step one of your, step one of your modeling procedure we have two energy storage components L and C either of the modes. So, you will have order to this mode, you have an order to in this mode, two different equations in this mode, two different equation in this mode. And then we make the combination. Identification of the variables, v g is one of variables, input variable, v c across the capacitance is another variable I L through the inductor is another variable.

Likewise in mode two we see across the capacitance which is same as output voltage, and the current through the inductor, these are the two state variables v g of course is 0 here. It is not connected by that anywhere we shall write in the variable. And, then the list of parameters we have L C R, we have L C and R in both the cases. Now, we are ready to write down the equations, the mode by mode. First mode during the mode D T, let us write down the equation. Two equations which are l d I l by d t which is equal to the voltage of across the inductor, the voltage has this point is nothing, but v g, and the voltage of this point is nothing but, v c. So, the voltage across the inductor is nothing but, v g minus v c. Then the next equations d v c by d t, we will do the simplification later.

The capacitor equation, differential equation is c d v c by d t is the nothing, but current through the capacitance which is the current through the inductor minus the current flowing through the load resistance R. Therefore, we can write it down as I L minus v c by R. Always use just only those variables which we list down the state variable input variable and the parameters, nothing less nothing more. Now, these are the two equations in mode d t when the switch S 1 is on. Likewise, two equations in mode 1 minus d t when switch S 2 is S 1 is off.

So, you see that L d I L by d t is equal to the voltage, and at this side of the inductor is 0, and it is started to the round by switch to voltage on this side is v c. So, you write it down 0 minus v c, and the capacitance equations, equation c d v c by d t equals to current I L through the inductor minus the current which flows through the load resistance part. So, we know how two equations, each we know put them in the state wise representation.

Each of the set of the equations you can put it in state wise representation as follows. You have I L dot v c dot, they represent the, they represent the derivatives of the state nothing but x dot, now you have a A matrix, the state, vector x plus the B matrix the input vector u. And the input factor is nothing but v g. Now, we just fill in the elements of the matrix. Look at the equation hear I L the co-efficient for IL in this equation, in the first equation here is 0. So, we putting 0 here.

The co-efficient of the c is minus 1 by l by talking by l on to the this part of the equation likewise for the second equation, The co-efficient of I L is 1 by c by taking the c into this part, this height minus 1by R C. so, this forms the A matrix the B matrix again looking back into the equations, you have the co-efficient of the v g which is 1 by to some amount of a step keeping looking it mentally, but you need not actually skip this steps. You can write them down, but with experience you can do things faster. And the second equation you see that there is no v g term in this second equation. Therefore, this becomes 0.

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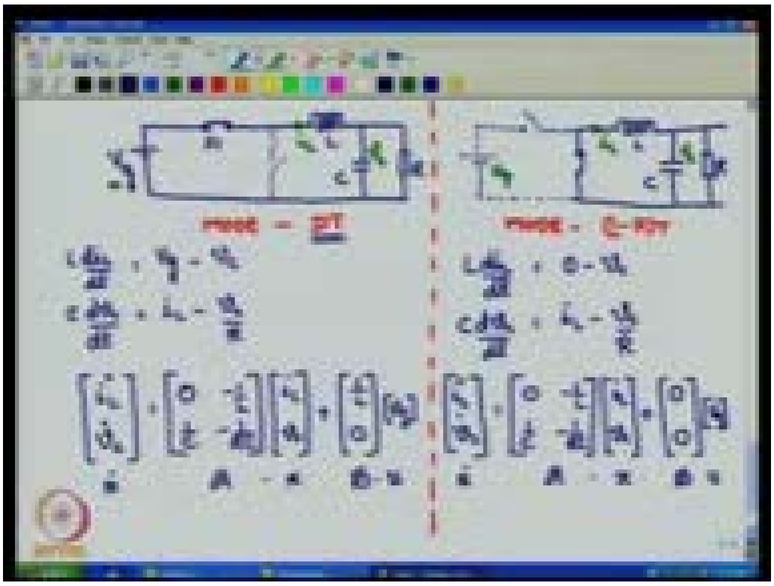

So, this becomes the state equation for mode, for that part of the circuit, for that part, segment of the circuit which is operative in mode where due to cycle is speed v t the time period is v t. Likewise, similarly, for the mode where the period of operation 1 minus b. We write down the state equation in a similar manner, you have I L dot the derivatives, we see dot the derivative, nothing but extra, that is equal to, we have a state characteristic matrix multiplied by the state vector x.

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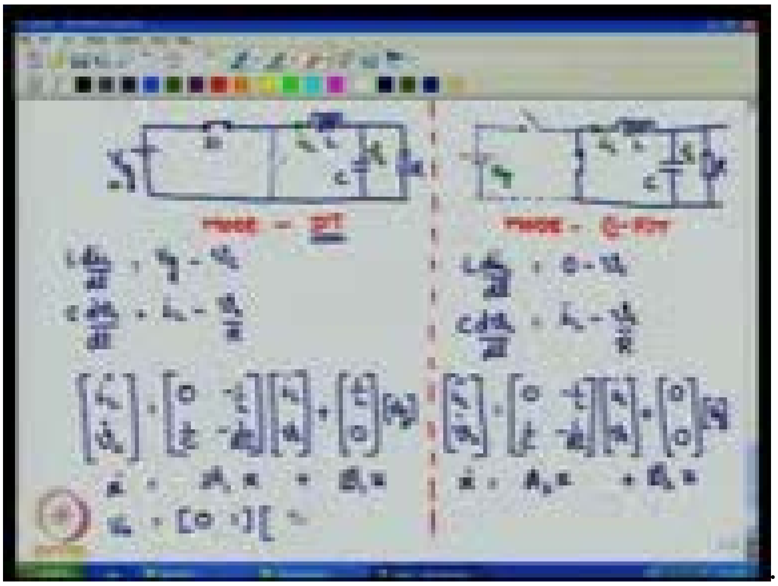

Therefore, this has to be A, Plus the input scaling matrix B, and the input itself v g u in this case. So, you can observe here that the first line of the equation, there is no I L component. Therefore, the co-efficient of I L is 0, and we populate the co-efficient of v c 0 minus 1 by L by taking l on this side the equation, and looking at equation two. Co-efficient of I L is 1 by c by taking c on this side co-efficient of e c is minus 1 by r c. That we write down 1 by c minus 1 by r c. So, this forms A matrix, and in the case of B matrix there is no input term coming into the picture here. Both are 0. So, this forms the state equation for these two modes, and we are going to call it x dot, because the state is same renaming it as A 1.

X plus B 1 u, and then we will use x dote equals A 2 x plus B 2 u, jus just to indicate that this is one state equation, and the other state equation, and we need to combine these two in some form, some manner, some kind of an average we need to do. Likewise in a similar manner the output equation, very simple ios nothing but V naught is our output, and V naught is nothing but V C.

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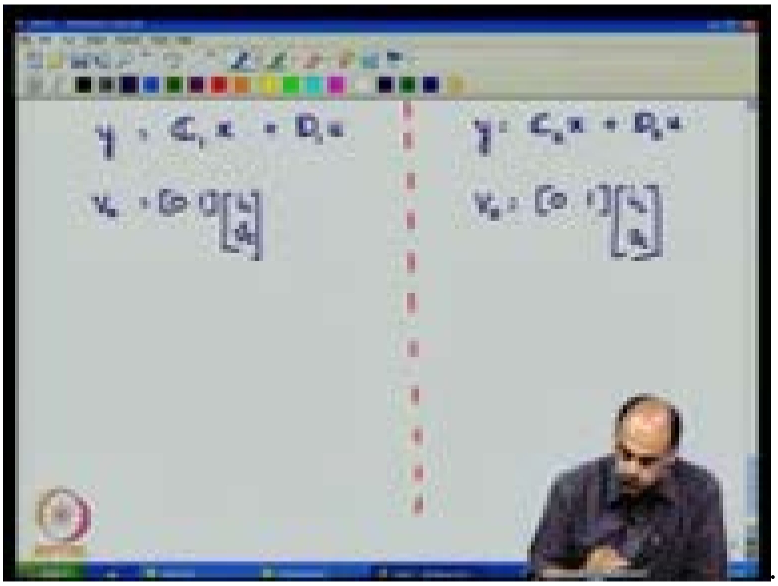

Therefore, the c matrix is nothing but 0,1. I think we shall use a fresh page for this purpose. Let me write in the line of (( )). The output is of the form y is equal to C1 x plus D 1 u, and in the second case is output, same output is C 2 x plus D 2 u. So V naught is the output, and we know that the state is, state vector in the I L and V C we need to calculate the C matrix plus we know that there is no input term directly connected to the output. So, we may as well not use it, so that is 0. And here we have v not which is equal to C matrix and the state i L V C. so, this is nothing but 0, and 1 because V C is directly coming to the output 0 and one, both are same, but still we denoted by these two output equation.

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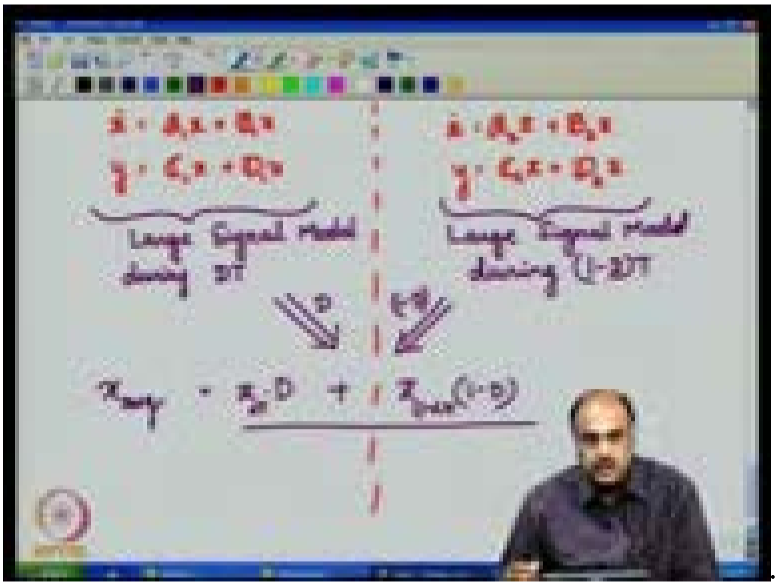

Now, how do we combine these two into a single common model. Now, this equation this site of equation during the period d t, and the set of equation during the period 1 minus D T are called the large signal models. A 1 x plus V 1 u x dot is equal to A 2 x plus B 2 u. Y equals C 1 x plus D one u. Y equals C 2 x plus D 2 u. So, these are called the large signal model during D T period of time, and this is the large signal model during 1 minus D T. So, what we now do is combine these two by averaging. Take this model average it by multiplying by D, take this model average it by multiplying by 1 minus T. So, that we get some kind of a state averaging. So, if I take for example, state x here, x into D, that is x during time D T plus x during time 1 minus D T into 1 minus T would give u a some kind of an average state x average.

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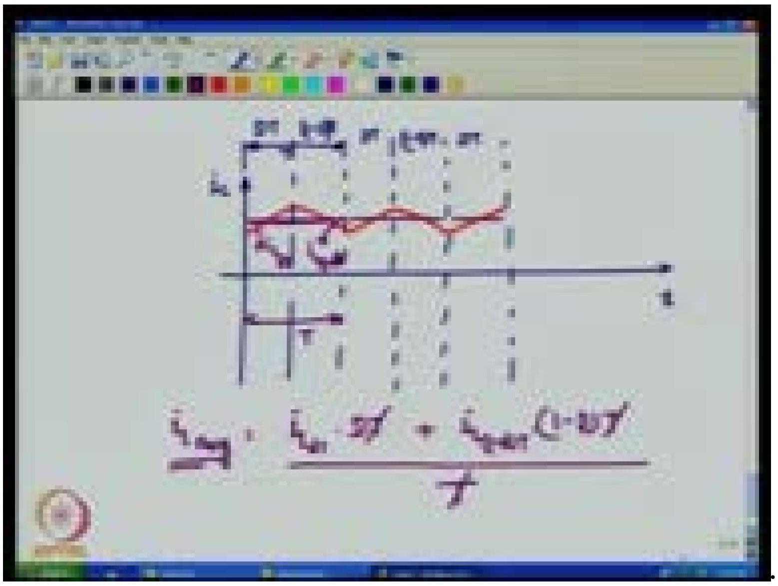

So, this kind of an averaging is done, this is actually an approximate what the basically happens is that the state lose some information, for example, if I take the simple case of an inductor, inductor current, the inductor current in the case of the buck converter is like this. Let me put the time axis T. this is time period D T, 1 minus D T. This whole is time period T again D T 1 minus D T, so on, it proceeds.

Now how does the inductor current I L look like? Period to period, so you will see that there is a rise portion, this you have studied during the when your studying, discussing the operation of the circuit. So, it rises with a slop of V D minus V C by L. Falls with the slope of V C by L, again rises, falls so on. Now, if you take one period the period is very small enough compare to the time constant involved L C time L R and R C time constant involved.

Then we could say that the average value of the state, this is actually a state I L. So, the average value of the state instead of writing it like this if you say this is the average, this is the average of this state, and during this period, this is the average of the state during this periods so on. Then we call this as the averaging of large signal. So, how do achieve the averaging of the large signal, we take the state value here multiply with D. So, let us say if this is I L during the T period, and if this is I L during 1 minus D T period.

Then what do we do? We said I L average equals I L during D T period into D T plus I L during 1 minus D T period into 1 minus D T divided by the time period which is the whole T. Therefore, we obtain I L D T into D I L 1 minus I L of the state into minus T. That would become the average value of the state during the one time period T, and that is the value.

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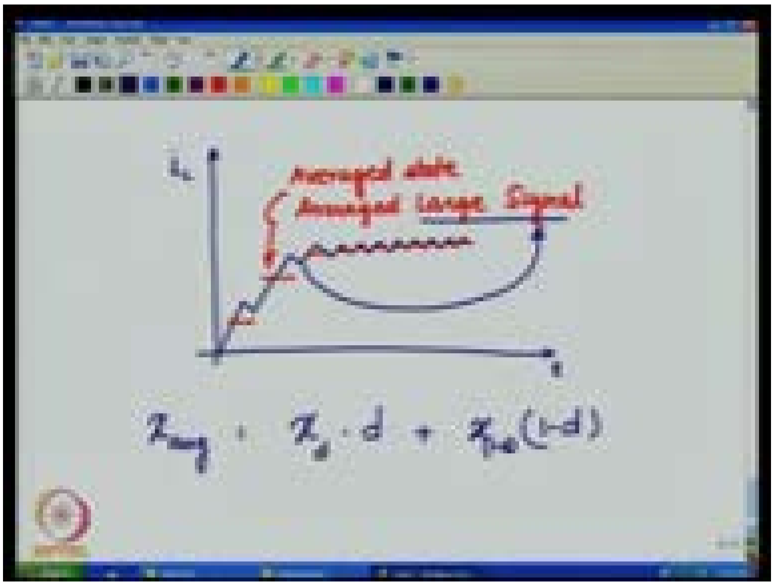

So, this is the approximation that we do, assuming that time period is small, so which would imply that if I have the inductor current dynamically changing with the respective time I L, if it proceeds in this fashion, and then comes to some kind of factors. So, we are making the approximation saying that we have an average like that, an average like that, average value like that, so on for every time interval. So, this red marked curve still indicating dynamic, but the time period being very small compared to the dynamics of the system, this is called the averaged state.

And we call it as the averaged large signal where this is the blue is actually the large signal under consideration. This is the large signal under consideration, thus any state if we take the averaged value is nothing but x the state during the period d, I am indicating now small d, we will explain why small d later on during the period d into d plus x.

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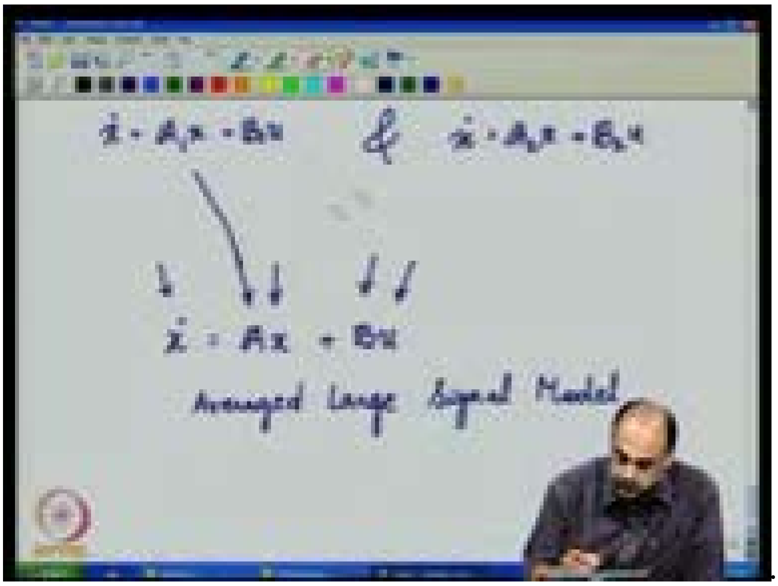

During this period 1 minus d into 1 minus d. This would become the average state within a period. And, this is call the averaged large signal state model. So, from these, from these two models, what we do is obtain a new state, new state representation x dot which is equal to A x plus B u, where each corresponds to the averaged value call the average large signal model.

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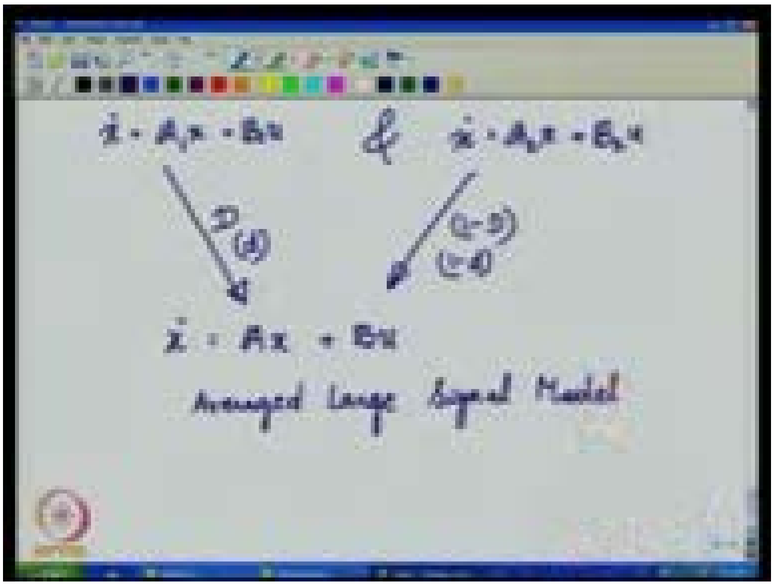

We shall continue with this obtaining the averaged large signal model from, from the two large signal model which is x dot is equal to A 1 x plus B 1 u and x do t is equal to A 2 x plus B 2 u. Now, these two models will be combined together, will be combined together in the this fashion by waiting it with D and 1 minus D and instead of upper case d, and upper case one minus d, we will be using in future d lower case 1 minus d lower case. The logic and the reason for the symbol change will be explained in the next class.

Thank you for now.