Switched Mode Power Conversion
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Lecture - 20
State Space representation - 1

Good day to all of you; so today in this session, we shall discuss and learn about the state space representation.

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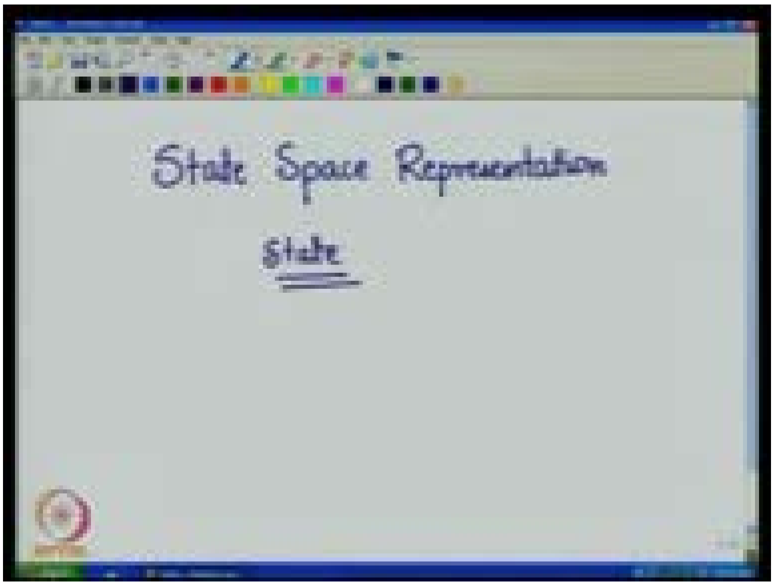

So, today's title would be state space representation. So, in the previous class, we discussed about the basics of modeling and how we represent any system which is, which may or may not be on linear system in a small signal manner, such that the small signal model enter is a linear model. So, today we will extend those principles and try to apply them on simple circuit to get a generic approach or generic method of obtaining a state space representation of the physical system. Then using this general concept we will apply to the VC, DC convertor circuit and see how we can get this state space representation of DC, VC convertors circuit.

VC, DC convertor circuit being slightly different from the normal continuous power physical systems in the sense that switched power, you have switches apart from their star LNCs. These switches make the power flow to the having switching pattern and this need to be addressed when we are trying to model the dynamic model of the VC, DC convertors. So, when we talk of state space modeling, we need to understand what state is and what it represents?

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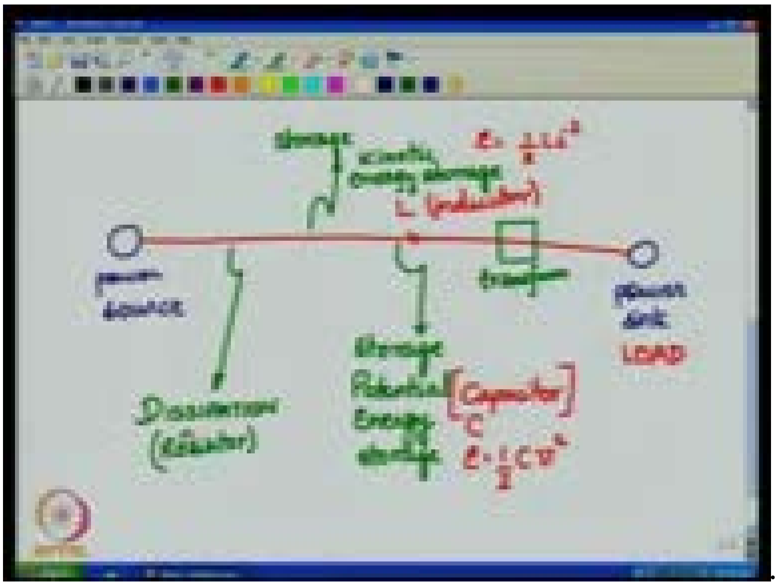

Therefore, we start from the energetic concepts, so if you have power source, a power source and we have power sink, so normally this power sink is what is known as load. We are familiar with that term and the energy has the power starts flowing from the power source to the power sinks, in this manner. So, when power starts flowing from the source to the sink or source to the load, this could be a battery, it could be DC power supply and this load could be resistance where any other equipment connected to this.

There are four possible actions events or that can happen; one of them one possible event that can happen even that is this dissipation. So, specific to the electric domain had dissipation implies it that this event happens through the component called the resistor. So, some of the power, which is flowing from the power source encounters the resistor and some of the power is lost within the resistor and gets converted to heat. Another thing that could happen is storage. The power can get the stored in the form, stored as energy in some components.

The storage can be into ways; one is called the kinetic storage, Kinetic energy storage and another type of storage is called the potential energy storage, energy storage. So, the kinetic energy storage happens with a component called L or the inductor in the electrical doming. You have equivalent components on the other domains like the inertia in case of the mechanical domain. So, here the energy is stored by virtue of the movement of the charges, by virtue of the flow of the current and the energy is given by half L i square were i is the current flowing through the inductor.

So, if you have an inductor L and a current flowing through it and that inductor is suppose to have an energy built up within it equal to half L i square. Likewise, you can have another type of storage called the potential storage, energy storage. There end this accomplished by a component called like a capacitor. Capacitor with, which is denoted by the symbol C and if there is a voltage v across the capacitor, the energy that is stored in the capacitor is given by half C v square. You know all these things are known, just trying to recap that these as the two mediums through which the energy will be stored, either the kinetic or potential.

So, apart from these three event there is one more possible event that can happen and that is a transformation. The transformation can be in, transformation can be in scale. For example, you could have, you could have voltage at one level here. The voltage will be transformed into another level at this point by means of a transformer or you could have an electrical domain energy getting transform to magnetic doming energy here. So, like this there could be energy form transformer transfer or there could be one of the power variables either potential or kinetic variable getting scaled accordingly to this transformer in a lost less way.

So, in an energy transformer case there is neither loss nor storage happening just you transform, this is what we want to indicate that in a typical power flow or energy flow from source to sink, these four a possibilities can occur, which is dissipation, kinetic storage, potential energy storage and are transformation of an energy are variable. Now, with this possibility, we have in a clear cut manner three important components.

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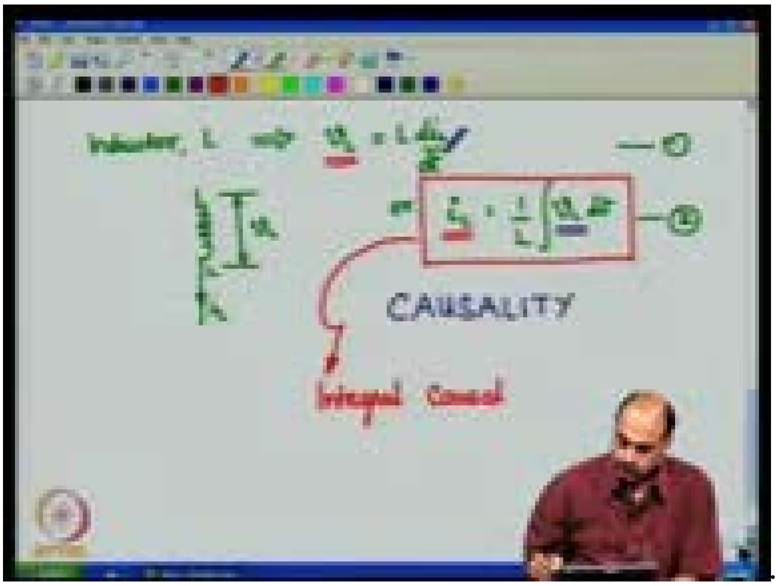

One as I was saying is the inductor, L as the symbol. Now, this inductor is governed by two possible governing equations, wherein you could have the voltage across the inductor. So, we have the symbol for the inductor, which is like this. Now, there is a voltage cause the inductor and we shall call it as v L and there is current through the inductor and we shall call that as i L. So, the voltage across the inductor and the current through the inductor are related by these the differential equation relationship.

V L is equal to L d i L by d t or you could say i L is given by 1 by L integral of v L d t. So, these are the two possible governing equations for the inductor, because I may say these two equations are the same except that have been rearranged. However, there is one significant difference and that is of of causality, causality. In the equation one, the cause or the independence variable is i L, the effect is the v L. The current through the inductor is the cause and the voltage across the inductor is the effect that in the second relationship, the voltage across the inductor is the cause.

The current through the inductor is the output effect in the relation from off with respect to the relationship to. It should be noted that in nature, nature always seems to prefer the causality where in the integral is integral causality. So, this is relationship that nature seems to prefer in almost all physical systems. This is the relationship, which we will most of the time we using for a developing the models. This is the integral causal, what this basically means is that causality which leads to an integral relationship.

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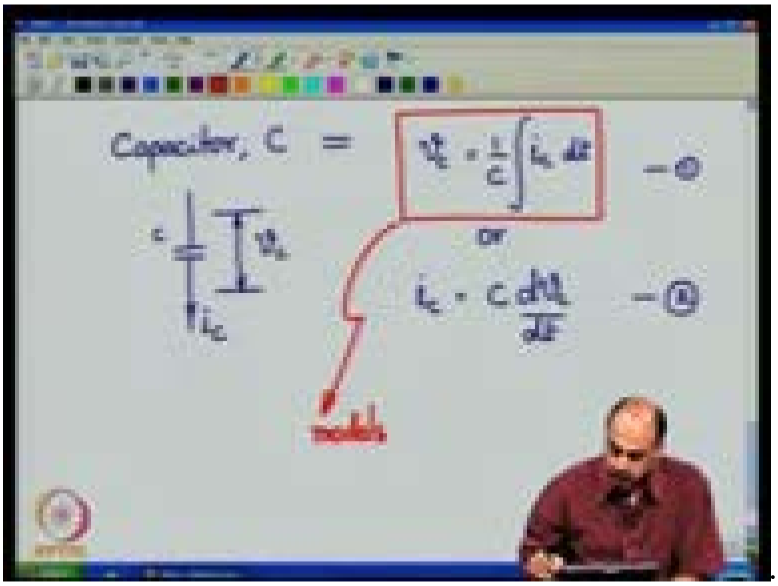

Likewise, in the case of capacitor with symbol C there in potential storage occurs, potential energy storage and it is given as half C v square. The capacitor is symbolically represented in this manner, you can have a voltage across the capacitor, and we will call it as v C and a current through the capacitance will call it as a i C causes the capacitance C. The governing equation for the capacitance also can be put into forms where you have the voltage has the effect output, which is equal to 1 by C integral of the current though the capacitance d t.

You could have the other way, which is the current through the capacitance as the effect and the cause being the voltage, so C d v C by d t. So, here to you have a two possibility with two different cause and effect. So, you could have the current has the cause, voltage is the effect. Voltage is the cause current has the effect. Here again nature prefer that casualty, which leads to an integral effect or an integration variables, because nature everything is a continuum, everything is continues. So, this is the one, which we will be used using for all the models without loss of generality.

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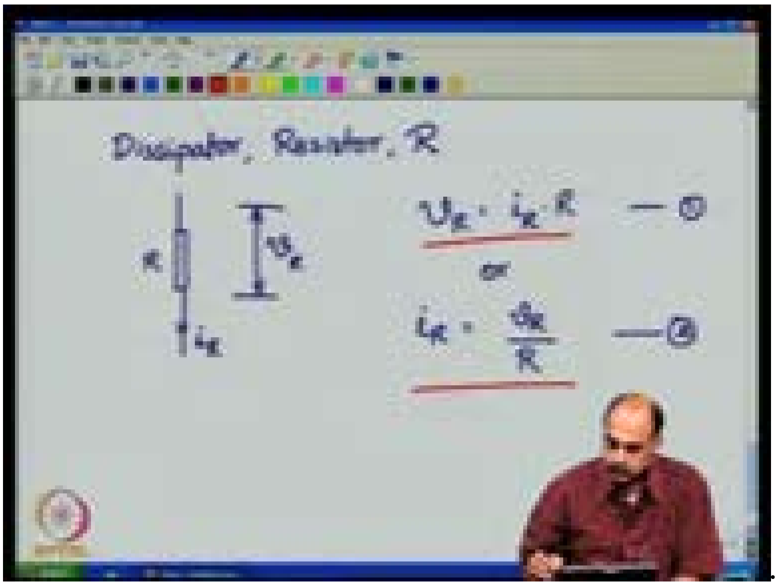

The third component that we saw was the dissipater, which is called the resistor in the electrical doming as a symbol R and it is denoted in this manner. Let us say it has a voltage that across that this resistance R has the voltage v R across it and the current through it is denoted i R. It also has two governing relationship v R, which is equal to i R into R. This is the first relationship, where in the current is the cause we are the voltage across the resistance is the effect. In the second relationship i R, which is the effect is a result of v R by R, where v R is the cause.

Now, here as there is no integral or an integral relationship an either of one of two, there is no prefer causality as such, that particular causality which is applicable for a given case is taken at that point of time. So, either this or this can be used in obtaining a model of the physical system. These are the three major components, which are used in almost all the circuits, electric circuits and we will see how they get in corporate into the model? How he get the state space model system in a systematic manner? Now, let us take up the case of obtaining the state of space of representation by means of an example, but first let us see what is it that we ultimately want?

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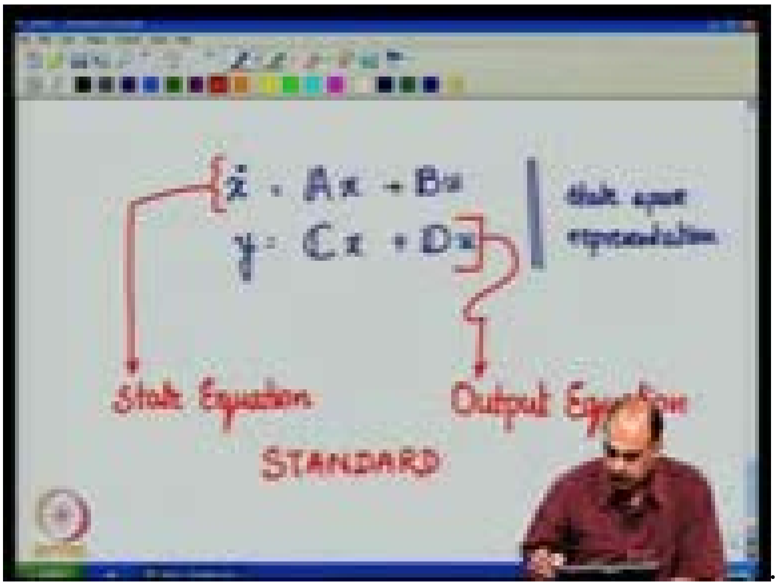

So, the state space that representation is given in this fashion. x dot, which is equal to A x plus B u and y, which is equal to C x plus D u. All physical systems all linear physical systems can be brought into this state space representation. This is also called the state, let me write it down here. This first equation is called the state equation. The second equation is called the output equation, so this is the standard form, which is used for representing all linear physical systems.

Now, here you see different variables, you see the variables x, you see the variables A B y C and D. x is called, x dot is nothing but the derivative of x, D x by D t is x dot. A is called the state matrix or the system state matrix, it holds the property of the system, character of the system, all the characteristic property get into this matrix A. B is called the input matrix or the parameter, gains related to scaling the input variable. y is called the output variable that is the controlled variable or the general variable that is presented out of the system.

This is obtained from the state variable and a scaling matrix C called the output matrix, directly from the input and scaling matrix D called the (( )) matrix. Now, this is the final state equation representation form but let us see, how we go about getting that one. Before that let us have some blocked or view of this state equation itself, such that we get a better understanding of the inner working of the system.

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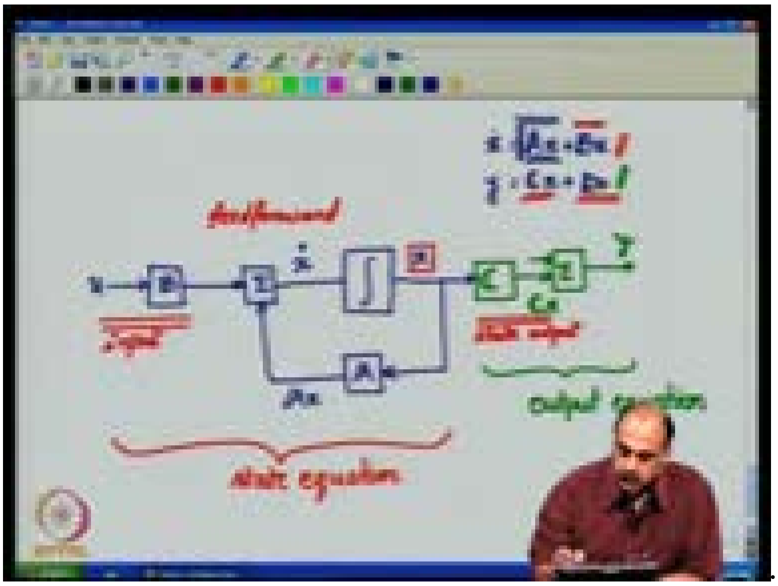

So, we started with x dot is equal to A x plus B u, y is equal to C x plus D u. So, this we said that was the state space representation, what does this implies? So, let us say we have a block, the central block in all the physical systems is the integrator. Integrator is something, which is so very much evident in almost all physical systems, which is modeling and the way the nature behaves, it is the central block even in modeling. The output of the integrator contains history, memory. It remembers the output the integrator have the memory effect, it can remember its past history and we call that one as x.

So, all the state variables should have, should be the output of integrate or should be input of memory blocks, only those qualified to be the state variables. So, that is very, very crucial, remember that a state variable has memory, it remembers it past evolution or immediate past history. By virtue of integrator being a memory element or memory block, it output its output will directly qualify for the being called a state variables.

So, if x is the output the integrator the input to the integrator is defiantly derivate of output, which is x dot. So, let us see what happens every systems has a local internal feedback. So, the state x is scaled through some parameters and you obtain x and this is added some (( )) with the input. You have the input u, which flows in it gets scaled by it means, the input scaling and we call that one B matrix. D represents the input scaling and it goes to the summer block, which act up with the output of the intergraded, scaled value and results in x dot, which is the derivative value of the state, which gets integrated on finally becomes the state vector.

Now, this is still it complete. This in fact here represents the state equation portion of the state space for representation. Now, continuing further the output x gets scaled by other parameters. Let us say C, will give you C x and to that you sum up add the input, which comes directly as we been forward term. Let us sum up at this point to give you the output y. So, if u see, this portion is called your output equation or the output equation portion of your state space representation. So, if you look at this block diagram, we see that the input u gets scaled by this scaling matrix B, set of gains parameters of the system. Then it gets added up the feedback portion A x becomes the derivative.

This derivative goes through an integrator, always an integrator becomes the state variable, which used for feeding back. The state variable get scaled by the matrix are the gain value C, and you gets C x. To this is added the direct feed forward portion of the input, scaled by D summed with C x you get the output y. So, every system can be represented in this form. You have an input portion has shown here, you have the input portion, which is indicated here in the equation.

You have an output portion directly from the states, state out put some combination of the state is indicated here. You have a portion of the input, with the portion of the input, let me call that one as that feed forward. Meaning it is the input is directly by passed and send directly to the output after suitable scaling. The system is actually avoided and it is taking the bypass and goes through the scaling de matrix D u and then comes out at the output.

So, this D u is actually represented here at this portion of the output equation. Then you have this portion that is the integral portion plus the feedback portion, the state which is feedback through the scale matrix and back again input the integrator. Now, this portion actually the dynamics of system and this is characteristics, which gives you the character of the system. Therefore we have what is called the characteristic matrix or the dynamic matrix indicating the dynamics of the system is represented by this A x portion of the equation.

So, these two provided for A x and B u provide, what is called as for scope the next evolution of the variable. So, x dot is actually giving you a prediction, an estimate, prediction the slope. It says this the derivative and the next value of x depends on this particular slope. It will value based on this particular prediction of the slow that is derivative, which actually is used by the integrator to achieve the obtain the next value of the state, of the next next contain value of the a state.

So, this is the format in which every systematically be reduced to and appropriately output into this forms. Input portion, the out portion, the dynamic portion, the format portion. In many cases you may not find out all the portions, in many cases you may not find the a forward portion at all, so which means D will be 0. Employing that this portion is not existing and in the state equation also this portion would be absent.

So, in many of the state equation where there is no feed forward portion you will just have this portion, this part of the representation where output equation is just reduced to y is equal to C x, this is also very very common. So, our job now is to bring any system weather electrical or otherwise into this format. V C, D C convertors are also to be brought into this format. Now, let us take up a very simple example, which all of you would be similar with, so let us say we have an input source, a resistor and a capacitance this form.

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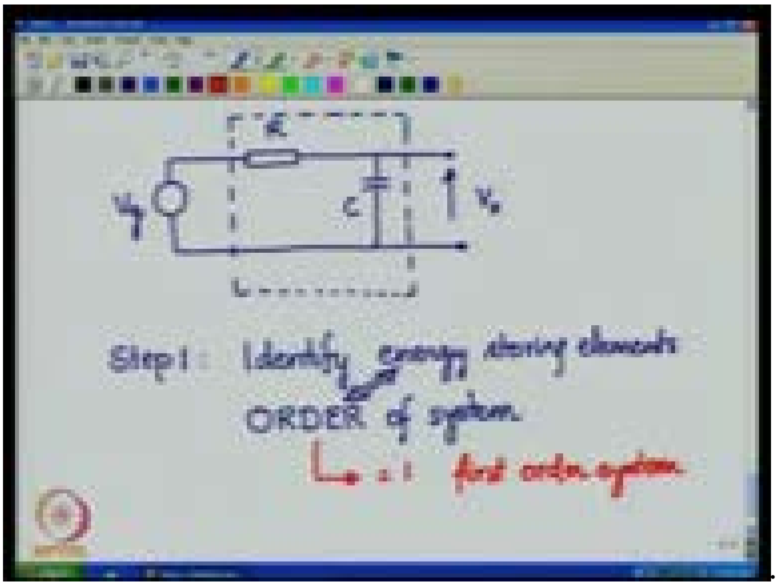

You have an R and you have a C, so the system actually is made up of this. This is our land under consideration or system under consideration. We have an input, we call it as V g and we have an output we call it as V naught. There is a current flowing through this particular circuit into the through the R in the capacitance. Now, let us say we want to represent this particular circuit in the state space form, how do we go about doing it?

So, step one; step one we identify the energy storing elements. So, the number of energy storing elements directly give you or direct measure, direct indicator of the number of integrals or integrators that you will have in your models. Because as we saw in few moments ago that you have three possible events apart from the transformation, which is dissipation, kinetic energy storage and potential energy storage. So, the kinetic energy storage in potential energy storage are linked directly to the inductor capacitor element in the respective bio domains.

The inductor the governing equations is the current through the integral, which is given 1 by L integral of voltage across the inductor into D t. The voltage across the capacitance the integral, which is given as 1 by C integral the current through the capacitance to the D t. So, these two are the integral relationship, which is preferred by nature. This integral occur only in energy storing elements as energy continues parameter. So, identify the number of energy storing element the number of the element storing elements is also called the order of the system.

The order of the system is called is directly connected with the number of energy storing elements in the system. So, in the above circuit as we see, we have only one dynamic component, one component, which is capable storing energy. Therefore, we have the order of the system equals 1. So, it is the first order system as as we see just only one energy storing element.

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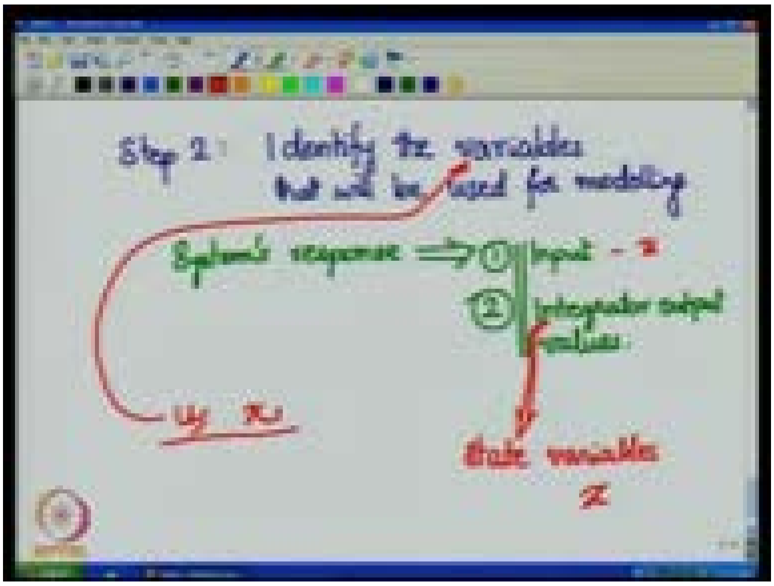

Now, next step; what you need to do is to identify step two. Identify the variables that will be used for modeling. Now, this is the important stage, which will lead which will lead you to a correct modeling. So, at this point it is, it may be appropriate to mention that any system, any system response any systems response is a dependant on 1. The input given to the system and also dependent on the integrator outputs, integrator output values, remember that integrator has memory. Even if you switch off the system, the integrator output will store the previous history or previous value.

The next, the next state of the value for the state of the system is a decided by not only the input, but also the values that are associated with the output of the memory devises or integrators in this specific case. So, it is good to use this items the input and the integrators output as the variables. The integrator output are called as we saw state variables. The integrator output are called the state variable x and the input we have been denoting it as u in our state variable representation. Therefore, we need to identify what is u and what is x? All the u's and all the x's, we need to identify and list down and they form the variables that will be used for the modeling.

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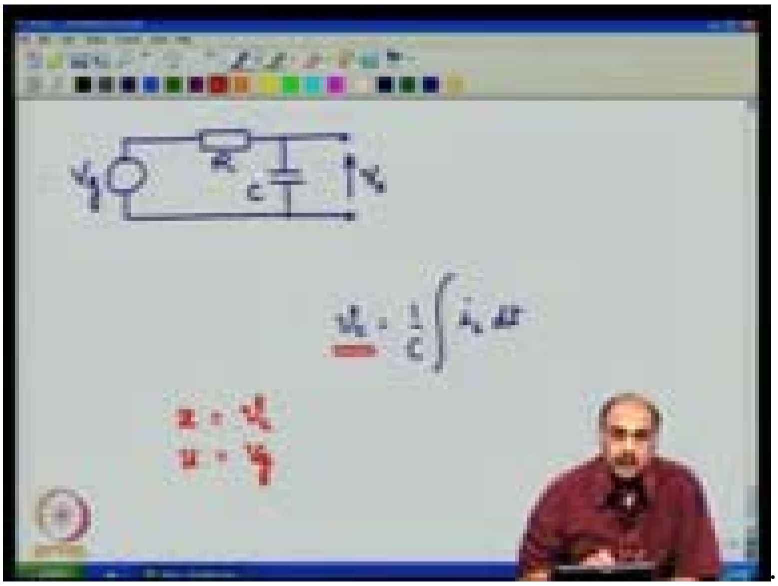

So, if you look at the circuit that we have taken up, which is a comprising of a source called V g and two components R and C and V naught. Now, we know that if we are having C, we would like to use the dynamic relationship v C, which is equal to 1 by C integral of i C, D t the integral equation. So, apparently we see the voltage across the capacitance the integrator, so this will directly qualify as been as one of the state variable. So, has this first order equation, as this the first order system, there will be only one equation that is sufficient to represent the whole system. So, you have x, the number of state variables is just 1, because it is just one energy storing element will be v C. Then u will be V g, the input in this case is V g, which is called the control input for this case.

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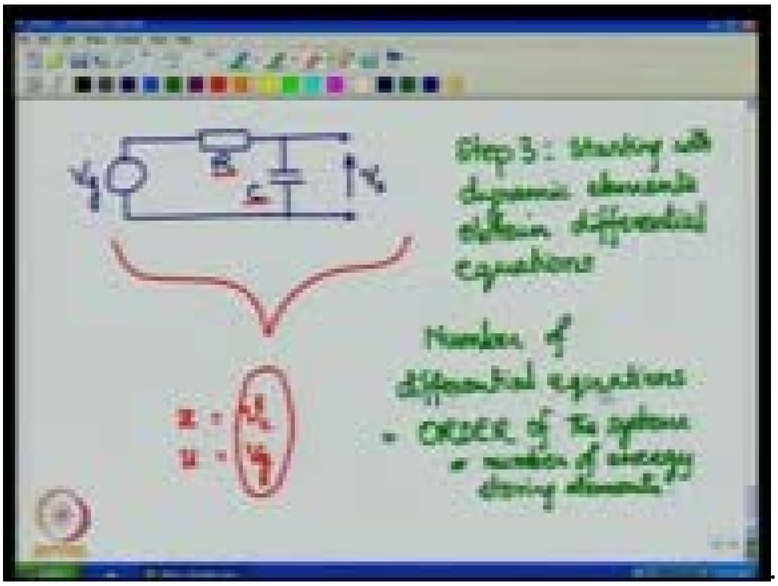

So, considering these two variables you have just these two variables the whole system has been represented with just this to these two variables. Plus the parameters the fixed or the constant parameters that define the system, this case R and C, these are the only variable system, we need to use that from to use only this two. Now, let us look at the the system based on these two variable only. Now, take start always with the dynamic equations.

So, now we come to step three; so step three is starting with dynamic element obtain different equation. This is the third step; starting with dynamic element, dynamic elements obtain differential equation, if it is mini energy storing element in the circuit. Here, it is the only one element storing, therefore we are starting with the dynamic element obtain one different equation.

So, the number of different equations the number of different equations will be equal to order of the system, of the system or the number of energy storing elements or number of memory elements, if it is in the case of that is very less. So, therefore in this specific case we have one energy storing element. Therefore, the order of the system is one, therefore you will have only one differential equation.

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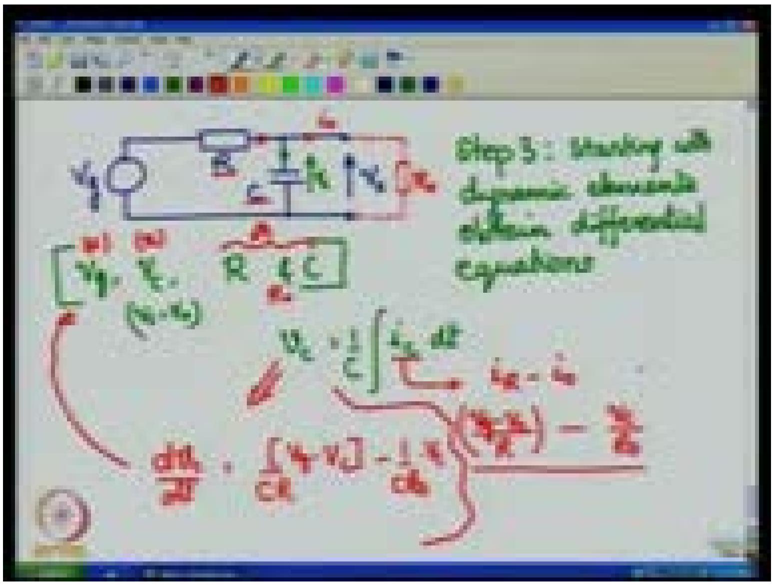

So, let us try to formulate the equation for this particular system in terms of variable V g and V naught as in terms of the variable V g and V C. VC is the voltage across the capacitor. Notice that V C is equal to V naught, but we will come to that come to this accept later, when we are developing the state equation completely. Then there are two parameters R and C, so we need to use only these. So, starting with the dynamic element were we have the voltage V C equals 1 by C integral of i C, d t.

i C is actually the current, which is flowing through the capacitance. The current through the capacitance I cannot use the symbol i C, variable i C because I have to restrict myself to variables in this group, in this group set. This is u the input variable, this is x. These two will go in to the A matrix, which represents the system character. Now, this has got to get replaced with that symbol set, so what is i C? Looking at the looking at the circuit we have i C is nothing but the current flowing through R minus the current that would have flown if there is I naught. But right now this is the open circuit.

Now, let us say u are connecting it to a load, an external load which is R naught, okay? Then i naught is nothing but V naught by R naught. So, let us replace this i C with symbol set here, which is nothing but i R minus i naught, which is nothing but V g minus V C by R minus i naught, which is nothing but V naught by R naught, but which is actually V C by R naught.

So, you see that in this equivalent value of i C, all the variables are been taken from this symbol side. Of course R naught is one more item that we have component we have added, it becomes the element amount R naught here. So, all the variables taken from the variables list are being used and it can be written as, this equation can now written as d v C by d t. Differentiating on both sides I have 1 by C or V g minus V C minus 1 by C R naught into V C. This would form a representation of this equation all in variables, which are actually taken from this list.

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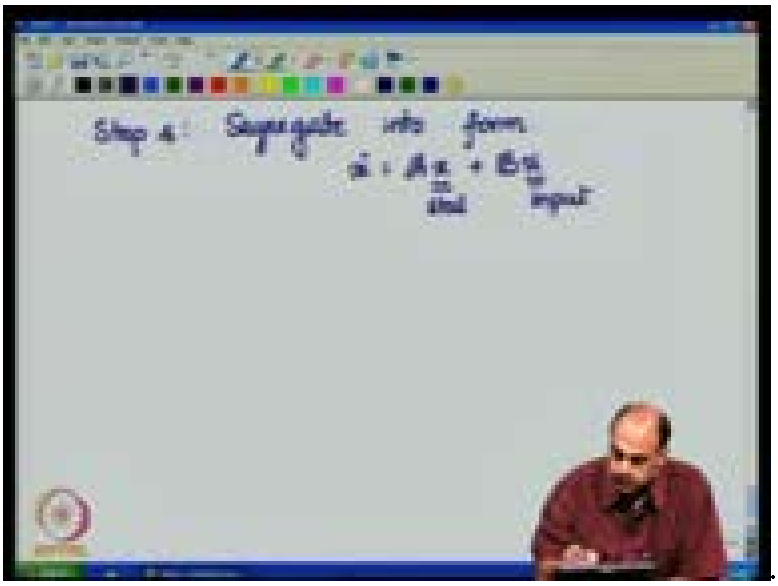

Next step; is segregate step four. Segregate into form x dot is equal to A x plus B u all the input states and the input this is the input and states.

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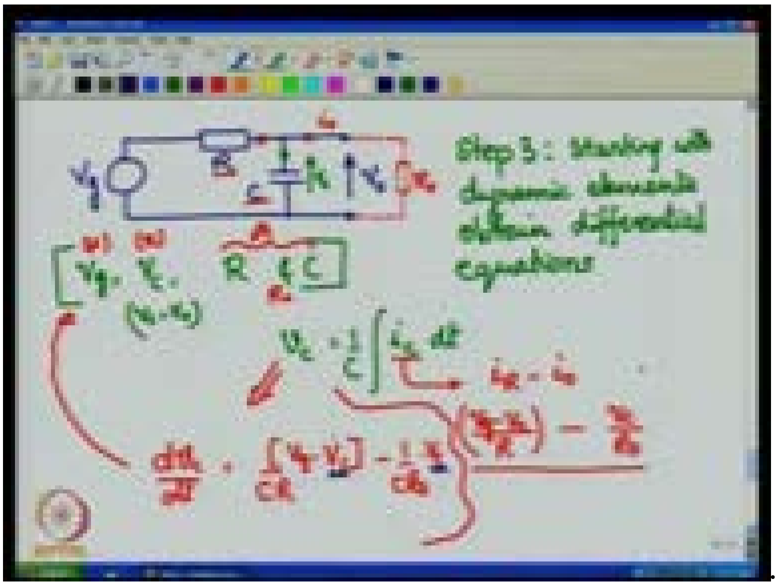

So, from this equation take all the portion, which is corresponding to states that is this portion into one portion and the inputs into the other portions.

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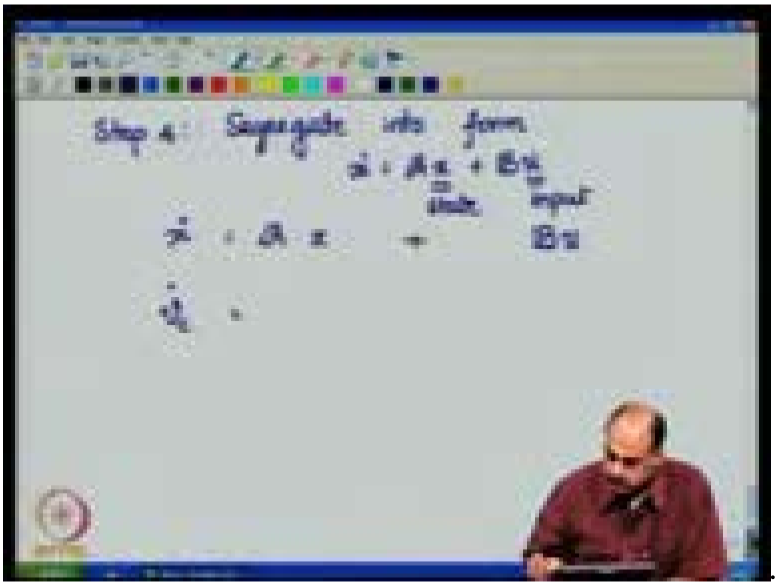

So, x dot is equal to Ax plus Bu can be written as x dot is nothing, but x is a state in this case is v C, v C dot that is the first portion of the equation that is this.

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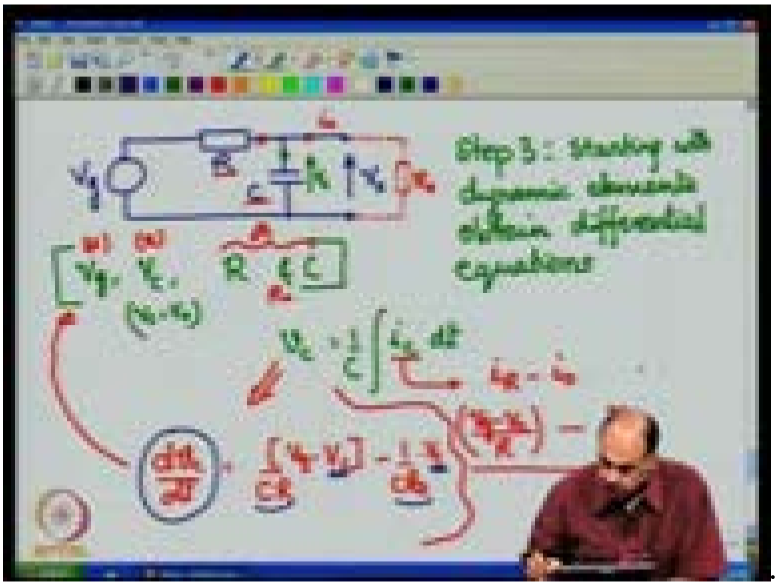

Then we are going to take the portions corresponding to the state variable 1 by 1 by R C and R naught C, 1 by R C minus minus 1 by R naught C both these together are multiplied by the state.

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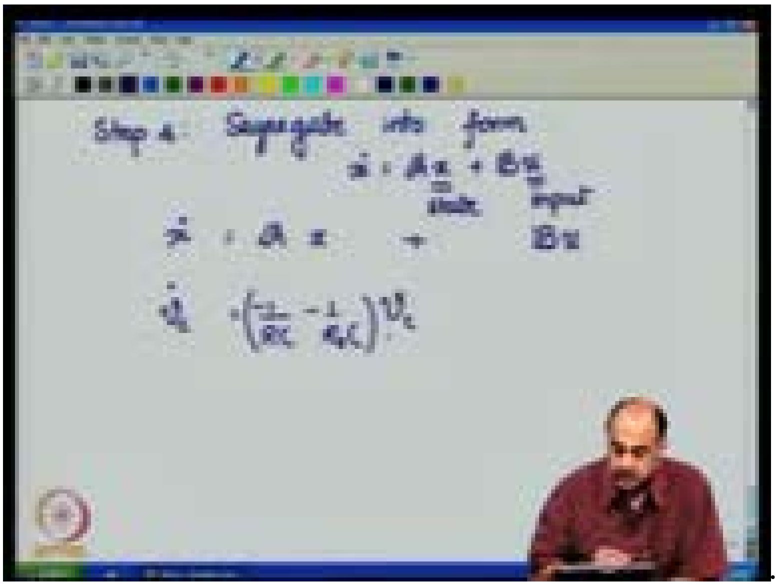

Then we take the coefficients (( )) the input variable V g in this case plus 1 by R C V g.

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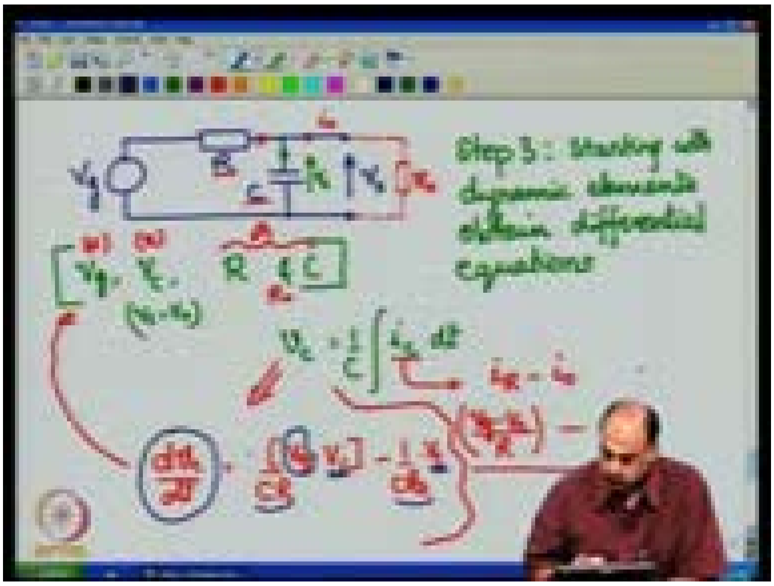

What is your output?

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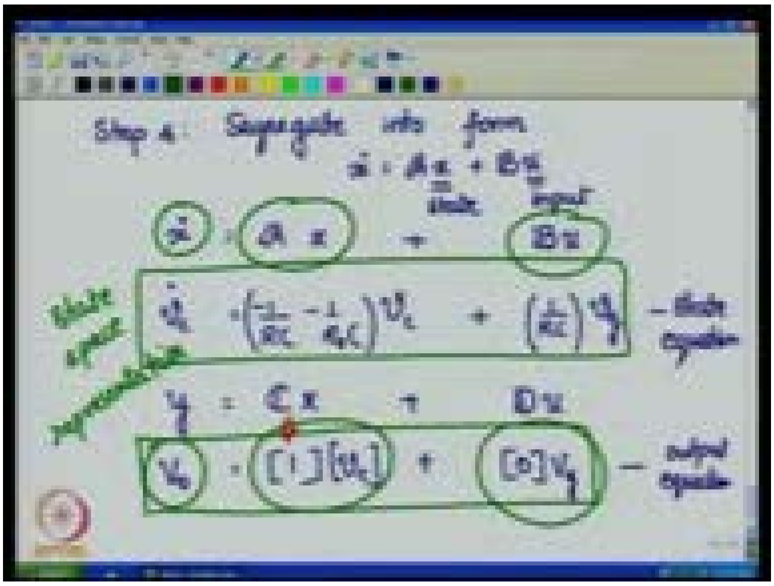

Now, this forms you are state equation this forms you are state equation portion of the state variable representation.

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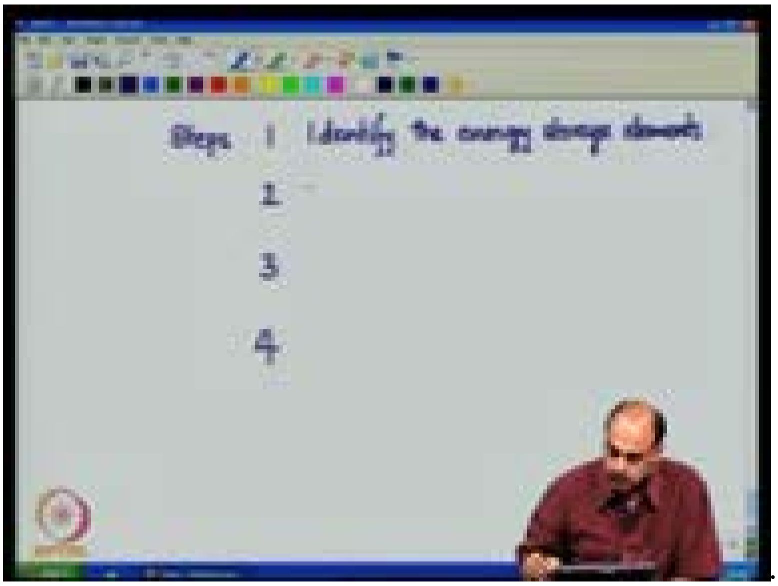

Output should be of the form y, which is equal to C x plus D u. If you look at this one why the output is V naught? Now, V naught represented as in the state scare form. Now, V C is our state variable, which is multiplied by 1, so C is nothing but 1 in this case. u is V g and V naught has no direct dependency on V g is only through R and C. So, there is no feet forward term, so this becomes 0, so this is your output equation.

So, if you look at these two terms, the two equations this together with this form the state space representation of the R C circuit we just saw. So, every such system, so every system whether it may be circuit or electromechanical system or mechanical system, any physical system for that matter, should be brought in this form. Representing dynamics called this state equation, and should be brought into this form, representing the output, which is the output equation together called state space representation.

So, this is how goes about obtaining the state space representation of a circuit, the simple R C circuit that we saw. Any circuit for that matter will, will follow the same approach followed by same approach as indicated that is steps; one, two, three, four. First step; is to identify identify the energy storage elements. This will give you a direct measure of the numbers are integrated. Therefore, the order of the system and therefore, the number of differential equations at are needed they are necessary to model the system.

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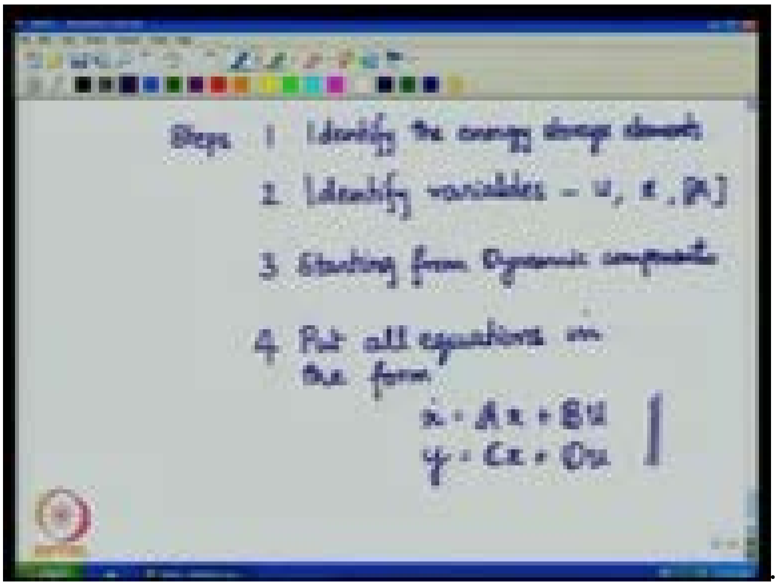

The second step; is to assumed as you see, as we saw in the case, the case of this simple circuit. We identified the variables for the R C circuit and that is what we need to do here in the as a second step, identify all the variables in the circuit identify variables. I will indicates it has u and x, meaning the input variables and state variables. So, state variables would definitely the voltage across the capacitors and the current through the inductors. So, the directly will be qualifying has state variables and the input variables whatever is input in the system.

These variables you need identify make a list and the parameters of the system and I will call that one has the A matrix parameters like the R C's the value of the L line constants, so on and so forth. Now, this list as to be made and you can use only variable from this list to make the equations and that is you start making the equations, starting from staring from dynamic components components. Make the equations one by one, if there are more than one dynamic component start with first one, may be a capacitance.

Then make the differential equation for that, next the pick up another dynamic component probably L. Then perform the dynamic differential equation for that and like that get all the differential equation, which matches the order of the system. Then put all the equations in the form x dot is equal A x plus B u, y is equal to C x plus D u. In this form you need represent the system and from there on you can do the controller design.

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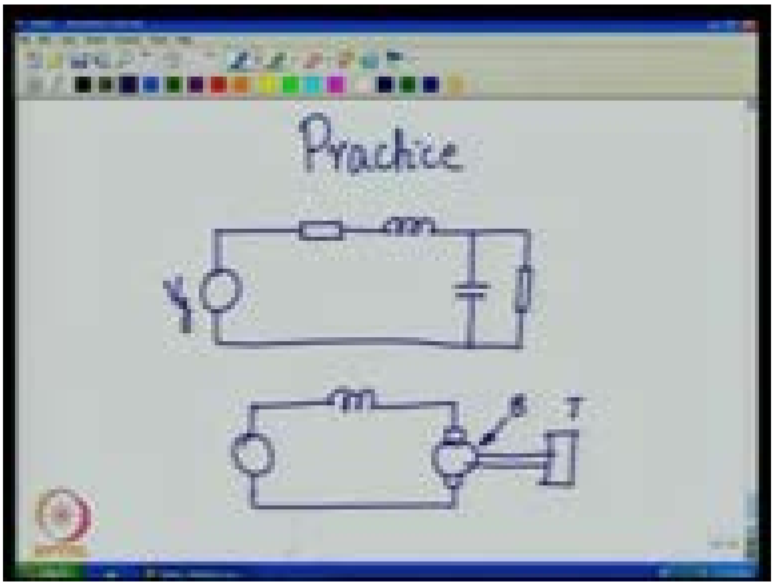

So, for practice I suggest that you try out many examples and one of the example could be simple RLC circuit. You see this is the second order system, you have two dynamic elements L and C therefore, order is two, you will have two different equations. Practice on this, probably you could also have motor and a system like simple DC motor system with mechanical shaft, inertia, bearing friction, B and then develop the relationship the state equations. Practice with as many equations and circuit as possible, and we will continue in the next class from here on.

Thank you.