Switched Mode Power Conversion
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Lecture - 19
Modeling DC-DC convertors

Good day to all of you. Today, we shall study about modeling switched mode power convertors. Though it is vast topic and we will not be able to cover in just one hour, we will be taking more, more classes. We will starts towards being able to model switched mode power convertors. Today, we will be discussing more on the modeling basics, so that is why the title of today's lecture is called modeling basics.

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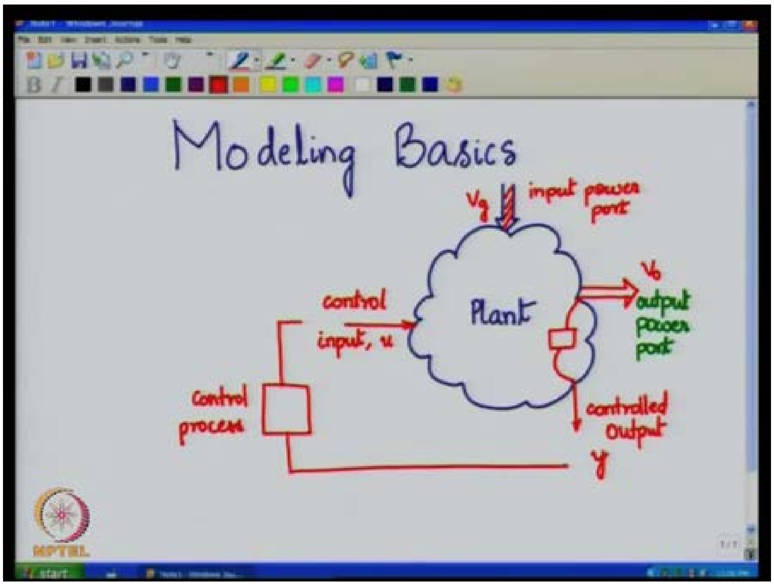

So, this is what we will be dealing in the major portion of today's lecture. Till now you have been discussing DC-DC convertors, the study state analysis, non idealities and to some extend the dynamic models of the various isolated and non isolated convertors. We shall now look at, how we try to control the switched mode power convertor? You have looked at the switched mode convertor as an open loop system, we need to control it develop controllers such that we get outputs with a certain performance pacification. So, that is the intent and towards this intent, we will develop the, evolve the subject. Before we go, before we start even trying to control the switched mode power convertor. We need to model them, the dynamic model which you have been familiarized by previous lectures.

We will dwell today in this class on trying to look at the generic way of developing these models and then apply it to the pacific DC DC convertors, so towards this goal I would like to bring your attention to one important aspect. In control system terminology what is a plant? The plant, this plant is what you have been discussing in the previous lectures on DC DC convertors, but let us bring it into a format, which is compatible for control systems design, it has an input energy port as shown here. Very important, I am going to hash it. This is an input power port energy port where in power is fed into this plant you have an output from the plant. We will call it as the output power port these two are the major energy or power level ports in the DC in any DC DC convertor.

The input is normally obtained from a, the input power supply DC power supply, which has been termed as V g. In the previous classes and the output, which is termed as V naught having a voltage V naught, so v g i g gives the input power V naught i naught the output power apart. From this there are two other ports not power ports, but signal ports which you would already be familiar with one is called the control input control input it is normally give assemble u and the sensed or controlled output. So, this controlled output as a symbol y, so internally this controlled output is generally linked to the actual power output by means of a sensing circuitry.

So, the sensing circuitry does the function of capturing the actual power signal converting it into compatible low low level signal voltage and giving it as signal with which we have named it as y. So, for example, in the case of a DC DC convertor the output voltage if you want to feed it back, you will be passing it through a device like an optocoupler or probably a hall sensor and the output of is a signal conditioned and brought out as the controlled output signal, which is what will be fed back.

So, with these, with these 4 ports, a controller is generally designed, which is basically interfacing with the control input and the control output. So, the control process is generally linking these two aspects of the plant ports so this is our control process which we will see what goes to make it up. Now, if we look at the case of specific convertor like the buck convertor now we know that.

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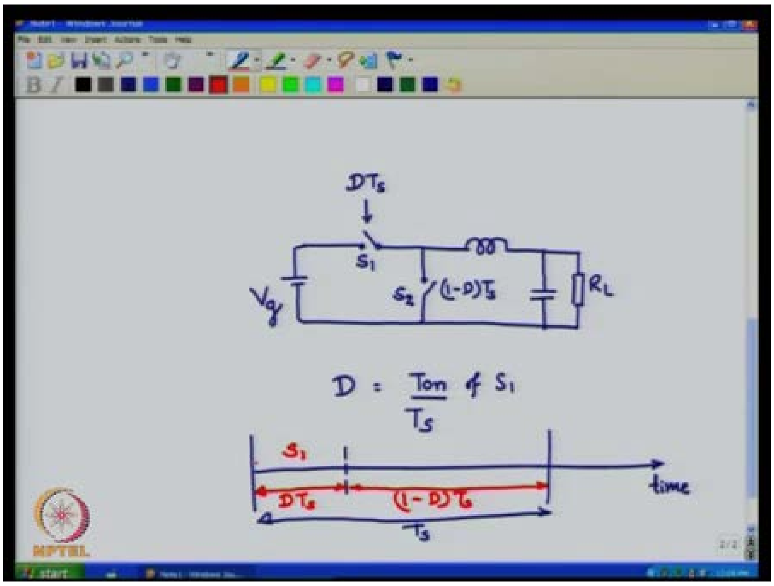

The buck convertor is of this form we indicate the switches in this manner, so the single pole double throw switches are replaced by two single pole single throw switches. You have an inductor, then a capacitor. So, this l c form the mechanism and followed by a load r l, now here at the input side. You have an unregulated or uncontrolled source V g you have switch S 1 and switch S 2 and these are mutually exclusive in the switching when this is when S 1 is on S 2 is off. When S 2 is on S 1 is off and you say that this is on for a period of time D T where T is the switching T S is the switching period and S 2 is on for a period 1 minus D T S where D is the on time of S 1 by T S.

The total switching period so to make it a bit more clear, if we say that time proceeds along this axis. We have this much time allocated for the switching period T S. Then we can say that this portion of the time the D is the fraction less than 1 is then or equal to 1. This fraction D T S and this fraction on minus D T S are the two mutually exclusive intervals of time during which time s one is on during D T S. S 2 is on during 1 minus D T S, now this particular circuit if we look at this, how do we map this circuit into the plant configuration that we have been looking at?

So, if we look at the plant, let us say that we have one input energy port we have one output energy or power port power output. So, what is the input and the output? So, if you look at this portion, this is the power input to the buck convertor, so that basically constitutes the input power. So, let us rewrite that as coming from here and the voltage here is V g the generated voltage and the output. You see is on this side and we shall bring that out in this fashion where the output here is V naught and the load is connected external to this plant, because it is decided by not the plant. But it is decided by the user now we have the input and output power port we need to define two more ports which is the control input.

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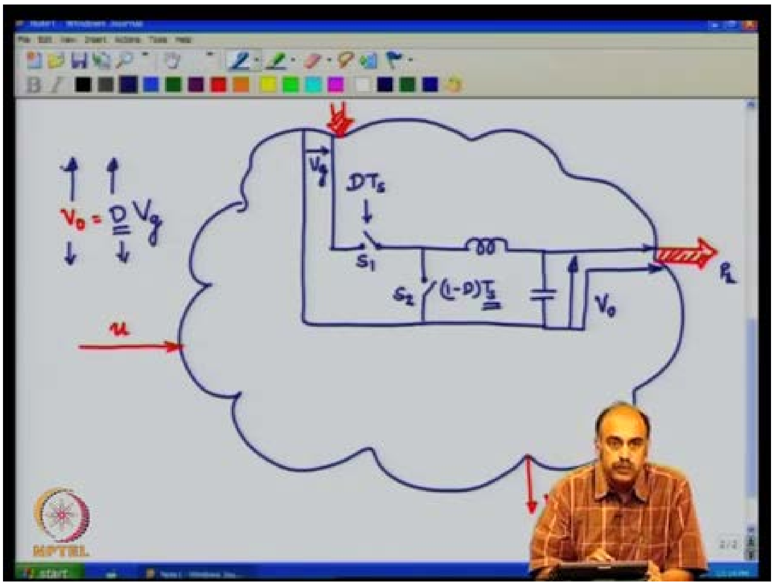

U and another port, which comes out of the plant, which is the controlled output y now what is the control input for this plant we have seen the study state analysis V naught is equal to D times V g. Now, D what is D D is basically the fraction of the time the switch S 1 and switch S 2 is on an a off within a period T S. So, D actually can be termed as a control input, if you reduce the value of D, then the output reduces. If you increase the value of D the output also increases, therefore the output in this case which is V naught.

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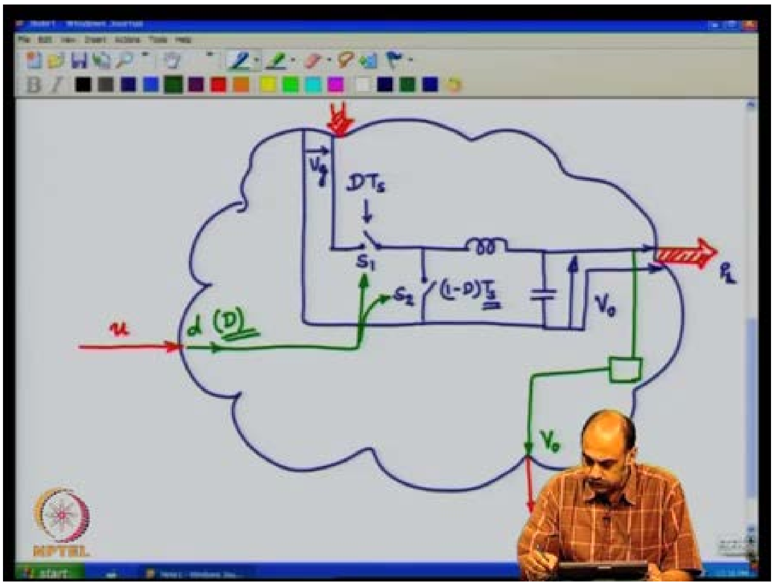

So, if we if we measure the output voltage signal condition it by passing it through appropriate approve coupled circuit and then pass it on to the output port. So, V naught is the output signal and this output signal can be controlled by input. What is that input D in the study state? We say it is upper case D, so this D which basically implies that you are in some manner trying to adjust the times for which these two switches are on and off. So, in affect it is actually one control input because these two switches are directly coupled they have only single variable D to control it and their mutual exclusive. So, in affect we have actually one control variable D and that is what is brought out as the plant control input. So, if you see this buck convertor is mapped into control plant blocked diagram controlled plant blocked diagram.

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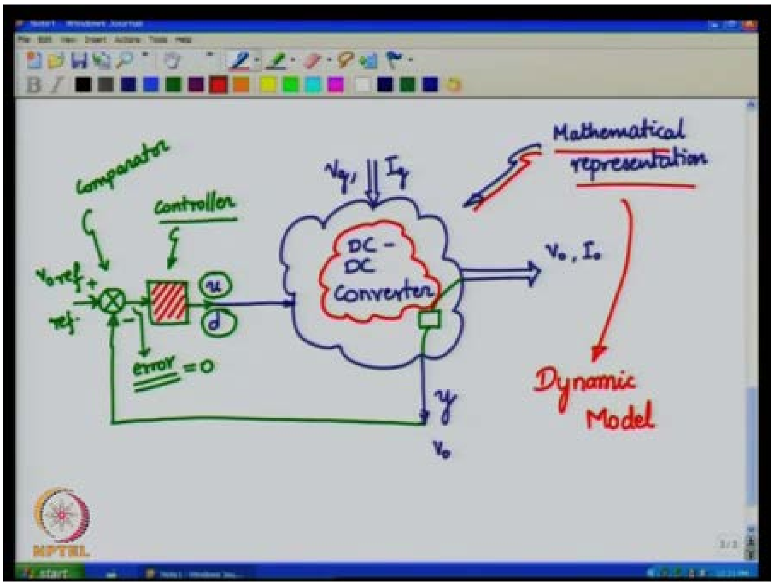

By saying that the control input is U, which in this case is D this lower case D is used to represent both variations plus steady state there is a controlled output U, which is to be controlled which in this case is V naught. We have the power inputs, the power input being obtain from a supply V g unregulated V g and i g being the input currents and an output power output, which is actually supplying a load I naught at a voltage V naught. So, this is our DC DC convertor, which needs to be modeled. Now, this DC DC convertor a practical circuit needs to mathematically represented mathematically represented.

In a way which is amenable to develop the controllers now the process of control is that we need to have a comparator and we need to have a controller. So, the basic minimum we need to have these two important components, this is the controller and a comparator so what is done with these two. So, the controlled output which is sensed, sensed and then given as an out controlled output signal low power. Low level signal information information signal is fed back to this comparator. Now, to this comparator we need to also give a reference, so it could be an output reference signal or a value which is given as a positive or in phase. To that reference we are subtracting the fed back signal to get the error.

So, what you get here is the error between the fed back signal and the reference and the objective of the controller is to give an appropriate value of control input. Whatever be the control input D D, in this case such that the error goes to 0, so that is the job of this controller. So, this is what we want to ultimately achieve, so in order to design this controller, we need to have knowledge of the DC DC converter in a mathematical form. That representation is called the dynamic model.

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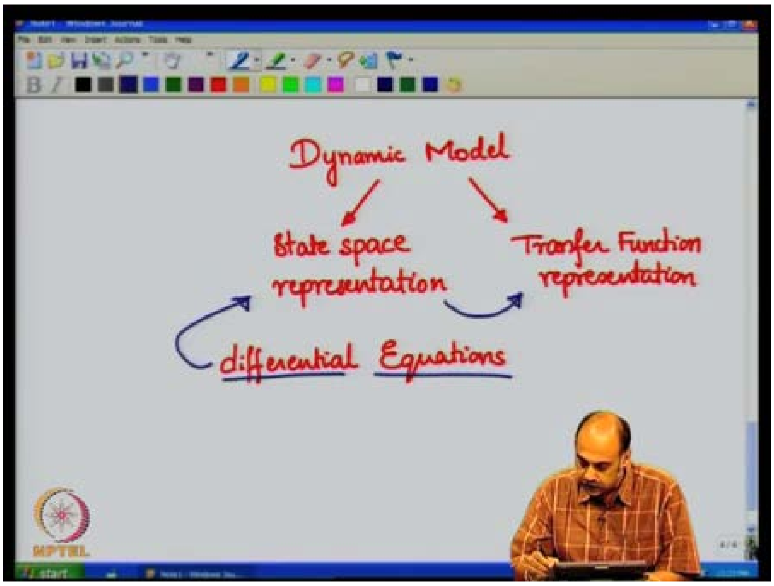

So, this dynamic model can be represented in two basic ways; the dynamic model the states space representation and the other transfer function representation. However, it may be pointed out at this point that when we develop the dynamic model of a physical system from the physics of this physics of its operation. We actually arrive at differential equations a set of differential equations set of differential equations, which are directly represented in this form called the state space representation and from the states space representation.

You can extract the transfer function representation, so you could say that the state space representation is the one which you will first obtain. Then convert it into transfer function representation for further analysis or you could continue to do your controller design in the state space representation itself anyway first let us see how we will obtain the state space representation of the system. the system in our case is the DC DC converter.

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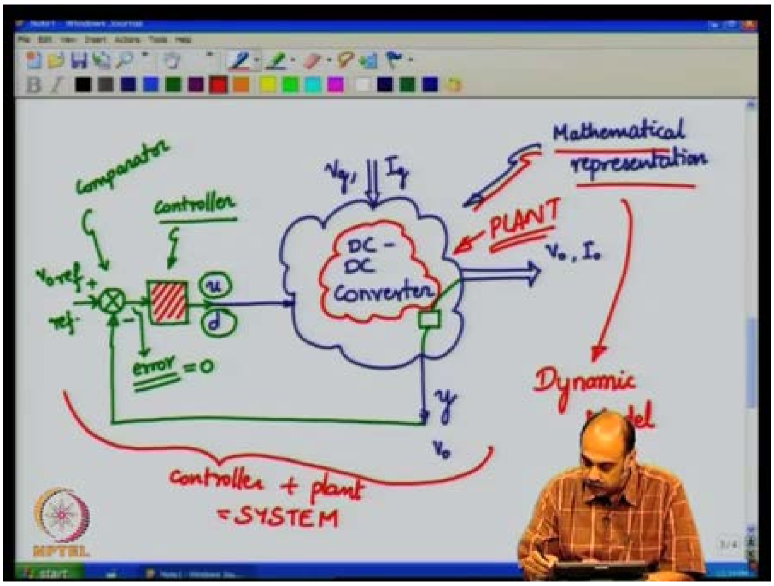

So, in control balance the whole the whole system including the controller and plant controller plus plant, we generally call it as the system the open. Loop uncontrolled part that we have here in blue is called the plant, so keep that in mind. Let us proceed for then trying to see how we can obtain?

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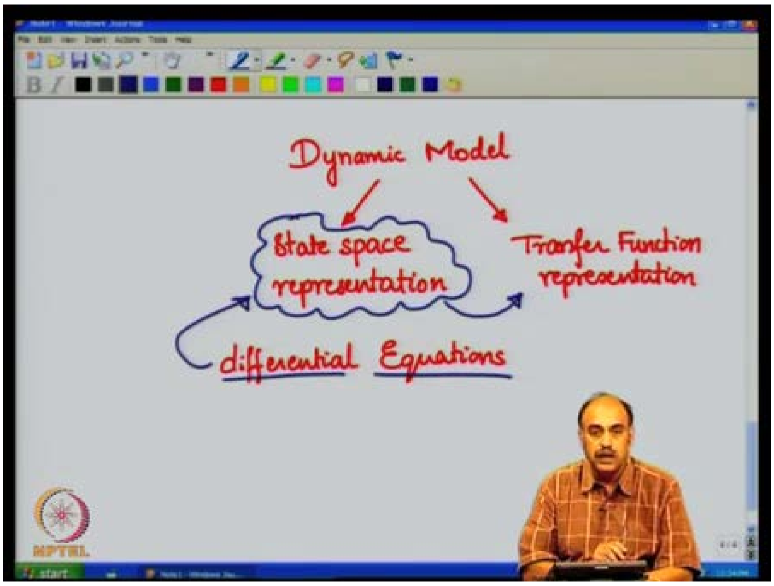

The state space representation of a given plant, so before we go into obtaining the state apace representation of a DC DC converter, because the DC DC converter is a switched power converter. It has switches which are non-linear I nature. In the in the sense that they have two states, so proportion principle may not be directly valid. We shall look at a generic method for first obtaining the state space representation of a general sis plant. Then we apply it to the specific and special case of DC DC converter. So, let us look at the case of a general plant.

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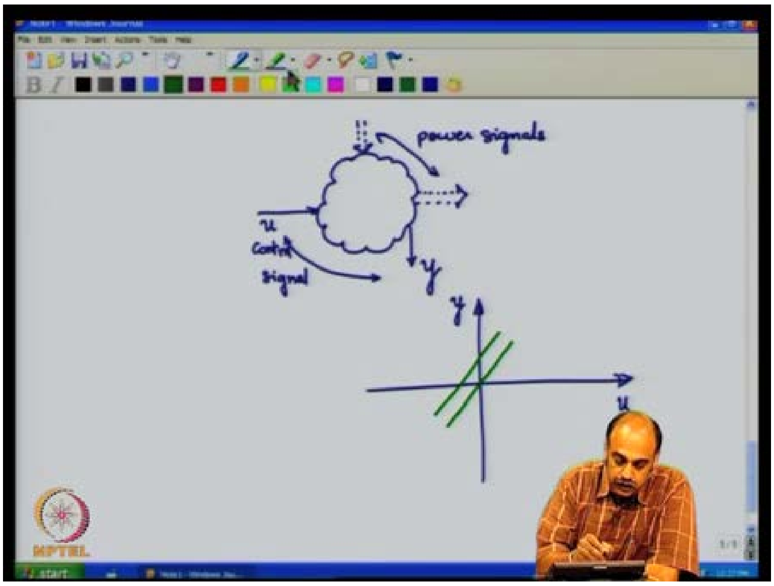

Wherein we have a controlled input u and control input u and a controlled output y, I will not be writing this power input and power output explicitly. Every time it is understood that these two ports are there, but to make the discussion faster these two will be understood to be present in the plant. I will be using only u and y from the control point of view these two are control signals these two are only control signals, they do not contain any power, whereas these two are power signals…

So, the job of the control signals is to control the flow of power in the power signals. So, even if I do not show these two ports there in mind that these two ports exist. They are part of the plant, it is only that I may want to draw your attention to this part well while we are talking on the controller design or the controller portions. Now, let us say that we have a plant now like this a black box, we do not know what is inside? The nature of the transfer curve, let us say input is on the x axis independent variable and the dependent variable y output is on the y axis can be linear as shown like this. It could be in this fashion or it could also be partly linear and and non linear with the saturation.

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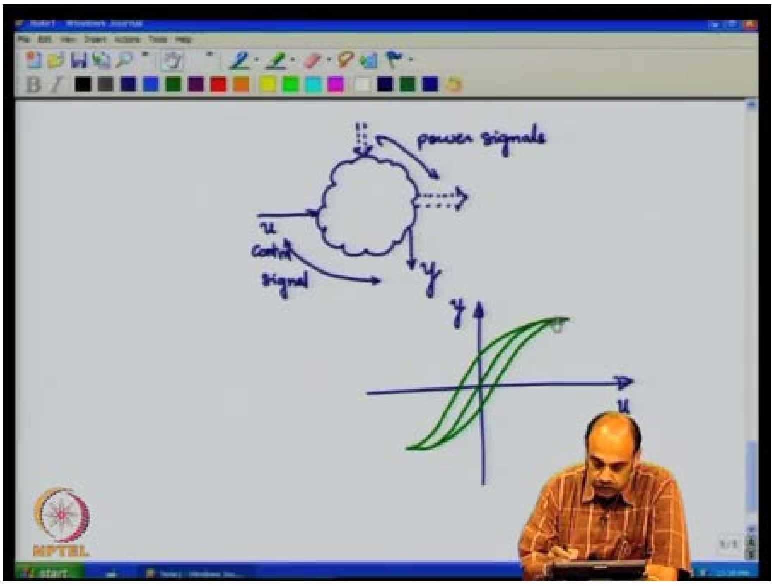

It is a non linear characteristic or it could also have memory hysteresis and various other characteristics. So, let us take a general non linear case and see, how we go about trying to model the system? So, here I would like to mention that all the control signal analysis is done for a linear system.

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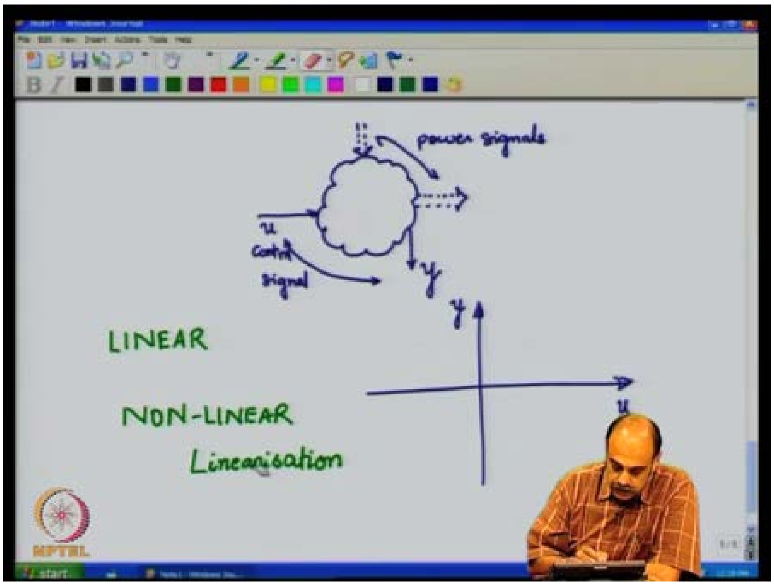
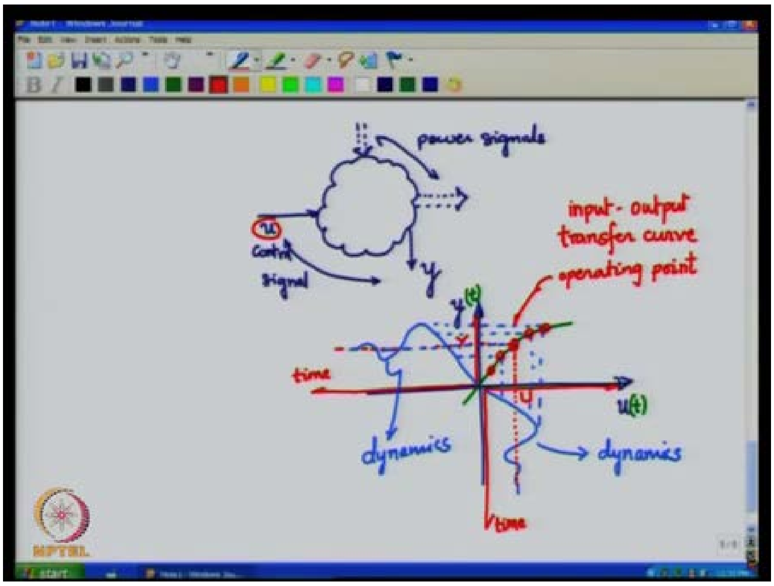


Analysis and design is done considering that the plant is a linear system. However, most of the systems are non linear in nature. Therefore, we need to perform a linearization, if we need to have an effective model of the plant, so we we will discuss these issues. (Refer Slide Time: 30:40)



Therefore, consider a general plant which is not linear and let us say for this input output transfer curve. There can be various operating points like that, let us say a specific operation point. Let me share it a solid the specific operating point shown like that and if it continues to stay in that specific operating point, then we say it is in the steady state. We say the study state values are upper case values like this. Now, this transfer curve as you see gives you an idea of the operating point.

This is the operating point, however this transfer curve does not give an idea of the evolution time evolution of the variables u and y. You should know that both u and y are functions of time both u and y are functions of time. So, let us say for example, that we would also like to see the time evolution of the two variables the input and the output variable let me put another graph oriented differently. This is u t along that axis and along another orthogonal axis. Let us say is time and you project this value of u along, now for this operating point to have reached this stable operating point, it would probably have gone through these various intermediate operating points over short.

May be then come back and then settled at this particular operating, operating point, so to see the time evolution, it probably may look like this. It may have started from 0 over short denominal operating point under shoot. Then probably settle down to this which will give this corresponding operating point as your seeing there. Likewise one could also have a time evolution of the output variable the y variable verses time on another graph with two orthogonal coordinates. Let us say if you were, if if you are interested in seeing the time evolution, it may look like this starting from 0 over shoots under shoots. Then settles down at the steady state value as it is shown by this operating point and this over shoots, and under shoots all corresponds to these various points.

I shown here, so this time evolution is the one which is of interest to us and that is what gives you an idea of the dynamics. So, dynamics of the input and the dynamics of the output and this is what we need to study and model, such that these dynamics are taken care by the control input, which we are showing here. So, we need to control the way the time signals evolve with time and ultimately settle in the study state point upper case u and upper case y. The design off course most of the time is going stay at the steady state point. Therefore, the design is done for the steady state operating point, which is what you have been discussing and studying till now in the previous classes.

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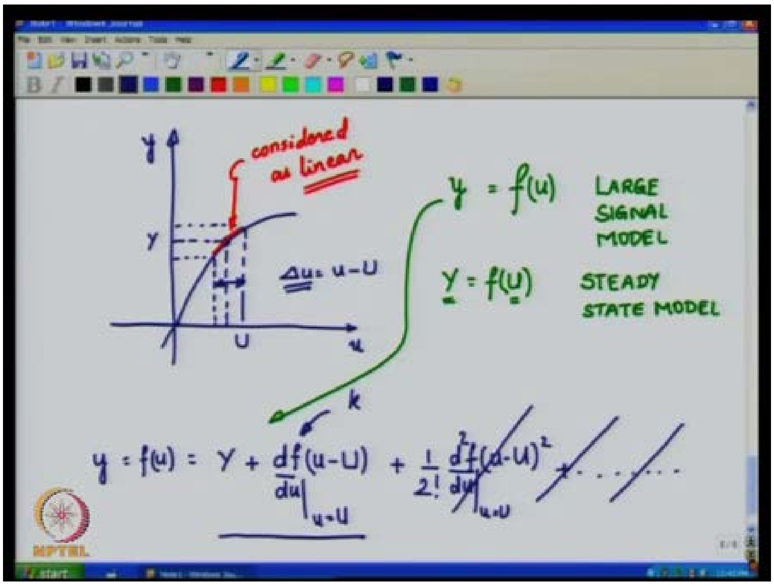

So, looking looking at this specific graph, so let us say that we have u and y input and output with the input output transfer cover like that. Let us we are interested and as specific operating point upper case u and y like this. Now, this is a non linear curve super portion principle not exactly valid for this particular curve. So, you have to treat it as a non-linear system the problem here we start designing and analyzing of control as non linear system is much more tedious and non intuitive. So, most of the time the controllers are design for linear system, but then if the controllers are designed for the linear systems and their plants are non-linear, how do they become compatible? So, we make a modification something like this, we call linearization. We take a small portion such has this for derivations of u and derivations expected.

Derivations of y we have a small portion segment of the transfer curve and this segment of the transfer curve can be considered as linear. So, if we say that the system is operating at this operating point than derivations in the neighborhood of operating point can be considered as linear and the controller can address or take of this derivation in the neighborhood of the operating point. Then a linear mathematical model and controller addressing the taking care of the a small derivation in the neighborhood of the operating point can be designed and built.

So, that is basically the approach that we will be using in building the controllers. Now, how do we go about saying that we have a linear model. Now, if if you look at the generic model you say the y is a function of the input variable u, now this is the general model for the complete transfer curve. Therefore, we call it as a large signal model because this model even though non linear handles the complete transfer curve u verses y of a particular system. Now, for this specific case for the specific case of the input u upper case u at the operating point u get a response output, which is upper case y and that is called the model for that specific operating point, which is in the equilibrium state.

This is called the steady state model, this called the steady state model. Notice that in the steady state model both the upper case input and the upper case response output they are in equilibrium. They do not vary with time they are constant with respect to the time or the d the the differential quantity would be 0. Now, let us consider the case the generic case y which is equal to f of u. We could use Taylor series and expand it about the operating point, so we know that the output y has to be y plus or minus something plus or minus some portion, which is given by the Taylor series as differential with respect to u the function f u minus upper case u the delta u plus 1 by 2 factorial second derivative u minus u square.

All these evaluated at all the derivatives evaluated at u equal to the operating point value plus so on, it is going. Now, if you if you take a very small deviation where is delta u, which is u minus the operating point. If you take a very small deviation about the operating point and consider them to be linear then we could say that because delta u is small u minus u is very small all second order and higher order terms can be neglected delta u square will be still smaller. We can take only this portion and as we say that it is linear the slope we can say is equal to a constant k.

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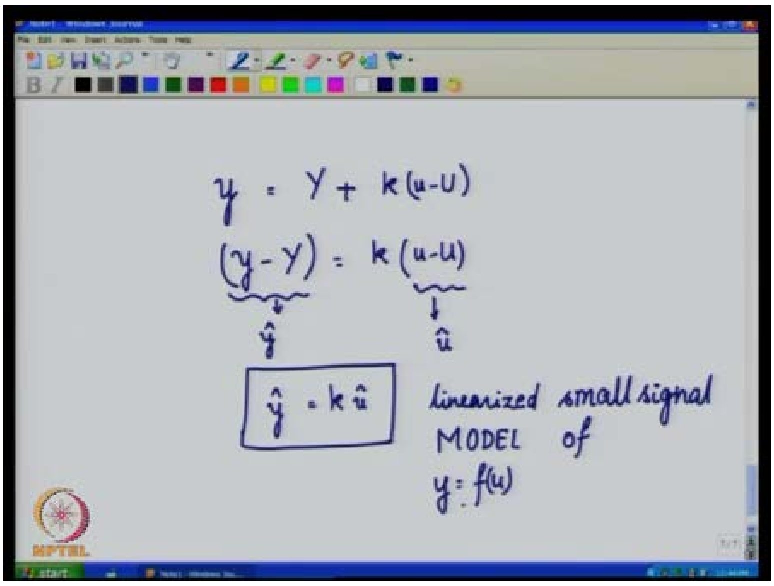

So, this becomes y becomes k times u minus r y minus y steady state value is now, this is the small signal deviation in the output. This is the small signal deviation in the input, so the small signal deviation the output. We will give it as a symbol y hat and the small signal deviation in input, we will give it as a symbol u hat. So, we say y hat is equal to some constant times u hat and this is a linear equation. This is the linearized small signal model of y is equal to f of u, which is the large signal model. So, you see that we have seen three ways of representing a particular system. Firstly you have the transfer input output transfer curve and this whole transfer curve can be represented as y is equal to f of u.

That is called the large signal model because it handles the whole large signal of u and y. It can address that one and then the equilibrium value the operating point value is given by this y is equal to f of u. This is called the steady state model, it does not have any derivatives and thirdly u have the small signal model. which is basically in the neighborhood of the operating point. So, in the neighborhood of operating point we take a very small deviation about the operating point. That is, small deviation is so small that we can consider that segment of the transfer curve as linear and thereby have a model which is a linear model, called the small signal model of y is equal to f u. Now, it is this small signal model, which is used for designing the controller. The controller handles these deviations the small signal deviations, the steady state operating point requirement is met by separately by a feed forward value.

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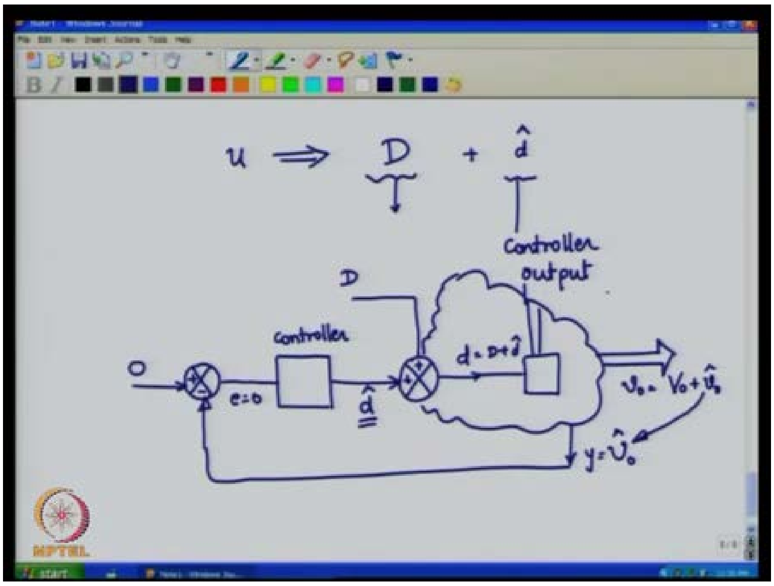

So, for example, in the case of a DC DC converter you could say the input u comprises of a steady state value of duty cycle plus a small signal deviation in the duty cycle. So, the steady state duty cycle is a constant which is given directly to the open up plant and the controller output, which handles the deviations about this steady state value is coming from is coming from the controller. So, the controller output provides this small signal corrections to the plant, so this is the approach that we will be taking even for the DC DC converter. Now, from this what we gain is that whatever may be the plant linear or non linear. The first step is to convert the plant into a linearized small signal model, so it is this linear small signal model that we further use for developing the controller design.

So, if you look at our controller, it would look something like this. Now, let us say we have a comparator here the feedback portion you have a controller as shown here and the output of the controller is actually given as the control input to the plant. In this case it is the duty cycle, so let us say that at the plant I have an adder here plus and plus and to this you give the steady state d. This goes in as d and let the controller output give the deviation. So, this becomes d plus d hat, which is actually going into the plant and controls the output y. which is now a small signal value V naught hat. So, out of the sensed signal from the actual power output, which is at V naught this is having the steady state component V naught.

So, let me write it in this following the V naught is containing the steady state portion plus small signal deviations above the steady state. So, this is the one which is fed back here and this is one which is fed back here. What should be the deviations? We want ultimately that the deviations should right die down should be 0 in the steady state, we do not want any deviation about the operating point. It should state the operating point, which means the reference for the deviations should be 0. So, if this error is equal to 0, then this d would represent the d corresponding to the deviations and this d represents the steady state value together. They are given to the converter and the power of course, coming from the input V g.

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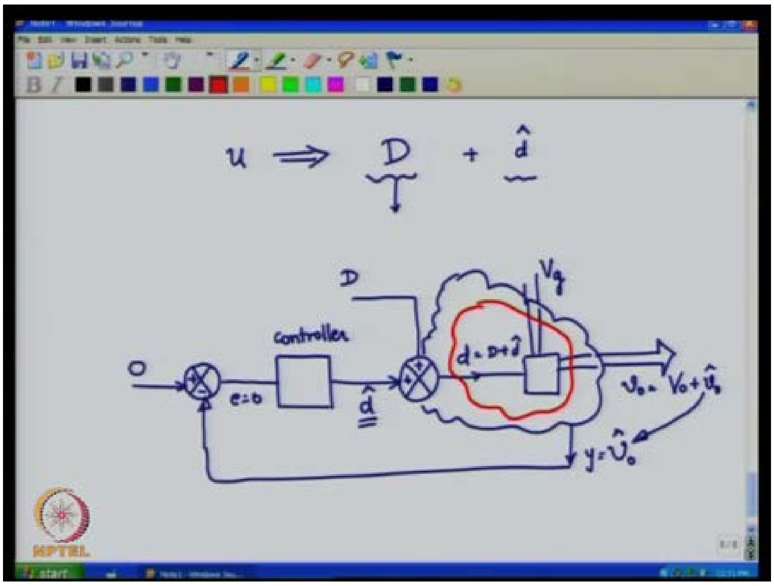

So, this is how the control block diagram will look like and this is how we will try to attack the problem. So, we try to develop the small signal model for the converter linearized and then try to see, how we go about designing the controller? So, that being the main objective, so in order to develop the small signal model of the controller, we adopt the following strategy any system. At least power electronic systems are composed of three basic or elementary components, which is R this is dissipative.

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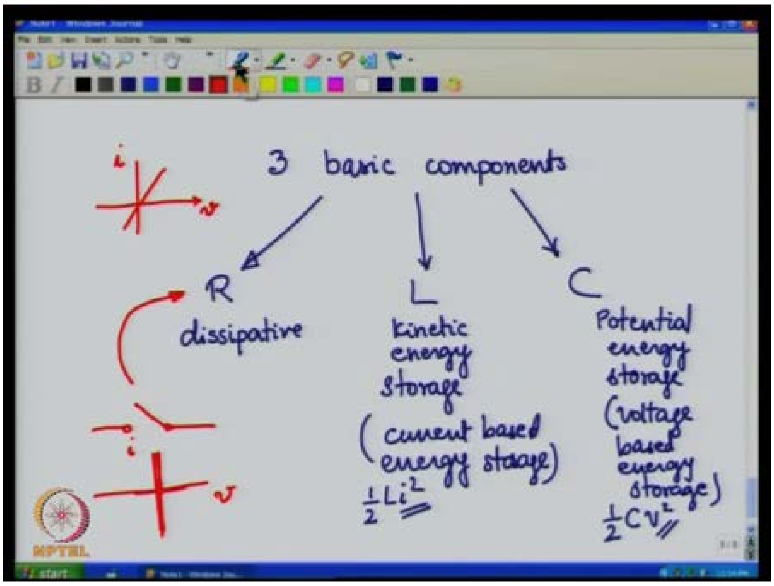

Second one we call it as the l kinetic storage, kinetic energy storage are current based energy storage. As you know that the energy stored in half L i square by virtue of the current flowing through the inductor l and third another energy storage component called c capacitance, which is potential energy storage component. This is actually a voltage based energy storage component as you know that the energy in the capacitance is half C V square. The energy is stored by virtue of a potential a charge within the capacitor. Now, these are the three basic components, which are used in power electronic equipments and to that we have one more component, which is the switch.

So, switch is actually an non linear version of a resister, a resister has an I V characteristic, which is a linear, whereas in the case of the switch it as I V characteristics, which is non-linear. So, when when the switches off, we have it along the x axis and when the switches on, we have the y axis as the operating points in this fashion. Therefore, we we need to address this component also which is the switch component also into our dynamic models.

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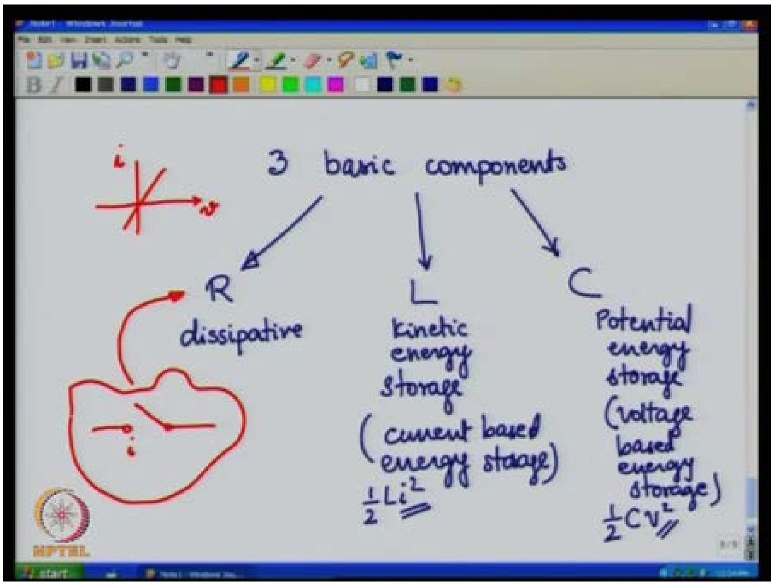

How we go about doing? This is what we will look at and study in the classes to come, so to summaries.

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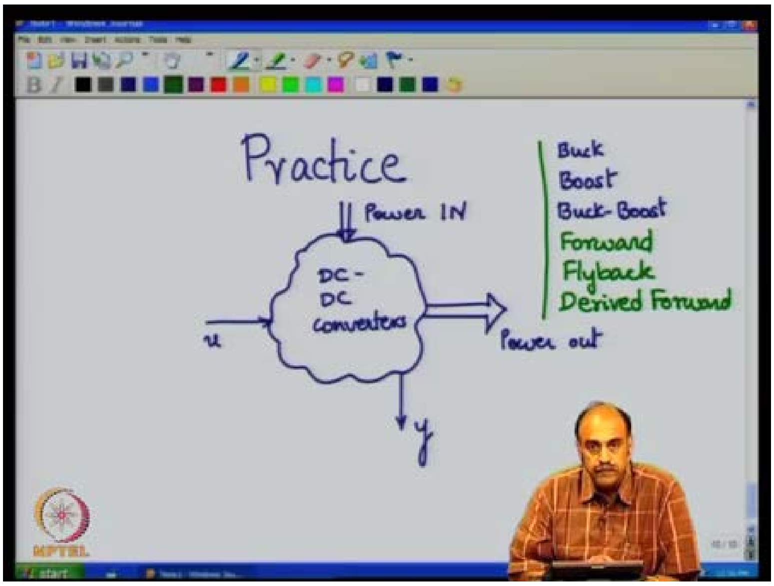

Now, what I would suggest now is that you get some practice on what are what are the inputs? What are the output control, output? What is the power output the power output? What is the power input of various DC DC convertors. What are the input variables? What are the output variables? What are the power in variables and power out variables of the three non isolated convertors. The isolated convertors like the forward fly back and the derived forward convertors like the push pull, half bridge, full bridge convertors.

So, for these convertors get some practice in trying to phrase your block diagram in this fashion. So, that it becomes compatible for controller design and understanding the controllers. In the next class, we shall proceed continue from where we left of today in trying to obtain these state space equations, the dynamic model for the DC DC convertors.

Thank you.